Fluid dynamical description of relativistic heavy ion collisions

Harri Niemi

J. W. Goethe University,
Frankfurt am Main

Initial Stages 2016
Search for QCD matter properties

Relativistic heavy ion collisions:
- Create small droplet of QCD fluid
- Extract limits for $\eta/s$, $\zeta/s$, ... from experimental data

Need a complete model:
- Initial particle production
- Fluid dynamical evolution
- Convert fluid to particle spectra
Fluid dynamics

- Fluid dynamics: power series in $\text{Re}^{-1} \sim |\pi^{\mu\nu}|/p$ and $\text{Kn} = \ell_{\text{micr}}/L_{\text{macr}}$.
- Applicability: $\text{Re}^{-1} \lesssim 1$ and $\text{Kn} \lesssim 1$.
- Sufficiently close to equilibrium and gradients are sufficiently small.

**Fluid dynamical limit:**

Dynamics of the system is entirely controlled by a few macroscopic functions, $p(T)$, $\eta(T)$, ... 

- All the microscopic information is integrated into these functions.
- In principle can be calculated from the underlying microscopic theory.
Initial states come in all shapes and sizes

Average over all events
Characterizing initial conditions

\[ \varepsilon_n e^{in\Phi_n} = \{ r^n e^{in\phi} \} \]

\[ \{ \cdots \} = \int dx dy e(x, y, \tau_0)(\cdots) \]

- \( \varepsilon_n \) eccentricity
- \( \Phi_n \) “participant plane” angle

\( n=2 \quad n=3 \quad n=4 \)

Fluid dynamics: \( \varepsilon_n, \Phi_n \rightarrow v_n, \Psi_n \)
(conversion efficiency depends on EoS, \( \eta/s, \cdots \))
Flow fluctuations

Gardim, Grassi, Luzum and Ollitrault,

Strong correlation between $v_{2/3}$ and $\varepsilon_{2/3}$, i.e. $v_n \sim C_n \varepsilon_n$

Relative fluctuations of $\varepsilon_n \rightarrow$ relative fluctuations of $v_n$ ($n = 2, 3$)

Probability distributions $P(v_n/\langle v_n \rangle) = P(\varepsilon_n/\langle \varepsilon_n \rangle)$
in peripheral collisions non-linear correlation.
shows no sensitivity to $\eta/s$. (Note: average $v_2$ scaled out)
Depend only on the initial state (good constraint)
\[ \eta/s(T) \text{ from } v_n \text{ data} \]

- \( \eta/s(T) \) parametrizations tuned to reproduce the \( v_n \) data at the LHC.
- No strong constraints to the temperature dependence (all give equally good agreement)
- Deviations mainly in peripheral collisions, where the applicability of the framework most uncertain.
Constraints for $\eta/s(T)$ from RHIC $v_n$ data

- Same $\eta/s(T)$ as at LHC.
- Simultaneous fit constraints temperature dependence

$\langle v_2^2 \rangle_{ev}$

$\eta/s = \text{param}1$

$\eta/s = \text{param}2$

$\eta/s = \text{param}3$

$\eta/s = \text{param}4$

$\langle v_2^2 v_4 \cos(4[\Psi_2 - \Psi_4]) \rangle_{ev}$

$\langle v_2^2 \rangle_{ev}$
Event-plane correlations

\[ \langle \cos(4(\Psi_2 - \Psi_4)) \rangle_{\text{SP}} \]

- LHC 2.76 TeV \( \text{Pb+Pb} \)
- \( \eta/s \)
- \( T \) [MeV]

- Already from the LHC data more constraints to \( \eta/s(T) \).
- Small hadronic viscosity needed to reproduce the data

\[ \langle \cos(k_1 \Psi_1 + \cdots + nk_n \Psi_n) \rangle_{\text{SP}} \equiv \frac{\langle v_1^k_1 \cdots v_n^{k_n} \cos(k_1 \Psi_1 + \cdots + nk_n \Psi_n) \rangle_{\text{ev}}}{\sqrt{\langle v_1^{2k_1} \rangle_{\text{ev}} \cdots \langle v_n^{2k_n} \rangle_{\text{ev}}}} \]
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Fluid dynamics in HI collisions
Rapidity dependent $v_2$

- Rapidity dependent $v_2$ gives constraints especially for low temperature (hadronic) $\eta/s$
- The data clearly favors large hadronic $\eta/s$

Temperature dependent $\eta/s$ and chemistry

- Microscopic calculations of hadronic $\eta/s(T)$: strong increase with decreasing $T$.
- EKRT (small) hadronic $\eta/s$ in chemically frozen hadron gas (PCE).
- Denicol et. al. hadrons in chemical equilibrim (CE)

Estimate $\eta/s(T)$ in chemical equilibrium:

$$\eta/s_{\text{CE}} = (\eta/s_{\text{PCE}}) \times \left( \frac{s_{\text{PCE}}}{s_{\text{CE}}} \right)$$

using $\eta_{\text{CE}} \sim \eta_{\text{PCE}}$ (Wiranata, Prakash, Huovinen, Wang, J.Phys.Conf.Ser. 535 (2014) 012017)
Bulk viscosity

- Bulk viscosity can be large near the QCD transition
- Large bulk viscosity affects the determination of $\eta/s$
- Helps to reduce average $p_T$ (important especially at LHC energies)
Beam Energy scan

- More constraints to the hadronic properties of the matter
- Important background in determining the QGP properties
- Here constant $\eta/s$ fitted separately for each $\sqrt{s}$

Evidence for temperature and/or net-baryon density dependence of $\eta/s$?

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Fluid dynamics in HI collisions
p+Pb collisions

- Can be described by using hydrodynamics
- Typically $\eta/s$ small $O(0.08)$
- Inconsistency with AA results with saturation based initial conditions $\eta/s \sim 0.20$

Is hydrodynamics valid?


Global fits and emulators

Hydrodynamical behavior: All the different system described by the same equation of state and transport coefficients:

\[ \eta/s(T, \{\mu_i\}), \zeta/s(T, \{\mu_i\}), p(T, \{\mu_i\}), \ldots \]

Different collisions at different collision energies probe different regions of temperature and densities.

Call for global analysis

Computationally very expensive
(mainly because many observables require event-by-event analysis)

Solution: Emulators

Global fits and emulators

- Calculate large number of collision events with random choices for the parameters (avoid huge number of runs)
- Emulator: (essentially) interpolate between the runs
- Start from the equal probability for each of the free parameters.
- Weight with the probability of describing the actual measurements
- Here separately for two different types of initial conditions.

Global fits and emulators

- Continuous parametrization of initial states (TRENTO) from KLN to Glauber wounded nucleon model
- Temperature dependent $\eta/s + \text{UrQMD}$
- $dN/dy$, $\langle p_T \rangle$ and $\nu_n$ (RHIC 200 GeV and LHC 2.76 TeV)


[nucl-th]
Constraining the equation of state
Similar work with emphasis in constraining EoS

Summary

- The magnitude, fluctuations and correlations of the flow coefficients in a wide variety of systems can be described using relativistic hydrodynamics.
- At least between some systems (top RHIC energy and the LHC) it is possible to find a consistent description (same EoS, transport coefficients).
- These findings suggest that we indeed create a small droplet of fluid in AA collisions, and that at least the minimum value of $\eta/s$ is small.
- Temperature dependence of $\eta/s$ is not yet so well constrained.
- Applicability limits of fluid dynamics still not known (does it work in pA?)
- Bulk viscosity is not yet well constrained, but apparently non-zero.
- Emulators make global statistical analysis feasible.
Transient Fluid Dynamics (Israel & Stewart)

\[ \pi^{\mu\nu} = 2\eta \nabla \langle \mu u^\nu \rangle \]

\[ \tau_{\pi} \frac{d}{d\tau} \pi^{\langle \mu \nu \rangle} + \pi^{\mu\nu} = 2\eta \nabla \langle \mu u^\nu \rangle - c_1 \tau_{\pi} \pi^{\mu\nu} \nabla \lambda u^\lambda - c_2 \tau_{\pi} \pi^{\langle \mu \sigma \nu \rangle \lambda} + \cdots \]

- Transient: \( \pi^{\mu\nu} \to 2\eta \nabla \langle \mu u^\nu \rangle + O(2) \) with timescale \( \tau_{\pi} \)
- \( \pi^{\mu\nu}/p \sim \text{Re}^{-1} \) inverse Reynolds number: measures deviations from equilibrium
- \( \tau_{\pi} \nabla \lambda u^\lambda \sim \text{Kn} \) Knudsen number: measures separation between microscopic scales (\( \tau_{\pi} \)) and macroscopic scales (\( \nabla \lambda u^\lambda = \text{volume expansion rate} \))
- \( O(1) \times O(1) = O(2) \) Second order fluid dynamics
- linearly stable and causal
- \( c_n \) e.g. from kinetic theory ([Denicol,Niemi,Molnar,Rischke,PRD85(2012)114047])

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Fluid dynamics in HI collisions
Converting fluid to particles (Freeze-out)

\[ e, u^\mu, \pi^{\mu\nu} \longrightarrow E \frac{dN}{d^3p} \]

- Standard Cooper-Frye freeze-out for particle \( i \)
  \[
  E \frac{dN}{d^3p} = \frac{g_i}{(2\pi)^3} \int d\sigma^\mu p_\mu f_i(p, x),
  \]
  where
  \[
  f_i(p, x) = f_{i,eq}(p, u^\mu, T, \{\mu_i\}) \left[ 1 + \frac{\pi^{\mu\nu} p_\mu p_\nu}{2T^2(e+p)} \right]
  \]
- Integral over constant temperature hypersurface (decoupling surface)
- Decays of unstable hadrons
(non)linear-response?

5-10 %

5−10 %

(b)

LHC 2.76 TeV Pb + Pb

55-60 %

55−60 %

(h)

LHC 2.76 TeV Pb + Pb

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Fluid dynamics in HI collisions