Full NLO corrections for DIS structure functions in the dipole factorization formalism

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Outline

- Introduction: what was known, and what was the problem?

- One-loop correction to the $\gamma^*_{T,L} \to q\bar{q}$ light-front wave-functions:

  Direct calculation

  G.B., *to appear in* arXiv:1606.xxxxx

- DIS at NLO in the dipole factorization

  Example: F_L case

  G.B., *in preparation*

Introduction

At low x_{Bj}, many DIS observables can be expressed within dipole factorization, including gluon saturation $\rightarrow$ rich phenomenology.

In particular: Dipole amplitude obtained from fits of HERA data for DIS structure functions in the dipole factorization at LO+LL with rcBK
Albacete *et al.*, PRD80 (2009); EPJC71 (2011)
Kuokkanen *et al.*, NPA875 (2012);
Lappi, Mäntysaari, PRD88 (2013)

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In the last 10 years, many theoretical (including numerical) progresses towards NLO/NLL accuracy for gluon saturation/CGC.

Obviously, DIS structure functions at NLO in the dipole factorization are required to push the fits beyond LO+LL accuracy.

DIS at NLO: general structure in dipole factorization
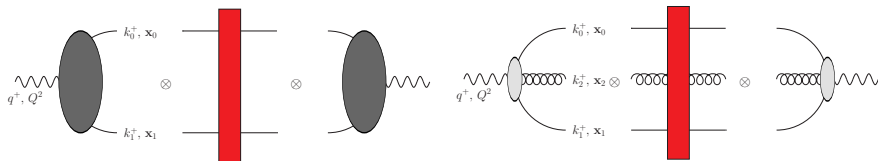
$$\sigma_{T,L}(Q^2, x_{Bj}) = \sum_{q\bar{q} \text{ states}} \left| \widetilde{\Psi}_{q\bar{q}}^{\gamma_{T,L}^*} \right|^2 \left[1 - \langle \mathcal{S}_{01} \rangle_0 \right]$$

$$+ \sum_{q\bar{q}g \text{ states}} \left| \widetilde{\Psi}_{q\bar{q}g}^{\gamma_{T,L}^*} \right|^2 \left[1 - \langle \mathcal{S}_{012} \rangle_0 \right] + O(\alpha_{em} \alpha_s^2)$$

With:

- $\widetilde{\Psi}_{q\bar{q}(g)}^{\gamma_{T,L}^*}$: LFWF for a $q\bar{q}(g)$ Fock state (in mixed space) inside an incoming γ^* (in momentum space)
- $\langle \mathcal{S}_{01(2)} \rangle_0$: dipole and tripole operators in the quasi-classical approximation

DIS at NLO: existing results

2 independent calculations have been performed for NLO corrections to photon impact factor and/or DIS cross-section:

1. Balitsky, Chirilli, PRD83 (2011); PRD87 (2013)
   Using covariant perturbation theory. Results provided as
   - Current correlator in position space
   - Impact factor for $k_\perp$ factorization $\rightarrow$ Good for BFKL phenomenology

2. G.B., PRD85 (2012)
   Using light-front perturbation theory. Results provided as
   - DIS structure functions in dipole factorization
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However, in both papers only the $q\bar{q}g$ contribution was calculated explicitly, whereas NLO corrections to the $q\bar{q}$ contribution were guessed. Methods used for that:

1. In Balitsky, Chirilli, PRD83 (2011):
   Matching with older vacuum results. (But not very clear to me.)
2. In G.B., PRD85 (2012):
   Unitary argument. But I realized later that it does not work...

Unitarity sum rule

Fock state decomposition of the physical state of an incoming γ:

$$|\gamma_{\text{phys}}\rangle = \sqrt{Z_\gamma}\Bigg[a_\gamma^\dagger\,|0\rangle + \sum_{l\bar{l}\ \text{states}}\ \Psi_{l\bar{l}}^\gamma\ \ b_l^\dagger\,d_{\bar{l}}^\dagger\,|0\rangle + \sum_{q\bar{q}\ \text{states}}\ \Psi_{q\bar{q}}^\gamma\ \ b_q^\dagger\,d_{\bar{q}}^\dagger\,|0\rangle$$
$$+ \sum_{q\bar{q}g\ \text{states}}\ \Psi_{q\bar{q}g}^\gamma\ \ b_q^\dagger\,d_{\bar{q}}^\dagger\,a_g^\dagger\,|0\rangle + \ \cdots\Bigg]$$

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$$
\begin{aligned}
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&\quad + \sum_{q\bar{q}g \text{ states}} \Psi_{q\bar{q}g}^\gamma \ b_q^\dagger \, d_{\bar{q}}^\dagger \, a_g^\dagger |0\rangle + \cdots \Bigg]
\end{aligned}
$$

Normalization of both the physical state and the Fock states implies:

$$
\frac{1-Z_\gamma}{Z_\gamma} = \sum_{\bar{l}l \text{ states}} \left|\Psi_{\bar{l}l}^\gamma\right|^2 + \sum_{q\bar{q} \text{ states}} |\Psi_{q\bar{q}}^\gamma|^2 + \sum_{q\bar{q}g \text{ states}} |\Psi_{q\bar{q}g}^\gamma|^2 + O(\alpha_{em}\,\alpha_s^2)
$$

Perturbative expansion $\Rightarrow$ at each order, one gets a new relation .

Unitarity sum rule

In particular, terms of order $\alpha_{em}\,\alpha_s$:

$$\left(1 - Z_\gamma\right)_{\alpha_{em}\,\alpha_s} = \left(\sum_{q\bar{q}\ \text{states}} |\Psi_{q\bar{q}}^\gamma|^2\right)_{\alpha_{em}\,\alpha_s} + \left(\sum_{q\bar{q}g\ \text{states}} |\Psi_{q\bar{q}g}^\gamma|^2\right)_{\alpha_{em}\,\alpha_s}$$

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$$\left(1-Z_\gamma\right)_{\alpha_{em}\,\alpha_s} \;=\; \left(\sum_{q\bar{q}\ \text{states}} \left|\Psi^\gamma_{q\bar{q}}\right|^2\right)_{\alpha_{em}\,\alpha_s} + \left(\sum_{q\bar{q}g\ \text{states}} \left|\Psi^\gamma_{q\bar{q}g}\right|^2\right)_{\alpha_{em}\,\alpha_s}$$

In the previous study (G.B., PRD85 (2012)):

I assumed that $\left(1-Z_\gamma\right)$ received no $\alpha_{em}\,\alpha_s$ contribution, in order to get $\left(\sum_{q\bar{q}\ \text{states}} \left|\Psi^\gamma_{q\bar{q}}\right|^2\right)_{\alpha_{em}\,\alpha_s}$ from $\left(\sum_{q\bar{q}g\ \text{states}} \left|\Psi^\gamma_{q\bar{q}g}\right|^2\right)_{\alpha_{em}\,\alpha_s}$

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$$\left(1-Z_\gamma\right)_{\alpha_{em}\,\alpha_s} = \left(\sum_{q\bar{q}\text{ states}}\left|\Psi^\gamma_{q\bar{q}}\right|^2\right)_{\alpha_{em}\,\alpha_s} + \left(\sum_{q\bar{q}g\text{ states}}\left|\Psi^\gamma_{q\bar{q}g}\right|^2\right)_{\alpha_{em}\,\alpha_s}$$

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However, there is a non-trivial (and finite) contribution to $\left(1-Z_\gamma\right)$ at order $\alpha_{em}\,\alpha_s$.

$\Rightarrow$ In this approach, not possible to get the $\left|\Psi^\gamma_{q\bar{q}}\right|^2$ at NLO from unitarity!

$\Rightarrow$ One-loop correction to $\Psi^\gamma_{q\bar{q}}$ has to be calculated independently

Calculation of the $\gamma_{T,L} \to q\bar{q}$ LF wave-functions at NLO

- Calculation done in Light-front perturbation theory for QCD+QED

- Cut-off $k_{\min}^+$ introduced to regulate the small k^+ (soft) divergences

- No collinear divergence can show up in this calculation ($Q^2 > 0$)

- UV divergences from various tensor integrals, but no UV renormalization at this order.
  $\Rightarrow$ UV divergences (and finite regularization artifacts) have to cancel at cross-section level
  $\Rightarrow$ Need a consistent UV regularization (not cut-off!)
  $\Rightarrow$ Use (Conventional) Dimensional Regularization, and pay attention to rational terms in $(D-4)/(D-4)$

- Convenient trick: Tensor reduction of transverse integrals (Passarino-Veltman)
  Allows to organize better the calculation (reduces the number of integrals to calculate and of Dirac structures) and show the cancellation of unphysical divergences already at the integrand level

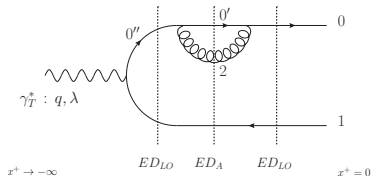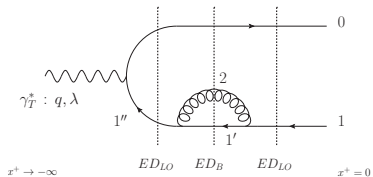# Diagrams for γ_T and γ_L LFWFs: self-energies

Diagram A

Diagram B

- Straightforward to calculate

- Clearly factors into LO wave-function times Form Factor

- DimReg prevents quadratic UV divergences to appear, only logarithmic ones remain

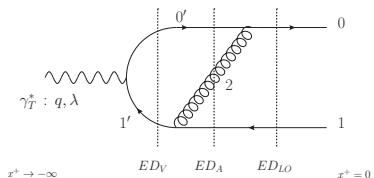- Contain not only log but also unphysical $\log^2$ soft divergences

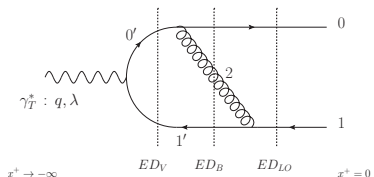# Diagrams for γ_T and γ_L LFWFs: vertex corrections

Diagram 1

Diagram 2

- By far the hardest to calculate

- Involves various tensor integrals in transverse-momentum as well as various Dirac structures

- Contain unphysical $\log^2$ soft divergences which cancel the ones of the previous graphs.

- In the γ_L case: contain unphysical power-like soft divergences.

- In the γ_T case: even after tensor reduction, still not proportional to the LO LFWF: one extra piece remain. However, it cancels between the diagrams 1 and 2.
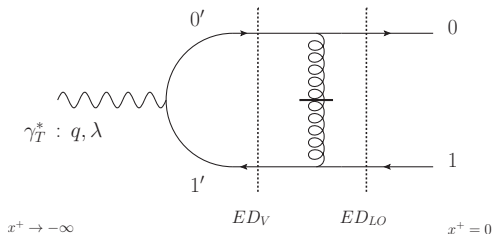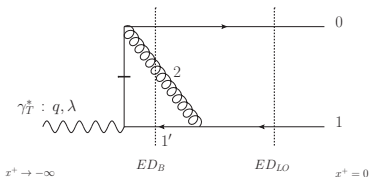
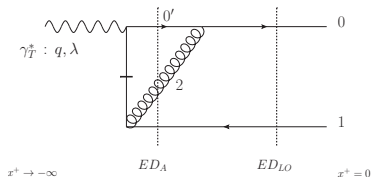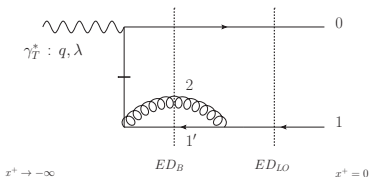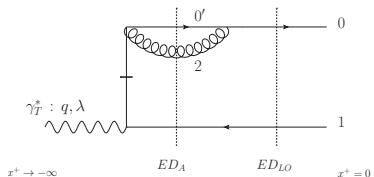# Diagrams for γ_T and γ_L LFWFs: vertex corrections

Diagram 3

- In the γ_T case: vanishes due to Lorentz symmetry

- In the γ_L case: non-zero, and cancels the unphysical power-like soft divergences of the other vertex correction graphs.

Diagrams for the $\gamma_T \to q\bar{q}$ LF wave-function only

Diagram A'   Diagram B'

Diagram 1'   Diagram 2'

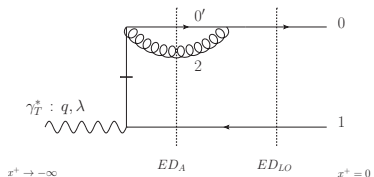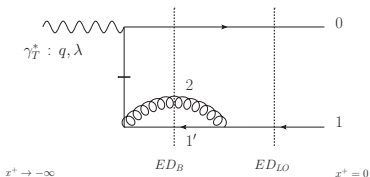# Diagrams for the $\gamma_T \to q\bar{q}$ LF wave-function only
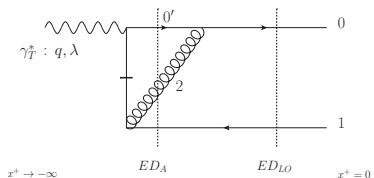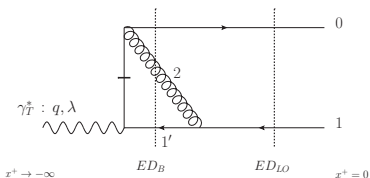

Diagram A'

Diagram B'

Diagram 1'

Diagram 2'

All four vanish due to Lorentz symmetry!

Results for NLO $\gamma_{T,L} \rightarrow q\bar{q}$ LFWFs in momentum space

$$\Psi_{q_0\bar{q}_1}^{\gamma_{T,L}^*} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right)\, \mathcal{V}^{T,L}\right]\, \Psi_{q_0\bar{q}_1, LO}^{\gamma_{T,L}^*} + \mathcal{O}(e\,\alpha_s^2)$$

$$\mathcal{V}^L = 2\left[\log\left(\frac{k_{min}^+}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right]\left[\Gamma\left(2 - \frac{D}{2}\right)\left(\frac{\overline{Q}^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2} - 2\log\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\overline{Q}^2}\right)\right]$$

$$+ \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O(D-4)$$

$$\mathcal{V}^T = \mathcal{V}^L + 2\left[\log\left(\frac{k_{min}^+}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right]\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\mathbf{P}^2}\right)\log\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\overline{Q}^2}\right) + O(D-4)$$

Notations: $\overline{Q}^2 \equiv \frac{k_0^+ k_1^+}{(q^+)^2}\, Q^2$,

and relative transverse momentum: $\mathbf{P} \equiv \mathbf{k}_0 - \frac{k_0^+}{q^+}\, \mathbf{q} = -\mathbf{k}_1 + \frac{k_1^+}{q^+}\, \mathbf{q}$

Results for NLO $\gamma_{T,L} \to q\bar{q}$ LFWFs in momentum space

$$\Psi^{\gamma^*_{T,L}}_{q_0\bar{q}_1} = \left[1 + \left(\frac{\alpha_s\,C_F}{2\pi}\right)\,\mathcal{V}^{T,L}\right]\,\Psi^{\gamma^*_{T,L}}_{q_0\bar{q}_1,LO} + \mathcal{O}(e\,\alpha_s^2)$$

$$\mathcal{V}^L = 2\left[\log\left(\frac{k^+_{min}}{\sqrt{k^+_0 k^+_1}}\right) + \frac{3}{4}\right]\left[\Gamma\left(2 - \frac{D}{2}\right)\left(\frac{\overline{Q}^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2} - 2\log\left(\frac{\mathbf{P}^2 + \overline{Q}^2}{\overline{Q}^2}\right)\right]$$

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Remark: results consistent with the ones of Boussarie, Grabovsky, Szymanowski and Wallon, arXiv:1606.xxxxx

Results for NLO $\gamma_{T,L} \rightarrow q\bar{q}$ LFWFs in mixed space

$$\widetilde{\Psi}_{q_0\bar{q}_1}^{\gamma_{T,L}^*} = \left[1 + \left(\frac{\alpha_s\, C_F}{2\pi}\right)\widetilde{\mathcal{V}}^{T,L}\right]\widetilde{\Psi}_{q_0\bar{q}_1,LO}^{\gamma_{T,L}^*} + \mathcal{O}(e\,\alpha_s^2)$$

$$\begin{aligned}
\widetilde{\mathcal{V}}^T &= \widetilde{\mathcal{V}}^L + O\,(D-4) \\
&= 2\left[\log\left(\frac{k_{min}^+}{\sqrt{k_0^+ k_1^+}}\right) + \frac{3}{4}\right]\left[\frac{\Gamma\left(2-\frac{D}{2}\right)}{(4\pi)^{\frac{D}{2}-2}} + \log\left(\frac{\mathbf{x}_{01}^2\,\mu^2}{4}\right) - 2\Psi(1)\right] \\
&\quad + \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + 3 + O\,(D-4)
\end{aligned}$$

- In mixed space: NLO corrections $\Rightarrow$ rescaling of the LO $\gamma_{T,L} \rightarrow q\bar{q}$ LFWFs by a factor independent of the photon polarization and virtuality !

- Leftover logarithmic UV and soft divergences to be dealt with at cross-section level.

From LFWFs to DIS cross-section
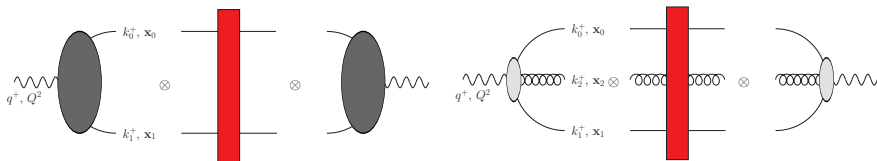
$$\sigma_{T,L}(Q^2, x_{Bj}) = \sum_{q\bar{q}\,\text{states}} \left| \widetilde{\Psi}_{q\bar{q}}^{\gamma_{T,L}^*} \right|^2 \left[1 - \langle \mathcal{S}_{01} \rangle_0 \right]$$

$$+ \sum_{q\bar{q}g\,\text{states}} \left| \widetilde{\Psi}_{q\bar{q}g}^{\gamma_{T,L}^*} \right|^2 \left[1 - \langle \mathcal{S}_{012} \rangle_0 \right] + O(\alpha_{em}\,\alpha_s^2)$$

$\widetilde{\Psi}_{q\bar{q}}^{\gamma_{T,L}^*}$ now known at NLO accuracy in Dim Reg.
$\Rightarrow$ Need to be combined with the $q\bar{q}g$ contribution
$\Rightarrow$ $\widetilde{\Psi}_{q\bar{q}g}^{\gamma_{T,L}^*}$ is required also in Dim Reg, in order to cancel UV divergences
as well as scheme dependent constants.

Example: γ_L total cross section at NLO

$$\sigma_L = 4N_c\,\alpha_{em}\,\mathrm{Re}\sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x_0}}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x_1}}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \frac{4Q^2}{q^+}\left(\frac{k_0^+}{q^+}\right)^2 \left(\frac{k_1^+}{q^+}\right)^2$$

$$\times \left\{ \delta(k_0^+ + k_1^+ - q^+) \left[\mathrm{K}_0\left(Qx_{01}\frac{\sqrt{k_0^+ k_1^+}}{q^+}\right) \right]^2 \left[1 + \frac{\alpha_s C_F}{\pi}\,\widetilde{\mathcal{V}}^L_{\mathrm{reg.}}\right]\left[1 - \langle \mathcal{S}_{01}\rangle_0\right] \right.$$

$$\left. + \frac{\alpha_s C_F}{\pi} \int_{k_{\min}^+}^{+\infty} \frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+) \int \frac{\mathrm{d}^2\mathbf{x_2}}{2\pi} \left[q\ \mathrm{term} + \bar{q}\ \mathrm{term} + \mathrm{leftover} \right] \right\}$$

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With:

$$q \text{ term} = \left[2 + \left(\frac{2k_2^+}{k_0^+}\right) + \left(\frac{k_2^+}{k_0^+}\right)^2\right]\left[\frac{\mathbf{x}_{20}}{x_{20}^2}\cdot\left(\frac{\mathbf{x}_{20}}{x_{20}^2} - \frac{\mathbf{x}_{21}}{x_{21}^2}\right)\right]$$

$$\times \left\{\left[\mathrm{K}_0(QX_{012})\right]^2\left[1 - \langle\mathcal{S}_{012}\rangle_0\right] - \left[\mathrm{K}_0\left(Qx_{01}\frac{\sqrt{(k_0^+ + k_2^+)k_1^+}}{q^+}\right)\right]^2\left[1 - \langle\mathcal{S}_{01}\rangle_0\right]\right\}$$

$$X_{012}^2 \equiv \frac{1}{(q^+)^2}\left[k_0^+ k_1^+ x_{01}^2 + k_0^+ k_2^+ x_{02}^2 + k_1^+ k_2^+ x_{12}^2\right] = \frac{q\bar{q}g \text{ form. time}}{2q^+}$$

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With:

$$\bar{q}\ \mathrm{term} = \left[2 + \left(\frac{2k_2^+}{k_1^+}\right) + \left(\frac{k_2^+}{k_1^+}\right)^2\right]\left[\frac{\mathbf{x}_{21}}{x_{21}^2}\cdot\left(\frac{\mathbf{x}_{21}}{x_{21}^2} - \frac{\mathbf{x}_{20}}{x_{20}^2}\right)\right]$$

$$\times \left\{\left[\mathrm{K}_0(QX_{012})\right]^2\left[1 - \langle\mathcal{S}_{012}\rangle_0\right] - \left[\mathrm{K}_0\left(Qx_{01}\frac{\sqrt{k_0^+(k_1^+ + k_2^+)}}{q^+}\right)\right]^2\left[1 - \langle\mathcal{S}_{01}\rangle_0\right]\right\}$$

$$X_{012}^2 \equiv \frac{1}{(q^+)^2}\left[k_0^+ k_1^+ x_{01}^2 + k_0^+ k_2^+ x_{02}^2 + k_1^+ k_2^+ x_{12}^2\right] = \frac{q\bar{q}g\ \mathrm{form.\ time}}{2q^+}$$

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$$\sigma_L = 4N_c\,\alpha_{em}\,\mathrm{Re}\sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi} \int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi} \int_0^{+\infty} dk_0^+ \int_0^{+\infty} dk_1^+ \frac{4Q^2}{q^+}\left(\frac{k_0^+}{q^+}\right)^2\left(\frac{k_1^+}{q^+}\right)^2$$

$$\times\left\{\delta(k_0^+ + k_1^+ - q^+)\left[\mathrm{K}_0\left(Qx_{01}\frac{\sqrt{k_0^+ k_1^+}}{q^+}\right)\right]^2\left[1+\frac{\alpha_s C_F}{\pi}\widetilde{\mathcal{V}}_{\mathrm{reg.}}^L\right]\left[1-\langle\mathcal{S}_{01}\rangle_0\right]\right.$$

$$\left.+\frac{\alpha_s C_F}{\pi}\int_{k_{\min}^+}^{+\infty}\frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\left[q\text{ term} + \bar{q}\text{ term} + \text{leftover}\right]\right\}$$

With:

$$\text{leftover} \;=\; \left[\left(\frac{k_2^+}{k_0^+}\right)^2 + \left(\frac{k_2^+}{k_1^+}\right)^2\right]\left[\mathrm{K}_0(QX_{012})\right]^2\left(\frac{\mathbf{x}_{20}\cdot\mathbf{x}_{21}}{x_{20}^2\,x_{21}^2}\right)\left[1-\langle\mathcal{S}_{012}\rangle_0\right]$$

$$X_{012}^2 \equiv \frac{1}{(q^+)^2}\left[k_0^+ k_1^+ x_{01}^2 + k_0^+ k_2^+ x_{02}^2 + k_1^+ k_2^+ x_{12}^2\right] = \frac{q\bar{q}g\text{ form. time}}{2q^+}$$

Example: γ_L total cross section at NLO

$$\sigma_L = 4N_c\,\alpha_{em}\,\mathrm{Re}\sum_f e_f^2 \int \frac{\mathrm{d}^2\mathbf{x}_0}{2\pi}\int \frac{\mathrm{d}^2\mathbf{x}_1}{2\pi}\int_0^{+\infty}\!\!dk_0^+\int_0^{+\infty}\!\!dk_1^+\;\frac{4Q^2}{q^+}\left(\frac{k_0^+}{q^+}\right)^2\left(\frac{k_1^+}{q^+}\right)^2$$

$$\times\left\{\delta(k_0^+ + k_1^+ - q^+)\left[\mathrm{K}_0\!\left(Qx_{01}\frac{\sqrt{k_0^+ k_1^+}}{q^+}\right)\right]^2\left[1 + \frac{\alpha_s C_F}{\pi}\widetilde{\mathcal{V}}^L_{\mathrm{reg.}}\right]\left[1 - \langle \mathcal{S}_{01}\rangle_0\right]\right.$$

$$\left. + \frac{\alpha_s C_F}{\pi}\int_{k_{\min}^+}^{+\infty}\frac{dk_2^+}{k_2^+}\,\delta(k_0^+ + k_1^+ + k_2^+ - q^+)\int\frac{\mathrm{d}^2\mathbf{x}_2}{2\pi}\left[q\text{ term} + \bar{q}\text{ term} + \text{leftover}\right]\right\}$$

With:

$$\widetilde{\mathcal{V}}^L_{\mathrm{reg.}} = \frac{1}{2}\left[\log\left(\frac{k_0^+}{k_1^+}\right)\right]^2 - \frac{\pi^2}{6} + \frac{5}{2}$$

UV and soft divergent terms have been moved from $\widetilde{\mathcal{V}}^L$ to the q and $\bar{q}$ terms, as well as a constant $1/2$ (rational term $(D-4)/(D-4)$)

BK/JIMWLK resummation

1. Assign $k_{\min}^+$ to the scale set by the target: $k_{\min}^+ = \frac{Q_0^2}{2x_0 P^-} = \frac{x_{Bj} \, Q_0^2}{x_0 \, Q^2} \, q^+$

2. Choose a factorization scale $k_f^+ \lesssim k_0^+, k_1^+$, corresponding to a range for the high-energy evolution $Y_f^+ \equiv \log\left(\frac{k_f^+}{k_{\min}^+}\right) = \log\left(\frac{x_0 \, Q^2 \, k_f^+}{x_{Bj} \, Q_0^2 \, q^+}\right)$

3. In the LO term in the observable, make the replacement

$$\langle \mathcal{S}_{012}\rangle_0 = \langle \mathcal{S}_{012}\rangle_{Y_f^+} - \int_0^{Y_f^+} dY^+ \left(\partial_{Y^+}\langle \mathcal{S}_{012}\rangle_{Y^+}\right)$$

with both terms calculated with the same evolution equation

4. Combine the second term with the NLO correction to cancel its $k_{\min}^+$ dependence and the associated large logs.

$\Rightarrow$ Works straightforwardly in the case of

- the naive LL BK equation
- the kinematically improved BK equation as implemented in
  G.B., PRD89 (2014)

  Should also work with the other implementation (Iancu *et al.*, PLB744 (2015)), but requires a bit more work.

Conclusions

- Direct calculation of $\gamma_{T,L} \to q\bar{q}$ LFWFs at NLO both in momentum and in mixed space

- Full NLO correction to F_L and F_T obtained from the combination of the $q\bar{q}$ and $q\bar{q}g$ contributions:
  UV Dim. Reg. used in both cases, in order to have the finite terms under control.

Phenomenology outlook : All ingredients soon available for fits at
                           NLO+LL accuracy, and hopefully NLO+NLL accuracy.

Theory outlook :   • Application of the NLO $\gamma_{T,L} \to q\bar{q}$ LFWFs to
                     calculate other DIS observables at NLO?
             • Extension to the case of massive quarks?
             • General method of calculation should be useful for
                most future NLO calculations in the CGC

# LO $\gamma_{T,L} \to q\bar{q}$ LFWFs in momentum space

$$
\begin{aligned}
\Psi^{\gamma_T^*}_{q_0\bar{q}_1,LO} &= (2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1} + \underline{k_0} - \underline{q})\, \delta_{\alpha_0,\,\alpha_1}\, \mu^{2-\frac{D}{2}}\, e\, e_f \\
&\times \left(\frac{-2k_0^+ k_1^+}{q^+\left[\mathbf{P}^2 + \overline{Q}^2 - i\epsilon\right]} \right)\; \overline{u}(0)\,\slashed{\epsilon}_\lambda(\underline{q})\, v(1)
\end{aligned}
$$

$$
\begin{aligned}
\Psi^{\gamma_L^*}_{q_0\bar{q}_1,LO} &= (2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1} + \underline{k_0} - \underline{q})\, \delta_{\alpha_0,\,\alpha_1}\, \mu^{2-\frac{D}{2}}\, e\, e_f \\
&\times \left(\frac{-2k_0^+ k_1^+}{q^+\left[\mathbf{P}^2 + \overline{Q}^2 - i\epsilon\right]} \right)\; \frac{Q}{q^+}\quad \overline{u}(0)\,\gamma^+\, v(1)
\end{aligned}
$$

LO $\gamma_{T,L} \to q\bar{q}$ LFWFs in mixed space

$$
\widetilde{\Psi}^{\gamma_T^*}_{q_0\bar{q}_1,LO} = 2\pi\,\delta(k_0^+ + k_1^+ - q^+)\,\delta_{\alpha_0,\,\alpha_1}\,e^{\,i\frac{\mathbf{q}}{q^+}\cdot(k_0^+\mathbf{x}_0 + k_1^+\mathbf{x}_1)}\,e\,e_f\,\mu^{2-\frac{D}{2}}\,(2\pi)^{1-\frac{D}{2}}
$$

$$
\times\,(-i)\left(\frac{\overline{Q}}{|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-1}\mathrm{K}_{\frac{D}{2}-1}\!\left(|\mathbf{x}_{01}|\,\overline{Q}\right)\varepsilon^i_\lambda\,\mathbf{x}^j_{01}
$$

$$
\times\left\{\left(\frac{k_0^+ - k_1^+}{q^+}\right)\delta^{ij}\,\overline{u_G}(0)\,\gamma^+ v_G(1) - \tfrac{1}{2}\,\overline{u_G}(0)\,\gamma^+\left[\gamma^i,\gamma^j\right]v_G(1)\right\}
$$

$$
\widetilde{\Psi}^{\gamma_L^*}_{q_0\bar{q}_1,LO} = 2\pi\delta(k_0^+ + k_1^+ - q^+)\,\delta_{\alpha_0,\,\alpha_1}\,e^{\,i\frac{\mathbf{q}}{q^+}\cdot(k_0^+\mathbf{x}_0 + k_1^+\mathbf{x}_1)}\,e\,e_f\,\mu^{2-\frac{D}{2}}\,(2\pi)^{1-\frac{D}{2}}
$$

$$
\times\,(-1)\left(\frac{\overline{Q}}{|\mathbf{x}_{01}|}\right)^{\frac{D}{2}-2}\mathrm{K}_{\frac{D}{2}-2}\!\left(|\mathbf{x}_{01}|\,\overline{Q}\right)\frac{2k_0^+ k_1^+}{(q^+)^2}\,Q\,\overline{u_G}(0)\,\gamma^+ v_G(1)
$$

Kinematics for Deep Inelastic Scattering (DIS)
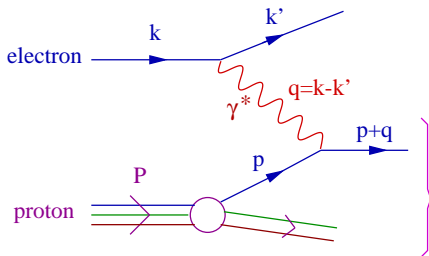
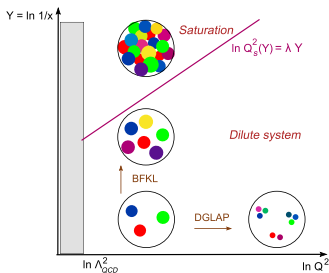
$$\frac{d\sigma^{ep \to e+X}}{dx_{Bj}\, d^2Q} = \frac{\alpha_{em}}{\pi x_{Bj} Q^2} \left[\left(1 - y + \frac{y^2}{2}\right) \sigma_T^{\gamma p \to X}(x_{Bj}, Q^2) + (1-y)\, \sigma_L^{\gamma p \to X}(x_{Bj}, Q^2) \right]$$

Photon virtuality: $Q^2 \equiv -q^2 > 0$

Bjorken x variable: $x_{Bj} \equiv \frac{Q^2}{2P \cdot q} \in [0, 1]$

Inelasticity: $y \equiv \frac{2P \cdot q}{(P+k)^2} \in [0, 1]$

Kinematical regimes of DIS

- For $Q^2 \to +\infty$: target more and more dilute due to DGLAP evolution.
  $\Rightarrow$ QCD-improved parton model more and more valid.

- For $x_{Bj} \to 0$: target more and more dense
  $\Rightarrow$ Linear BFKL evolution eventually breaks down, as well as parton picture.

  Onset of nonlinear collective effects: Gluon saturation!

Dilute-dense processes at high-energy

High energy scattering:

$\quad$ projectile : momentum $q^\mu \simeq \delta^{\mu+} q^+$

$\quad\quad$ target : momentum $P^\mu \simeq \delta^{\mu-} P^-$

$\Rightarrow$ Mandelstam s variable: $s \simeq 2P^- q^+$

Eikonal approximation: Take the high-energy limit $s \to +\infty$ and drop power-suppressed contributions.

Semi-classical approximation: At weak coupling g, dense target $\to$ random classical background field $\mathcal{A}_a^\mu(x) = O(1/g)$.

In the semi-classical approximation, the eikonal limit can be obtained by an infinite boost $P^- \to +\infty$ of the target field $\mathcal{A}_a^\mu(x)$. Hence:

- Only the $\mathcal{A}_a^-$ component is relevant
- Infinite Lorentz dilation: $\mathcal{A}_a^\mu(x)$ independent of x^-
- Infinite Lorentz contraction: $\mathcal{A}_a^\mu(x) \propto \delta(x^+)$ $\quad$ (shockwave)

Eikonal dilute-dense scattering

Recipe for *dilute-dense* processes at high-energy,
following Bjorken, Kogut and Soper (1971):

- Decompose the projectile on a Fock basis at the time $x^+ = 0$, with
  appropriate Light-Front wave-functions.

- Each parton *n* scatters independently on the target via a light-like
  Wilson line $U_{\mathcal{R}_n}(\mathbf{x}_n)$ through the target:

$$U_{\mathcal{R}_n}(\mathbf{x}_n) = \mathcal{P}_+ \exp \left[ig \int dx^+ \, T^a_{\mathcal{R}_n} \, A^-_a(x^+, \mathbf{x}_n) \right]$$
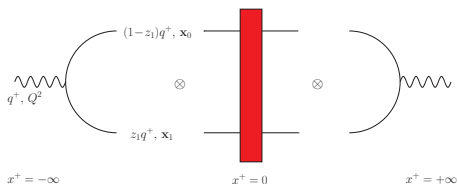
  with $\mathcal{R}_n = A$, F or $\bar{F}$ for g, q or $\bar{q}$ partons.

- Include final-state evolution of the projectile remnants.

Comments:

1. Light-cone gauge $A^+_a = 0$ strongly recommended!

2. At this stage, no apparent dependence on s ...
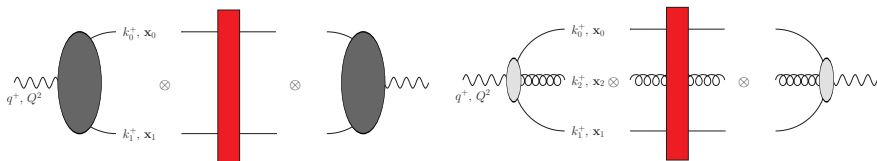
Dipole factorization for DIS at LO

$$\sigma_{T,L}^{\gamma p \to X}(x_{Bj}, Q^2) = \frac{4 N_c \, \alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int d^2\mathbf{x}_0 \, d^2\mathbf{x}_1 \int_0^1 dz_1$$
$$\times \, \mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1, Q^2) \left[1 - \langle \mathcal{S}_{01} \rangle_\eta \right]$$

Bjorken, Kogut, Soper (1971); Nikolaev, Zakharov (1990)

Dipole operator: $\qquad \mathcal{S}_{01} = \frac{1}{N_c} \mathrm{Tr}\left(U_F(\mathbf{x}_0) \, U_F^\dagger(\mathbf{x}_1) \right)$

η: regulator of rapidity divergence of light-like Wilson lines $U_F(\mathbf{x}_n)$
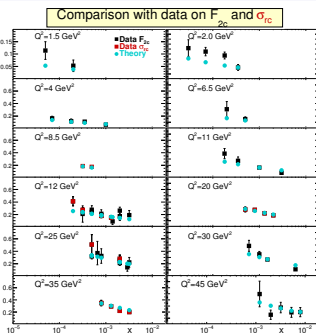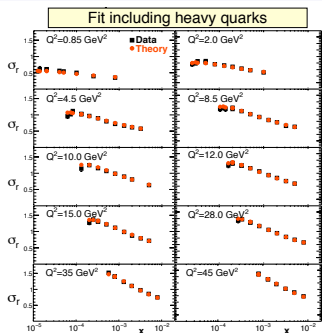
DIS at NLO: general structure and real corrections

$$\sigma_{T,L}^{\gamma p}(Q^2, x_{Bj}) = 2 \frac{2 N_c \, \alpha_{em}}{(2\pi)^2} \sum_f e_f^2 \int d^2\mathbf{x}_0 \int d^2\mathbf{x}_1 \int_0^1 dz_1$$

$$\times \left\{ \left[\mathcal{I}_{T,L}^{q\bar{q},LO}(x_{01}, z_1, Q^2) + \mathcal{O}(\alpha_s C_F) \right] \left[1 - \langle \mathcal{S}_{01} \rangle_0 \right] \right.$$

$$\left. + \frac{2\alpha_s C_F}{\pi} \int_{z_{min}}^{1-z_1} \frac{dz_2}{z_2} \int \frac{d^2\mathbf{x}_2}{2\pi} \, \mathcal{I}_{T,L}^{q\bar{q}g}(\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, z_1, z_2, Q^2) \left[1 - \langle \mathcal{S}_{012} \rangle_0 \right] \right\}$$

with $z_n = k_n^+ / q^+$ and $z_{min} = \frac{x_{Bj}}{Q^2} \frac{Q_0^2}{x_0}$.

G.B. (2012)

DIS phenomenology

Fits of the reduced DIS cross-section σ_r and its charm contribution σ_{rc} at HERA data with numerical solutions of the running coupling BK equation.

Albacete, Armesto, Milhano, Quiroga, Salgado (2011)

see also: Kuokkanen, Rummukainen, Weigert (2012);

Lappi, Mäntysaari (2013); . . .

Very good fit, but require a big rescaling of Λ_{QCD} as extra parameter, to slow down the BK evolution.