Determining QGP initial conditions and medium properties via Bayesian model-to-data analysis

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Initial Stages | May 25, 2016

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Woods-Saxon, Glauber modeling aspects generally well accepted

Useful to separate cross sections and entropy deposition map, i.e. \( dS/dy \sim f(T_A, T_B) \) where \( T \) is the nuclear thickness. *The mapping \( f \) is a 2D surface.*
Parametrizing the initial conditions

Generalized mean ansatz:

\[
\frac{dS}{d^2r \, dy} \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}
\]

Thickness [fm^{-2}]

Pb+Pb 2.76 TeV

\[ T_{\text{min}} < T < T_{\text{max}} \]

\[ -1 < p < 1 \]

\[ p = \infty \]

- maximum
- arithmetic: \[ \frac{T_A + T_B}{2} \]
- geometric: \[ \sqrt{T_A T_B} \]
- harmonic: \[ \frac{2T_A T_B}{T_A + T_B} \]

minimum

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Parametrizing the initial conditions

Generalized mean ansatz:

$$\frac{dS}{d^2r \ dy} \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}$$

Pb+Pb 2.76 TeV

- $T_{\text{min}} < T < T_{\text{max}}$
- $-1 < p < 1$
- $p = 1$

Maximum: $\frac{T_A + T_B}{2}$
Geometric: $\sqrt{T_A T_B}$
Harmonic: $\frac{2T_A T_B}{T_A + T_B}$

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Parametrizing the initial conditions

Generalized mean ansatz:

\[
\frac{dS}{d^2r \ dy} \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}
\]

- **Arithmetic:** \( p = 0 \)
- **Geometric:** \( p = 1 \)
- **Harmonic:** \( p = \infty \)

\[
T_{\text{min}} < T < T_{\text{max}} \quad -1 < p < 1 \quad p = 0
\]
Parametrizing the initial conditions

Generalized mean ansatz:

\[
\frac{dS}{d^2 r \, dy} \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}
\]

\begin{align*}
\text{Pb+Pb 2.76 TeV} \\
T_{\text{min}} < T < T_{\text{max}} \\
-1 < p < 1 \\
p = -1
\end{align*}

\begin{align*}
p = \infty & \quad \text{maximum} \\
\frac{T_A + T_B}{2} & \\
\sqrt{T_A T_B} & \quad \text{geometric} \\
2 \frac{T_A T_B}{T_A + T_B} & \quad \text{harmonic} \\
\end{align*}

minimum
1) Calc participants: $P_{\text{coll}}(b) = 1 - \exp[-\sigma_{gg}T_{pp}(b)]$, $\int 2\pi b\, db\, P_{\text{coll}}(b) = \sigma_{\text{NN}}^{\text{inel}}$

2) Build participant density: $T_A(x, y) = \sum_{i=1}^{N_{\text{part},A}} \gamma_i T_p(x - x_i, y - y_i)$, $\gamma \sim \Gamma(k, 1/k)$

3) Parametrize entropy deposition: $dS/dy \propto \left(\frac{T_A^0 + T_B^0}{2}\right)^{1/p}$
Compare parametrization to existing IC models

- Wounded nucleon model
  \[ \frac{dS}{dy \, d^2r_\perp} \propto \tilde{T}_A + \tilde{T}_B \]

- EKRT model \[ \text{PRC 93, 024907 (2016)} \]
  after brief free streaming phase
  \[ \frac{dE_T}{dy \, d^2r_\perp} \sim \frac{K_{\text{sat}}}{\pi} p_{\text{sat}}^3 (K_{\text{sat}}, \beta; T_A, T_B) \]

- KLN model \[ \text{PRC 75, 034905 (2007)} \]
  \[ \frac{dN_g}{dy \, d^2r_\perp} \sim Q_{s,\text{min}}^2 \left[ 2 + \log \left( \frac{Q_{s,\text{max}}^2}{Q_{s,\text{min}}^2} \right) \right] \]
Modern event-by-event hybrid model

- **TRENTo initial conditions**
  Moreland, Bernhard, Bass, PRC 92, no. 1, 011901 (2015)
  - norm: entropy normalization
  - $p$: entropy deposition parameter
  - $k$: proton-proton multiplicity fluctuations
  - $w$: Gaussian nucleon width

- **HotQCD equation of state**
  Bazavov, et. al. PRD 90, 094503 (2014)

- **iEBE-VISHNU hydrodynamics**
  Shen, Qiu, Song, Bernhard, Bass, Heinz, Comp. Phys. Comm. 199, 61 (2016)
  - $\eta/s$ min: shear viscosity minimum
  - $\eta/s$ slope: shear viscosity slope
  - $\zeta/s$ norm: bulk viscosity normalization
  - $T_{sw}$: hydro-to-urqmd switching temp

- **UrQMD hadronic afterburner**
The challenge of rigorous model-to-data comparison

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Observable</th>
</tr>
</thead>
<tbody>
<tr>
<td>shear viscosity</td>
<td>identified yields</td>
</tr>
<tr>
<td>bulk viscosity</td>
<td>identified mean $p_T$</td>
</tr>
<tr>
<td>pre-equilibrium flow</td>
<td>flow cumulants</td>
</tr>
<tr>
<td>nucleon width</td>
<td>mode mixing observables</td>
</tr>
<tr>
<td>hadronization temp</td>
<td>event plane decorrelations</td>
</tr>
<tr>
<td>p+p fluctuations</td>
<td>HBT interferometry</td>
</tr>
</tbody>
</table>

Testing a single set of parameters requires $\mathcal{O}(10^4)$ hydro events...and evaluating eight different parameters five times each requires $5^8 \times 10^4 \approx 10^9$ hydro events.

That’s roughly $10^5$ computer years!
Solution: Bayesian methodology

Model Parameters - System Properties
- initial conditions (e.g. nucleon width)
- QGP & HRG medium (e.g. $\eta/s$)

Physics Model
- TRENTo IC
- iEBE-VISHNU

Experimental Data
- ALICE flow and spectra

Gaussian Process Emulator
- non-parameteric interpolation
- fast surrogate for full model

Markov chain Monte Carlo (MCMC)
- random walk through param. space weighted by posterior probability

Bayes' Theorem:
posterior $\propto$ likelihood $\times$ posterior

Posterior Distribution
- probability distribution for true values of model parameters

calc events on Latin hypercube
after many steps, MCMC equilibrates to
Calibrating the model: before and after

- Top: run model ($\times 10^4$ events) at each design point ($\times 300$ evals)
- Bottom: emulator predictions for 100 samples from the posterior
Calibrated to identified particles

Entropy deposition parameter

Generalized mean parametrization:

$$dS/dy \propto \left( \frac{T_A^p + T_B^p}{2} \right)^{1/p}$$
Random Gamma nucleon weights:

\[ P(\gamma | k) = \frac{k^k \gamma^{k-1} e^{-k\gamma}}{\Gamma(k)} \]

\[ T_A(x, y) = \sum_{i=1}^{N_{\text{part},A}} \gamma_i \bar{T}_p(x - x_i, y - y_i) \]
Calibrated to identified particles

Nucleon width

Gaussian nucleon thickness:

\[ T_p(x, y) = \frac{1}{2\pi w^2} e^{-\frac{x^2+y^2}{2w^2}} \]
Shear viscosity parametrization:

$$(\eta/s)(T) = (\eta/s)_{\text{min}} + (T - T_c)(\eta/s)_{\text{slope}}$$
Running the model with high probability parameters

- Choose high probability model parameters from Bayesian posterior (right)
- Run full hybrid model using high probability parameters (bottom)

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<th>Initial condition</th>
<th>QGP medium</th>
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<tbody>
<tr>
<td>norm 120.</td>
<td>( \eta/s ) min 0.08</td>
</tr>
<tr>
<td>( \rho ) 0.0</td>
<td>( \eta/s ) slope 0.85 GeV(^{-1})</td>
</tr>
<tr>
<td>( k ) 1.5</td>
<td>( \zeta/s ) norm 1.25</td>
</tr>
<tr>
<td>( w ) 0.43 fm</td>
<td>( T_{sw} ) 0.148 GeV</td>
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Yields \( dN/dy, dN_{ch}/d\eta \)

Mean \( p_T \) [GeV]

Flow cumulants \( v_n \{2\} \)
Conclusions

Initial condition properties
- Yields, mean $p_T$ and flows impose strong constraints on IC.
- Entropy deposition mimicked by $dS/dy \sim \sqrt{T_A T_B}$
- Data strongly prefers small nucleon width $w \approx 0.4–0.6$ fm!
- A+A collisions weakly sensitive to p+p mult. fluctuations
- Preferred initial conditions similar to EKRT, IP-Glasma

Hydrodynamic transport properties
- First quantitative credibility interval on $(\eta/s)(T)$!
- Data prefer non-zero bulk viscosity
- Hydro-to-micro $T_{sw}$ determined by relative species yields

TRENTo is publicly available at [qcd.phy.duke.edu/trento](http://qcd.phy.duke.edu/trento)

More in the pre-print arXiv:1605.03954
Computer experiment design

Maximin Latin hypercube
- Random, space-filling points
- *Maximizes* the minimum distance between points → avoids gaps and clusters
- Uniform projections into lower dimensions

This work:
- 300 points across 8 dimensions
- 8 centrality bins
- $O(10^7)$ events total
TRENTo 3D, work in progress...

- Extend to forward/backward rapidities while maintaining mid-rapidity result:
  \[ s(x_\perp, \eta) = s(x_\perp, \eta = 0) \cdot f(x_\perp, \eta) \]

- Parametrize \( f(x_\perp, \eta) \) by first few cumulants,

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<td>( \gamma(x_\perp) )</td>
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- Reconstruct \( f(\eta) \) by \( \mathcal{F}^{-1} \) cumulant generating function,

  \[ \mathcal{F}^{-1} \exp(i\mu k - \frac{\sigma^2}{2} k^2 + i\gamma k^3 - \kappa k^4) \]
Extend to forward/backward rapidities while maintaining mid-rapidity result:

\[ s(x_, \eta) = s(x_, \eta = 0) \cdot f(x_, \eta) \]

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Comparing to the IP-Glasma model

- IP-Glasma: multi-stage dynamical model, simple analytic mapping unknown.
- Analyze effective mapping via eccentricity harmonics $\varepsilon_n$ (left).

Work ongoing: determine IP-Glasma effective mapping for direct comparison with TRENTo parametrization
**Entropy deposition parameter** $p = 0$, nucleon width $w = 0.5$ fm, p+p fluctuation factor $k = 1.6$, normalization varied with energy but not collision system.

**Good description of particle production at all energies, self consistent p+A and A+A multiplicities**