Colored particle-in-cell simulations for heavy-ion collisions

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INSTITUTE FOR THEORETICAL PHYSICS

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FWF Der Wissenschaftsfonds.

TU WIEN

Introduction

Goal: Simulate heavy-ion collisions in the color-glass-condensate (CGC) framework with finite nucleus thickness. Possible with colored particle-in-cell (CPIC).

Access to lower collision energies, break boost-invariance!

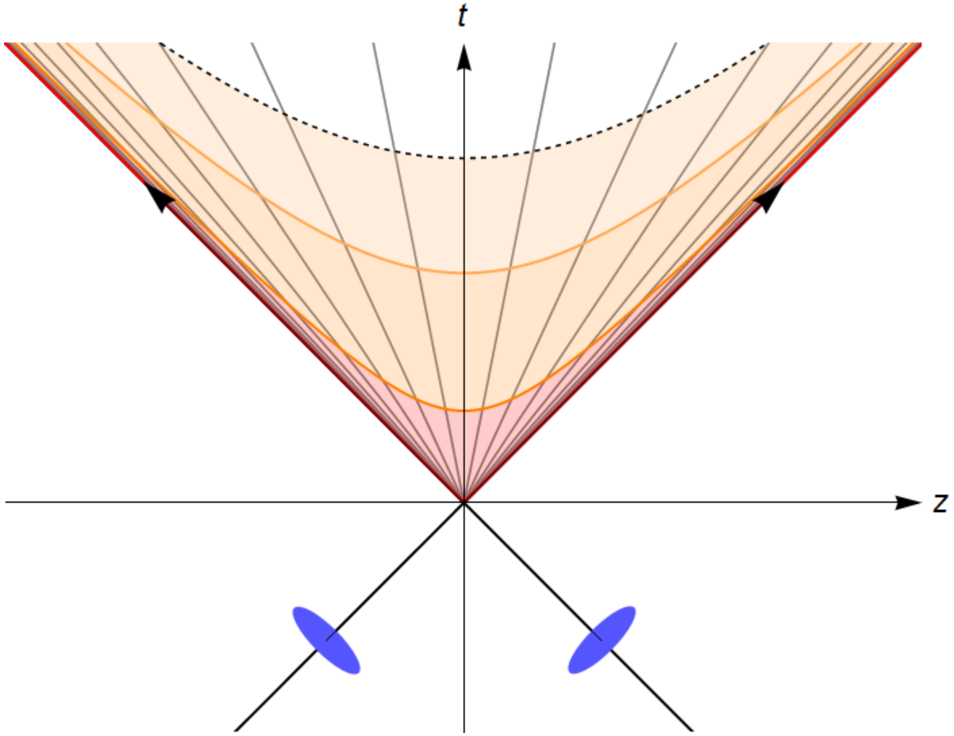
- Formation and evolution of the quark-gluon-plasma (QGP)?
- How does the QGP become isotropic and thermalized?
- **What is the role of boost-invariance?**

Various experiments with wide range of the gamma factor γ*:*

- LHC (ALICE) @ CERN: Pb+Pb with **~2.76 TeV** per nucleon pair. ($\gamma \approx 2700$)
- RHIC @ BNL: Au+Au with **~200 GeV** ($\gamma \approx 100$)
- RHIC beam energy scan: **~7.7 – 62.4 GeV** ($\gamma \approx 4 - 30$)

Need to go beyond boost-invariant approximation. → Simulations with “thick” nuclei!

Boost-invariant CGC collision



- CGC: Separation of hard and soft degrees of freedom, weak coupling
- Color currents of the nuclei restricted to the light cone and infinitely thin
- Analytical solutions exist for everything except the forward light cone
- Fields in the forward light cone are independent of rapidity. Reduction from 3D+1 to 2D+1
- Need to solve 2D+1 source-free Yang-Mills equations in the forward light cone with special initial conditions on the light cone

$$D_\mu F^{\mu\nu}(\tau, x_T) = 0$$

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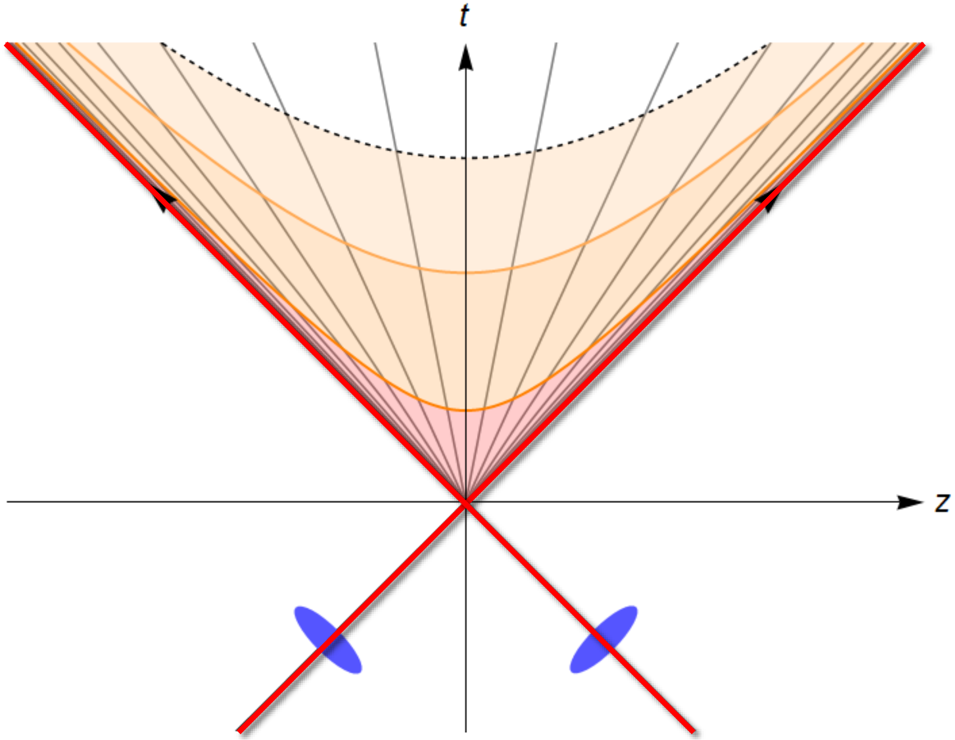

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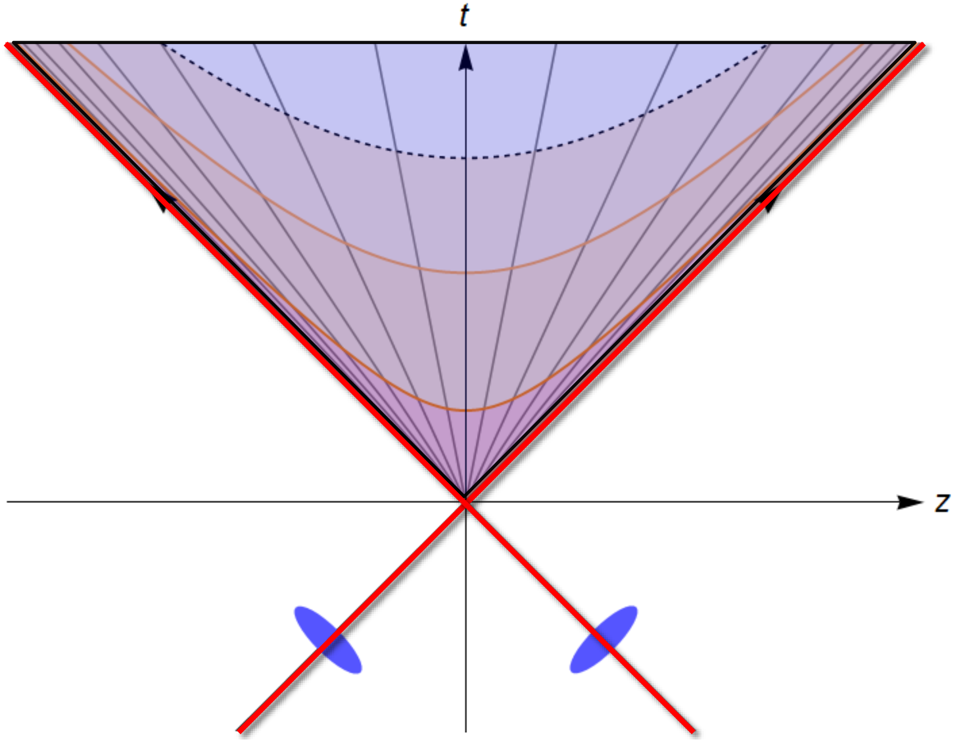

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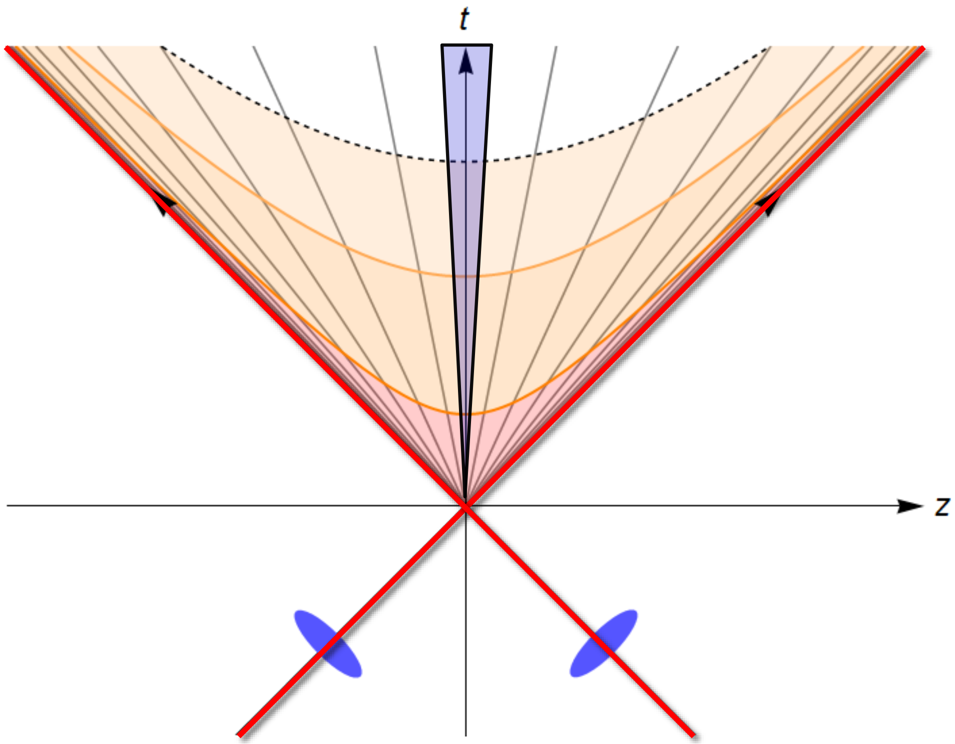

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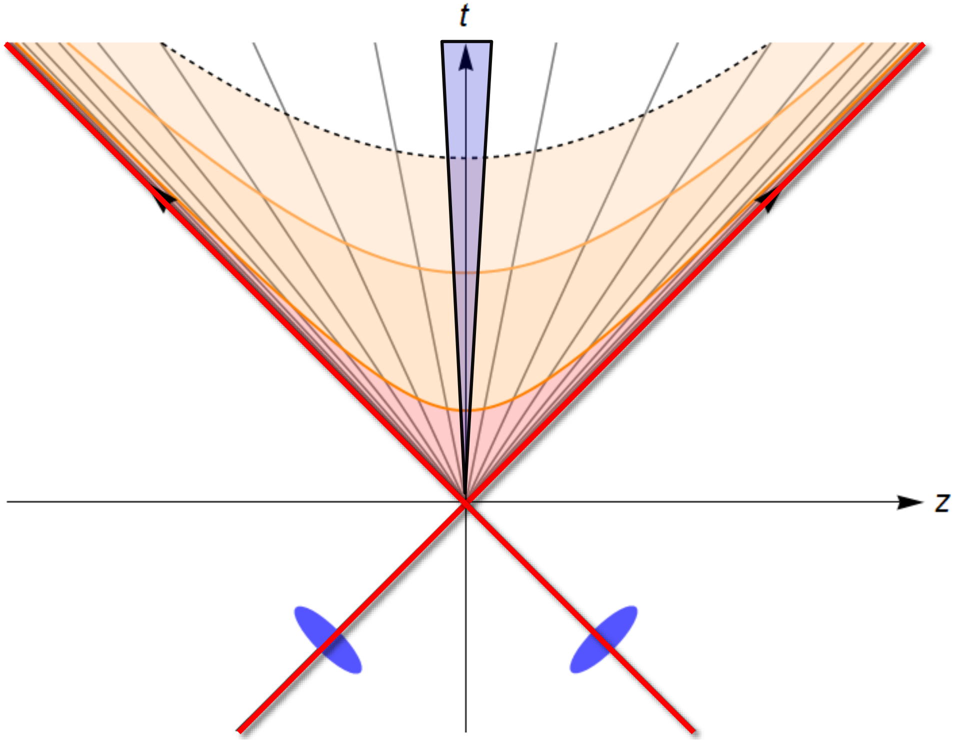

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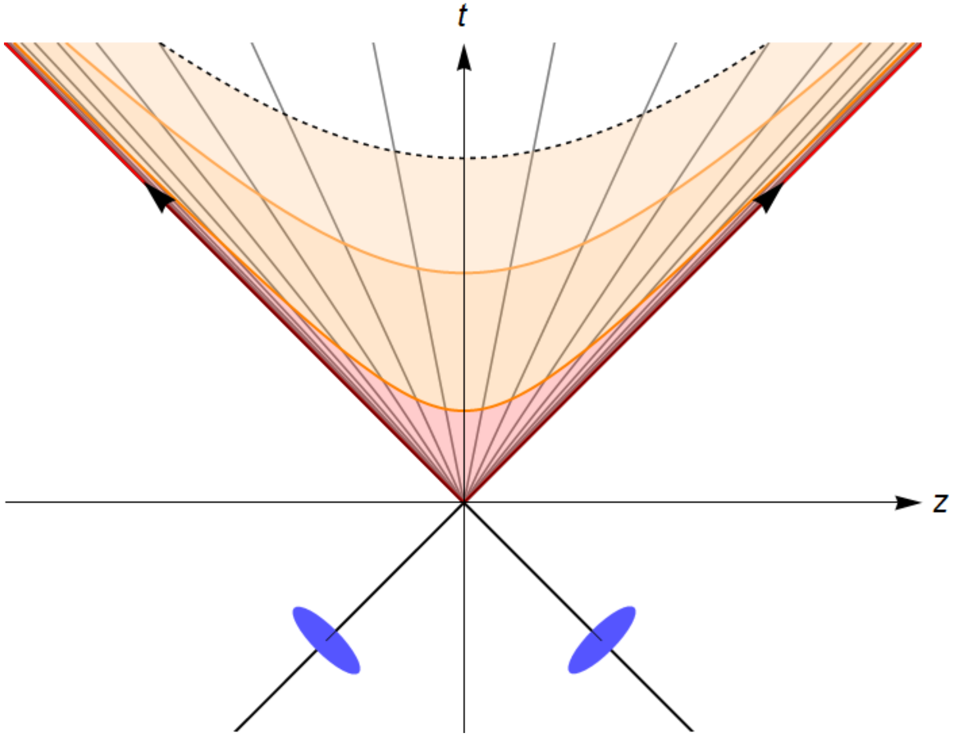
Finite nucleus thickness



- Extended color currents need to be taken into account.
- Fields depend on rapidity.
- Need to solve full 3D+1 Yang-Mills equation with currents.

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Colored particle-in-cell (CPIC) provides a framework to numerically solve the field and current equations on a lattice.

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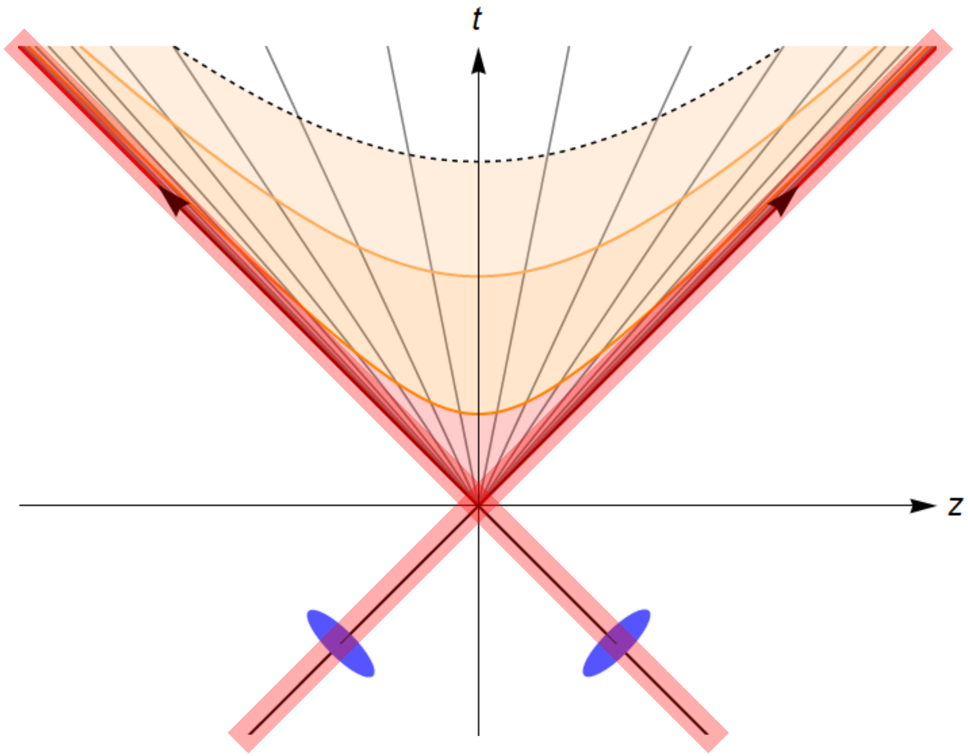

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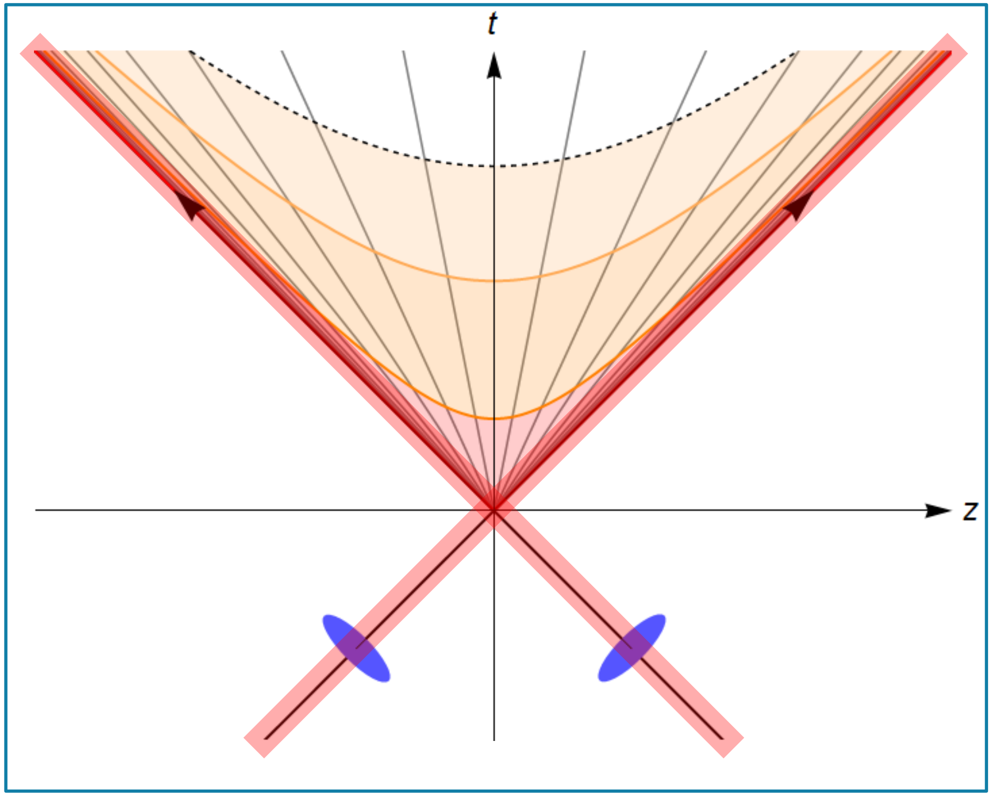

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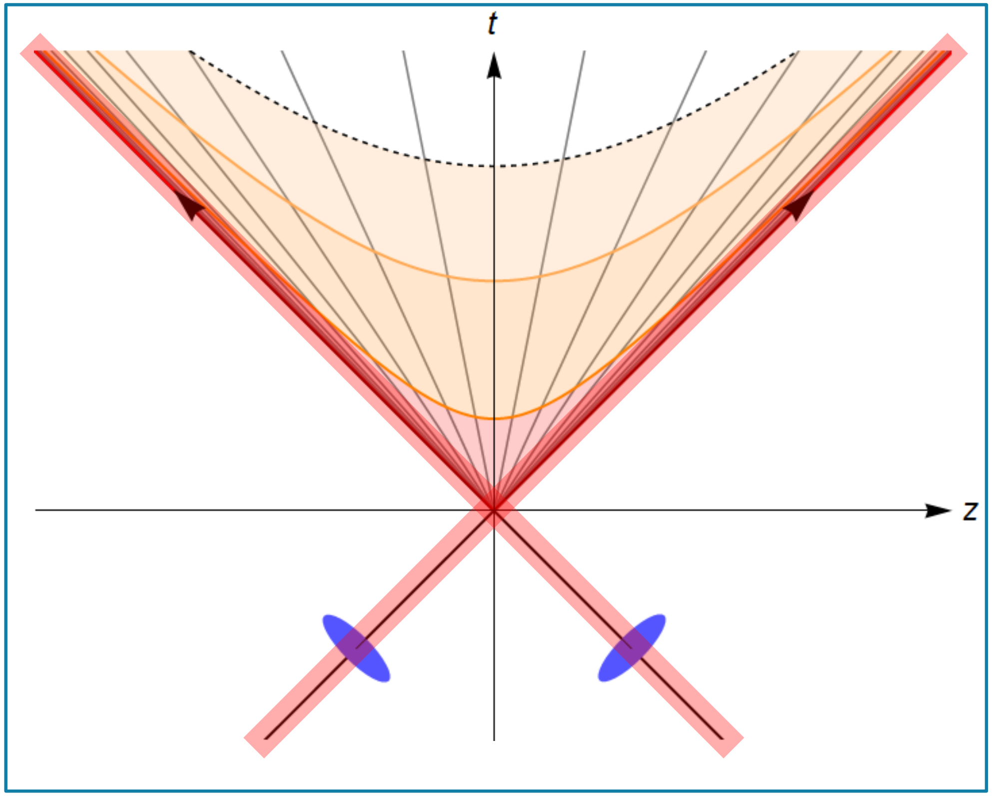

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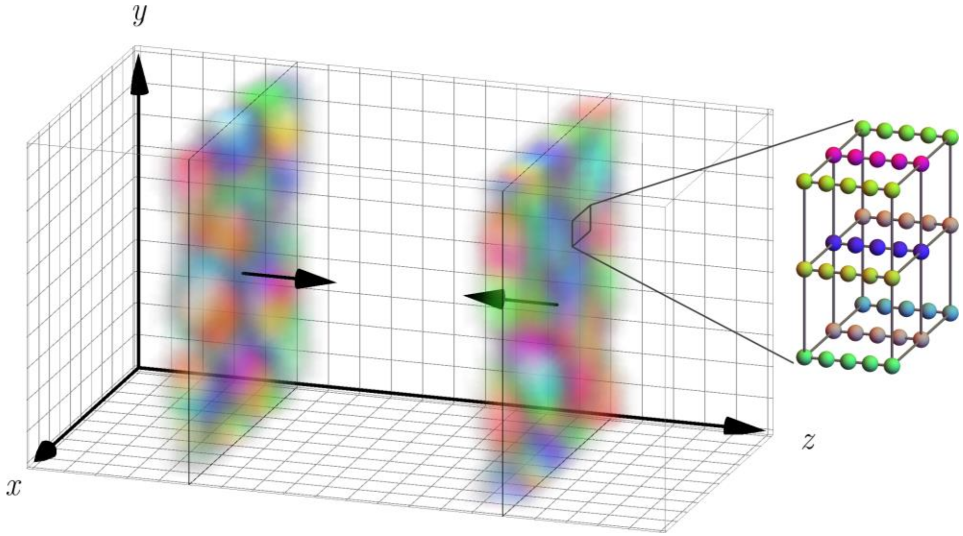
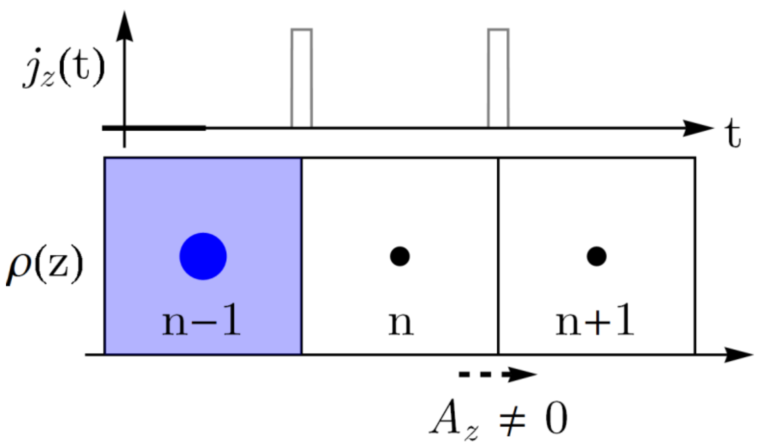
Colored particle-in-cell (CPIC) method



Nearest-grid-point method (NGP)



CPIC: non-Abelian generalization of the particle-in-cell method from plasma physics.

[A. Dumitru, Y. Nara, M. Strickland: PRD75:025016 (2007)]

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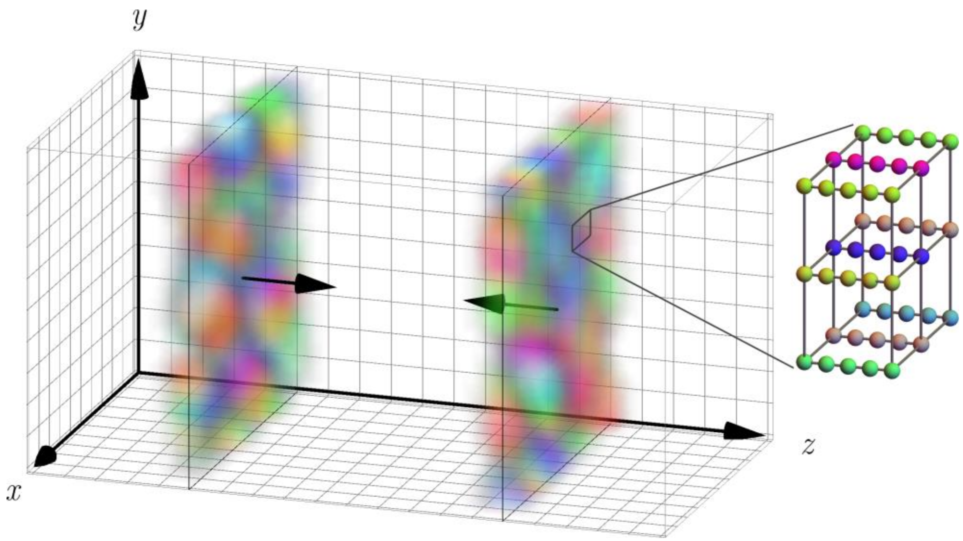

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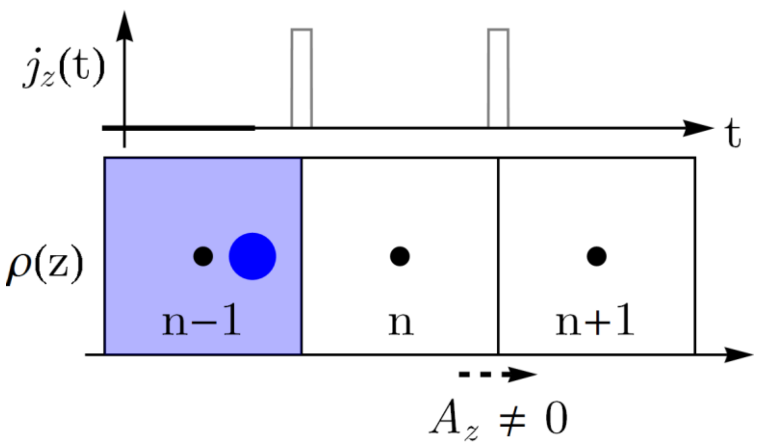

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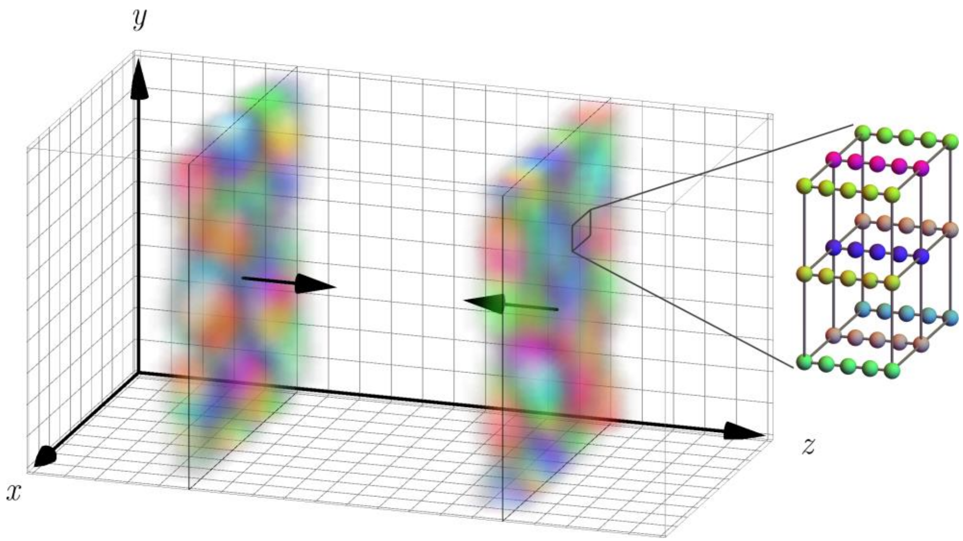

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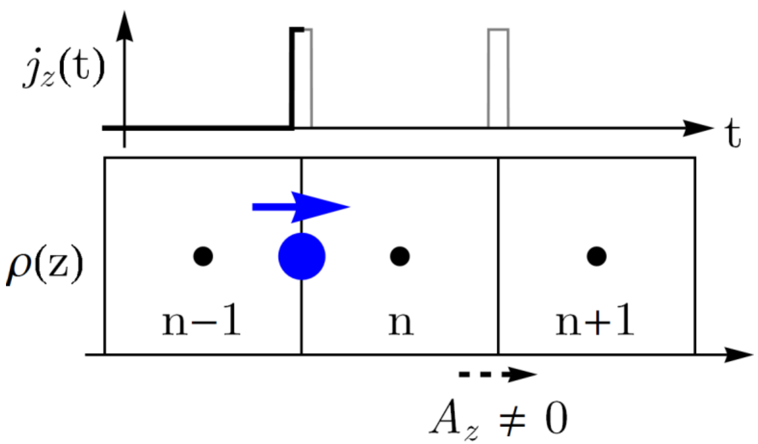

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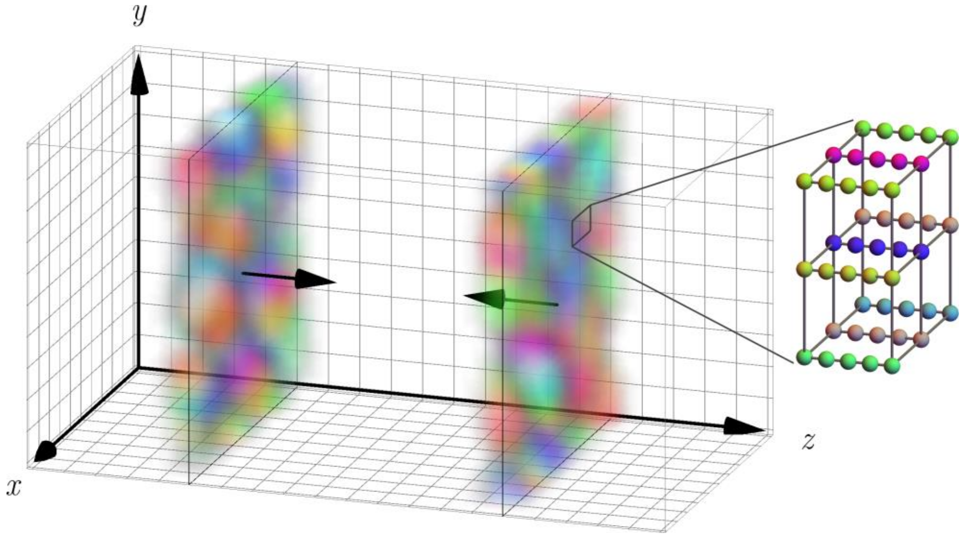

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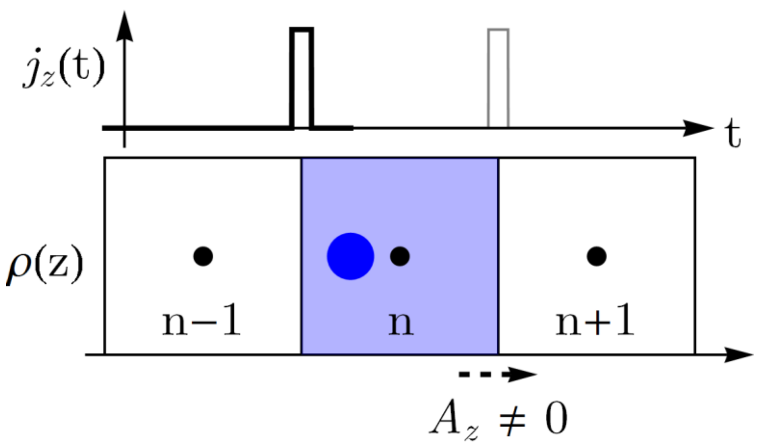

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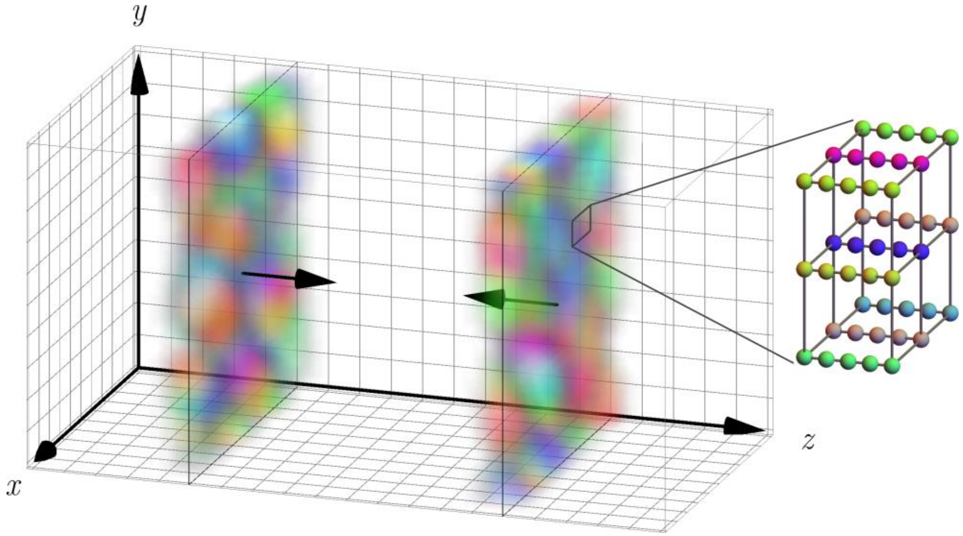

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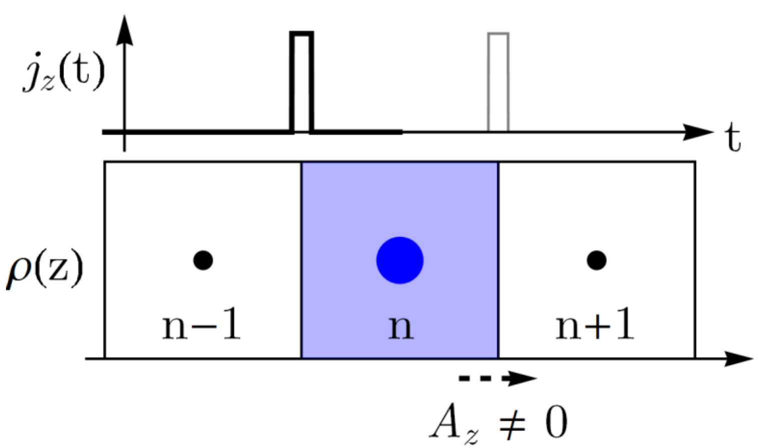

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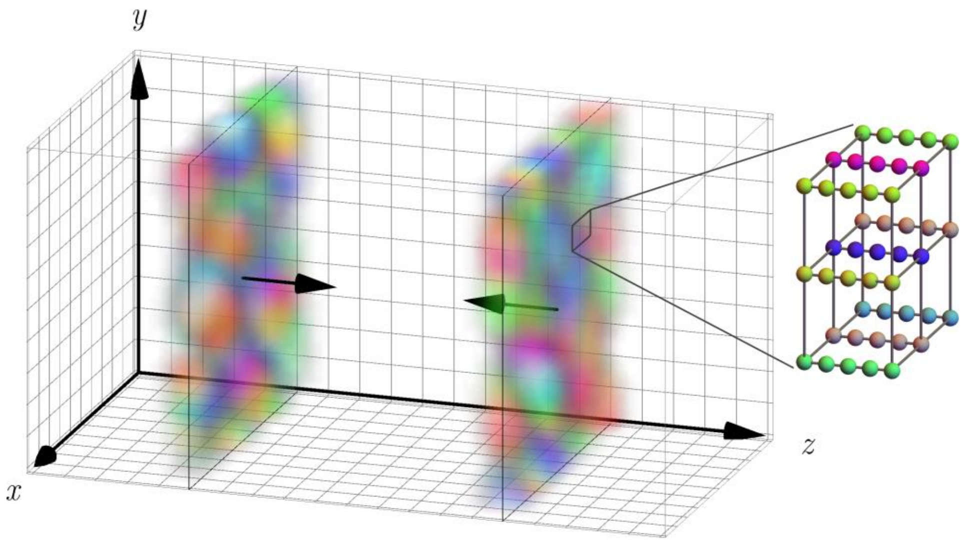

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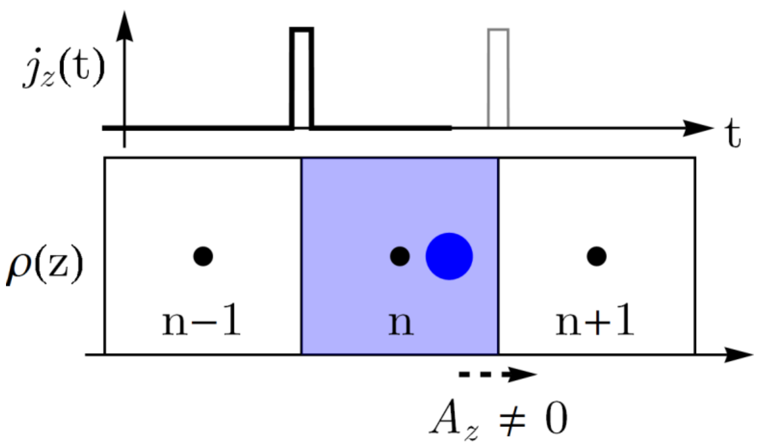

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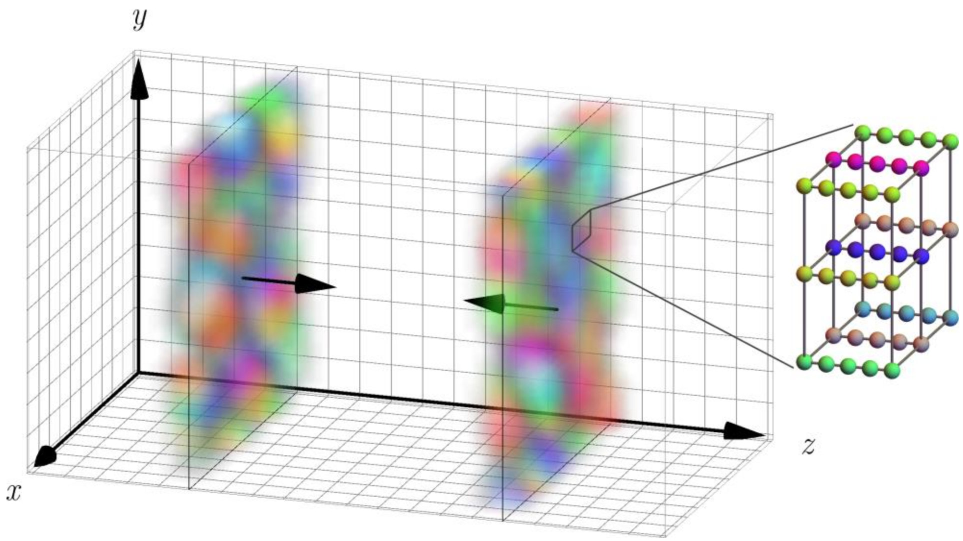

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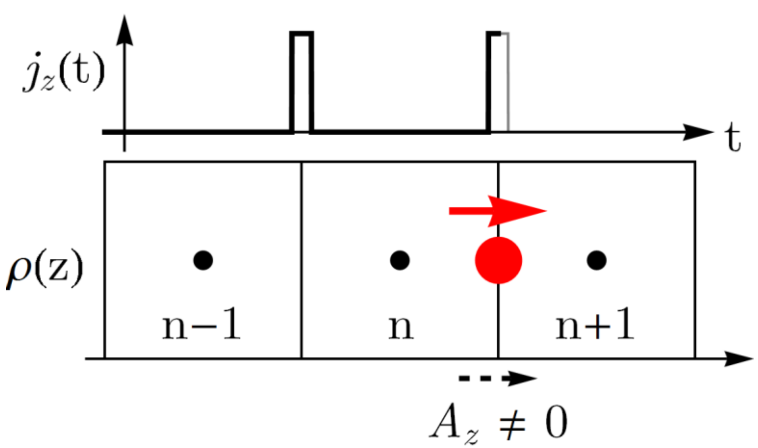

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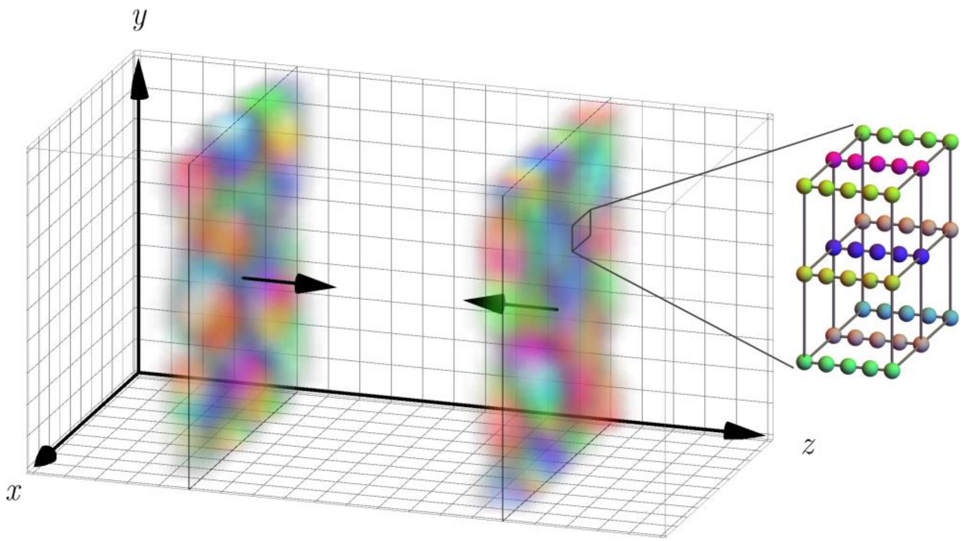

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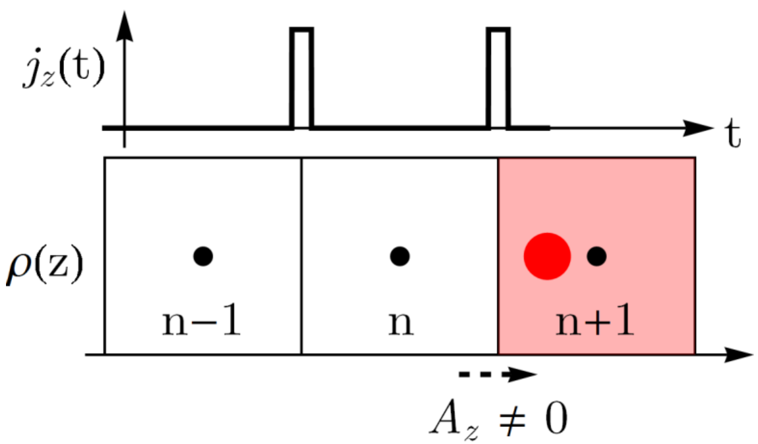

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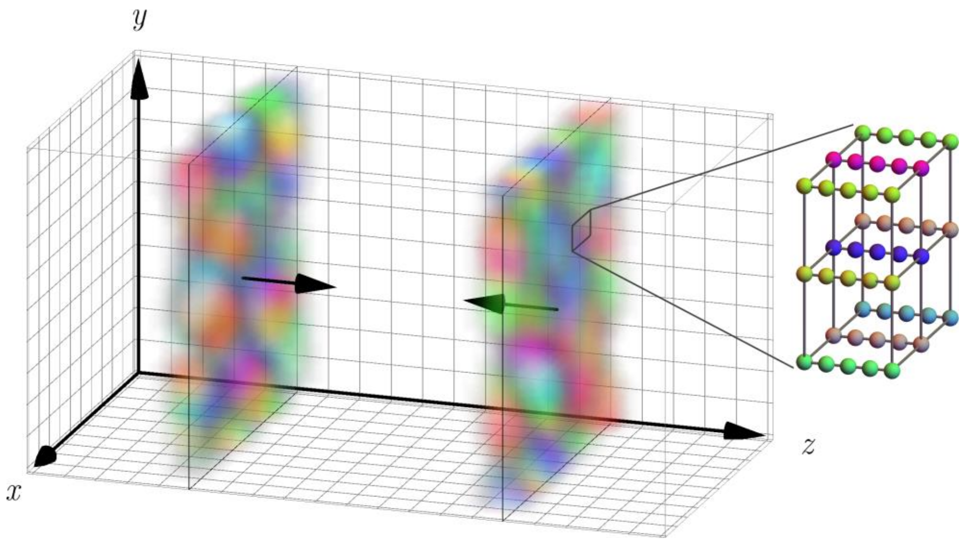

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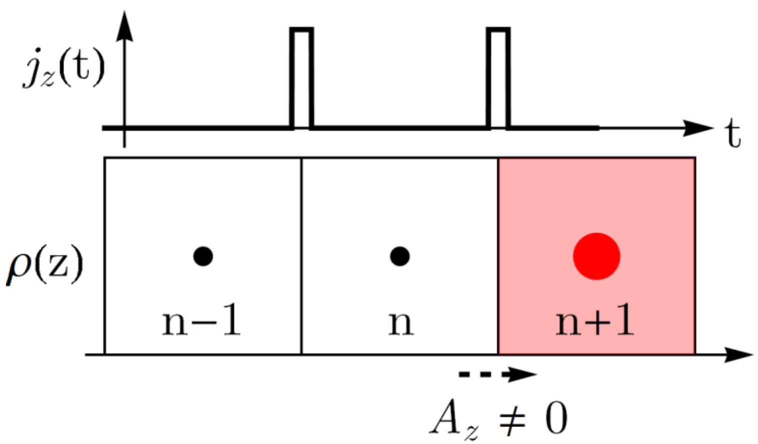

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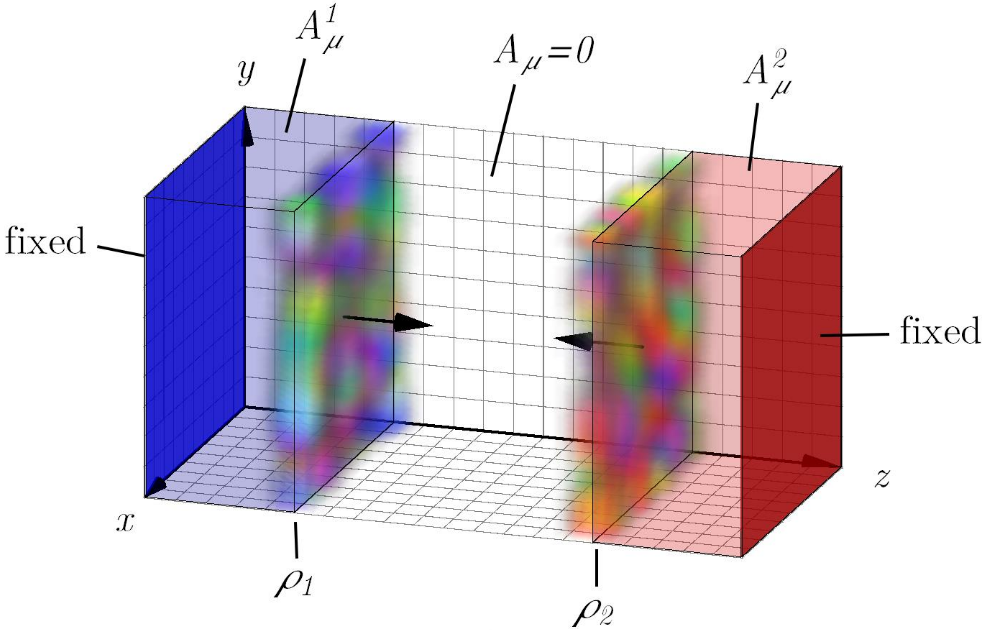

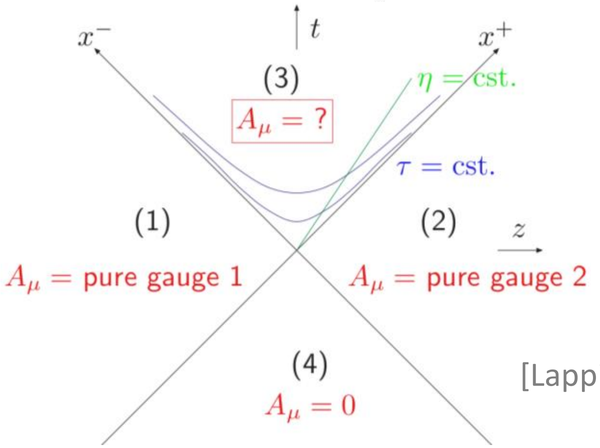

[Lappi, Phys.Lett. B643 (2006) 11-16]

- Temporal gauge ($A_0 = 0$) suitable for numerical time evolution.
- Asymptotically pure gauge "trails" behind nuclei.
- Fixed boundary conditions on the longitudinal boundaries are required.
- Random charge densities $\rho_{(1,2)}$ are sampled from **McLerran-Venugopalan (MV)** model.

[McLerran, Venugopalan: PRDD49 (1994) 3352-3355]

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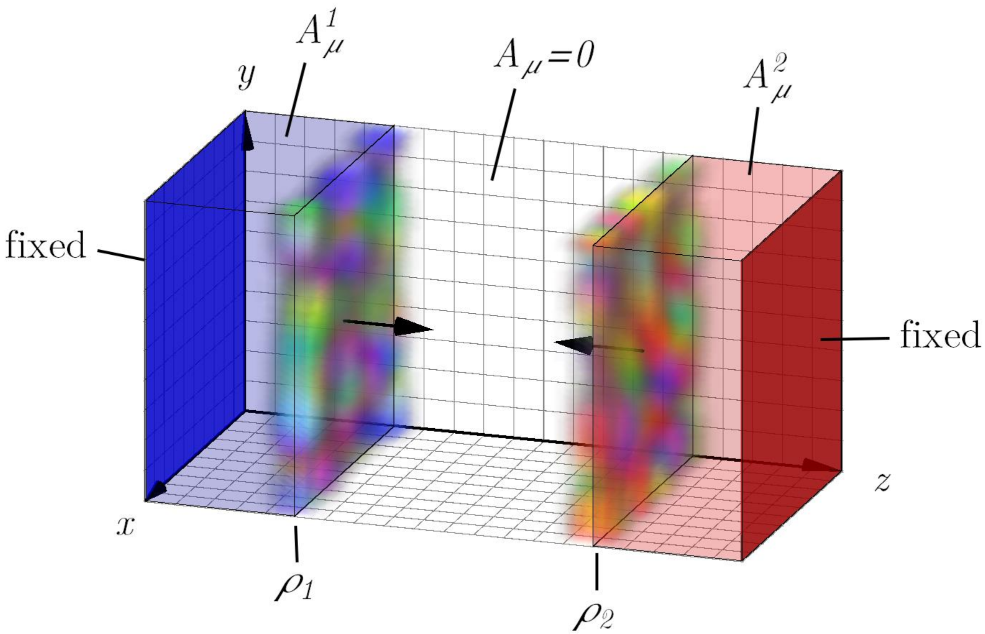

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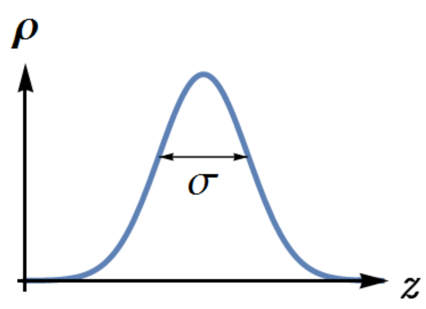


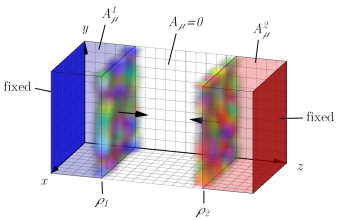
finite longitudinal thickness

$$\langle \rho^a(x_T)\rho^b(y_T)\rangle = g^2\mu^2\delta^{(2)}(x_T - y_T)$$

MV parameter $\mu \approx 0.5$ GeV (Au)

Simulation overview

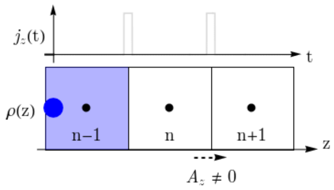
1. Initialize random charges and fields of two colliding nuclei.



2. **Simulation cycle**:

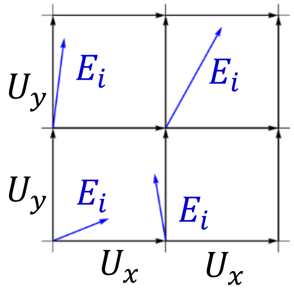
   a. Move particles and apply parallel transport.

   b. Generate currents from particle movement.



   c. Evolve fields in time with currents as input.

   d. Compute observables ($T_{\mu\nu}$, ε, p_L, p_T, …).



3. Average over many random events.

→ $\langle T_{\mu\nu} \rangle$, $\langle \varepsilon \rangle$, $\langle p_L \rangle$, $\langle p_T \rangle$,…

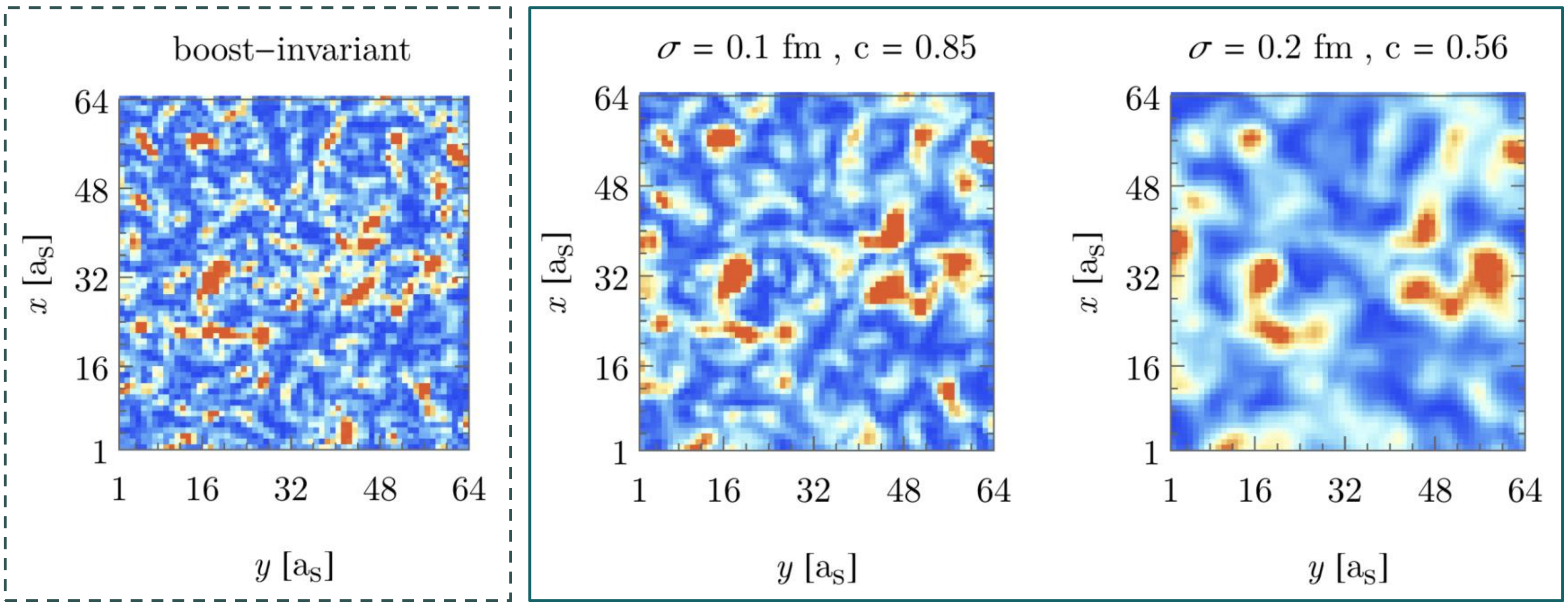
Numerical results

Au-Au collision in the MV model, SU(2)

Comparison to boost-invariant results

- Check validity of simulation results with finite nucleus thickness by comparing to analytical boost-invariant results.
- Compare boost-invariant Glasma initial conditions to simulated fields and vary thickness parameter σ.

Energy density component $\mathrm{tr}E_L^2(x_T)$ in the transverse plane at $\eta = 0$.



analytic result

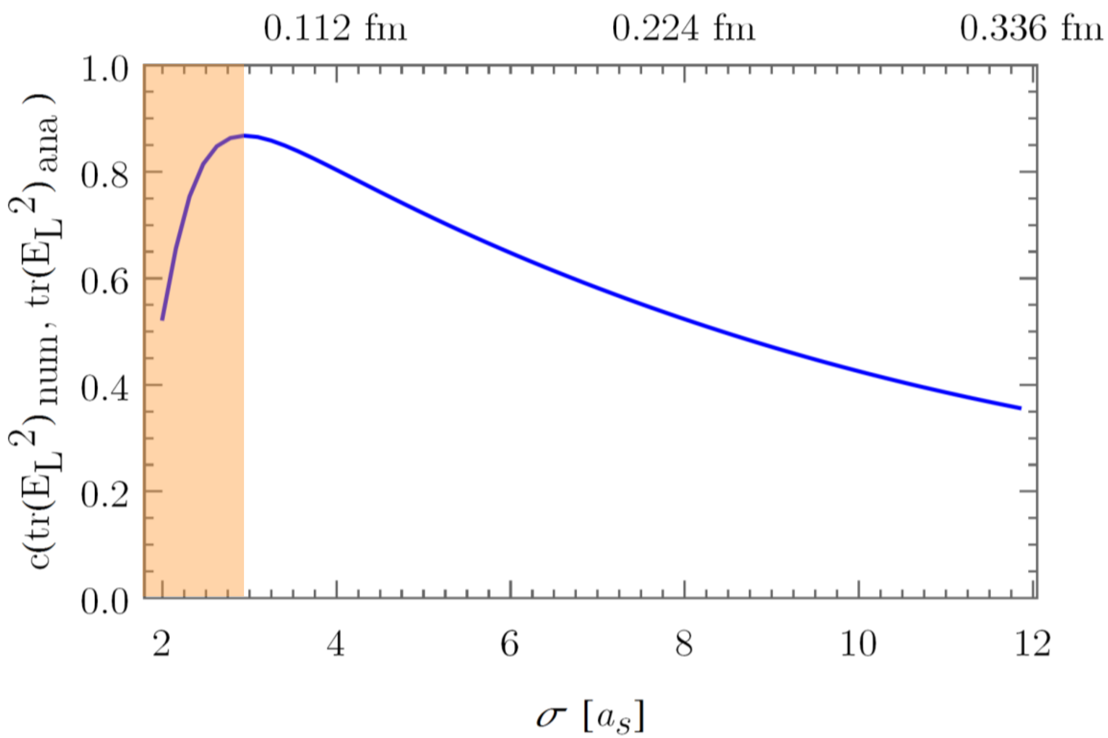
simulation result

Comparison to boost-invariant results

- Check validity of simulation results with finite nucleus thickness by comparing to analytical boost-invariant results.
- Compare boost-invariant Glasma initial conditions to simulated fields and vary thickness parameter σ.

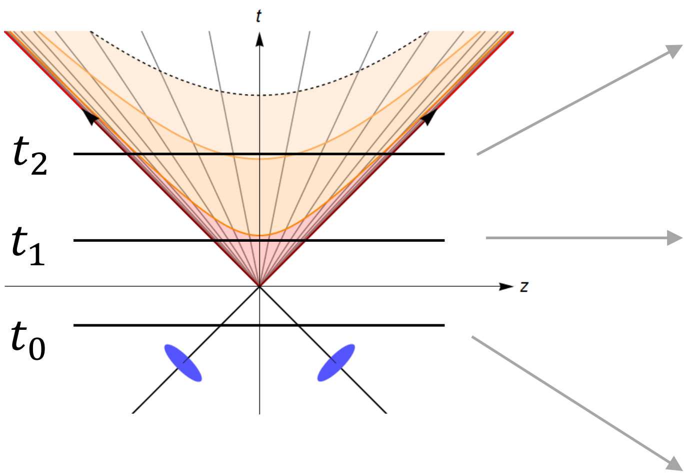
- Compute correlation between analytic and numerical results as a function of σ.
- Thick nuclei: low correlation
- Thin nuclei: high correlation
- Numerical instabilities prohibit very thin nuclei.

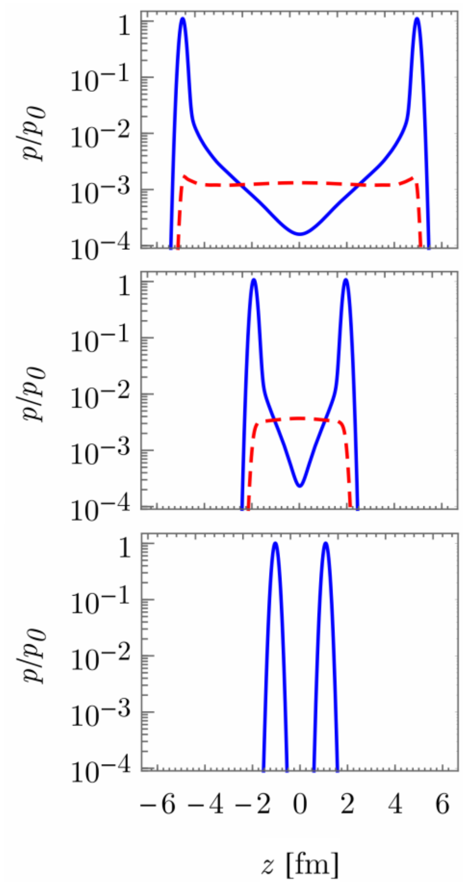
(but it’s just a question of lattice sizes)



Pressure anisotropy (1)

- Compute longitudinal and transverse pressure $p_L(z)$ and $p_T(z)$ as a function of the longitudinal coordinate z.





$t_2 = +5$ fm/c
(late times)

$t_1 = +2$ fm/c
(after collision)

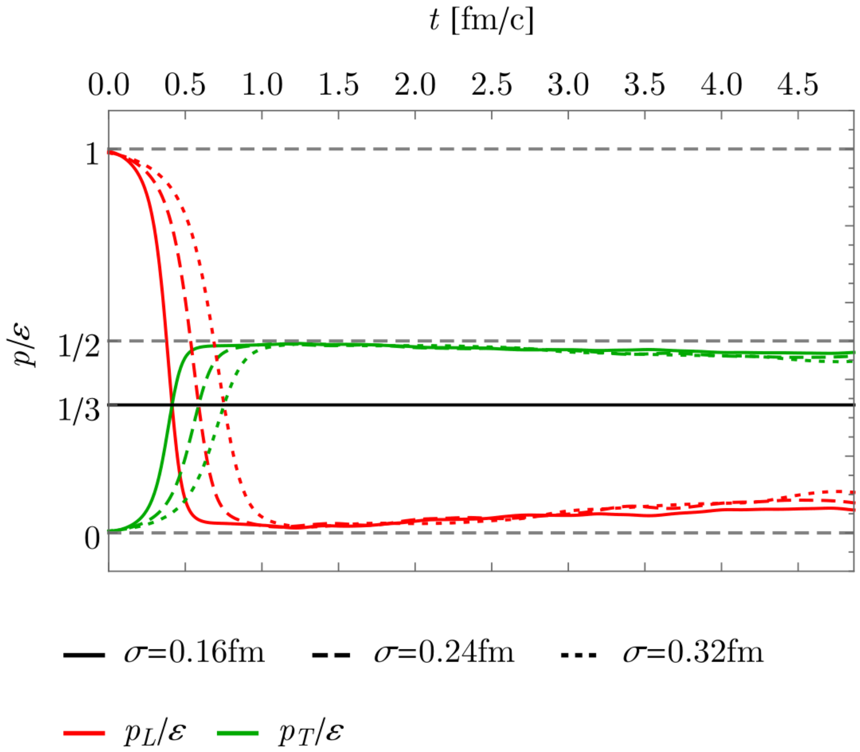
$t_0 = -1$ fm/c
(before collision)

→ Pronounced pressure anisotropy

Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after ~ 0.1 fm/c to a few fm/c.
- Boost-invariance breaking perturbations drive system towards isotropization. [Epelbaum, Gelis, PRL 111 (2013) 232301]. Finite thickness breaks boost-invariance.

“Thick” nuclei ($\gamma \sim 20 - 40$)

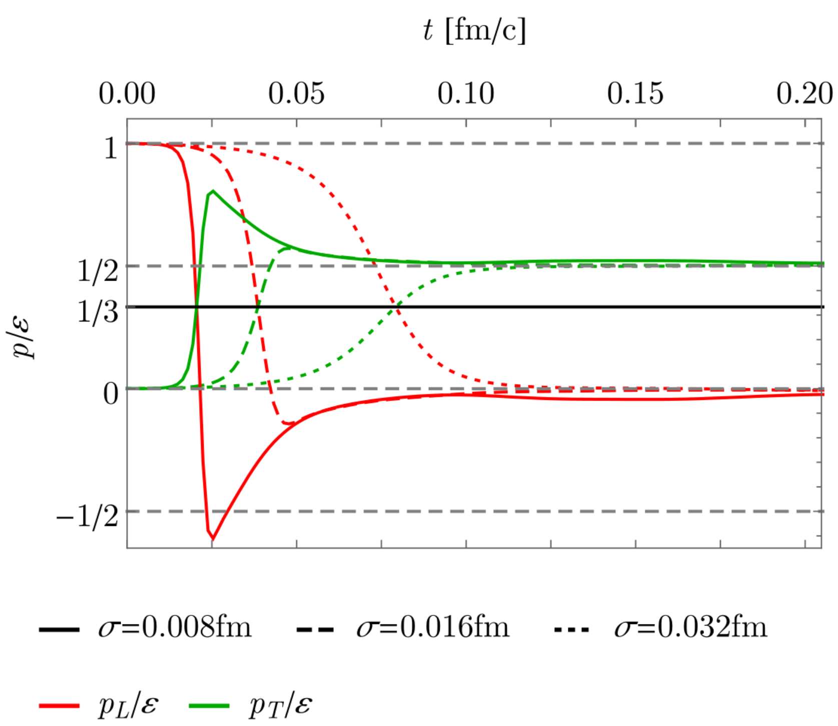


- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thick nuclei: pronounced pressure anisotropy (free-steaming).
- Slight movement towards isotropization visible, but it is too slow.
- Negative longitudinal pressures?

Pressure anisotropy (2)

- **Isotropization**: initial pressure anisotropy should vanish after ~ 0.1 fm/c to a few fm/c.
- Boost-invariance breaking perturbations drive system towards isotropization. [Epelbaum, Gelis, PRL 111 (2013) 232301]. Finite thickness breaks boost-invariance.

"Thin" nuclei ($\gamma \sim 200 - 1000$)

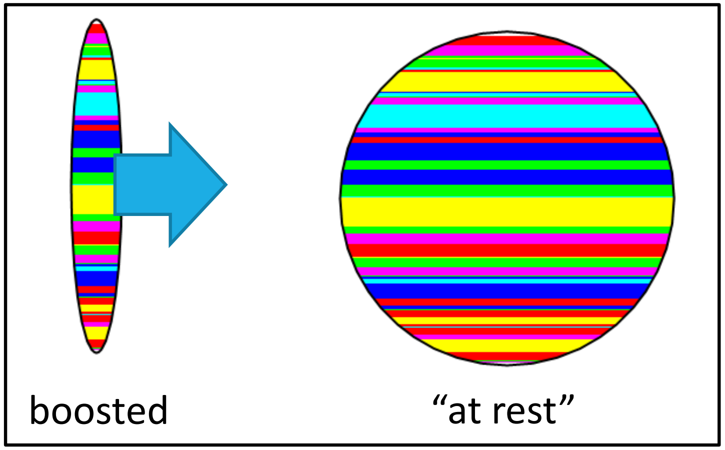


- Analyze pressure to energy density ratio in the central region at $\eta = 0$.
- Thin nuclei: negative longitudinal pressures
- Observables always influenced by presence of the nuclei at early times.

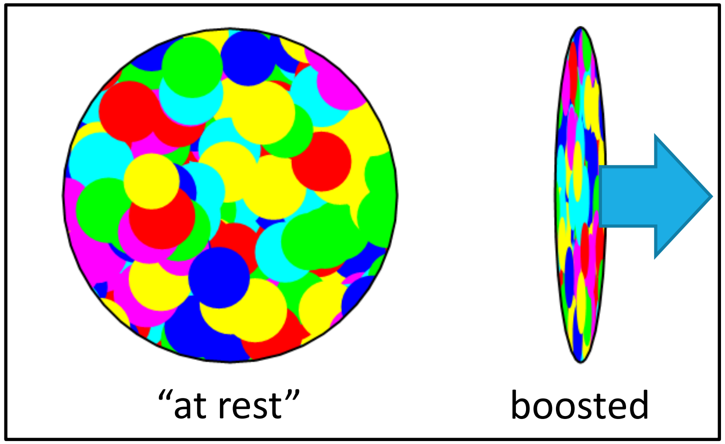
Longitudinal structure

- Initial conditions are still missing **random longitudinal structure**.
- Longitudinal randomness…
  - leads to higher energy density in the glasma.
    [Fukushima, PRD 77 (2008) 074005]
  - could provide boost-invariance breaking perturbations.
- Possible consequence: **faster isotropization times? → future work!**

Current implementation



Longitudinal randomness

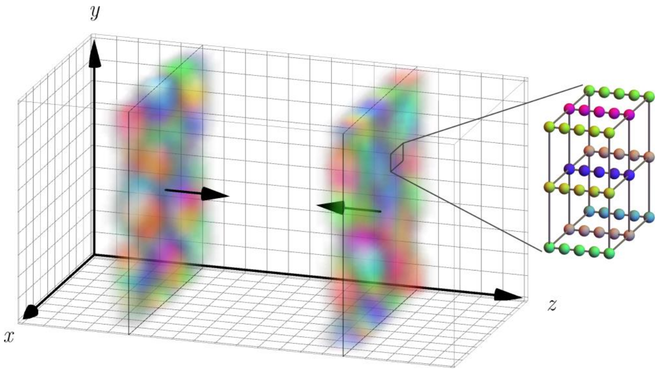


Conclusions

- Simulating CGC collisions in 3D+1 with finite nucleus thickness in the laboratory frame using CPIC is viable.
- Boost-invariant results reproduced in the limit of thin nuclei.
- We observe a pronounced pressure anisotropy after the collision.
- Observed isotropization too slow.
- **Future**: Study effects of initial conditions with random longitudinal structure on isotropization.

arXiv:1605.07184 [hep-ph]
open source: https://github.com/openpixi

Thank you for your attention!



arXiv:1605.07184 [hep-ph]

open source: https://github.com/openpixi

Backup slides

Pressure in comoving frame

- Pressure components $p_L(t,z)$ and $p_T(t,z)$ computed in laboratory frame.
- Rapidity dependence?
- Transform $p_L(t,z)$ to $\bar{p}_L(\tau,\eta)$.

$$\bar{p}_L(\tau,\eta) = p_L(\tau,\eta)\cosh^2\eta + \varepsilon(\tau,\eta)\sinh^2\eta - 2S_L(\tau,\eta)\cosh\eta\sinh\eta$$

$$S_L \equiv 2\mathrm{tr}\left(\vec{E}\times\vec{B}\right)_z$$

"Thick" nucleus ($\gamma \sim 20$)