

# Thermodynamics and kinetics of the Gribov-Zwanziger plasma

Wojciech Florkowski

IFJ PAN, Cracow & UJK Kielce, Poland

in collaboration with:

R. Ryblewski (INP PAN, Cracow), N. Su (Frankfurt U.), K. Tywoniuk (ICC, Barcelona U.)  
and Viktor Begun (UJK, Kielce)

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- RHIC and LHC HIC data suggest that QGP is a strongly-interacting dissipative fluid  
⇒ relativistic dissipative fluid dynamics (equation of state?, transport coefficients?)
- equation of state (at high  $T$ )
  - lattice QCD  
(M. Cheng et al., PRD 77, 014511 (2008); S. Borsanyi et al., JHEP 11, 077 (2010); S. Borsanyi et al., JHEP 07, 056 (2012))
  - re-summed perturbation theory  
(J.-P. Blaizot, E. Iancu, Phys. Rept. 359, 355 (2002); J. O. Andersen, M. Strickland, Annals Phys. 317, 281 (2005); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); J. Hong, D. Teaney, PRC 82, 044908 (2010); N. Su, Commun. Theor. Phys. 57, 409 (2012); )
- transport coefficients  
(W. Israel, J. M. Stewart, Ann. Phys. (NY) 118, 341 (1979); A. Muronga, PRC 69, 034903 (2004); A. El et al., PRC 81, 041901(R) (2010); G. S. Denicol et al., PRL 105, 162501 (2010); A. Jaiswal, PRC 87, 051901(R) (2013))

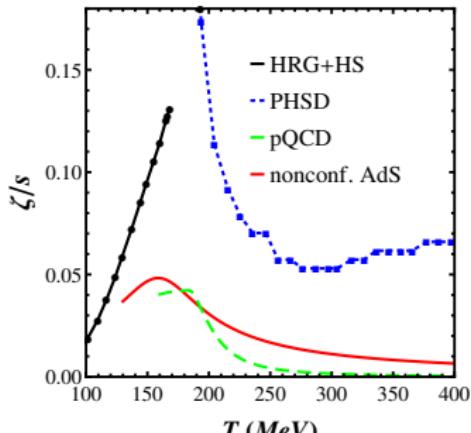
$$\begin{aligned}\dot{\Pi} + \frac{\Pi}{\tau_\Pi} &= -\frac{\zeta}{\tau_\Pi} \theta - \delta_{\Pi\Pi} \Pi \theta + \lambda_{\Pi\pi} \pi^{\mu\nu} \sigma_{\mu\nu} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_\pi} &= 2 \frac{\eta}{\tau_\pi} \sigma^{\mu\nu} + 2 \pi^{\langle\mu}_{\gamma} \omega^{\nu\rangle\gamma} - \delta_{\pi\pi} \pi^{\mu\nu} \theta - \tau_{\pi\pi} \pi^{\langle\mu}_{\gamma} \sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi} \Pi \sigma^{\mu\nu}\end{aligned}$$

$\zeta \rightarrow$  bulk viscosity       $\eta \rightarrow$  shear viscosity

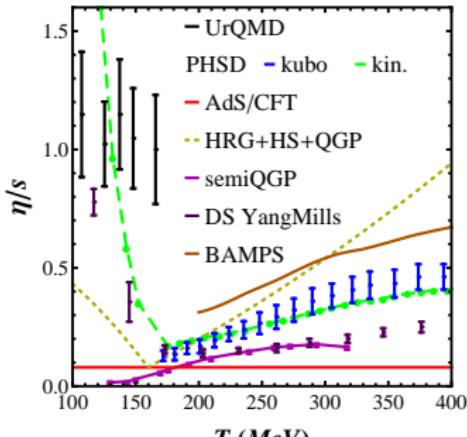
# Motivation

- transport coefficients (cont'd)

- pQCD (P. B. Arnold et al., JHEP 11, 001 (2000); P. B. Arnold et al., JHEP 05, 051 (2003); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); P. B. Arnold et al., PRD 74, 085021 (2006);)
- functional renormalization techniques (M. Haas et al., PRD90, 091501 (2014); N. Christiansen et al., (2014), arXiv:1411.7986 (hep-ph))
- low energy theorems (D. Kharzeev, K. Tuchin, JHEP 09, 093 (2008); F. Karsch et al., PLB 663, 217 (2008);)
- lQCD (H. B. Meyer, PRL 100, 162001 (2008); H. B. Meyer, PRD 76, 101701 (2007);)
- $\mathcal{N} = 4$  supersymmetric YM plasma with broken conformal symmetry (G. Policastro et al., PRL 87, 081601 (2001); P. Benincasa et al., NPB 733, 160 (2006); A. Buchel, PRD 72, 106002 (2005); S. I. Finazzo et al., JHEP 02, 051 (2015)))
- HRG+HS (G.P. Kadam, H. Mishra, NPA 934 (2014) 133-147))
- parton-hadron-string dynamics transport approach (V. Ozvenchuk, O. Linnyk, M. I. Gorenstein, E. L. Bratkovskaya and W. Cassing, PRC 87, no. 6, 064903 (2013)))
- ...



around  $T_C$   $\zeta$  and  $\eta$  are comparable



# Gribov's confining gluons

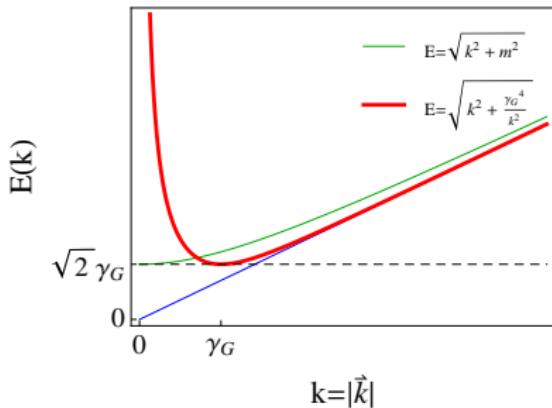
- Gribov quantization of YM theory - fixing the infrared (IR) residual gauge transformations remaining after Faddeev-Popov procedure



a new scale  $\gamma_G$  that leads to an  
IR-improved dispersion relation for gluons  
(Coulomb gauge)  
(V. Gribov, NPB 139, 1 (1978);  
D. Zwanziger, NPB 323, 513 (1989))

$$E(\mathbf{k}) = |\mathbf{k}| \quad \rightarrow \quad E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}}$$

- attracted a lot of attention recently  
(G. Burgio, M. Quandt, H. Reinhardt, PRL 102, 032002 (2009);  
N. Su, K. Tywoniuk, PRL 114, 161601 (2015);  
D. E. Kharzeev, E. M. Levin, PRL 114, 242001 (2015);  
...)



- reduction of the physical phase space due to the large energy cost of the excitation of soft gluons



essential feature of the confinement  
(V. Gribov, NPB 139, 1 (1978);  
R.P. Feynman, NPB 188, 479 (1981);  
D. Zwanziger, NPB 485, 185 (1997); )

# Covariant setup

local rest frame

comoving frame

$$E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{\mathbf{k}^2}} \quad \rightarrow \quad E(k \cdot u) = \sqrt{(k \cdot u)^2 + \frac{\gamma_G^4}{(k \cdot u)^2}}$$

$k = (|\mathbf{k}|, \mathbf{k})$  four-momentum of perturbative gluons

## thermodynamics of the system of confining gluons

(D. Zwanziger, PRL 94, 182301 (2005);)

(W. F., R.Ryblewski, N. Su, K. Tywoniuk, arXiv:1504.03176;)

$$\varepsilon = g_0 \int \frac{d^3 k}{(2\pi)^3} E(\mathbf{k}) f(\mathbf{k}) \quad \rightarrow \quad \varepsilon = \int dK E(k \cdot u) f(x, k)$$

$$P = \frac{g_0}{3} \int \frac{d^3 k}{(2\pi)^3} |\mathbf{k}| \frac{\partial E(\mathbf{k})}{\partial |\mathbf{k}|} f(\mathbf{k}) \quad \rightarrow \quad P = \frac{1}{3} \int dK \frac{(k \cdot u)^2}{E(k \cdot u)} \left( 1 - \frac{\gamma_G^4}{(k \cdot u)^4} \right) f(x, k)$$

$$\int dK (\dots) = g_0 \int \frac{d^3 k}{(2\pi)^3 k^0} (k \cdot u) (\dots)$$

$$g_0 = 2(N_c^2 - 1) \quad (SU(N_c))$$

# GZ plasma vs lQCD data — fixing $\gamma_G$

in high-T limit  $\gamma_G \rightarrow \frac{d}{d+1} \frac{N_c}{4\sqrt{2}\pi} g^2 T$

D. Zwanziger, PRD 76, 125014 (2007)

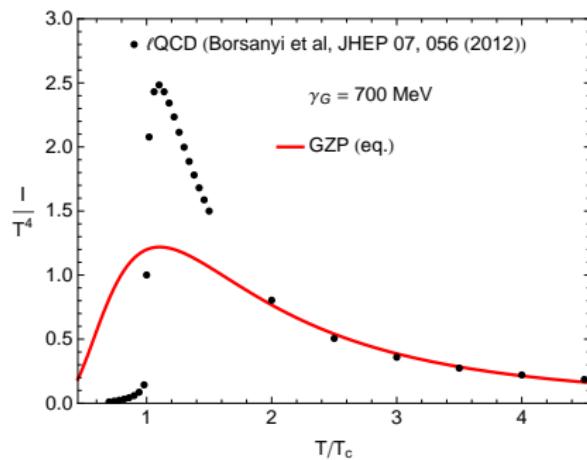
$$T \approx (2 - 4) T_c \quad \Rightarrow \quad \gamma_G \approx \text{const.}$$

$\gamma_G(T)$  derived from the gap equation with running coupling from lQCD

J. O. Andersen et al., PRL 104, 122003 (2010);

K. Fukushima, N. Su, PRD 88, 076008 (2013);

W. F., R. Ryblewski, N. Su, K. Tywoniuk, arXiv:1504.03176



# (0+1)D Bjorken symmetry implementation

- consider the case of a **transversely homogeneous boost-invariant** system
- assume the **Bjorken flow** of matter in longitudinal direction (boost-invariance)  
(J. D. Bjorken, PRD 27, 140 (1983);)

$$u^\mu = (t/\tau, 0, 0, z/\tau)$$

- introduce convenient boost-invariant variables  
(A. Bialas et al., NPB 296, 611 (1988);)

$$v = k^0 t - k_{\parallel} z$$

$$w = k_{\parallel} t - k^0 z$$

- EOM follow from the conservation of  $T^{\mu\nu}$

$$f = f(\tau, w, k_{\perp})$$

- within the assumed symmetries  $T^{\mu\nu}$  has the anisotropic form  
(W. F., R. Ryblewski, PRC 85, 044902 (2012);  
M. Martinez et al., PRC 85, 064913 (2012);)

$$T^{\mu\nu} = (\varepsilon + P_{\perp}) u^\mu u^\nu - P_{\perp} g^{\mu\nu} + (P_{\parallel} - P_{\perp}) z^\mu z^\nu$$

$$z^\mu = (z/\tau, 0, 0, t/\tau)$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow$$

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = 0$$

(A. Muronga, PRC 69, 034903 (2004);  
R. Baier et al., PRC 73, 064903 (2006);)

$$\frac{d\varepsilon}{d\tau} + \frac{1}{\tau} (\varepsilon + P_{GZ} + \Pi - \pi) = 0 \quad \Leftrightarrow \quad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{||}}{\tau} = 0$$

## ● dissipative fluxes

$$\pi = \frac{4}{3} \frac{\eta}{\tau}$$

$$\pi = \frac{2}{3} (P_{||} - P_{\perp})$$

$$\Pi = -\frac{\zeta}{\tau}$$

$$\Pi = P - P_{GZ} = \frac{1}{3} (P_{||} + 2P_{\perp}) - P_{GZ}$$

(Navier-Stokes dissipative fluid dynamics)

$$\frac{de}{d\tau} + \frac{\varepsilon + P_{||}}{\tau} = \int dK E(\tau, w, k_{\perp}) \frac{\partial f(\tau, w, k_{\perp})}{\partial \tau}$$

- (0+1)D kinetic equation in RTA

P. L. Bhatnagar et al., Phys. Rev. 94, 511 (1954);  
 G. Baym, PLB 138, 18 (1984);  
 G. Baym, NPA 418, 525C (1984);)

$$\frac{\partial f(\tau, w, k_{\perp})}{\partial \tau} = \frac{f_{GZ}(\tau, w, k_{\perp}) - f(\tau, w, k_{\perp})}{\tau_{\text{rel}}(\tau)}$$

- satisfied as long as Landau matching condition is satisfied  $\varepsilon_{GZ} = \varepsilon$

- formal solution

NPA 916, 249 (2013);  
 PRC 88, 024903 (2013);  
 PRC 89, 054908 (2014);)

$$f(\tau, w, k_{\perp}) = f_0(w, k_{\perp}) D(\tau, \tau_0) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{\text{rel}}(\tau')} D(\tau, \tau') f_{GZ}(\tau', w, k_{\perp})$$

$$\varepsilon = \int dK E(\tau, w, k_{\perp}) f(\tau, w, k_{\perp})$$

$$P_{||} = \int dK \frac{w^2}{\tau^2 E(\tau, w, k_{\perp})} \left[ 1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

$$P_{\perp} = \int dK \frac{k_{\perp}^2}{2 E(\tau, w, k_{\perp})} \left[ 1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

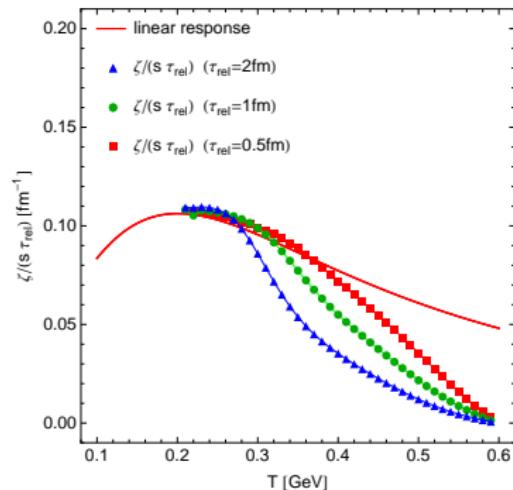
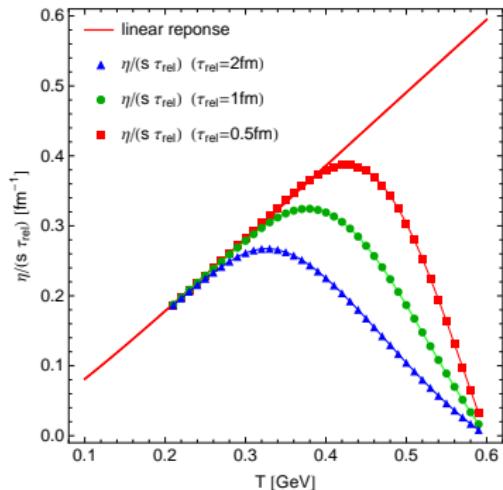
$$D(\tau_2, \tau_1) = \exp \left[ - \int_{\tau_1}^{\tau_2} \frac{d\tau}{\tau_{\text{rel}}(\tau)} \right]$$

close to equilibrium one can use the linear response approximation  $f \approx f_{\text{GZ}} + \delta f + \dots$ ,  $y = |\mathbf{k}|/\gamma_G$

$$\zeta(T, \gamma_G) = \frac{g_0 \gamma_G^5}{3\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^\infty dy \left[ c_s^2 - \frac{1}{3} \frac{y^4 - 1}{y^4 + 1} \right] f_{\text{GZ}}(1 + f_{\text{GZ}})$$

$$\eta(T, \gamma_G) = \frac{1}{10} \frac{g_0 \gamma_G^5}{3\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^\infty dy \frac{(y^4 - 1)^2}{y^4 + 1} f_{\text{GZ}}(1 + f_{\text{GZ}})$$

# Results: Bulk and shear viscosity



# Results: $\zeta/\eta$ dependence

$$\frac{\zeta}{\eta} = 15 \left( \frac{1}{3} - c_s^2 \right)^2$$

- photon gas coupled to hot matter

(S. Weinberg, *Astrophys. J.* 168, 175 (1971);)

## scalar theory

(A. Hasoya et al., *Ann. Phys. (N.Y.)* 154, 229 (1984);  
R. Horsley, W. Schoenmaker, *NPB* 280, 716 (1987);)

## weakly-coupled QCD (large- $T$ limit)

(P.B. Arnold et al., *JHEP* 0011 (2000) 001;  
P.B. Arnold et al., *JHEP* 0305 (2003) 051;  
P.B. Arnold et al., *PRD* 74 (2006) 085021;)

$$\frac{\zeta}{\eta} = \kappa \left( \frac{1}{3} - c_s^2 \right)$$

- strongly-coupled nearly-conformal gauge theory plasma using gauge theory–gravity duality (large- $T$  limit)

( $\kappa = 2$ ,  $\kappa = 4.558 - 4.935$ )

(P. Benincasa et al., *NPB* 733, 160 (2006);  
A. Buchel, *PRD* 72, 106002 (2005);)

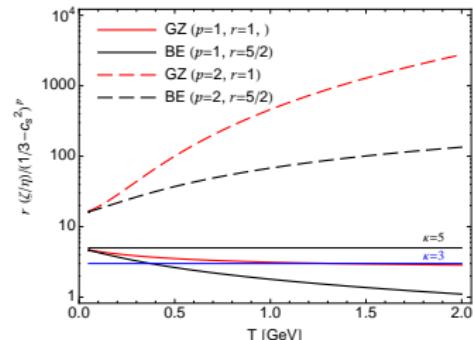
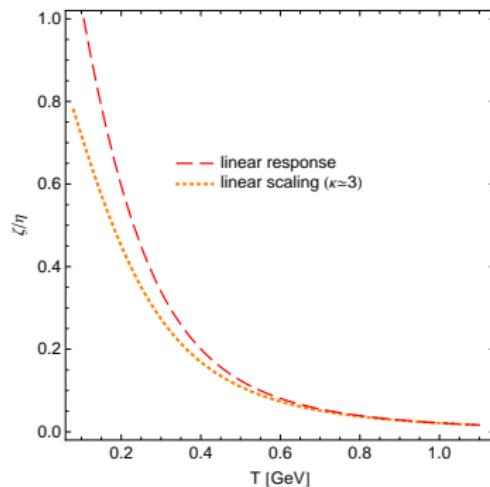
- **Gribov's plasma of confining gluons**

( $\kappa = 3$ )

(W. F., R. Ryblewski, N. Su, K. Tywoniuk arXiv:1504.03176;  
W. F., R. Ryblewski, N. Su, K. Tywoniuk, arXiv:1509.01242;)

different from massive BE gas !

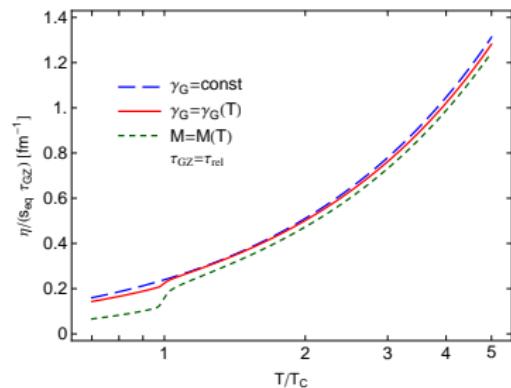
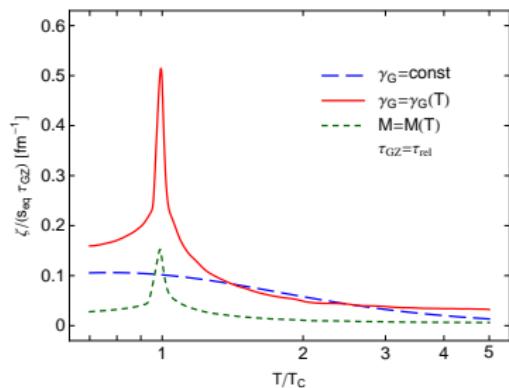
(C. Sasaki, K. Redlich, *PRC* 79, 055207 (2009);)



# Results: Bulk viscosity for temperature dependent Gribov parameter

V. Begun, WF, R. Ryblewski, arXiv:1602.08308

$\gamma_G$  fitted from the lattice results for the entropy density  
the bag pressure  $B(T)$  added for thermodynamic consistency



## Summary

- a dynamic and non-equilibrium description of a plasma consisting of confining gluons (obtained from the Gribov quantization of SU(3) YM theory) introduced for the first time
- the expressions for the shear and bulk viscosities of the Gribov-Zwanziger plasma were derived
- $\zeta/\eta$  T-scaling which is in line with the strong-coupling methods results was found
- inclusion of the temperature dependent  $\gamma_G$ , evidence for divergent bulk viscosity at the phase transition

## Outlook

- use our formula in the hydrodynamic simulations