Equilibration in weak coupling approaches

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Objectives

1. Hydrodynamics extremely successful for description of soft physics observables in A+A collisions
   - provide equilibration time and initial conditions for hydro

2. Develop a microscopic understanding of how QGP is created from the high-energy collision of heavy nuclei
   - Energy density of initial state corresponds to temperature scale on the order~4 $T_c$ (LHC)
     - weak-coupling methods can work
General picture at weak-coupling

CGC:
Colliding nuclei

Glasma flux tubes

Over-occupied plasma

Min-jets + soft bath

Equilibrium

Time

Strong fields

Quasi particles

Classical-statistical lattice gauge theory

Effective kinetic theory

Hydro

Consistent theoretical description requires combination of different weak-coupling methods.
Before the collision

Nuclei described in terms of distribution of color charges carried by the quarks & gluons inside

Color glass condensate: (McLerran, Venugopalan D49 (1994) 2233-2241)

Saturation of the gluon density occurs when phase-space density becomes non-perturbatively large

High-energy nuclei feature a non-perturbatively large number of gluons $\sim 1/\alpha_s$, with a typical momentum $Q_s(\sqrt{s})$
Early times \(0 < Q_s \tau < 1\)

Strong boost invariant classical fields \(E^\eta, B^\eta\) created immediately after the collision

Decoherence of classical fields occurs on a time scale \(1/Q_s\)

- leading order \((\alpha_s)\) dynamics

well studied analytically and numerically in classical-lattice gauge theory

\(\text{(c.f. talk by R. Fries)}\)

classical fields remain boost invariant
Early times ($0 < Q_s \tau < 1$)

Strong boost invariant classical fields $E^\eta, B^\eta$ created immediately after the collision.

Decoherence of classical fields occurs on a time scale $1/Q_s$.

- Next-to-leading order ($\alpha_s$) corrections break boost invariance.

-> Plasma instabilities lead to an increase of longitudinal pressure.

(Epelbaum, Gelis PRL 111 (2013) 232301, Berges, Schenke, SS, Venugopalan NPA 931 (2014) 348-353)
Classical regime \((1 < Q_s \tau < \alpha_s^{-3/2})\)

Different initial anisotropies \(\xi_0\)

Classical field interactions are not sufficiently strong to restore isotropy beyond \(1/Q_s\)

\[-\text{anisotropy of the plasma increases again}\]

No sign of plasma instabilities playing a significant role at later times \(Q_s \tau > 1\)

(Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)
Classical regime \( (1 < Q_s \tau < \alpha_s^{-3/2}) \)

Classical field evolution \((\tau > 1/Q_s)\) of hard modes can be accurately described in terms of weakly interacting quasi particles

\[ \rightarrow \text{classical particle/field duality} \]

Effective kinetic description

(Arnold, Morre, Yaffe JHEP 0301 (2003) 030)

\[ \frac{df}{dt} = -C_{2\leftrightarrow2}[f] - C_{1\leftrightarrow2}[f] \]

Gluon spectrum: \(\tau \alpha f(p)\)

Kurkela, Zhu
PRL 115 (2015) 182301

\[ \text{long. momentum } \tau \alpha p_z \]

Berges, Boguslavski, SS, Venugopalan
PRD 89 (2014) 074011

\[ \rightarrow \text{Effective kinetic description (AMY) can be used to study dynamics of hard modes from } \tau > 1/Q_s \text{ all the way to equilibration} \]
Bottom-up scenario

Equilibration process beyond $\tau \sim 1/Q_s$ occurs as a three step process
(Baier et al. PLB 502 (2001) 51-58)

Need to switch from classical Yang-Mills to kinetic description to describe approach to equilibrium

Kurkela arXiv:1601.03283

Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 074011
Quantum regime \((Q_s \tau > \alpha_s^{-3/2})\)

Inelastic processes dominate and lead to a radiative break-up

\[\implies\text{mini-jets loose all their energy to soft thermal bath}\]

Kurkela, Lu PRL 113 (2014) 182301

equilibration \(\leftrightarrow\) (mini-) jet quenching
Quantum regime ($Q_s \tau > \alpha_s^{-3/2}$)

Inelastic processes dominate and lead to a radiative break-up

$\rightarrow$ mini-jets loose all their energy to soft thermal bath

Soft bath heating up due to energy deposited by mini-jets

$\rightarrow$ longitudinal pressure rises and system isotropizes

Kurkela, Lu PRL 113 (2014) 182301

Kurkela, Zhu PRL 115 (2015) 182301
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Kurkela, Lu PRL 113 (2014) 182301
Kurkela, Zhu PRL 115 (2015) 182301

- smooth matching to hydro-dynamics (no free parameters)
Equilibration process

Clear understanding of the dynamics in the weak-coupling limit

CGC colliding nuclei

Glasma flux tubes

over-occupied plasma

min-jets + soft bath

equilibrium

$Q_s \tau_{\text{Inst}} \sim \log^2(\alpha_s^{-1})$

$Q_s \tau_{\text{Eq}} \sim \alpha_s^{-13/5}$

$Q_s \tau_{\text{Quant}} \sim \alpha_s^{-3/2}$

classical-statistical lattice gauge theory

eff. kinetic theory

Developed the tools to compute equilibration process from combination of weak-coupling methods
Beyond weak coupling

Extrapolate leading order weak coupling description to physical values $\alpha_s=0.3$

Even though distinctions become less clear, basic mechanism remains the same.

Kurkela, Zhu PRL 115 (2015) 182301
Onset of hydrodynamics

Smooth matching to second order viscous hydrodynamics on a time scale $\sim 1$ fm/c.

Pressure evolution matches for $P_L/P_T \sim 1/5$

$\rightarrow$ Equilibration takes much longer than $\sim 1$ fm/c for expanding system

Initial conditions for hydro from weak-coupled equilibrium dynamics

(c.f. talk by Aleksas Mazeliauskas)
Exploring pre-equilibrium dynamics in small systems?

- Small systems provide a unique laboratory to probe early time dynamics.

(SS arXiv:1601.01177)
Early time dynamics in p+A

Initial state immediately after the collision ($\tau=0^+$)

(SS, Schenke, Venugopalan PLB 747 (2015) 76-82)

$\rightarrow$ Significant momentum space correlations $v_2$ at $\tau=0$ due to production mechanism.

$\rightarrow$ No odd harmonics for gluons without final-state interactions.

Initial spectrum symmetric under $k_T\leftrightarrow-k_T$
Early time dynamics in p+\(A\)

Classical Yang-Mills evolution after the collision — includes re-scattering of produced gluons

(SS, Schenke, Venugopalan PLB 747 (2015) 76-82)
Early time dynamics in p+A

Classical Yang-Mills evolution after the collision — includes re-scattering of produced gluons

(SS, Schenke, Venugopalan PLB 747 (2015) 76-82)
Early time dynamics in p+A

Classical Yang-Mills evolution after the collision — includes re-scattering of produced gluons

- Clear modifications of correlations due to pre-equilibrium dynamics on a time scale ~0.4 fm/c

Quantitative understanding requires theoretical description which consistently takes into account initial state and final state effects.
Conclusions & Outlook

Development of leading order weak-coupling approach to equilibrium

- first results look promising in terms of phenomenology

Ultimate goal: Qualitative insights -> Quantitative framework

- Screening in anisotropic plasmas
- Quarks & chemical evolution
- 3D structure of Glasma fields

Several applications beyond bulk phenomenology

- small systems
- electromagnetic & hard probes
- sphaleron transitions & chiral magnetic effect

(Mace, SS, Venugopalan PRD93 (2016) 074036 ; Mueller, SS, Sharma in preparation)
Backup
The ridge in p+p, p+Pb & Pb+Pb

p+p @ 7 TeV  p+Pb @ 5.02 TeV  Pb+Pb @ 2.76 TeV
Gluon spectra
Evolution in classical regime

Dynamics in the scaling regime consists of longitudinal momentum broadening — not strong enough to completely compensate for red-shift due to longitudinal expansion.

Transverse spectrum shows thermal-like $1/p_T$ behavior up to $Q_S$.

(Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)
Self-similarity

Dynamics can be entirely described in terms of universal scaling exponents $\alpha=-2/3$, $\beta=0$, $\gamma=1/3$ and scaling function $f_S$ extracted from simulations

$$f(p_T, p_z, \tau) = (Q\tau)\alpha f_S((Q\tau)\beta p_T, (Q\tau)\gamma p_z)$$

(Berges, Boguslavski, SS, Venugopalan PRD 89 (2014) 074011; 89 (2014) 114007)
Quantum regime
Equilibration process
Quantum regime

- Numerical study of effective kinetic description (AMY 2<->2 + eff. coll. 1<->2)

Kurkela & Zhu PRL 115 (2015) 18, 182301

Clear observation of the three distinct stages of “bottom up” scenario

Hydrodynamic behavior on time scales ~1 fm/c when extrapolating weak coupling description to $\alpha_s = 0.3$

-> Viscous hydrodynamics applicable already for large pressure anisotropies
Expanding scalars & Universality
Expanding scalar field theory

Consider massless N-component scalar field theory with quartic self-interaction

\[ S[\varphi] = \int d^4x \sqrt{-g(x)} \left( \frac{1}{2} (\partial_{\mu} \varphi_a) g^{\mu\nu} (\partial_{\nu} \varphi_a) - \frac{\lambda}{24N} (\varphi^2)^2 \right) \]

in a longitudinally expanding setup.

Even though perturbatively there is no preference for small angle scattering, one at least expects elastic processes to dominate

\[ \sim \lambda^2 \]

-> Energy conservation
& Particle number conservation

(Berges, Boguslavski, SS, Venugopalan PRL114 (2015) 061601)
Comparison of simulations

Comparison of longitudinally expanding Yang-Mills and scalar field theory in the classical regime of high occupancy

Evolution of the single particle spectrum

Scalar theory shows three distinct scaling regimes at soft ($\sim p^{-5}$), intermediate ($\sim 1/p_T$) and hard momenta ($\sim \text{const}$)

$\rightarrow$ Common $\sim 1/p_T$ scaling regime

(Berges, Boguslavski, SS, Venugopalan PRL114 (2015) 061601)
Universality far from equilibrium

Scaling exponents and scaling functions agree in the inertial range of momenta, where both theories show $1/p_T$ behavior

(Berges, Boguslavski, SS, Venugopalan PRL 114 (2015) 061601)
Sphalerons & CME
Chern-Simons number Histograms

Generated early \((10<Qt<20)\)

Generated later \((50<Qt<60)\)

\[\Delta N_{CS}\]

Probability: \(P(\Delta N_{CS})\)

-> Early times dominate generation of axial charge imbalance
Quantifying the sphaleron rate

Extract auto-correlation function of the Chern-Simons number

\[
\frac{1}{V} \left\langle \left( N_{CS}(t + \delta t) - N_{CS}(t) \right)^2 \right\rangle = \Gamma_{sph}^{eq} \delta t
\]

**Equilibrium:**

**Glasma:**

-> Simple probabilistic picture not applicable in the Glasma — clear non-Markovian emerge from the auto-correlation function

Mace, SS, Venugopalan  
Quantifying the sphaleron rate

Define non-equilibrium sphaleron rate by the early rise of the auto-correlation function

\[ \Gamma_{sph}^{neq}(t) = \left( \frac{(N_{CS}(t + \delta t) - N_{CS}(t))^2}{V \delta t} \right)_{Q_s \delta t < 10} \]

**Non-equilibrium sphaleron rate**

- Sizeable contribution from field strength fluctuations
- Strong time dependence observed — rate is largest at early times and decreases rapidly as a function of time

Do we understand the non-equilibrium sphaleron rate?

Since in equilibrium $\Gamma_{sph}$ is controlled by modes on the order of the magnetic screening scale $\sim \sqrt{\sigma}$, one should really compare $\Gamma_{sph}/\sigma^2$.

**Characteristic scales**

**Sphaleron rate** $\Gamma_{sph}/\sigma^2$

-> Separation of scales emerges dynamically

-> Non-equilibrium sphaleron rate dominated by modes on the order of the magnetic screening scale
Chiral Magnetic Effect

Evolution of axial/vector charges during and after sphaleron transition in the presence of magnetic field

-> Based on real-time lattice simulation with dynamical fermions

Axial charge \( j_a^0 \)
Vector charge \( j_v^0 \)

(Mueller, SS, Sharma in preparation)