Gravitational waves from oscillons after inflation

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Based on
Oscillons

spatially localized, oscillatory scalar field configurations with large amplitude

can be extremely long-lived \( \rightarrow \) can survive many thousands of oscillations!

radiate energy \( \rightarrow \) live long but not forever!

often tend to be spherical (depends on potential!)
Oscillons are spatially localized, oscillatory scalar field configurations with large amplitude. They have the following characteristics:

- can be extremely long-lived → can survive many thousands of oscillations!
- radiate energy → live long but not forever!
- often tend to be spherical (depends on potential!)
Oscillons

When do they form?

• generic feature of scalar field theories where the potential opens up away from the minimum
  (e.g. plateau-like inflation models with a minimum, axion monodromy inflation, hybrid-like models...)

• necessary condition: potential must be shallower than quadratic around the minimum for some $\Delta \phi$

• form during non-linear, oscillatory phase e.g. during preheating after inflation
possible (observable) consequences

- affect expansion history $\rightarrow$ delay of reheating
- production of gravitational waves
possible (observable) consequences

- affect expansion history $\rightarrow$ delay of reheating
- production of gravitational waves
Using the program LATTICEEASY:

\[ V(\phi) = V_0 \left(1 - \frac{\phi^p}{v^p}\right)^2, \quad \text{with} \quad v \ll m_{Pl} \]

\[ \ddot{\phi}(t, \vec{x}) - \frac{\vec{\nabla}^2}{a^2} \phi(t, \vec{x}) + 3H \dot{\phi}(t, \vec{x}) + \frac{\partial V}{\partial \phi} = 0 \]

\[ H^2 = \frac{1}{3m_{Pl}^2} \left\langle V + \frac{\dot{\phi}^2}{2} + \frac{1}{2a^2} \left| \vec{\nabla} \phi \right|^2 \right\rangle \]

+ Vacuum fluctuations
Using the program LATTICEEASY:

\[ V(\phi) = V_0 \left(1 - \frac{\phi p}{v p}\right)^2, \quad \text{with} \quad v \ll m_{Pl} \]

Simultaneously with \( ds^2 = -dt^2 + a^2(t)(\delta_{ij} + h_{ij})dx^i dx^j \):

\[ \ddot{h}_{ij} + 3H \dot{h}_{ij} - \frac{1}{a^2} \nabla^2 h_{ij} = \frac{2}{m_{Pl}^2 a^2} \Pi_{TT}^{ij}, \]

where \( \Pi_{TT}^{ij} = [\partial_i \phi \partial_j \phi]^T \) satisfies \( \partial_i \Pi_{TT}^{ij} = \Pi_{TT}^{ii} = 0 \)

The spectrum of gravitational wave energy per logarithmic momentum interval is

\[ \Omega_{GW} h^2 \equiv \frac{h^2}{\rho_c} \frac{d \rho_{GW}}{d \ln k} \]

with

\[ \rho_{GW} = \frac{1}{4\sqrt{m_{Pl}^2}} \left\langle \dot{h}_{ij}(x) \dot{h}_{ij}^*(x) \right\rangle_N. \]
Inflation along the hill

\[ V(\phi) = V_0 \left( 1 - \frac{\phi^p}{v^p} \right)^2, \quad v \ll m_{\text{Pl}} \text{ “small-field hilltop”} \]

**slow-roll inflation**
- universe inflates while \( \phi \) rolls away from the hilltop towards \( \phi = v \)
- end of inflation: \( \eta(\phi) = m_{\text{Pl}}^2 V,\phi/\dot{\phi} \simeq -1 \)
- once \( p \geq 4 \) and \( v \) are chosen, \( V_0 \) is determined by CMB observations
Inflation along the hill

\[ V(\phi) = V_0 \left(1 - \frac{\phi p}{v p}\right)^2, \quad v \ll m_{Pl} \text{ “small-field hilltop”} \]

**preheating**

- typically non-linear
- two phases (strong dependence in \( v \), weaker on \( p \)):
  - tachyonic preheating: growth of IR modes; most efficient for very small \( v \)
  - tachyonic oscillations: growth of modes around a certain scale \( \rightarrow \) oscillon formation
Lattice & model setup:

<table>
<thead>
<tr>
<th>Model</th>
<th>$L H_i$</th>
<th>$v/m_{Pl}$</th>
<th>$V_0/m_{Pl}^4$</th>
<th>$p$</th>
<th>$\langle \phi_i \rangle / v$</th>
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<tr>
<td>hilltop inflation</td>
<td>0.01</td>
<td>$10^{-2}$</td>
<td>$10^{-19}$</td>
<td>6</td>
<td>0.08</td>
<td>$2.49 \times 10^{-9}$</td>
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$t = 573.48/m, a = 1.47$
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$t = 5543.5/m, a = 4.29$
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$t = 18924.4/m, a = 8.9$
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$t = 38040/m, a = 13.81$

![Graph showing $\Omega_{GW}$ vs. $k/(aH_i)$](image-url)
Lattice simulations
Gravitational waves from oscillons after single-field hilltop inflation

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$t = 57155.5/m, a = 17.94$
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![Lattice simulation diagram](image1)

![Graph of $\Omega_{GW}$ vs. $k/(aH_i)$](image2)
Inflation orthogonal to the hill ("hybrid-like" inflation)

\[ V(\phi, \chi) = V_0 \left( 1 - \frac{\phi p}{v p} \right)^2 + V_{\text{inf}}(\phi, \chi), \quad v \ll m_{\text{Pl}} \]

- universe inflates as \( \chi \) rolls along the valley
- inflation is ended by a tachyonic instability in \( \phi \)
- \( p \geq 2, \ v \) and \( V_0 \) are free!
- CMB observables are determined by \( V_{\text{inf}} \)!
Inflation orthogonal to the hill ("hybrid-like" inflation)

\[ V(\phi, \chi) = V_0 \left(1 - \frac{\phi_p}{v_p}\right)^2 + V_{\text{inf}}(\phi, \chi), \quad v \ll m_{\text{Pl}} \]

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Lattice simulations

Gravitational waves from oscillons after a 2\textsuperscript{nd} order phase transition

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<td>0.01</td>
<td>$10^{-2}$</td>
<td>free</td>
<td>6</td>
<td>0</td>
<td>0</td>
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\begin{figure}
\centering
\includegraphics[width=\textwidth]{GW_spectrum}
\end{figure}

- aLIGO O5
- $V_0 \approx (110 \text{ TeV})^4$
- $V_0 \approx (200 \text{ TeV})^4$
- $V_0 \approx (375 \text{ TeV})^4$

$\Omega_{GW,0} h^2$

10\textsuperscript{-11} to 10\textsuperscript{-9}

$f$ (Hz) from 1 to 100
We have investigated the gravitational wave production from oscillons in a potential

$$V = V_0 \left(1 - \frac{\phi p}{v_p}\right)^2,$$

with $v \ll m_{Pl}$, $p = 6$

**If $\phi$ is the inflaton ($V_0$ constrained):**
- production of a pronounced GW peak at frequencies $f \sim 10^{10}$ Hz
- continuous growth of the peak until the end of the simulations
- stronger signal compared to previous studies (which assumed a symmetric potential)

**If $\phi$ is a waterfall field ($V_0$ unconstrained):**
- observable prediction for current and future experiments
- e.g. if $(100 \text{ TeV})^4 \lesssim V_0 \lesssim (400 \text{ TeV})^4$ → peak at $10 \text{ Hz} \lesssim f \lesssim 70 \text{ Hz}$ → potentially observable by aLIGO O5
- $\Omega_{GW,0} h^2$ and $f$ depend on the expansion history → might also be pushed into sensitivity region of BBO and DECIGO

**ToDo:**
- impact of $p$, $v$?
- multiple fields?
Backup
\[ V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \quad \text{with } v \ll m_{Pl} \]

\[ N_\ast \text{ e-folds} \]
before \( \phi_{\text{end}} \)
\[ \eta(\phi_{\text{end}}) = -1 \]

Tachyonic oscillations:
- periodic entering into the tachyonic region \((\partial^2 V/\partial \phi^2 < 0)\)
  - \(\rightarrow\) interplay between growth of the \(\phi_\kappa\) around \(|k_{\text{peak}}|\) and damping due to Hubble friction
- For \(v \gtrsim 10^{-1} m_{Pl} \rightarrow\) strong damping
- For \(10^{-5} m_{Pl} < v < 10^{-1} m_{Pl} \rightarrow\) fluctuations eventually grow non-linear \(\rightarrow\) system eventually develops localized bubbles which oscillate between the two minima \(\phi = \pm v\), typically separated by a distance \(\lambda_{\text{peak}} \sim 2\pi/k_{\text{peak}}\)
\[ V(\phi) = V_0 \left(1 - \frac{\phi^6}{v^6}\right)^2, \quad \text{with } v = 10^{-2} m_{Pl} \]
Field spectra

\[ \frac{P_\phi}{v^2} = \frac{1}{(aH_i)^2} \]

- For \( a = 1.47 \)
- For \( a = 4.29 \)
- For \( a = 8.9 \)
- For \( a = 13.81 \)
- For \( a = 17.94 \)