# Initial conditions for hydrodynamics from weakly coupled pre-equilibrium evolution

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*L. Keegan, A. Kurkela, A.M. and D.Teaney, arXiv:1605.04287* (perturbations)
A. Kurkela, Y. Zhu, Phys. Rev. Lett. 115, 182301 (2015) (uniform background)

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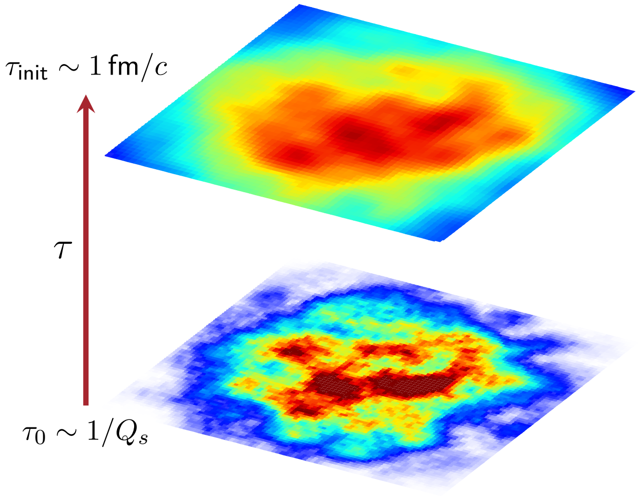
# Pre-equilibrium evolution




mean $\langle p_T \rangle$ , bulk viscosity $\xi/s$, preflow

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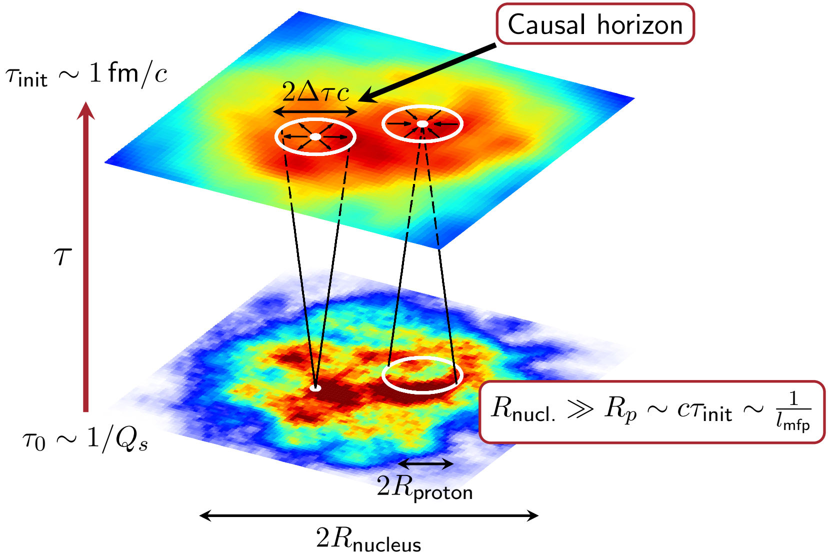


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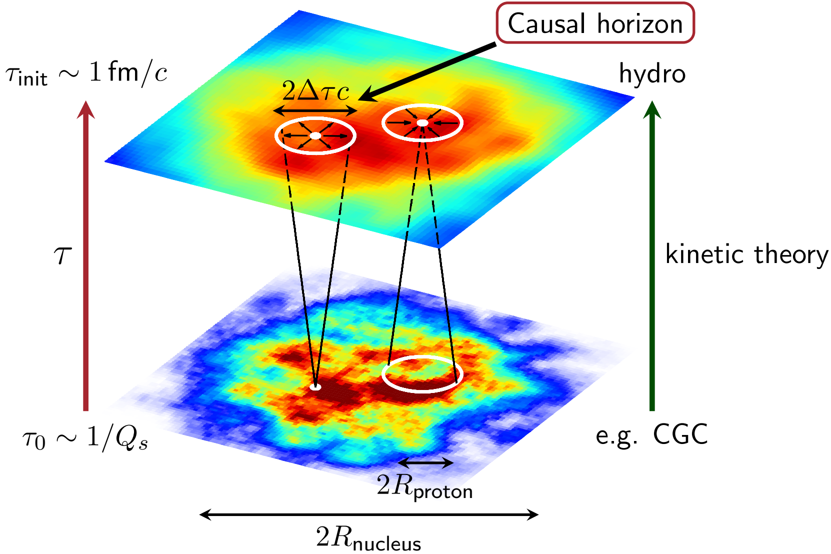
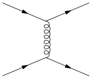


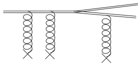

mean $\langle p_T \rangle$ , bulk viscosity $\xi/s$, preflow

# Kinetic theory for perturbations

## Effective kinetic theory Arnold, Moore, Yaffe (2003)

$$\partial_\tau f + \frac{\boldsymbol{p}}{|p|}\cdot\nabla f - \underbrace{\frac{p_z}{\tau}\partial_{p_z} f}_{\text{Bjorken expansion}} = -\underbrace{\mathcal{C}_{2\leftrightarrow 2}[f]} - \underbrace{\mathcal{C}_{1\leftrightarrow 2}[f]}\,,$$





LO acccurate in $\alpha_s$, here $\alpha_s \approx 0.3$.

## Gluon distribution function

$$f = \underbrace{\bar{f}_{\boldsymbol{p}}}_{\text{uniform background}} + \underbrace{\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}} e^{i\boldsymbol{k}_\perp\cdot\boldsymbol{x}_\perp}}_{\text{transverse perturbations}}\,.$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z})\bar{f}_{\boldsymbol{p}} = -\mathcal{C}[\bar{f}] \qquad \text{background}$$

$$(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z} + \frac{i\boldsymbol{p}_\perp\cdot\boldsymbol{k}_\perp}{p})\delta f_{\boldsymbol{k}_\perp,\boldsymbol{p}} = -\delta\mathcal{C}[\bar{f},\delta f] \qquad \text{perturbation}$$

*Study energy $\delta e$ and momentum $g^x$ perturbations.*

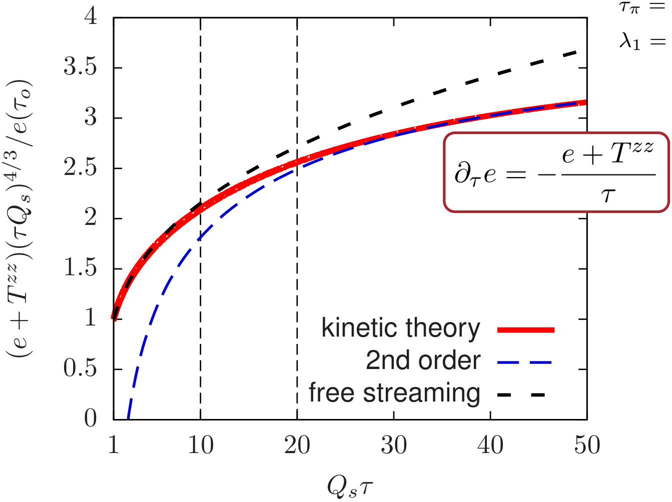
# Background approach to hydro

Hydro constitutive equation: $T^{zz}(e) = \frac{1}{3}e - \frac{4}{3}\frac{\eta}{\tau} - \frac{8}{9}\frac{\tau_\pi\eta - \lambda_1}{\tau^2}$.

$\eta/s = 0.62$

$\tau_\pi = 5.1\eta/sT$

$\lambda_1 = 0.8\eta\tau_\pi$




*Kinetic theory smoothly interpolates between free streaming and hydrodynamics.*

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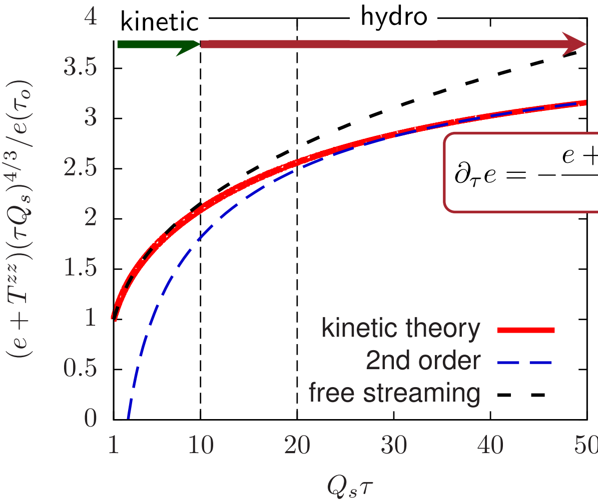


$$\partial_\tau e = -\frac{e + T^{zz}}{\tau}$$

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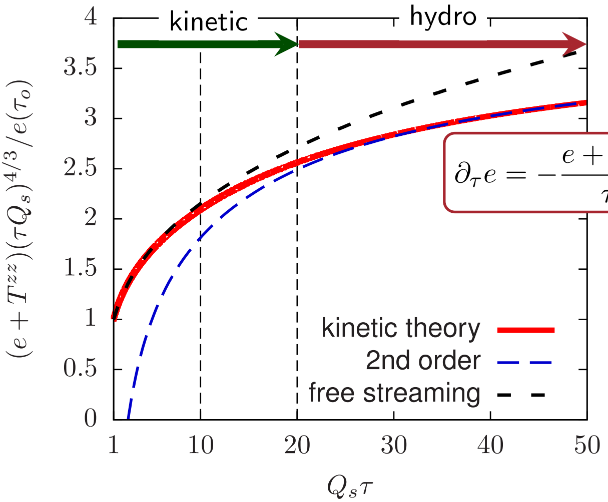


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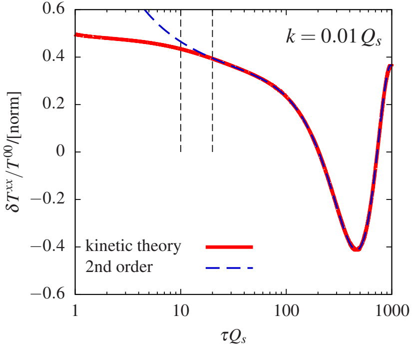
*Kinetic theory smoothly interpolates between free streaming and hydrodynamics.*

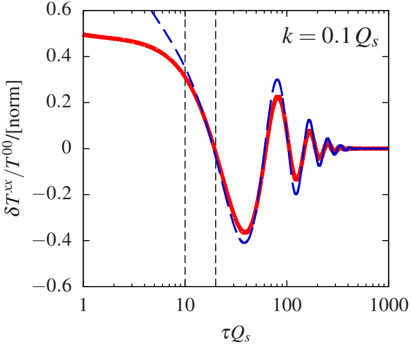
# Equilibration of perturbations

$$\delta T^{xx} = \underbrace{\frac{\delta e}{e}}_{\text{energy}} \left[\tfrac{1}{3}e + \tfrac{1}{3}\eta\tau_\pi k^2 + \tfrac{\eta}{2\tau} - \tfrac{2(\lambda_1 - \eta\tau_\pi)}{9\tau^2}\right] - \underbrace{\frac{ikg^x}{e}}_{\text{momentum}} \left[\eta - \tfrac{1}{\tau}\left(\tfrac{\eta^2}{2e} + \tfrac{\eta\tau_\pi}{2} - \tfrac{2}{3}\lambda_1\right)\right]$$

$k \sim 1/R_{\text{nucleus}}$

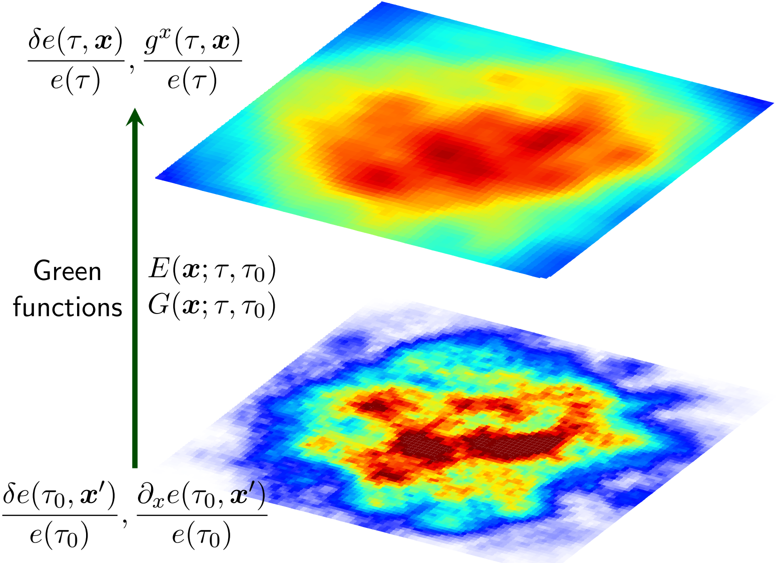



$k \sim 1/R_{\text{proton}}$



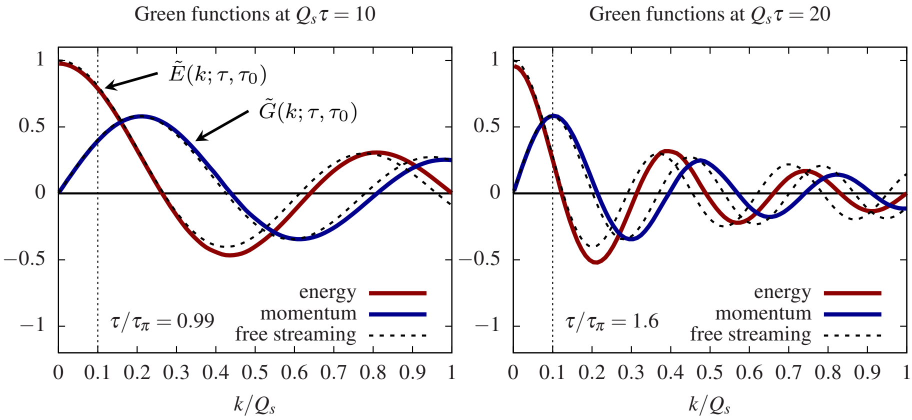

*Perturbations hydrodynamizes together with background* $Q_s\tau \sim \{10, 20\}$.

## Kinetic theory response functions to initial perturbations and gradients.



# Green functions in $\boldsymbol{k}$ space

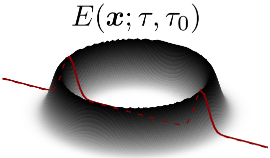
$$\frac{\delta e(\tau,k)}{e(\tau)} \equiv \underbrace{\tilde{E}(k;\tau,\tau_0)}_{\text{energy resp.}} \times \frac{\delta e(\tau_0,k)}{e(\tau_0)}, \qquad \frac{g^x(\tau,k)}{e(\tau)} \equiv -i \underbrace{\tilde{G}(k;\tau,\tau_0)}_{\text{momentum resp.}} \times \frac{\delta e(\tau_0,k)}{e(\tau_0)}.$$



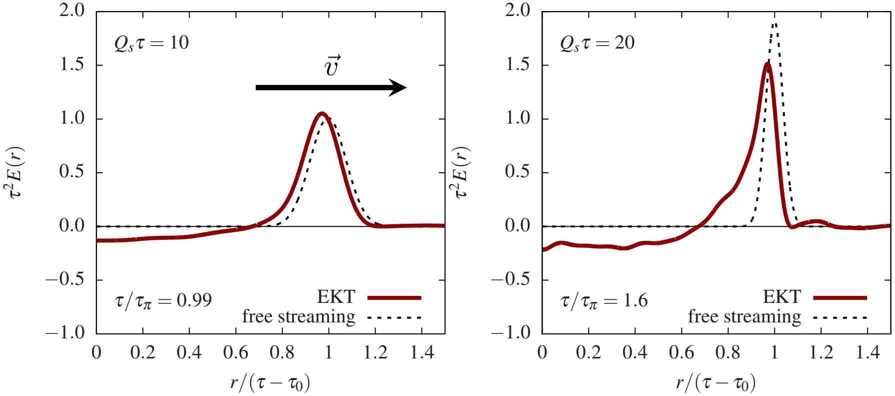

*Now Fourier transform $\tilde{E}(k,\tau,\tau_0)$ to coordinate space.*

# Energy Green function in coordinate space

Convolve energy perturbations with response kernel.



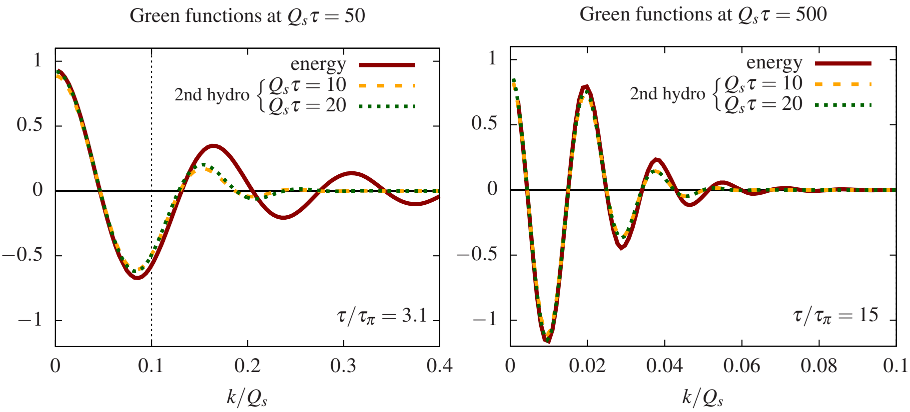

$$\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)} = \int d^2\boldsymbol{x}' E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_0) \times \frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}$$




*Continue evolution in hydro.*

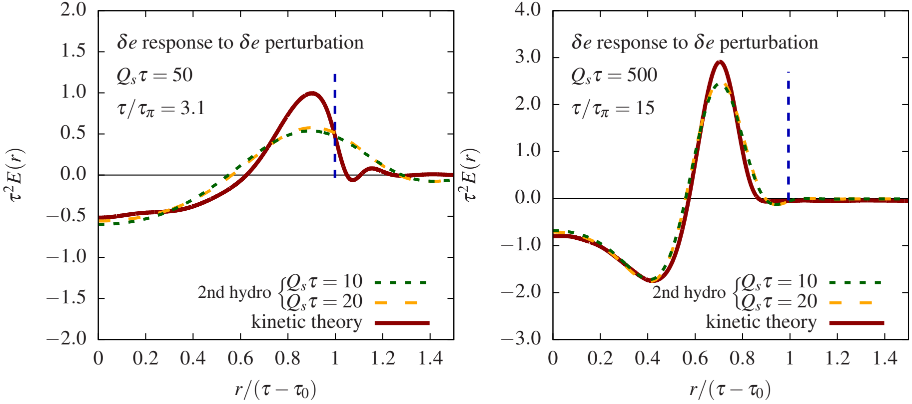
# Comparison with hydro at late times

Initialize hydro with $\delta e$ and $g^x$ at $\tau Q_s = \{10, 20\}$.




*Little sensitivity to hydro initialization time.*

# Energy Green functions at late times



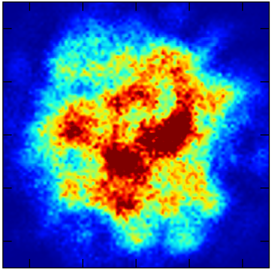

*Complete transition to hydrodynamic response at late times.*

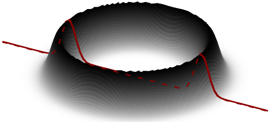
# Conclusion

Effective kinetic theory:

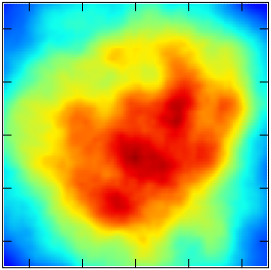
- Studied equilibration of transverse $\delta e$ and $g^x$ perturbations.
- Showed transition to hydro constituent equations $T^{ij}(e, \delta e, \vec{g})$.
- Constructed pre-equilibrium response functions.

$$\int d^2\boldsymbol{x}' \underbrace{\frac{\delta e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}, \frac{\vec{\nabla} e(\tau_0, \boldsymbol{x}')}{e(\tau_0)}} \times \underbrace{E(|\boldsymbol{x} - \boldsymbol{x}'|; \tau, \tau_0)} = \underbrace{\frac{\delta e(\tau, \boldsymbol{x})}{e(\tau)}, \frac{\vec{g}(\tau, \boldsymbol{x})}{e(\tau)}}$$





Pre-equilibrium smearing and preflow generation



# Outlook

- Quarks: chemical equilibration.
  - Non-equilibrium photon production.
- Collision kernels: $\alpha_s \approx 0.3$ NLO corrections, anisotropic screening.
- Hydro: full heavy ion simulations with kinetic theory pre-equilibrium.
- Holography: response functions in the strong coupling limit.

# Backup

# Expanding system

Varying saturation scale $Q_s(\boldsymbol{x}_\perp) \sim Q_s + \delta Q_s e^{i\boldsymbol{k}_\perp \cdot \boldsymbol{x}_\perp}$.

$$\delta f_{\boldsymbol{k}_\perp, \boldsymbol{p}}(\tau_0) = -\frac{\delta Q_s}{Q_s} p \, \partial_p \bar{f}_{\boldsymbol{p}},$$

Equation of motion for perturbations

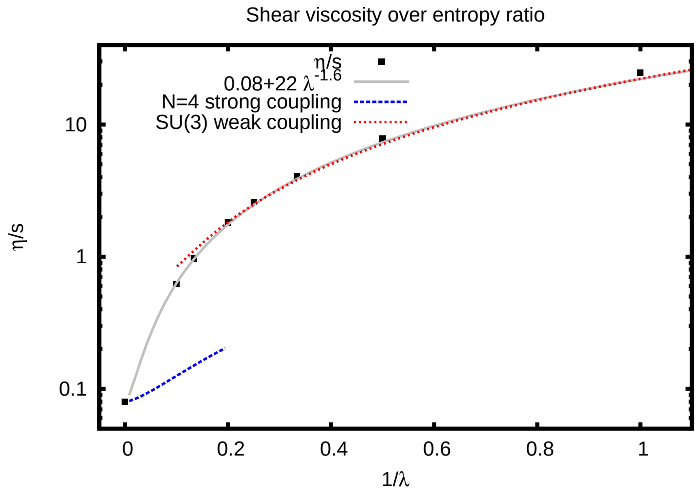
$$\partial_\tau \delta e(\tau, k) + ikg^x(\tau, k) = -\frac{\delta e(\tau, k) + \delta T^{zz}(\tau, k)}{\tau},$$
$$\partial_\tau g^x(\tau, k) + ik\delta T^{xx}(\tau, k) = -\frac{g^x(\tau, k)}{\tau},$$

For asymptotically small $k \to 0$

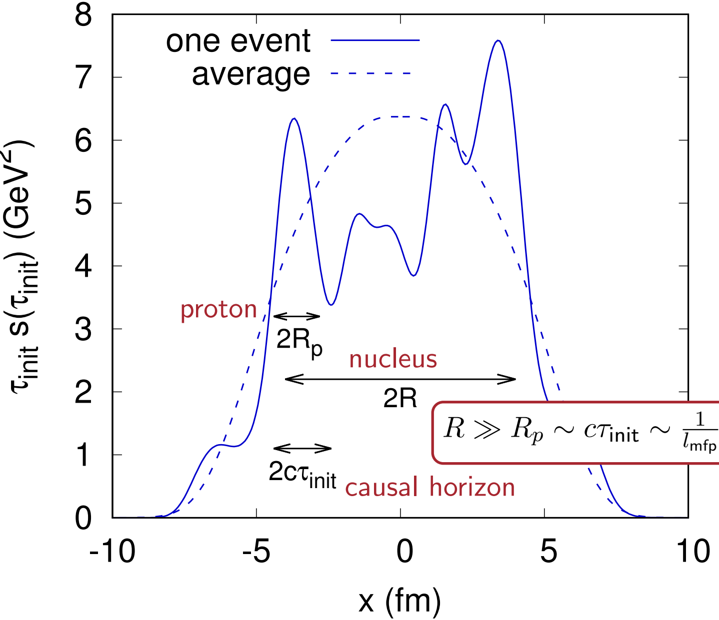
- $\delta e/(e + T^{xx}) = \text{const.}$ (to all orders in gradient expansion)
- $v^x = \frac{1}{2}\tau \times [-\partial_x \delta e/(e + T^{xx})]$, see also Vredevoogd and Pratt (2009)

# Equilibration in weak and strong coupling

L. Keegan, A. Kurkela, P. Romatschke, W. van der Schee and Y. Zhu (2015)



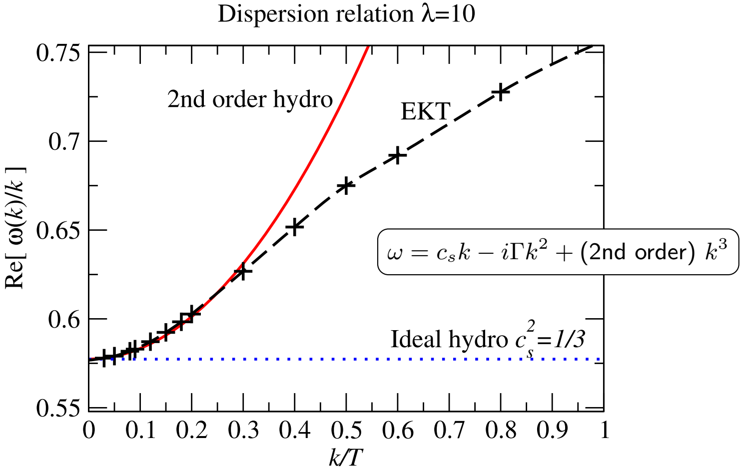
# Initial perturbations and preflow




$R \gg R_p \sim c\tau_{\text{init}} \sim \frac{1}{l_{\text{mfp}}}$

# Fluctuations around equilibrium

## Standing waves of temperature $T(x) = T_0 + \delta T e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$



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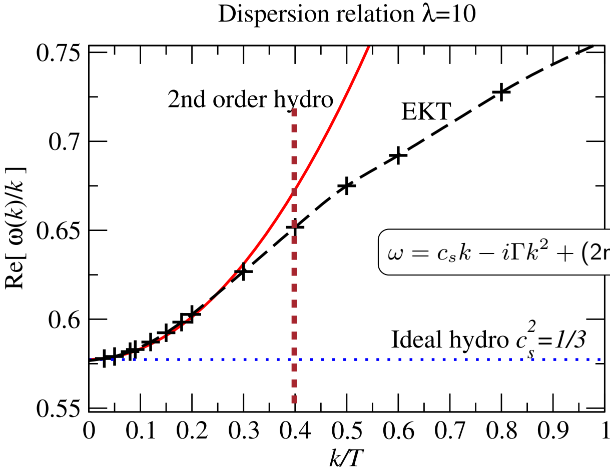


$\omega = c_s k - i\Gamma k^2 + \text{(2nd order) } k^3$

*Hydro description applies for* $k < 0.4T \sim 1/R_{\text{proton}}$