Early time dynamics: Pressure and 3+1D Flow From Classical Gluon Fields

RAINER J FRIES

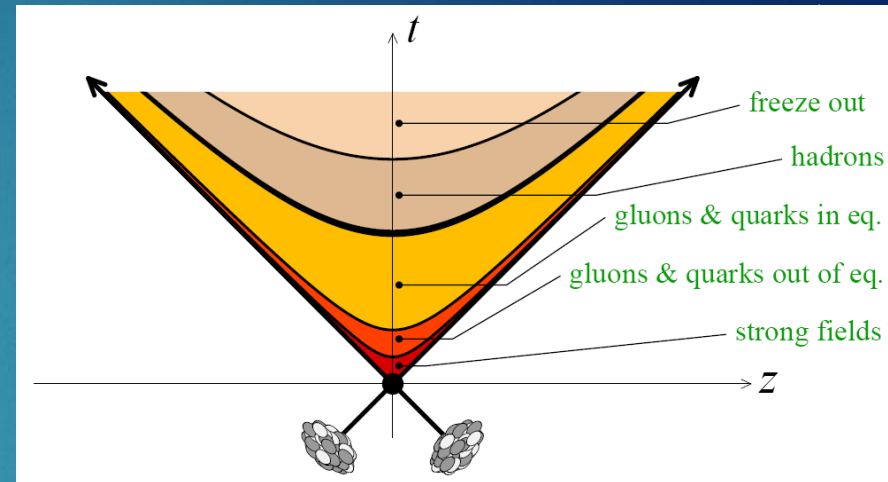TEXAS A&M UNIVERSITY

Little Bang: Macroscopic Evolution

▶ Initial nuclear wave functions

▶ Strong Longitudinal gluon fields

▶ Local equilibration/isotropization

▶ QGP/HG fluid close to local equilibrium

▶ Hadron gas in the kinetic regime and freeze-out

▶ Consensus Model for *Phase II* (times ≥ τ_{th}): fluid dynamics and hadronic transport.

▶ No consensus on *Phase I*: early time dynamics and equilibration.
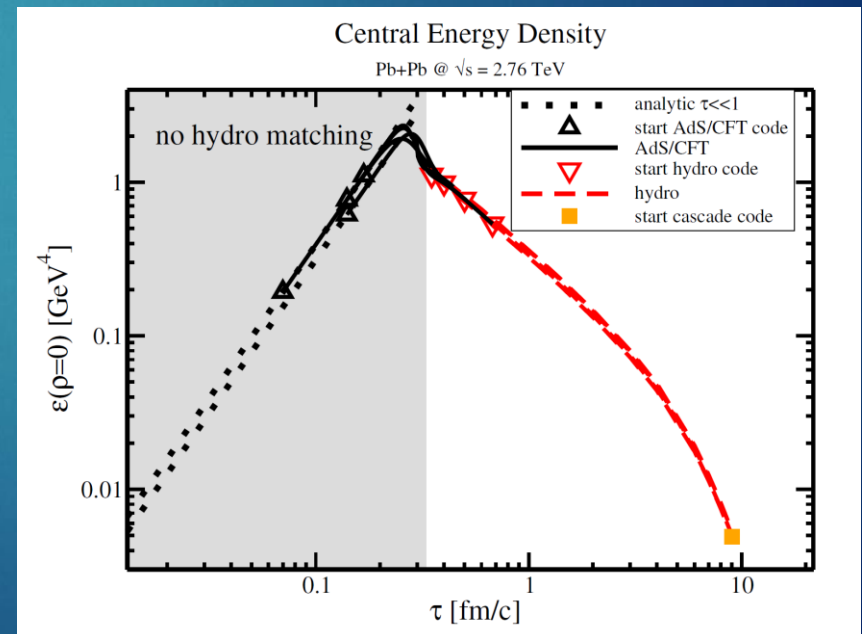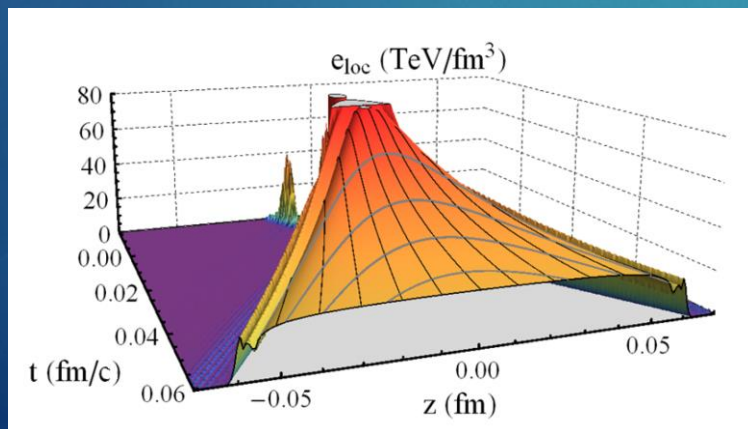
Early Time Evolution I

- ▶ Large body of work on early time dynamics

  - ▶ Phenomenological models

  - ▶ Weak coupling limit

  - ▶ Color glass condensate

- ▶ Recently much progress on strong coupling scenarios

Chesler, Kilbertus, van der Schee (2015)
van der Schee, Schenke (2015)
van der Schee, Romatschke, Pratt (2013)
...

Early Time Evolution II

▶ Initial wave function can be calculated approximately using Color Glass picture.

▶ Overlap mechanism of nuclear fields known (in the light cone limit) = boundary conditions for further time evolution.   Kovner, McLerran, Weigert (1995)

▶ Dominance of longitudinal (chromo) electric and magnetic fields ("flux tubes")   McLerran, Lappi (2006)
RJF, Kapusta, Li (2006)
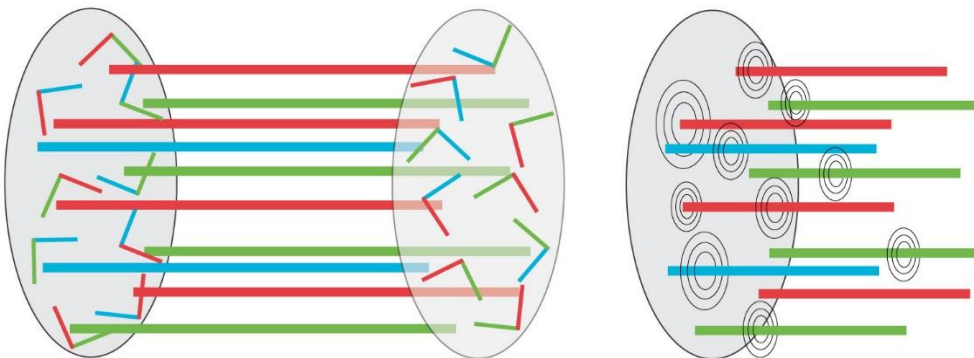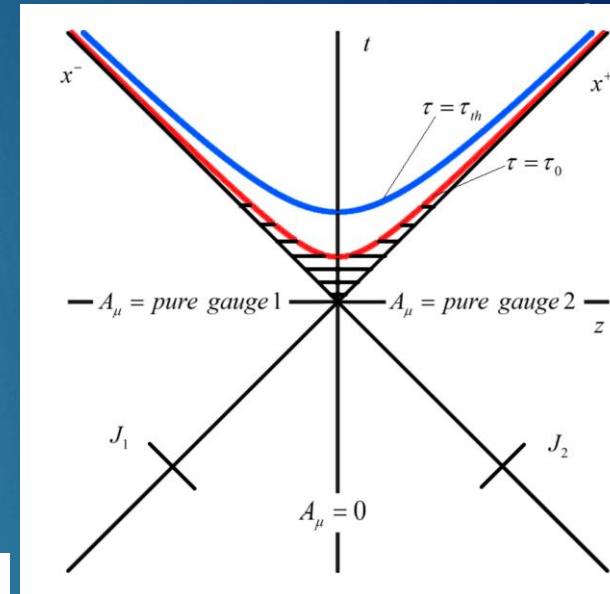
Classical MV Approximation

▶ Nuclei/hadrons at asymptotically high energy:

　　▶ Saturated gluon density ~ $Q_s^2 \rightarrow$ scale $Q_s \gg \Lambda_{QCD}$.

　　▶ Low-*x* wave function = classical gluon field with large-x partons as sources.

▶ Solve Yang-Mills equations $[D^\mu, F^{\mu\nu}] = J^\nu$ for gluon field $A^\mu(\rho)$.

　　▶ Source = light cone current *J* (2-D color charge distributions ρ).

　　▶ Calculate observables O(ρ) from the gluon field $A^\mu(\rho)$.

　　▶ ρ from *Gaussian* color fluctuations of a color-neutral nucleus.

$$\left\langle \rho_i^a(x) \right\rangle = 0$$

$$\left\langle \rho_i^a(x_1)\rho_j^b(x_2) \right\rangle = \frac{g^2}{N_c^2 - 1} \delta_{ij}\delta^{ab}\lambda_i(x_1^\mp)\delta\left(x_1^\mp - x_2^\mp\right)\delta^2\left(\mathbf{x}_{1T} - \mathbf{x}_{2T}\right) \qquad \mu_i = \int dx^\mp \lambda_i(x^\mp)$$

Solving Yang-Mills for $\tau \geq 0$

▶ Numerical Solutions → Successful Phenomenology
Krasnitz, Nara, Venugopalan (2003); Lappi (2003)
... ; Schenke, Tribedy, Venugopalan (2012)

▶ Here: Analytic approach, expansion of gauge field in time τ

$$A(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{(n)}(x_\perp)$$

RJF, Kapusta, Li (2006)
Chen, Fries, Kapusta, Li (2015)

$$A_\perp^i(\tau, x_\perp) = \sum_{n=0}^{\infty} \tau^n A_{\perp(n)}^i(x_\perp)$$

▶ Recursive solution:

$$A_{(n)} = \frac{1}{n(n+2)} \sum_{k+l+m=n-2} \left[D_{(k)}^i, \left[D_{(l)}^i, A_{(m)} \right] \right]$$

$$A_{\perp(0)}^i(x_\perp) = A_1^i(x_\perp) + A_2^i(x_\perp)$$

$$A_{(0)}(x_\perp) = -\frac{ig}{2} \left[A_1^i(x_\perp), A_2^i(x_\perp) \right]$$

$$A_{\perp(n)}^i = \frac{1}{n^2} \left(\sum_{k+l=n-2} \left[D_{(k)}^j, F_{(l)}^{ji} \right] + ig \sum_{k+l+m=n-4} \left[A_{(k)}, \left[D_{(l)}^i, A_{(m)} \right] \right] \right)$$

▶ 0th order (boundary conditions at $\tau = 0$): Immediately leads to longitudinal fields

$$F_{(0)}^{+-} = ig \left[A_1^i, A_2^i \right]$$

$$F_{(0)}^{21} = ig \varepsilon^{ij} \left[A_1^i, A_2^j \right]$$

Energy Momentum Tensor at $O(\tau^2)$

▶ Minkowski Components

$$\vec{S} = \vec{E} \times \vec{B}$$

$$T_f^{\mu\nu} = \begin{pmatrix} \varepsilon_0 + O(\tau^2) & \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) \\ \alpha^1 \cosh\eta + \beta^1 \sinh\eta & \varepsilon_0 + O(\tau^2) & O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta \\ \alpha^2 \cosh\eta + \beta^2 \sinh\eta & O(\tau^2) & \varepsilon_0 + O(\tau^2) & \alpha^2 \sinh\eta + \beta^2 \cosh\eta \\ O(\tau^2) & \alpha^1 \sinh\eta + \beta^1 \cosh\eta & \alpha^2 \sinh\eta + \beta^2 \cosh\eta & -\varepsilon_0 + O(\tau^2) \end{pmatrix}$$

▶ 0th order: Color Capacitor $\varepsilon_0 = \frac{1}{2}(E_0^2 + B_0^2)$

▶ 1st order: Poynting vector → Transverse Flow

RJF, Kapusta, Li (2006)
Chen, RJF (2013)
Chen, RJF, Kapusta, Li (2015)

$$\alpha^i = -\frac{\tau}{2} \nabla^i \varepsilon_0$$

$$\beta^i = \frac{\tau}{2} \varepsilon^{ij} \left([D^j, B_0] E_0 - [D^j, E_0] B_0 \right)$$

Like hydrodynamic flow, gradient of transverse pressure $p_T = \varepsilon_0$; even in rapidity.

Beyond hydro; odd in rapidity

Early Energy Density and Pressure

▶ Event-by-event calculations (semi-analytic) $\rho_1, \rho_2 \rightarrow A_1^i, A_2^i$

▶ Here: event averages with MV model, $\langle \rho_1^2 \rangle \sim \mu_1, \langle \rho_2^2 \rangle \sim \mu_2$
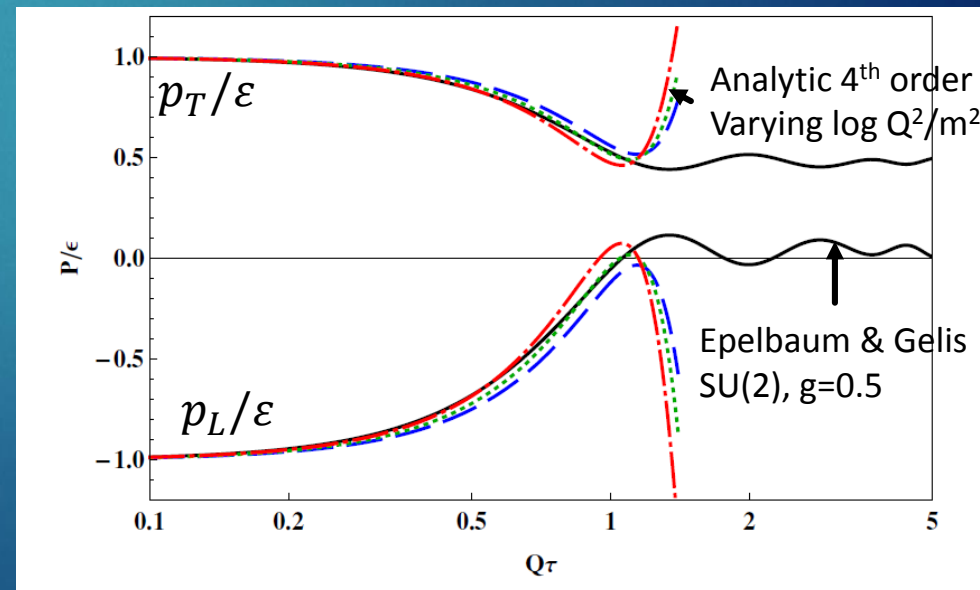
  ▶ Initial energy density

$$\varepsilon_0 = \frac{g^6 N_c \left(N_c^2 - 1\right)}{8\pi} \mu_1 \, \mu_2 \ln^2 \frac{Q^2}{m^2}$$

Lappi (2006)

▶ Order by order in time $\tau \rightarrow$ Analytic results for early time evolution of pressure and energy density, consistent with numerical evaluations

▶ Simplified pocket formulas

  ▶ $\varepsilon = \varepsilon_0 \left(1 - \frac{1}{2}(Q\tau)^2 + O(\tau^4)\right)$

  ▶ $p_T = \varepsilon_0 \left(1 - (Q\tau)^2 + O(\tau^4)\right)$

  ▶ $p_L = \varepsilon_0 \left(1 - \frac{3}{2}(Q\tau)^2 + O(\tau^4)\right)$
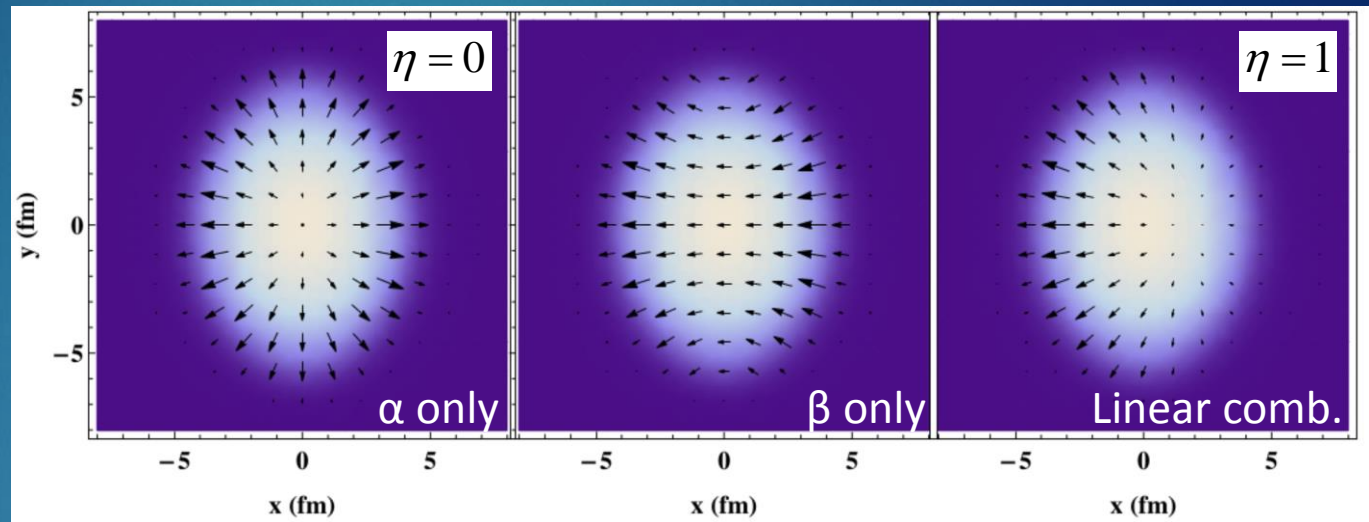
  Chen, RJF, Kapusta, Li (2015)

Analytic 4th order
Varying log Q^2/m^2

Epelbaum & Gelis
SU(2), g=0.5

▶ Transverse Poynting vector: Transverse plane

Chen, RJF (2013)
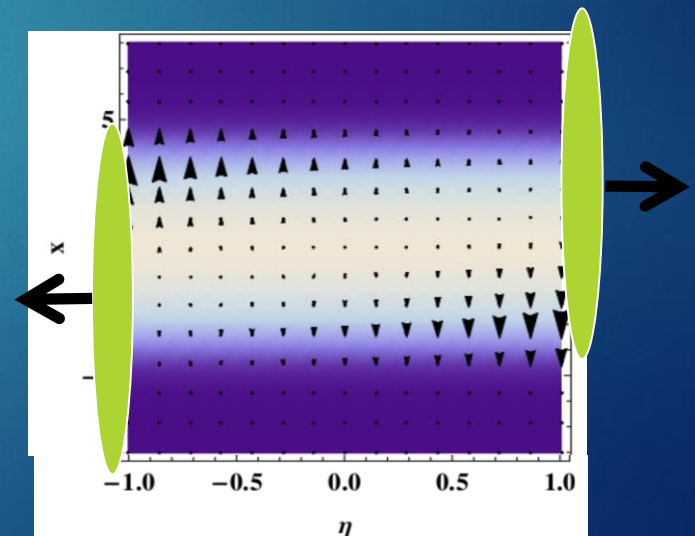
Pb+Pb, b=6 fm

▶ Transverse Poynting vector: Event Plane

(long. component suppressed)

▶ Radial and elliptic flow

▶ Rapidity-odd directed flow

▶ Angular momentum

Origin of Flow in YM

▶ Initial longitudinal fields E_0, B_0 → transverse fields through QCD versions of Ampere's, Faraday's and Gauss' Law.

    ▶ Here *abelian* version for simplicity.

▶ Gauss' Law at fixed time t

    ▶ Difference in long. flux → transverse flux

    ▶ *rapidity-odd* and *radial*

▶ Ampere/Faraday as function of t:

    ▶ Decreasing long. flux → transverse field

    ▶ *rapidity-even* and *curling* field

▶ Full classical QCD at $O(\tau^1)$:

$$E^i = -\frac{\tau}{2}\left(\sinh\eta\left[D^i, E_0\right] + \cosh\eta\,\varepsilon^{ij}\left[D^j, B_0\right]\right)$$

$$B^i = \frac{\tau}{2}\left(\cosh\eta\,\varepsilon^{ij}\left[D^j, E_0\right] - \sinh\eta\left[D^i, B_0\right]\right)$$
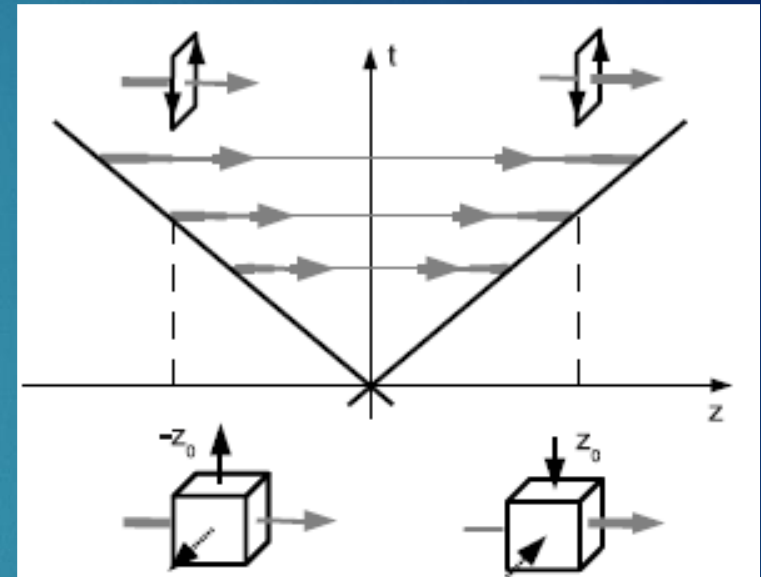
Figure 1: Two observers at $z = z_0$ and $z = -z_0$ test Ampère's and Faraday's Laws with areas a^2 in the transverse plane and Gauss' Law with a cube of volume a^3. The transverse fields from Ampère's and Faraday's Laws (black solid arrows) are the same in both cases, while the transverse fields from Gauss' Law (black dashed arrows) are observed with opposite signs. Initial longitudinal fields are indicated by solid grey arrows, thickness reflects field strength.
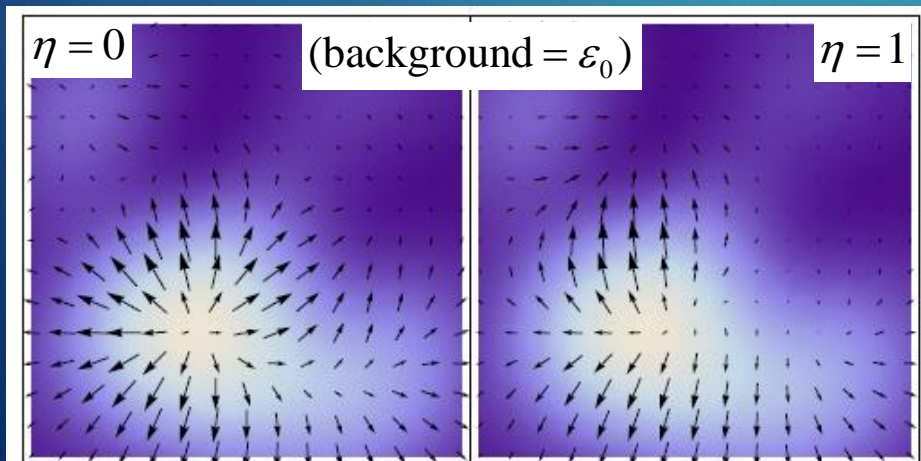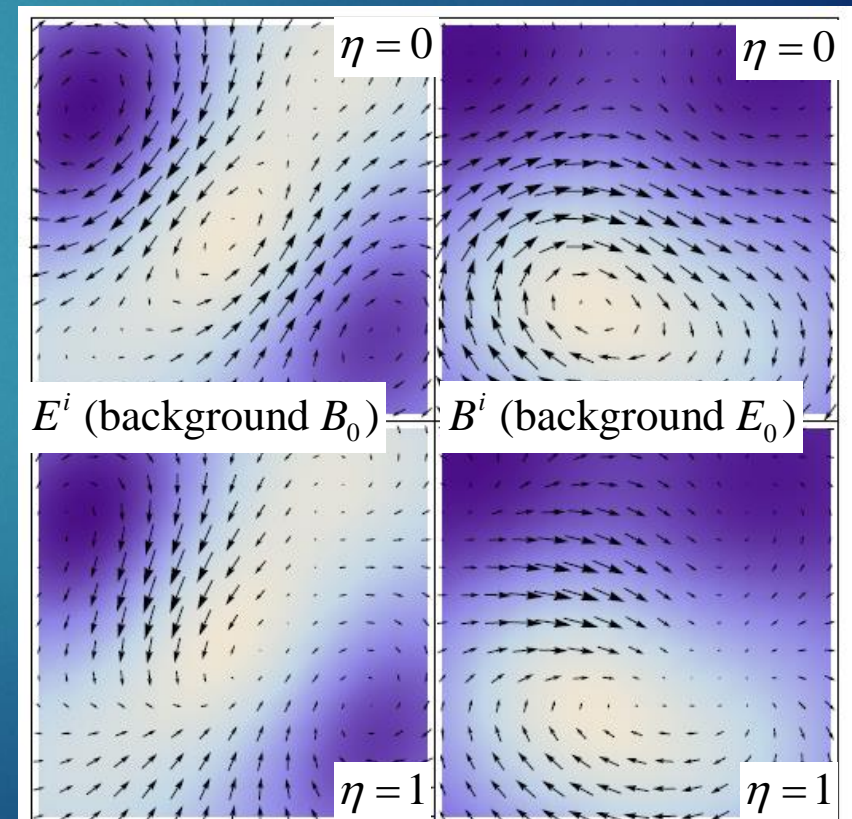
Chen, RJF (2013)

Field And Flow Patterns

▶ Transverse Poynting vector for random initial fields (abelian example).

▶ $\eta = 0$: "Hydro-like" flow from large to small energy density

▶ $\eta \neq 0$: Quenching/amplification of flow due to the underlying field structure.

Transverse Fields

Energy Flow

Chen, RJF (2013)

$\eta = 0$ (background $= \varepsilon_0$) $\eta = 1$

$\eta = 0$ $\eta = 0$

E^i (background B_0) B^i (background E_0)

$\eta = 1$ $\eta = 1$

▶ No equilibration in classical YM.

▶ Pragmatic solution: interpolate between clYM and viscous fluid dynamics, enforce conservation laws.

$$T^{\mu\nu} = T_f^{\mu\nu} r(\tau) + T_{pl}^{\mu\nu} (1 - r(\tau))$$

$$\partial_\mu T^{\mu\nu} = 0 \qquad \partial_\mu M^{\mu\nu\lambda} = 0 \qquad M^{\mu\nu\lambda} = x^\mu T^{\nu\lambda} - x^\nu T^{\mu\lambda}$$

▶ Direct matching = rapid thermalization assumption

$$r(\tau) = \Theta(\tau_0 - \tau)$$
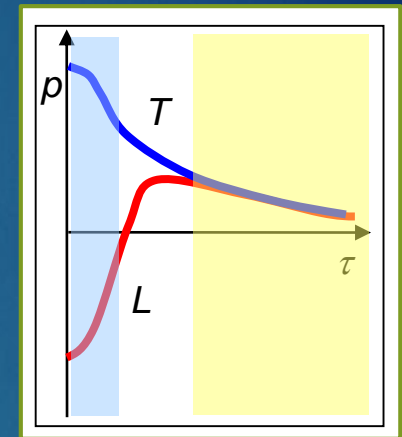
  ▶ Mathematically equivalent to imposing smoothness condition on all components of $T_{\mu\nu}$ (a la Schenke et al.).

  ▶ Decompose YM energy momentum tensor into hydro fields at some matching time τ_0

$$T_f^{\mu\nu} = (\varepsilon + p + \Pi)u^\mu u^\nu - (p + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Yang Mills in Terms of Hydro Fields

▶ We keep viscous stress even if it is large (otherwise we violate conservation laws at $\tau = \tau_0$).

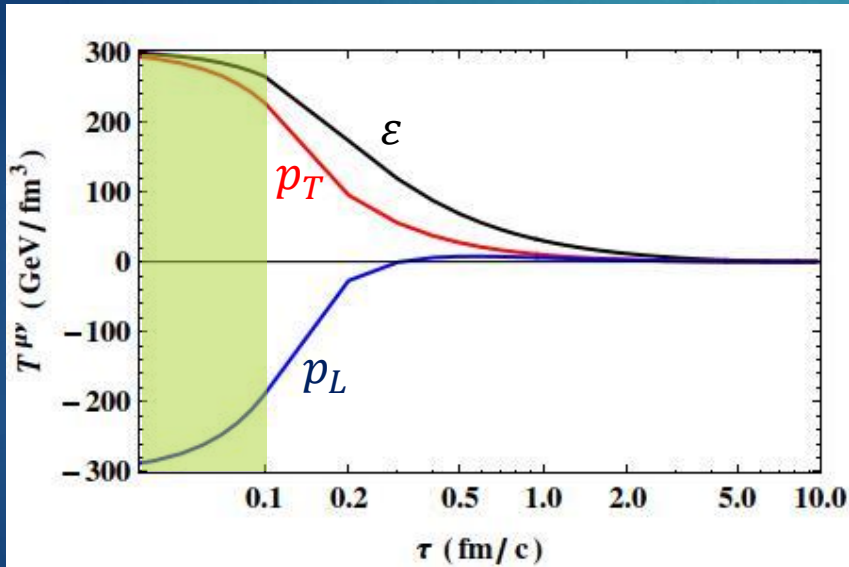▶ Viscous stress extracted from YM generally follows Navier-Stokes behavior.

▶ Samples
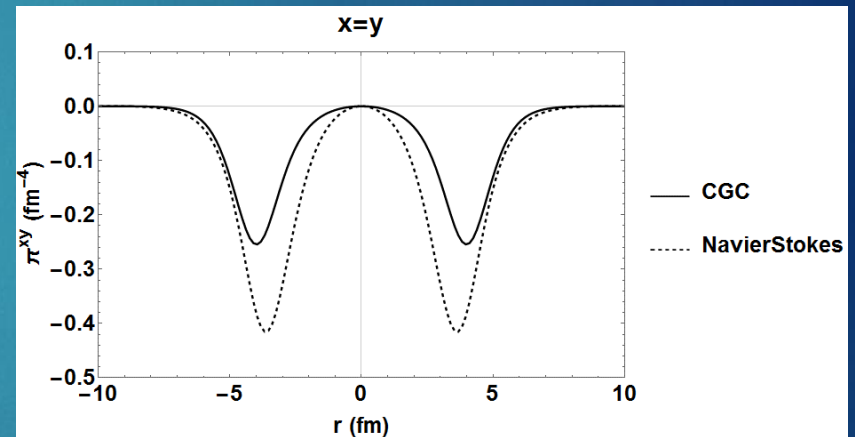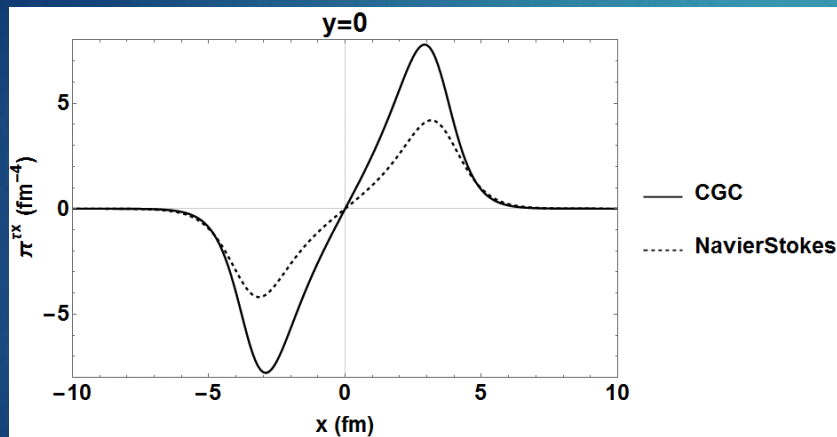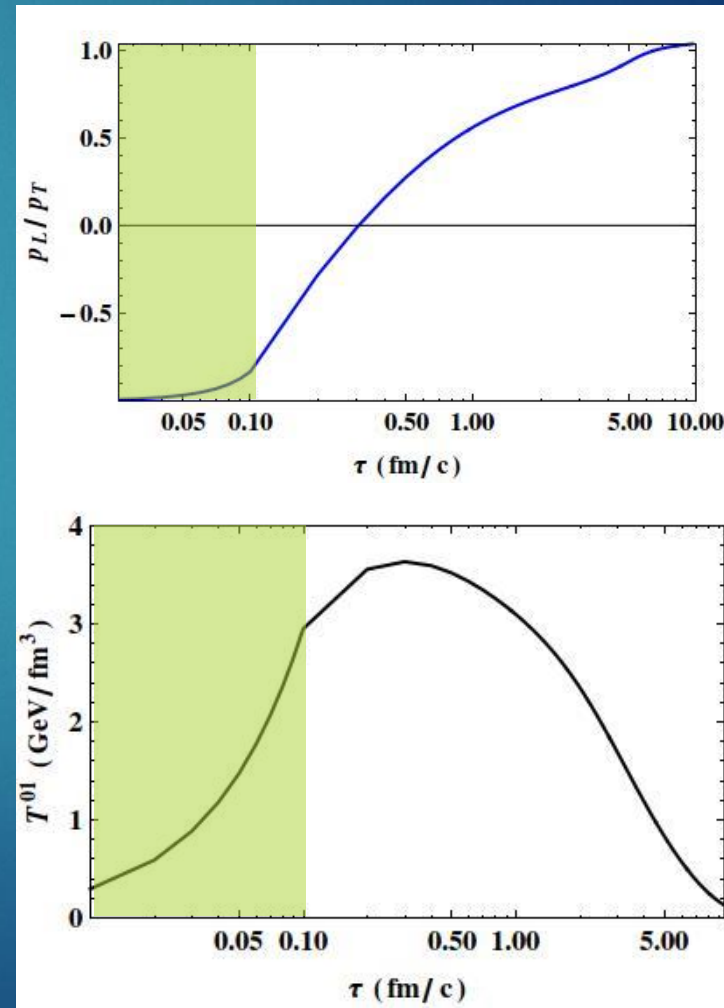

▶ Expected?

▶ Now feed into viscous fluid dynamics

Hydro Evolution

▶ Pressure and flow fields evolve smoothly

    ▶ Here Pb+Pb, b = 6 fm, not fitted to a particular energy, $\eta/s = 1/4\pi$.
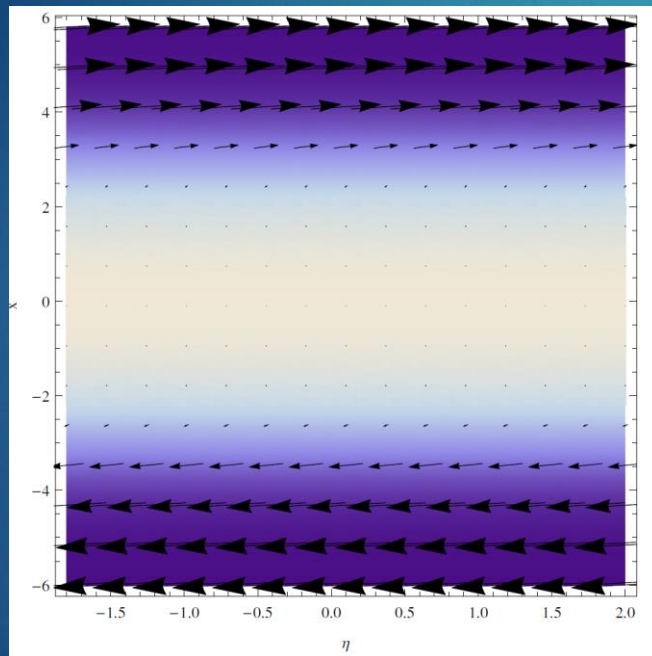
    ▶ Match at $\tau = 0.1$ fm.

▶ Not tuned to data.

Gauss Law vs Shear Viscosity
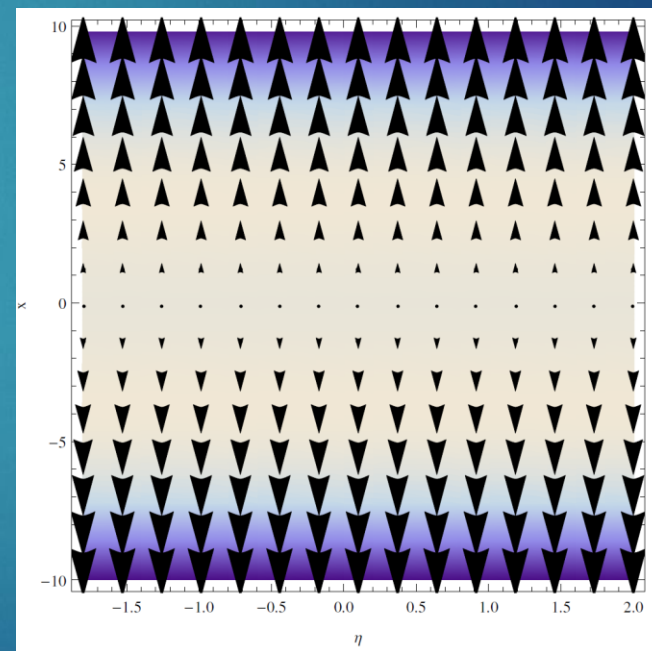
▶ Angular momentum realized as shear flow, not rotation (due to boost invariance)

▶ $\eta - x$ (event plane) velocity vector in Milne coordinates:

$\tau = 0.1$ fm

$\tau = 9.6$ fm

▶ Shear flow decreases rapidly in the hydro evolution.

Gauss Law vs Shear Viscosity

▶ Angular momentum conserved locally but not globally (sources on the light cone!)

▶ Yang-Mills phase: Gauss Law builds up shear flow.

▶ Viscous fluid dynamics works against the gradient.

$$\frac{\partial v_z}{\partial t} = \frac{\eta}{\varepsilon + p} \frac{\partial^2 v_z}{\partial x^2}$$
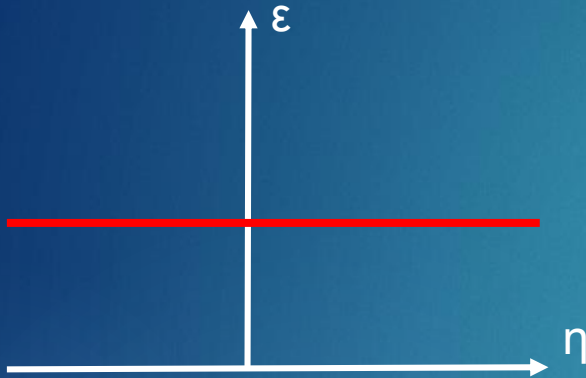
▶ Discontinuity in the time evolution.

▶ Missing microscopic information could be vital.

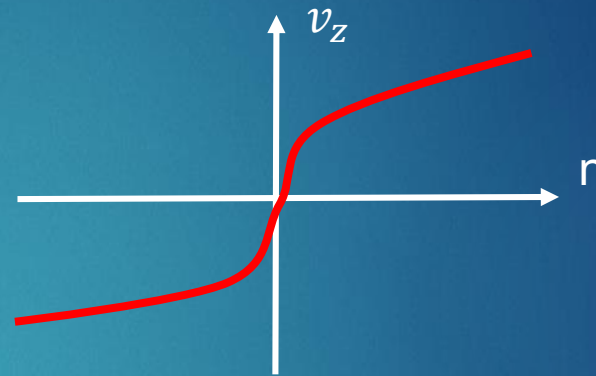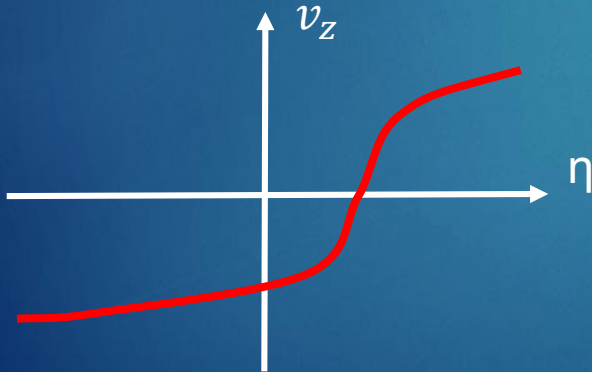▶ Conclusive study of angular momentum needs to break boost invariance.
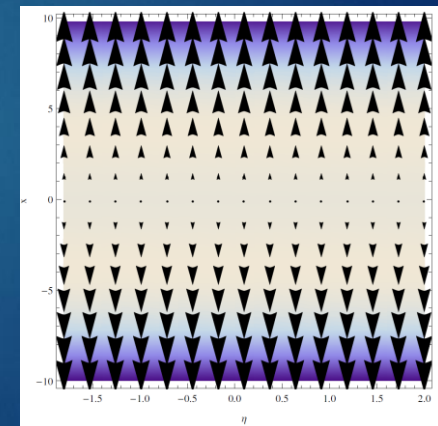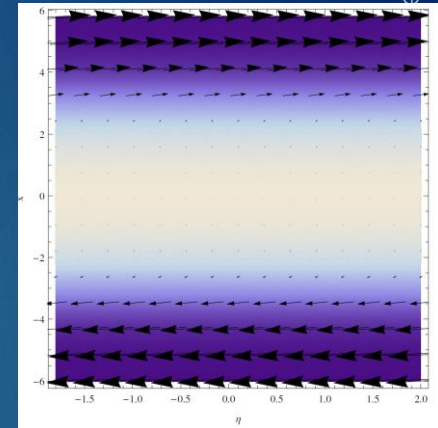
On Boost Invariance

▶ Naïve boost invariance:

▶ This is also allowed:

▶ Boost invariance does not have to pick the lab frame for the node $v_z = 0$.

▶ An asymmetry between ρ_1 and ρ_2 can lead to a shift of the node away from the lab frame.

▶ Locally determined for each point in the transverse plane.

Beyond Boost Invariance?

▶ Real nuclei are slightly off the light cone.

▶ Classical gluon distributions calculated by Lam and Mahlon.

Lam, Mahlon (2000)

▶ Using approximations valid for $R_A/\gamma \ll 1/Q_s$ we estimated the rapidity dependence of the initial energy density ε_0.

Ozonder, RJF (2014)

RHIC Au−Au
$\sqrt{s} = 200$ GeV
— Woods−Saxon
• 3dMVn
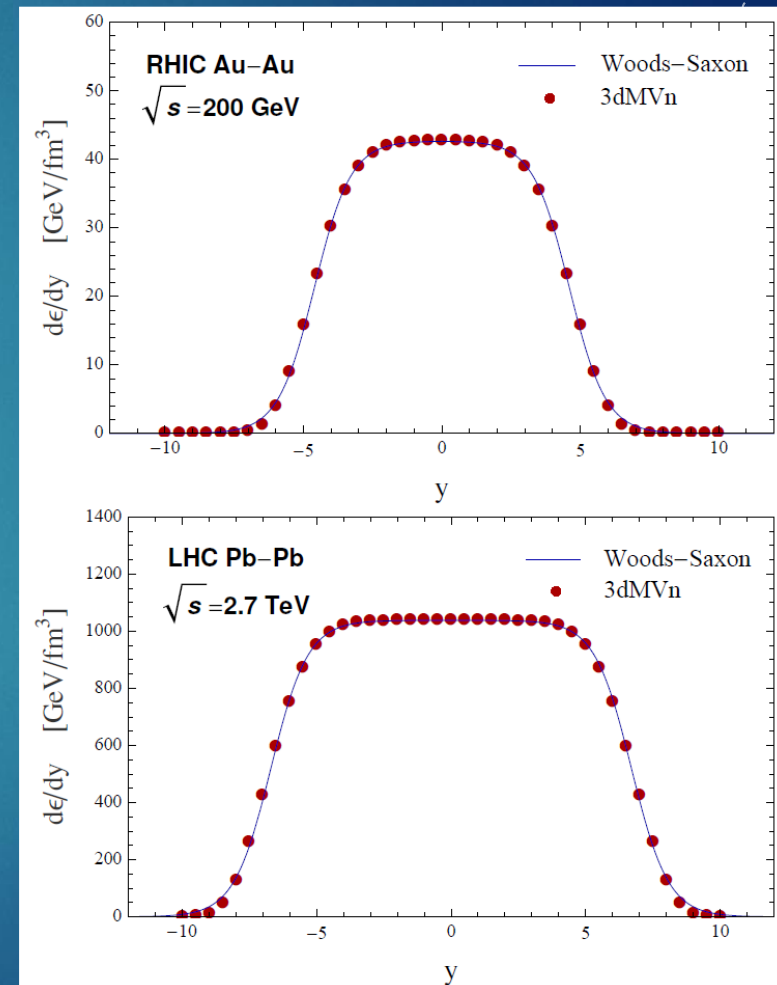
LHC Pb−Pb
$\sqrt{s} = 2.7$ TeV
— Woods−Saxon
• 3dMVn

▶ Generic arguments: convergence radius of the recursive solution ~$1/Q_s$.

▶ "Weak field" approximation to Yang Mills

$$A^{\mathrm{LO}}(\tau,\mathbf{k}_\perp) = \frac{2A_{(0)}(\mathbf{k}_\perp)}{k_\perp \tau} J_1(k_\perp \tau)$$
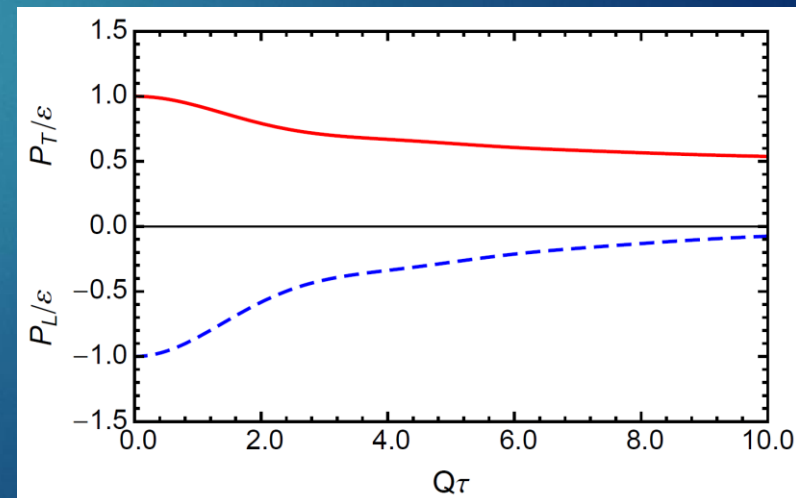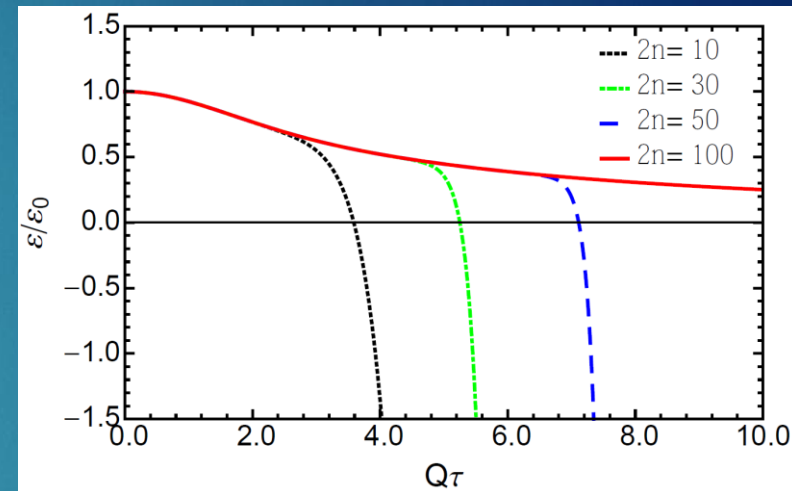
$$A_\perp^{i\,\mathrm{LO}}(\tau,\mathbf{k}_\perp) = A_{\perp(0)}^i(\mathbf{k}_\perp) J_0(k_\perp \tau)$$

  ▶ Can be rederived from the recursive solution.

▶ Resumming $(Q\tau)^k$ terms: semi-closed form

$$\mathcal{A} = \varepsilon_0 + \frac{2\varepsilon_0}{\ln^2(Q^2/m^2)} \mathsf{G}_A(Q\tau)$$
$$- \frac{\varepsilon_0}{\ln(Q^2/m^2)}(Q\tau)^2 \left[{}_3F_4\left(1,1,\frac{3}{2};2,2,2,2;-(Q\tau)^2\right) \right]$$

Li, Kapusta, 1602:09060
See Poster

Summary

▶ (Semi)-analytic results available for clQCD/MV early time evolution.

▶ Order by order in powers of time, usually good up to ~ $1/Q_s$.

▶ Partial resummations available.

▶ Flow driven by QCD Gauss/Ampere/Faraday.

▶ Rapidity odd (but boost invariant) contributions.

▶ Matching to hydrodynamics: smooth evolution of energy density and pressure.

▶ Discontinuities of other important aspects.

▶ Beyond boost-invariance: work in progress.

Backup

Texas 3+1 D Fluid Code

▶ KT for fluxes, 5th order WENO for spatial derivatives, 3rd order TVD Runge Kutta for time integration.

▶ Bulk and shear stress, vorticity

▶ Gubser and Sod-type tests:
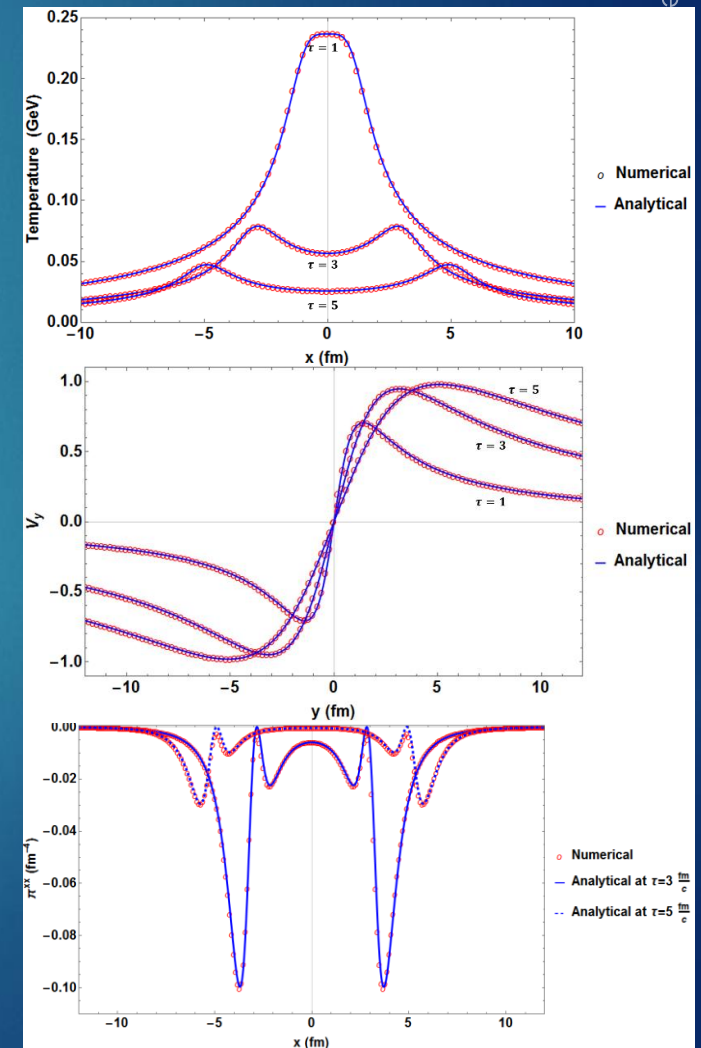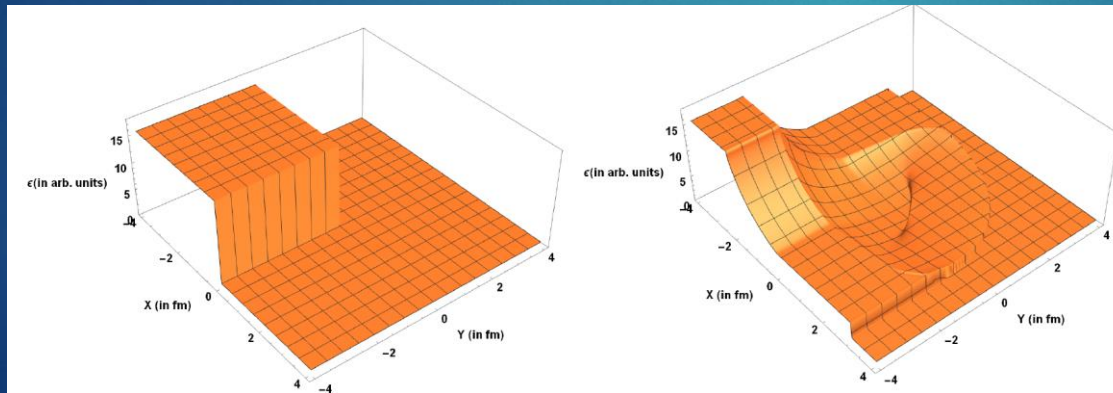
Matching to Hydrodynamics

▶ Hydro fields in Minkowski components