Parameter inference for compact binaries with the gravitational-wave observatory Advanced LIGO

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Gravitational-wave astrophysics

Fundamentally new way to learn about the **Universe**:  

• Is **General Relativity** in the correct theory of **Gravity**?

• What happens when **matter** is compressed to **nuclear densities**?

• What are the properties of the population of **compact objects**? Especially the ones we **cannot see**?

• Is the mechanism that generates **gamma-ray bursts** a **compact binary coalescence**?
Compact Binary Coalescence

- **Intrinsic** parameters: primary and secondary masses and spins (and eccentricity)

- **Extrinsic**: time, sky-position, distance, orientation, reference phase

Credit: LIGO
The GW150914 observation:

• How do we extract the **astrophysics**?
Gravitational waveform models

- **2 models** of the **signal** as a proxy for systematic errors:
  - **Double-aligned-spin model** (SEOBNRv2_ROM, [Taracchini, et al., 2014, Pürrer, 2014])
  - **Single-precessing-spin model** (IMRPhenomPv2, [Hannam et al. Phys. 2014])
Gravitational waveform models

- 2 models of the signal as a proxy for systematic errors:
  - Double-precessing-spin model (*SEOBNRv3, [Pan et al., 2014, Babak et al., 2016]*)
Masses from the inspiral and ringdown

- Chirp mass: \[ M = \frac{(m_1 m_2)^{3/5}}{(m_1 + m_2)^{1/5}} \]
- Total mass: ringdown
- Mass ratio: \[ q = \frac{m_1}{m_2} \]

(with total spin)
Effects of spins

- 2 spin vectors
  - **Magnitude**: orbital hang-up
  - Mis-alignment: precession and modulations

![Graph showing strain over time](image)
Effects of spins

- 2 spin vectors
  - Magnitude: orbital hang-up
  - **Mis-alignment**: precession and modulations
Parameter Estimation

• We want the **posterior** probability of parameters $\vec{\lambda}$, given the data $\vec{x}$. With Bayes' theorem:

$$p(\vec{\lambda}|\vec{x}, M) = \frac{p(\vec{\lambda}|M) p(\vec{x}|\vec{\lambda}, M)}{p(\vec{x}|M)}$$

• Fit a **model** to the data (**noise** and **signal** models)

• Build a **likelihood** function

• Specify **prior** knowledge

• **Numerically** estimate the resulting **distribution** (**sampling algorithms**)

SPINSpiral [van der Sluys, Raymond, et al. 2008], LALInference [Veitch, Raymond, et al., 2015]
Parameter Estimation

- We want the **posterior** probability of parameters $\tilde{\lambda}$, given the data $\tilde{x}$. With Bayes' theorem:

$$p(\tilde{\lambda}|\tilde{x}, M) = \frac{p(\tilde{\lambda}|M) \cdot p(\tilde{x}|\tilde{\lambda}, M)}{p(\tilde{x}|M)}$$

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SPINSpiral [van der Sluys, Raymond, et al. 2008], LALInference [Veitch, Raymond, et al., 2015]
Likelihood

• How close is the \textit{remainder} to the \textit{mean}?

• Assumptions: \textit{gaussianity} and \textit{stationarity}
Parameter Estimation

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*LALInference* [Veitch, Raymond, et al., 2015]
GW150914

- 2 models as a proxy for systematic errors:
  - **Double-precessing-spin** model (*SEOBNRv3*)
  - **Single-precessing-spin** model (*IMRPhenomP*)

\[
m_1 = 35.4^{+5.0}_{-3.4} \, M_{\odot}
\]

\[
m_2 = 28.9^{+3.3}_{-4.3} \, M_{\odot}
\]

[LIGO-Virgo Collaboration, 2016]
GW150914

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\]

\[
m_2 = 28.9^{+3.3\pm0.3}_{-4.3\pm0.3}\, M_\odot
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- Errors:
  - Signal strength
  - Model inaccuracies

[LIGO-Virgo Collaboration, 2016]
GW151226

- 2 models as a proxy for systematic errors:
  - **Double-aligned-spin** model (*SEOBNRv2_ROM*)
  - **Single-precessing-spin** model (*IMRPhenomP*)

\[
m_1 = 14.2^{+8.3}_{-3.7} \, M_\odot
\]

\[
m_2 = 7.5^{+2.3}_{-2.3} \, M_\odot
\]

[LIGO-Virgo Collaboration, 2016]
LVT151012

- 2 models as a proxy for systematic errors:
  - **Double-aligned-spin** model \((SEOBNRv2-ROM)\)
  - **Single-precessing-spin** model \((IMRPhenomP)\)

\[
m_1 = 23_{-6}^{+18} \, M_\odot
\]

\[
m_2 = 13_{-5}^{+4} \, M_\odot
\]

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[LIGO-Virgo Collaboration, 2016]
Remnant black hole

- Final values fitted from Numerical Relativity simulations
  - Final mass:
    \[ M_f = 62.2^{+3.7}_{-3.4} M_\odot \]
  - Final (dimensionless) spin:
    \[ a_f = 0.68^{+0.05}_{-0.06} \]
  - \(~3\) solar mass radiated!
Remnant black hole

- Final values fitted from Numerical Relativity simulations

- Final mass:

\[ M_f = 20.8^{+6.1}_{-1.7} \, M_\odot \]

- Final (dimensionless) spin:

\[ a_f = 0.74^{+0.06}_{-0.06} \]

- \(~1\) solar mass radiated

[LIGO-Virgo Collaboration, 2016]
Remnant black hole

• Final values fitted from Numerical Relativity simulations
  
  • Final mass:
  
  \[ M_f = 35^{+14}_{-4} \text{ M}_\odot \]
  
  • Final (dimensionless) spin:
  
  \[ a_f = 0.66^{+0.1}_{-0.09} \]
  
  • \textbf{~1.5 solar mass} radiated!

[LIGO-Virgo Collaboration, 2016]
Distance - inclination

[LIGO-Virgo Collaboration, 2016]
Distance - inclination

- **Degeneracies in extrinsic** parameters, strain $h$:

$$h = -\frac{1 + \cos^2(\iota)}{2d} F_{j+}(\text{R.A.}, \text{dec}, \psi) H_+$$

$$+ \frac{\cos \iota}{d} F_{j\times}(\text{R.A.}, \text{dec}, \psi) H_{\times}$$

3 angles for the orientation:

(R.A., dec, ψ)

Intrinsic waveform:

$H_{+\times}(m_1, m_2, \vec{S}_1, \vec{S}_2)$

- **Sampling in LALInference**

[Raymond, Farr, 2014]

[LIGO-Virgo Collaboration, 2016]
Distance - inclination

GW151226

LVT151012

[LIGO-Virgo Collaboration, 2016]
Were the black-holes spinning?

- Weak constrains on **spin magnitude**
- Very weak constrains on **spin orientation**
- Due to Almost **equal-mass, face-off** binary
  [Raymond, 2012]
  [LIGO-Virgo Collaboration, 2013]

[LIGO-Virgo Collaboration, 2016]
Were the black-holes spinning?

GW151226

LVT151012

[LIGO-Virgo Collaboration, 2016]
Were the black-holes spinning?

GW151226 $a_1$  
LVT151012 $a_1$

[LIGO-Virgo Collaboration, 2016]
Observing run 1 to Observing run 2 and beyond

- Merging binary black holes exist in a **broad mass range**
- New access to **black holes spins** (GW151226 **at least** one black-hole spinning)
- Measured **masses** and **spins** consistent with both:
  - **Isolated binary evolution** (more aligned spins)
  - **Dynamical formation** (more misaligned spins)
- **Statistical** errors dominate waveform **systematical** errors