On bound state effects in dark matter freeze-out[1,2]

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[1] Seyong Kim and ML, 1602.08105; 1609.00474.

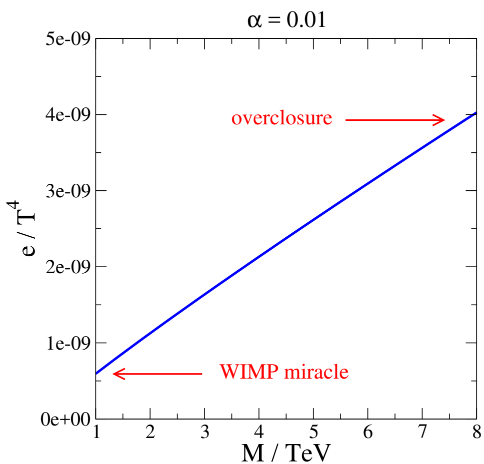
[2] Supported by the SNF under grant 200020-155935.

Standard WIMP paradigm appears to be in trouble (?)

Equate the Hubble rate with the co-annihilation rate:

$$H \sim n\langle \sigma v \rangle \;\Leftrightarrow\; \frac{T^2}{m_{\rm Pl}} \sim \Big(\frac{MT}{2\pi}\Big)^{3/2} e^{-M/T} \frac{\alpha^2}{M^2} \overset{\alpha\sim 0.01}{\Rightarrow} T \sim \frac{M}{25}\;.$$

Compare $e \equiv M n$ at the freeze-out with radiation $\sim T^4$:



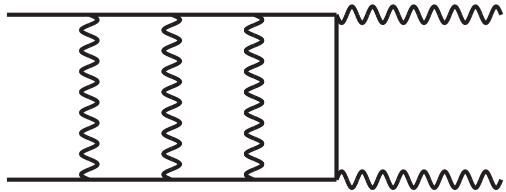
LHC pushes up lower bound on M, so there is a danger "overclosure"

$\Rightarrow$

refine model (e.g. α), or refine computation?

Could efficient decays help to avoid overclosure?

Indeed co-annihilating particles with $v \ll 1$ interact "strongly".



In particular the "Sommerfeld effect"[3] is widely discussed.[4]

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[3] L.D. Landau and E.M. Lifshitz, *Quantum Mechanics, Non-Relativistic Theory*, Third Edition, §136; V. Fadin, V. Khoze and T. Sjöstrand, *On the threshold behavior of heavy top production*, Z. Phys. C 48 (1990) 613.

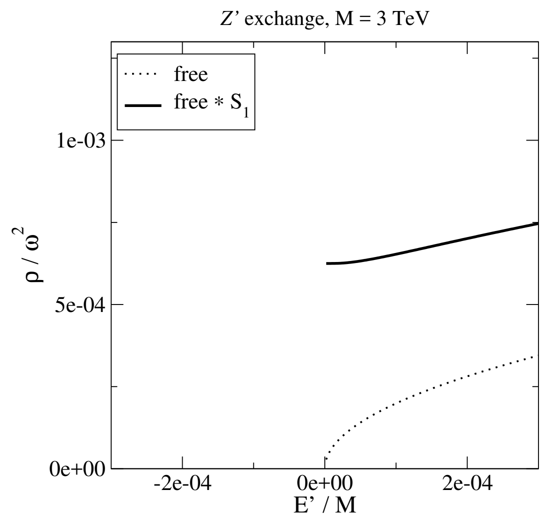
[4] e.g. J. Hisano, S. Matsumoto, M. Nagai, O. Saito and M. Senami, *Non-perturbative effect on thermal relic abundance of dark matter*, hep-ph/0610249; J.L. Feng, M. Kaplinghat and H.-B. Yu, *Sommerfeld Enhancements for Thermal Relic Dark Matter*, 1005.4678.

Rapid summary of the Sommerfeld effect

For attractive s-wave interaction:

$$S_1 = \frac{X_1}{1 - e^{-X_1}} , \quad X_1 = \frac{g^2 C_\mathrm{F}}{4v} .$$

Corresponding “spectral function” ($E' \equiv \omega - 2M \equiv Mv^2$):



What happens around and below the threshold?

Perhaps there could be bound states?[5]

This sounds exotic, but we are interested in **rare processes** where two dilute particles come together, i.e. $|\partial_t n| \sim e^{-2M/T}$. In bound states they are "already" together, with a less suppressed Boltzmann weight, because of a binding energy $\Delta E > 0$:

$$|\partial_t n_{\text{bound}}| \sim e^{-(2M-\Delta E)/T} \; .$$

If $T \lesssim \Delta E$, this contribution dominates the co-annihilation rate.

[5] e.g. B. von Harling and K. Petraki, *Bound-state formation for thermal relic dark matter and unitarity*, 1407.7874.

General spectral representation of the annihilation rate

Let θ, η annihilate DM and DM'. Like in the optical theorem, the rate is contained in an imaginary part of a 4-particle operator:[6]

$$\mathcal{O} = \frac{i c_1 \alpha^2 \, \theta^\dagger \eta^\dagger \, \eta \theta}{M^2} + \ldots \, .$$

Through a linear response analysis, this yields

$$\langle \sigma v \rangle = \frac{4 c_1 \alpha^2}{M^2 n_{\text{eq}}^2} \underbrace{\frac{1}{\mathcal{Z}} \sum_m e^{-E_m/T} \langle m | \theta^\dagger \eta^\dagger \, \eta \theta | m \rangle}_{\left(\frac{MT}{\pi}\right)^{3/2} e^{-2M/T} \int_{-\Lambda}^{\infty} \frac{\mathrm{d}E'}{\pi} \, e^{-E'/T} \, \rho(E')} \quad .$$

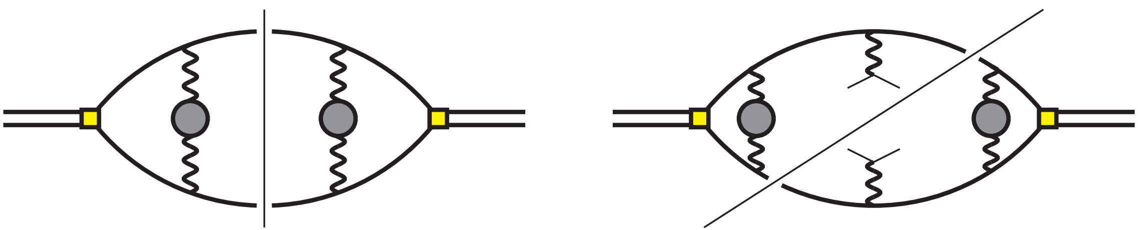
[6] G.T. Bodwin, E. Braaten and G.P. Lepage, *Rigorous QCD analysis of inclusive annihilation and production of heavy quarkonium*, hep-ph/9407339.

Spectral fcn can be solved from a Schrödinger equation

$$
\begin{aligned}
\Big[-\frac{\nabla^2}{M} + V(r) - i\,\Gamma(r) - E'\Big] G(E';\mathbf{r},\mathbf{r}') &= \delta^{(3)}(\mathbf{r}-\mathbf{r}')\ , \\
\lim_{\mathbf{r},\mathbf{r}'\to\mathbf{0}} \operatorname{Im} G(E';\mathbf{r},\mathbf{r}') &= \rho(E')\ .
\end{aligned}
$$

$$
V(r) - i\,\Gamma(r) = g^2 \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \Big(1 - e^{i\mathbf{k}\cdot\mathbf{r}}\Big)\, i\Delta_{00\mathrm{T}}(0,k)\ .
$$

The width represents real scatterings, present in a plasma:

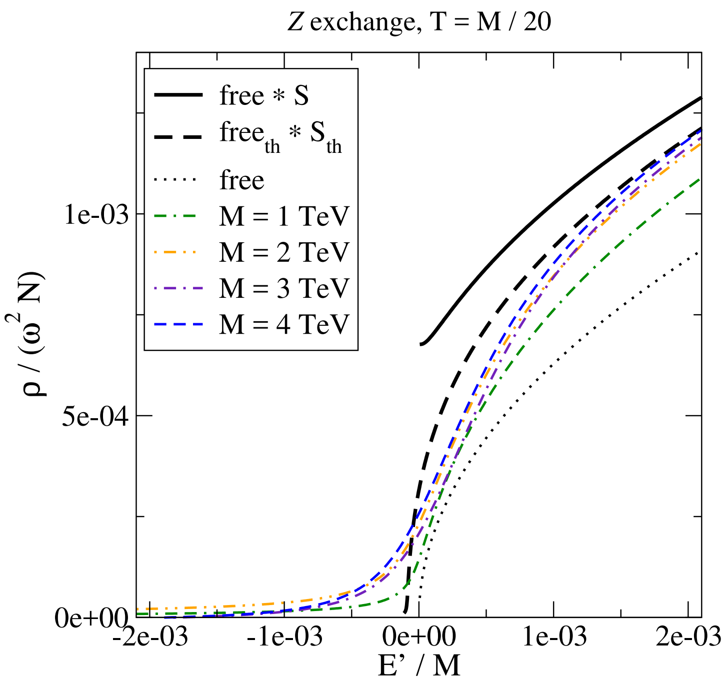


$\sim V(r)$ $\qquad\qquad$ $\sim \Gamma(r)$

In a nutshell

- Compute thermal (full or HTL) gauge field self-energy
- Determine corresponding time-ordered propagator
- Fourier-transform for potential and width
- Solve for $\rho(E') = \mathrm{Im}\, G(E'; \mathbf{0}, \mathbf{0})$
- Laplace-transform with weight $e^{-E'/T}$ for $\langle \sigma v \rangle$

Z exchange: no bound states are found



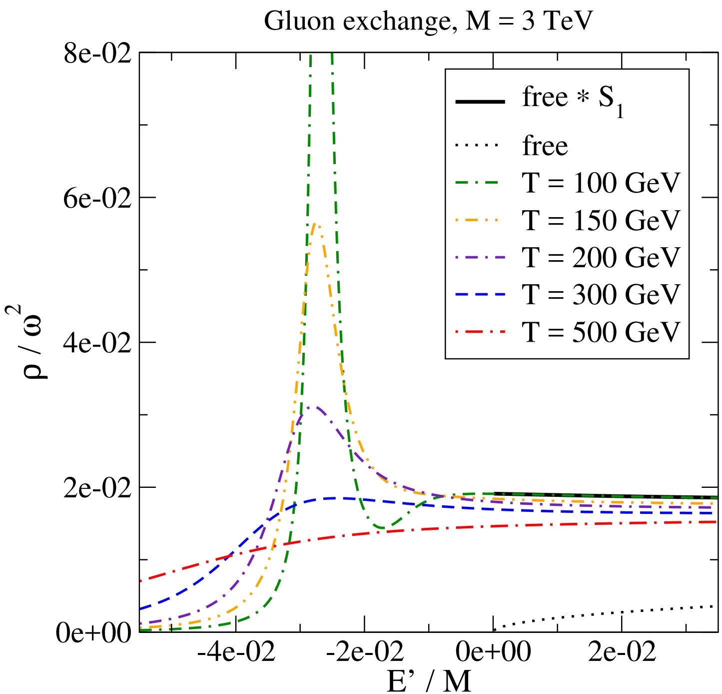
$$\alpha \equiv \frac{g_1^2 + g_2^2}{16\pi} \approx 0.01 \,,$$

screening by m_Z

$\simeq$ 91 GeV + Debye mass .

Average $\int \mathrm{d}E' \, e^{-E'/T} \ldots \Rightarrow S$ works with $\sim 1\%$ errors.

Gluon exchange between gluinos:[7] bound states persist and boost the Sommerfeld estimate by a factor 4...80



$$m_{\rm D}^2 \equiv 2 g_s^2 T^2 \; ,$$

$$\alpha_3 \equiv \frac{3 g_s^2}{4\pi} \; .$$

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[7] e.g. J. Ellis, F. Luo and K.A. Olive, *Gluino Coannihilation Revisited*, 1503.07142.

Summary

- Weak interactions: we confirm the Sommerfeld effect, but it gets slightly screened and smoothed by thermal corrections.

- Strong interactions: in addition to the Sommerfeld effect, the co-annihilation rate is much enhanced by bound states.

- Bound states are very sensitive to temperature, but the method proposed is strong enough to decide when they melt.

- Model-specific studies are needed for definite conclusions.