

# Studying generalised dark matter interactions with extended halo-independent methods.

Felix Kahlhoefer

TeV Particle Astrophysics

CERN

12-16 September 2016

Based on

**arXiv:1607.04418** with Sebastian Wild

# Motivation

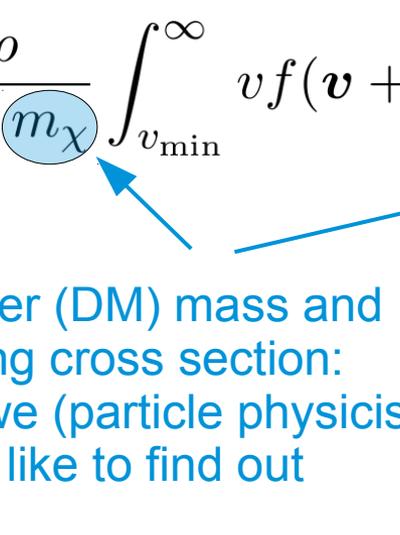
- Differential event rates in direct detection experiments are given by

$$\frac{dR}{dE_R} = \frac{\rho}{m_T m_\chi} \int_{v_{\min}}^{\infty} v f(\mathbf{v} + \mathbf{v}_E(t)) \frac{d\sigma}{dE_R} d^3v \quad \text{with} \quad v_{\min}(E_R) = \sqrt{\frac{m_T E_R}{2 \mu_{T\chi}^2}}$$



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- How reliably can we get to the particle physics properties of DM from future direct detection signals in the face of astrophysical uncertainties?



# There are many models of DM

- There are many plausible models for the interactions of DM beyond standard spin-independent (SI) and spin-dependent (SD) scattering:

- **DM with an Anapole moment (AM):**

$$\mathcal{A} \bar{\chi} \gamma^\mu \gamma^5 \chi \partial^\nu F_{\mu\nu}$$

This is the only possible interaction of Majorana fermions with photons.

- **DM with a magnetic dipole moment (MDM):**

$$\mathcal{D}_{\text{magn}} \bar{\chi} \sigma^{\mu\nu} \chi F_{\mu\nu}$$

This leads to long-range interactions due to the massless photon exchange.

- **DM with a dark magnetic dipole moment (DMDM):**

$$\mathcal{D}_{\text{magn}} \bar{\chi} \sigma^{\mu\nu} \chi F'_{\mu\nu}$$

Same as the the MDM but for a massive hidden photon (  $1/q^2 \rightarrow 1/M^2$  ).



# Astrophysical uncertainties

- While these models yield widely different expressions for the scattering cross sections, it is always possible to factorise the result as

$$\frac{d\sigma}{dE_R} = \frac{d\sigma_1}{dE_R} \frac{1}{v^2} + \frac{d\sigma_2}{dE_R}$$

- Introducing the first and second velocity integral

$$g(v_{\min}) = \int_{v_{\min}}^{\infty} \frac{1}{v} f(\mathbf{v} + \mathbf{v}_E(t)) d^3v, \quad h(v_{\min}) = \int_{v_{\min}}^{\infty} v f(\mathbf{v} + \mathbf{v}_E(t)) d^3v$$

we can then write the differential event rates simply as

$$\frac{dR}{dE_R} = \frac{\rho}{m_T m_\chi} \left( \frac{d\sigma_1}{dE_R} g(v_{\min}) + \frac{d\sigma_2}{dE_R} h(v_{\min}) \right)$$

- A useful observation is that the two halo integrals are related by integration by parts:

$$h(v) = - \int_{v_{\min}}^{\infty} v^2 g'(v) dv = [-g(v) v^2]_{v_{\min}}^{\infty} + \int_{v_{\min}}^{\infty} 2v g(v) dv$$



# A halo-independent approach

- In order not to make any assumptions on the DM velocity distribution, we can parameterise the velocity integral in a very general way.
- Specifically, we assume that  $g(v_{\min})$  is a piecewise-constant function with very many steps ( $N > 30$ ):

$$g(v_{\min}) = g_j \quad \text{for } v_{\min} \in [v_j, v_{j+1}]$$



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- One can then construct a matrix  $D_{ij}$  such that the expected number of events in bin  $i$  ( $R_i$ ) is given by a simple matrix multiplication:

$$R_i = \sum_j D_{ij} g_j$$

$$D_{ij} = G_{ij} + \sum_k F_{kj} H_{ik}$$

$$G_{ij} = \frac{\kappa \rho}{2m_T m_\chi} \int_{E_j}^{E_{j+1}} \frac{d\sigma_1}{dE_R} \epsilon(E_R) \left[ \text{erf} \left( \frac{E_{i+1} - E_R}{\sqrt{2}\Delta E_R} \right) - \text{erf} \left( \frac{E_i - E_R}{\sqrt{2}\Delta E_R} \right) \right] dE_R$$

$$H_{ij} = \frac{\kappa \rho}{2m_T m_\chi} \int_{E_j}^{E_{j+1}} \frac{d\sigma_2}{dE_R} \epsilon(E_R) \left[ \text{erf} \left( \frac{E_{i+1} - E_R}{\sqrt{2}\Delta E_R} \right) - \text{erf} \left( \frac{E_i - E_R}{\sqrt{2}\Delta E_R} \right) \right] dE_R$$



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- We can then find the velocity integral that maximises the likelihood:

$$-2 \log \mathcal{L} = 2 \sum \left[ R_i + B_i - N_i + N_i \log \frac{N_i}{R_i + B_i} \right]$$

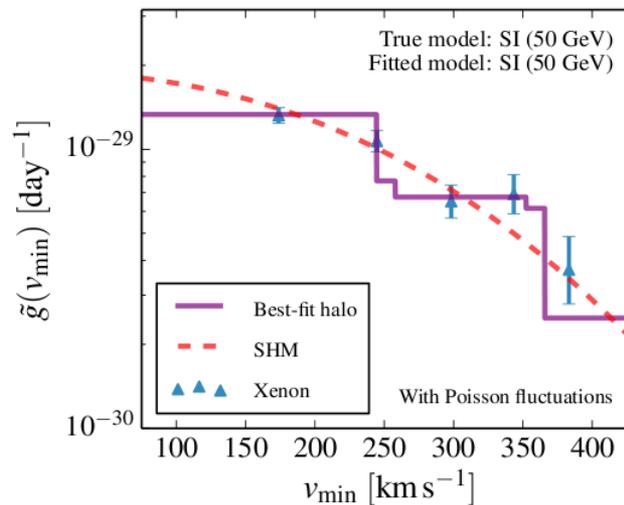
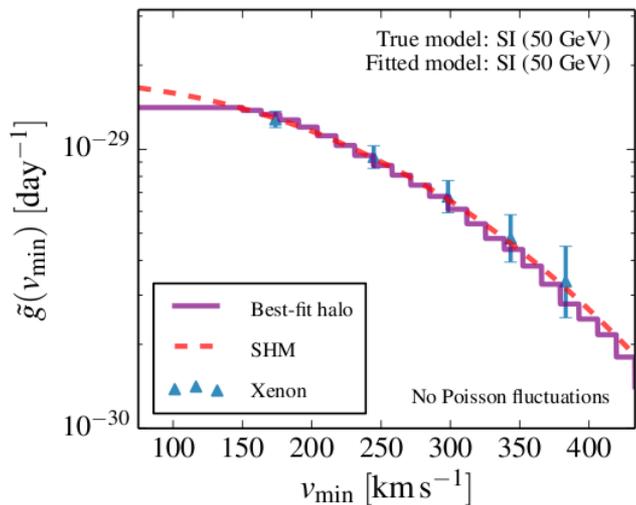
$R_i$ : Predicted signal in bin  $i$

$B_i$ : Predicted background in bin  $i$

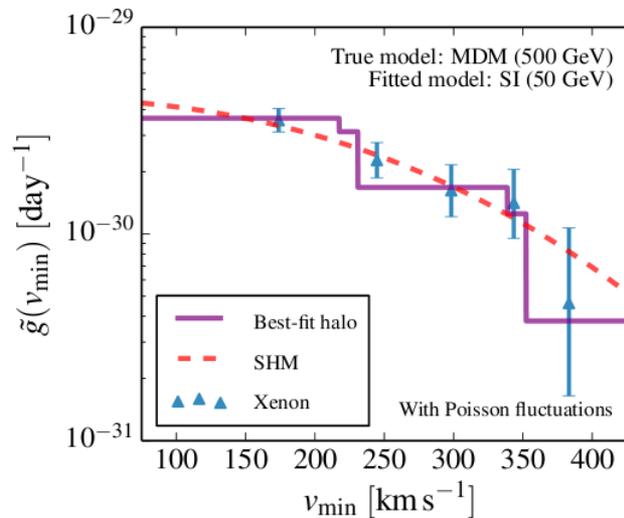
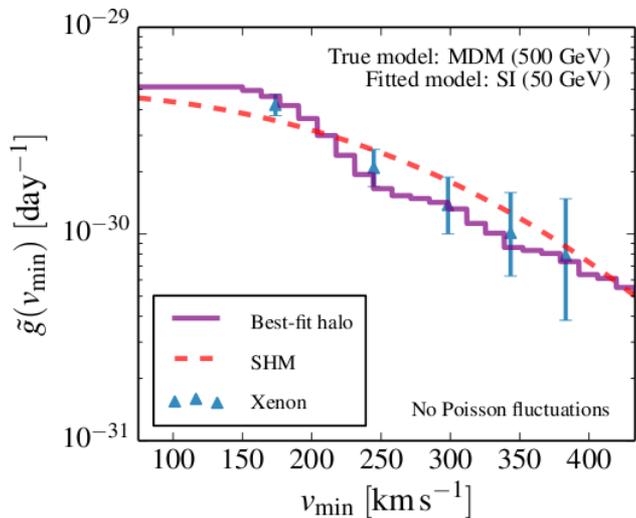
$N_i$ : Observed events in bin  $i$



# Examples for a single xenon experiment



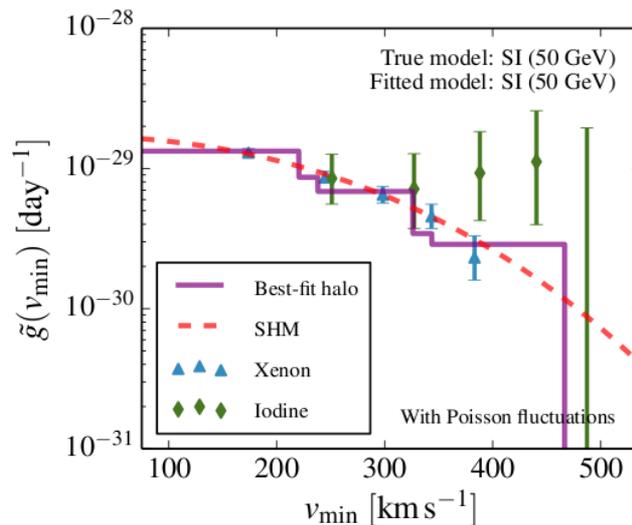
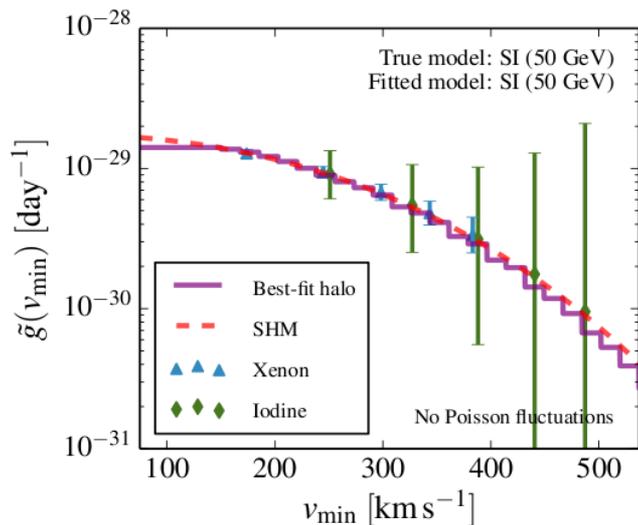
true model  
=  
fitted model



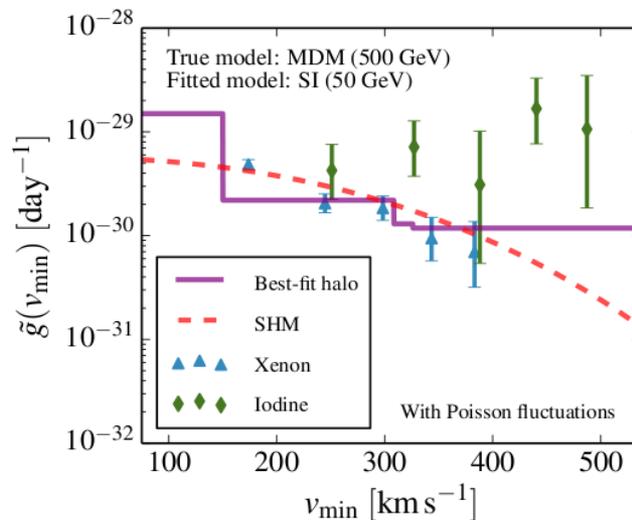
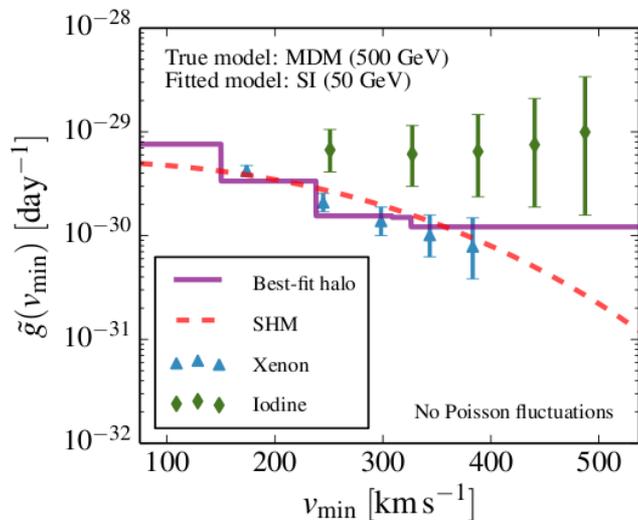
true model  
 $\neq$   
fitted model



# Examples for xenon+iodine experiments



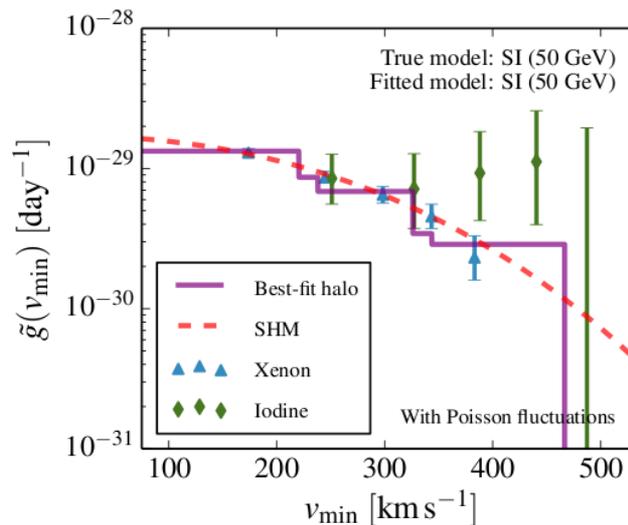
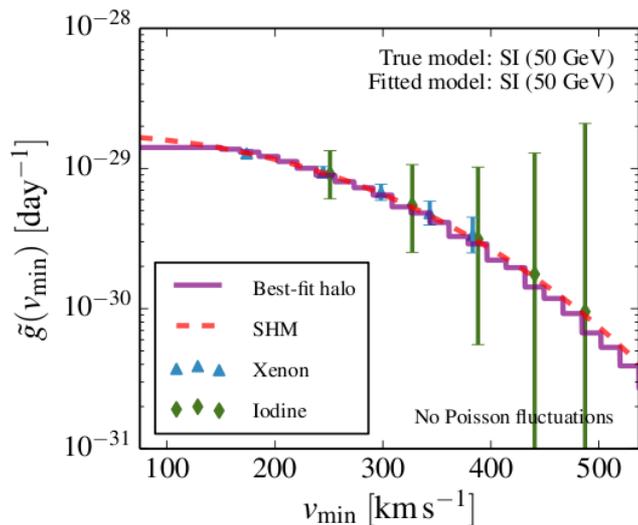
true model  
=  
fitted model



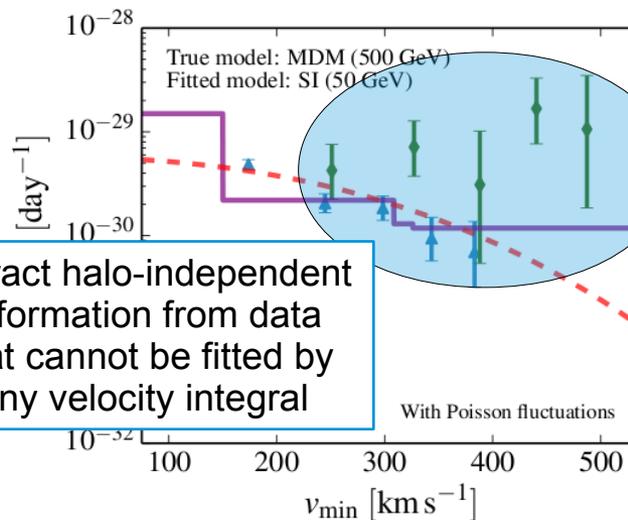
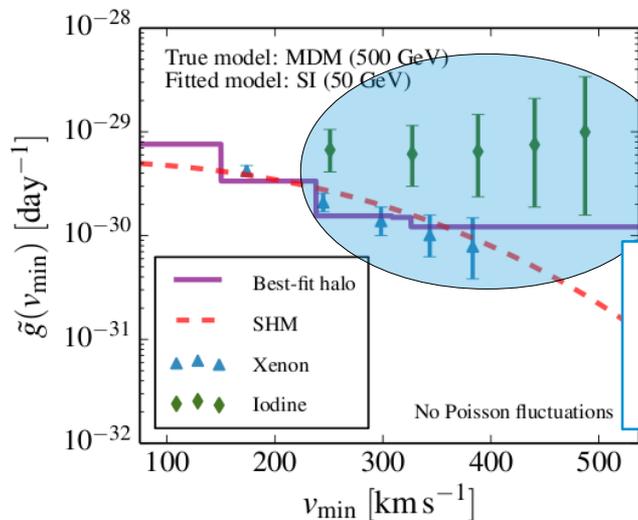
true model  
≠  
fitted model



# Examples for xenon+iodine experiments



true model  
=  
fitted model



true model  
≠  
fitted model

Extract halo-independent  
Information from data  
that cannot be fitted by  
any velocity integral



# Quantifying p-values

- For an assumed true DM model and a chosen DM model for the fit, we can then scan over both the velocity integral and the DM mass to maximise the likelihood, or equivalently minimise  $x_0 = -2 \log L$ .
- The distribution of  $x_0$ , called  $\zeta(x_0)$ , does not exactly follow a  $\chi^2$ -distribution, so we need a Monte Carlo simulation to determine  $p$ -values.
- We run this Monte Carlo simulation using the best-fit DM model from above to generate bin counts and then maximise the likelihood. From this maximum likelihood we define  $x_1 = -2 \log L$ .

- Using the distribution of  $x_1$ , called  $\zeta(x_1)$ , we can then calculate the  $p$ -value of  $x_0$ :

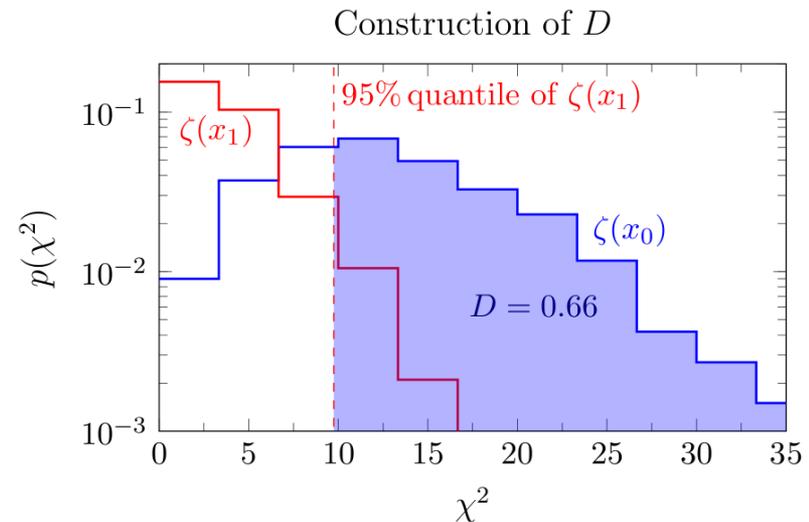
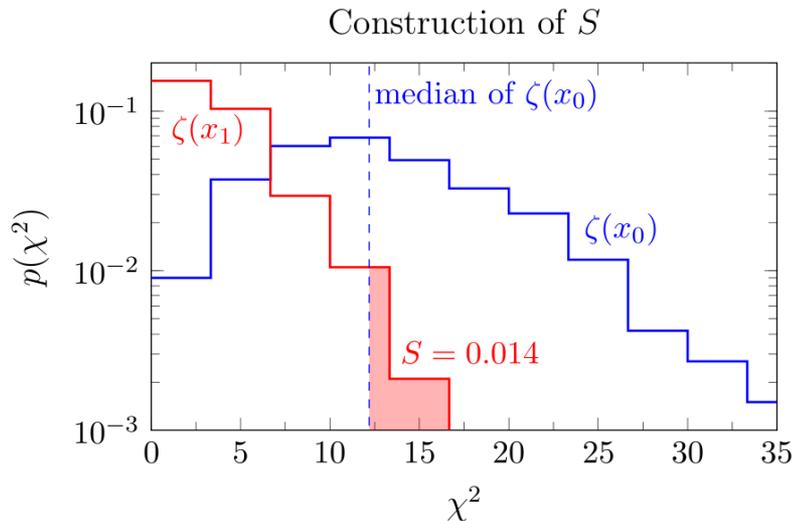
$$p = \int_{x_0}^{\infty} \zeta(x_1) dx_1$$

- This quantifies the probability that a value larger than  $x_0$  can result from random fluctuations of the data if the fitted model were to correspond to the true model of nature.



# Similarity of DM models

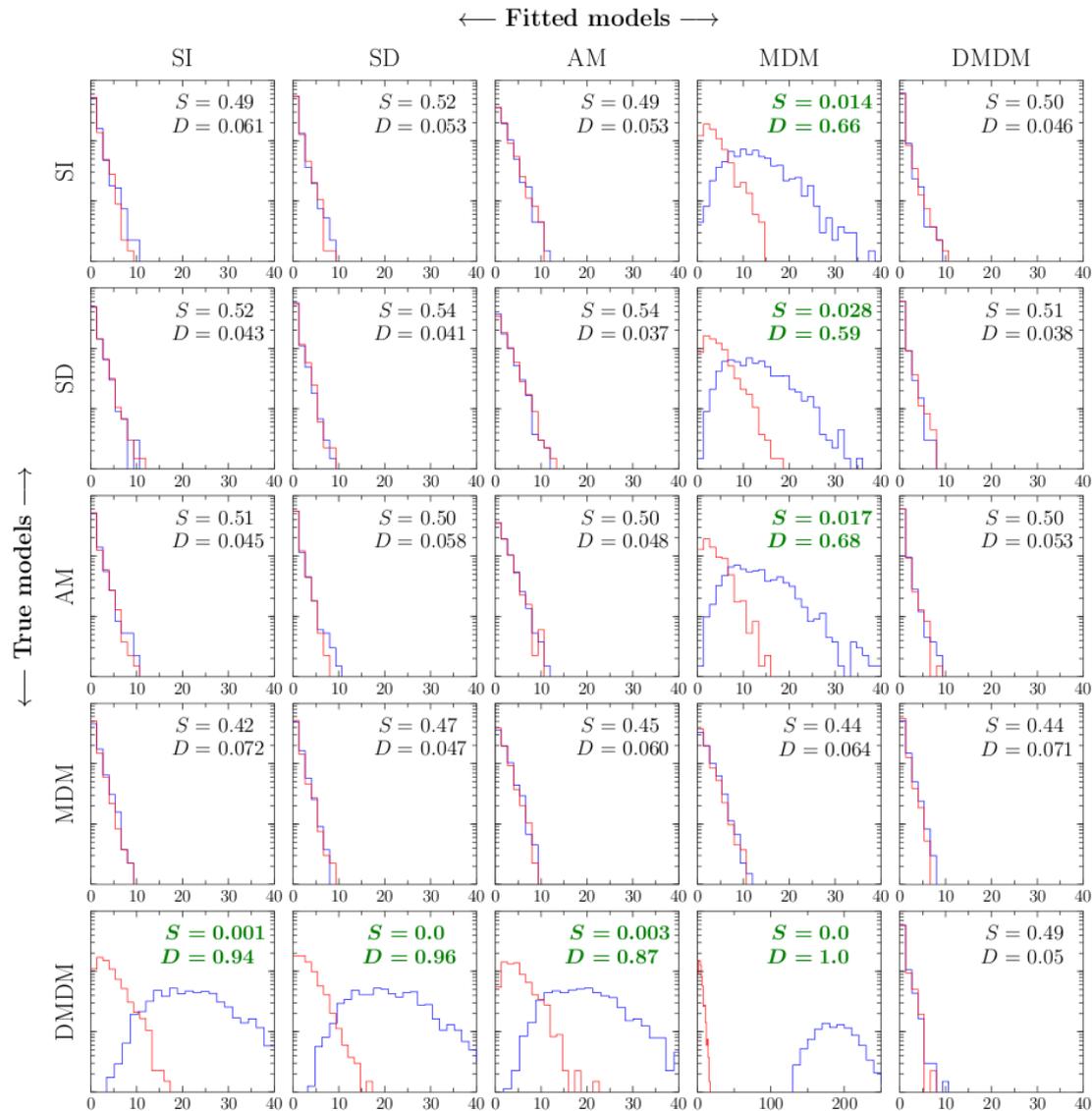
- If the fitted DM model is similar/identical to the true DM model, the distributions  $\zeta(x_0)$  and  $\zeta(x_1)$  will be very similar, otherwise they will differ.
- To look at all realisations at once, we define



- The similarity,  $S$ , as the  $p$ -value of a typical realisation of the assumed DM model (i.e. the median of  $\zeta(x_0)$ )
  - The distinguishability,  $D$ , as the fraction of realisations of the assumed DM model, having a  $p$ -value smaller than 0.05.
- For small  $S$  and large  $D$ , the fitted model can likely be ruled out by data.



# Results for a single xenon experiment

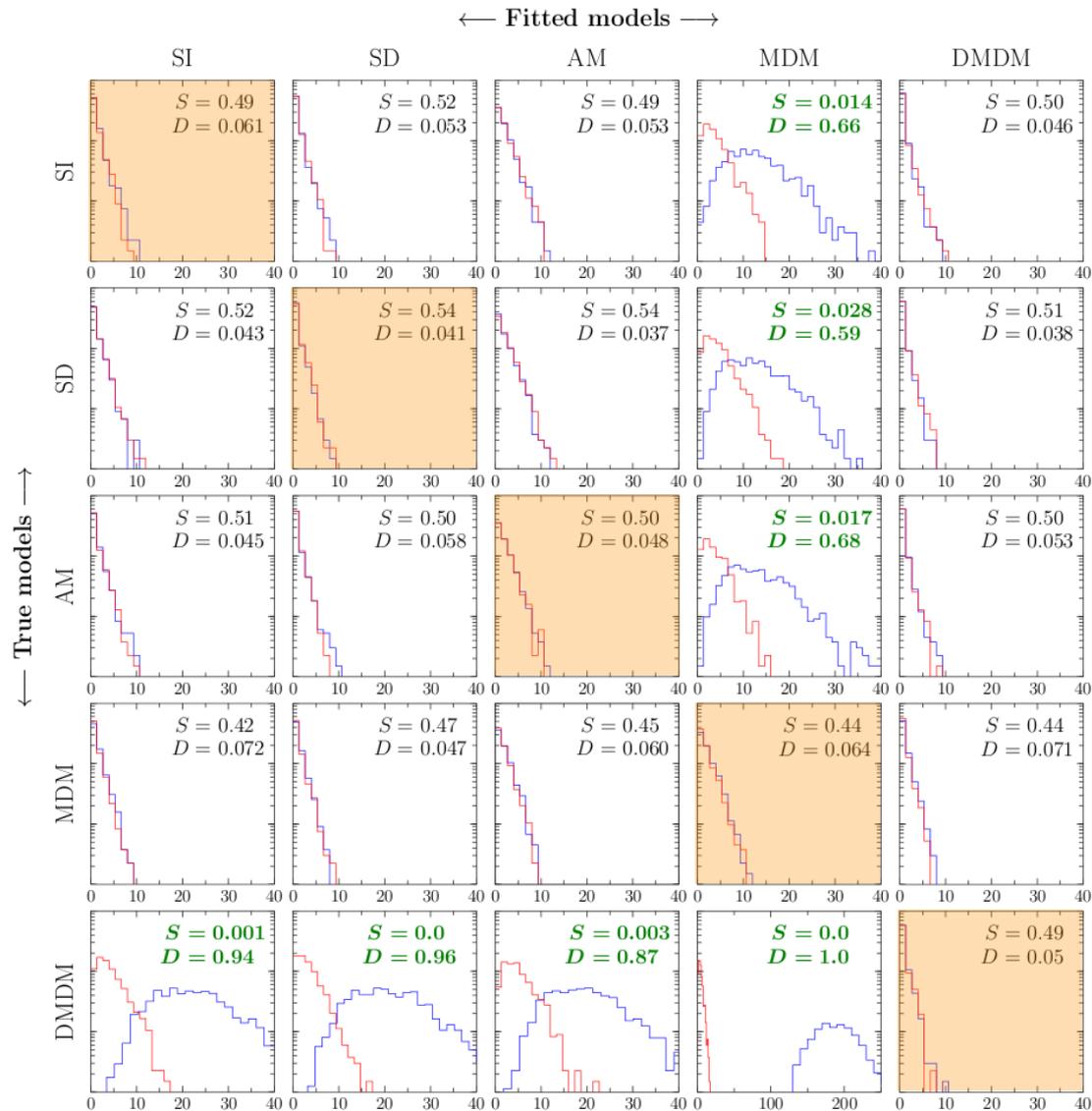


$S$ : p-value of a typical realisation

$D$ : fraction of realisations that are excluded at 95% CL



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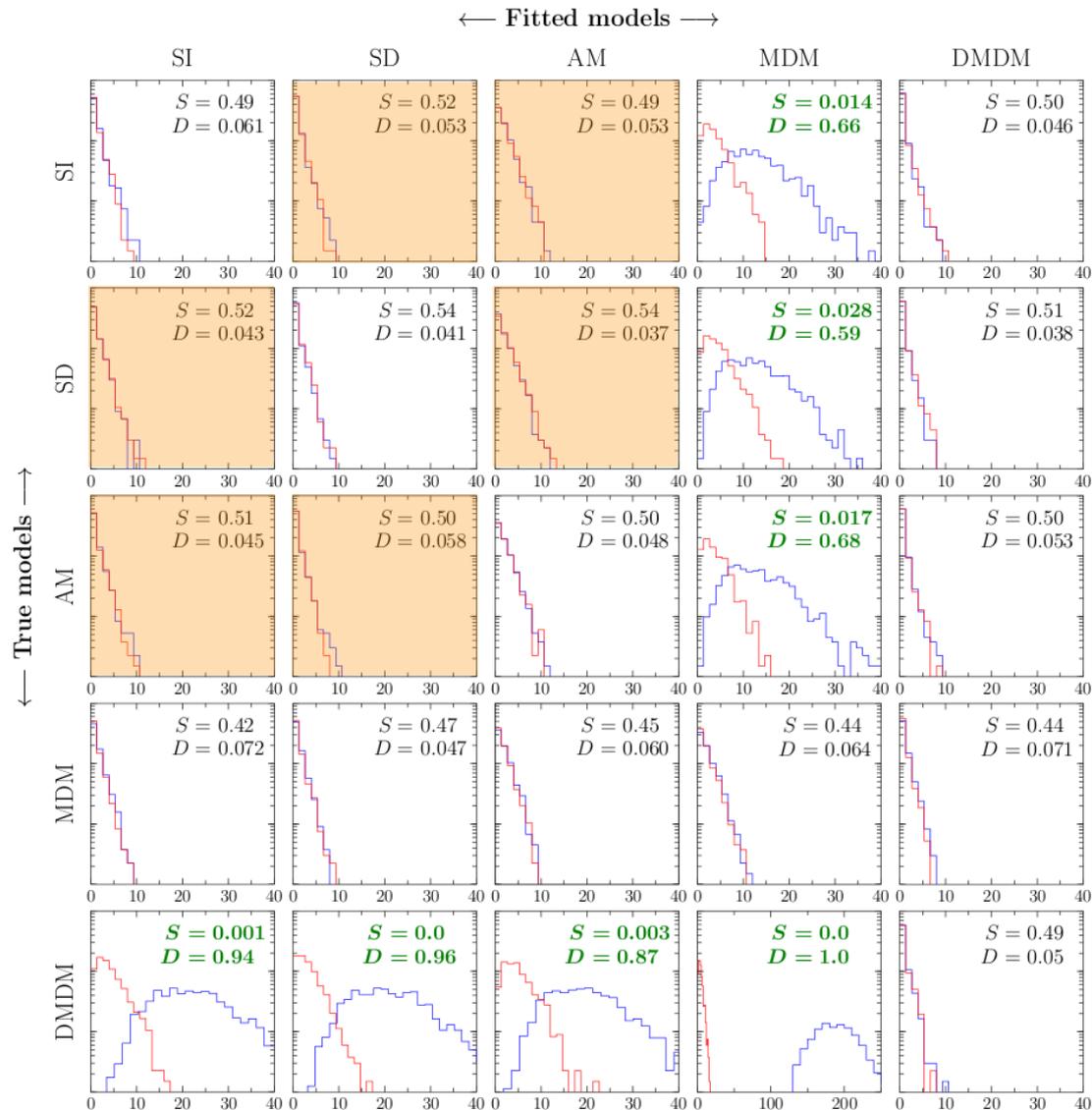
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true model  
=  
fitted model

$S \sim 0.5$   
 $D \sim 0.05$



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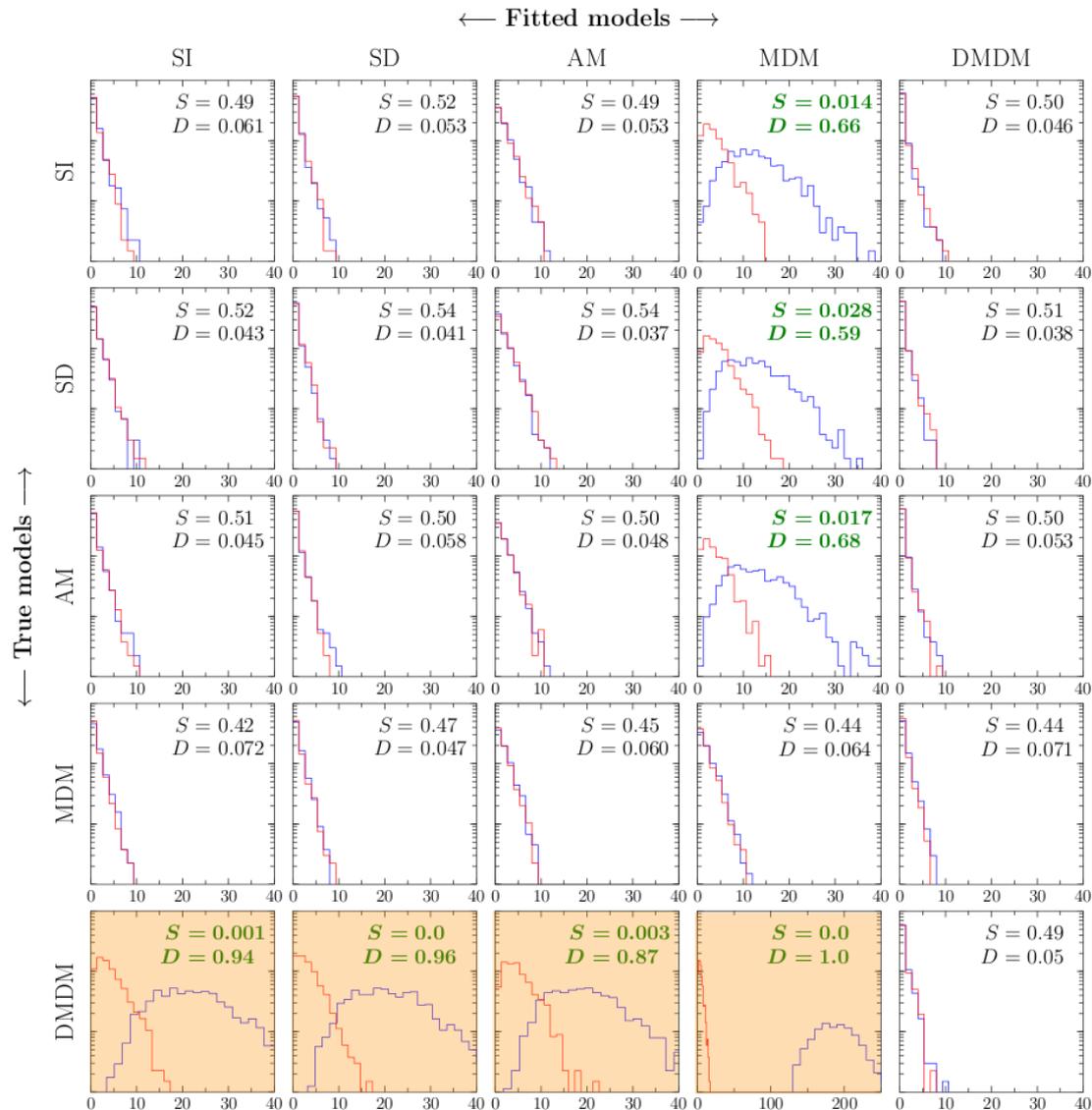
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- We find that SI, SD and anapole moment interactions cannot be distinguished with a single xenon experiment.
- For SI and SD this is unsurprising (only the form factors differ).
- For anapole interactions the reason is that (for xenon nuclei) the interactions differ only in their dependence on the velocity integral.



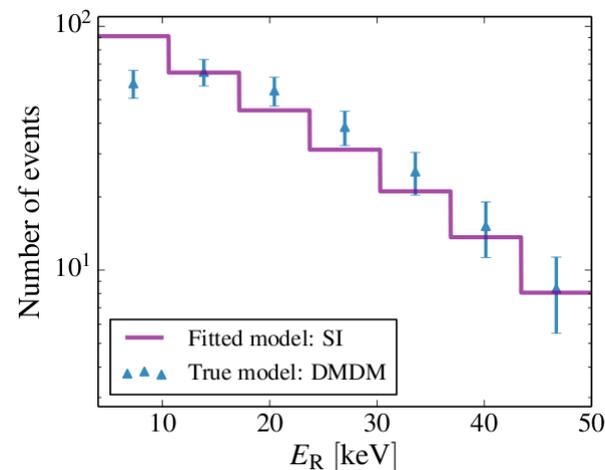
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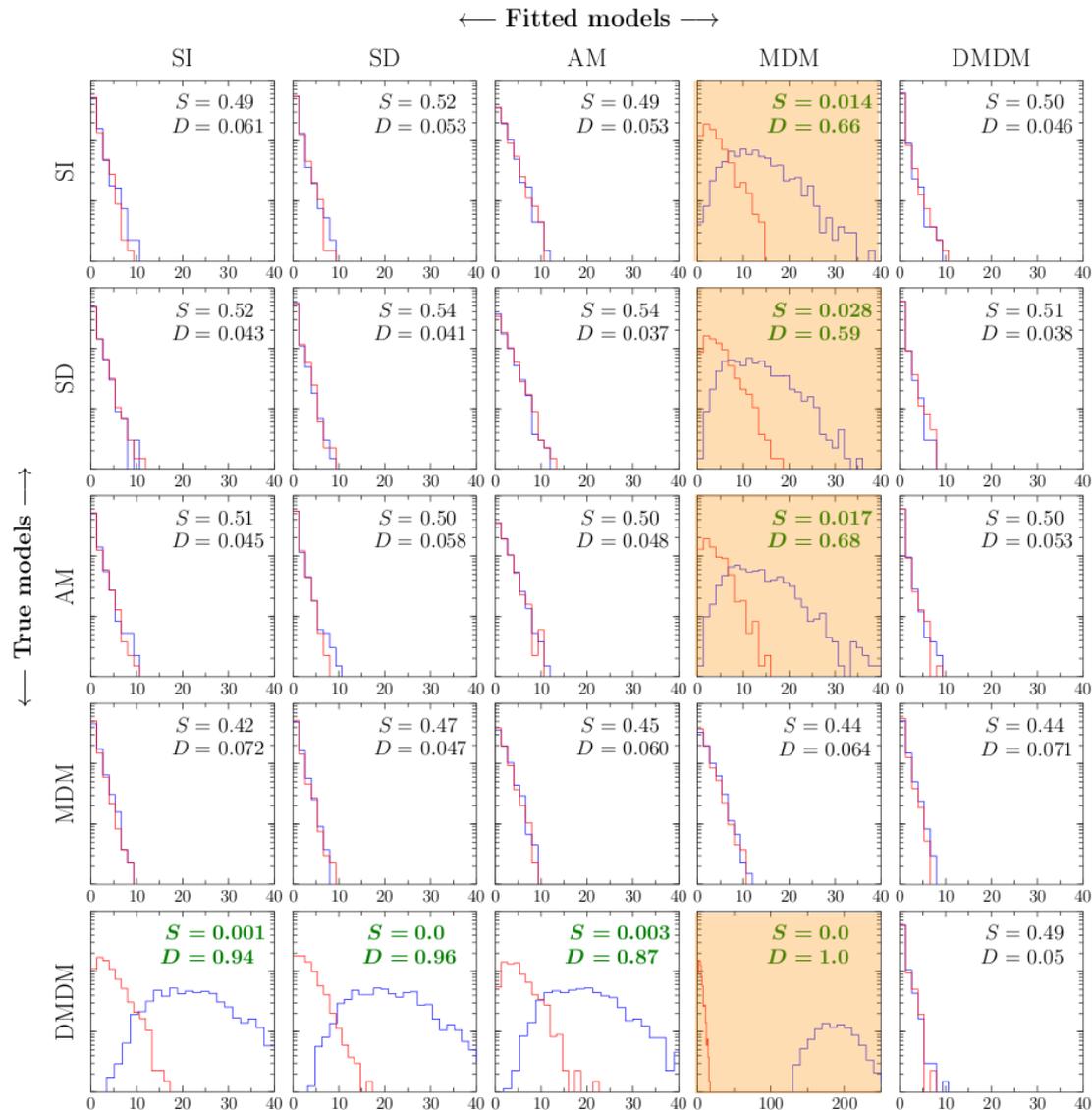
➤ If DM has a dark magnetic dipole moment, the recoil spectrum has a maximum.



➤ This cannot be fitted by any other interaction.



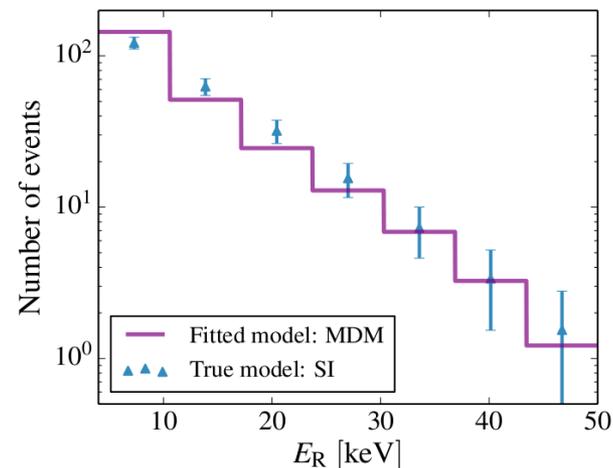
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➤ If DM has a magnetic dipole moment, the recoil spectrum falls very steeply.



➤ This gives a bad fit to any other interaction.



# Adding more experiments

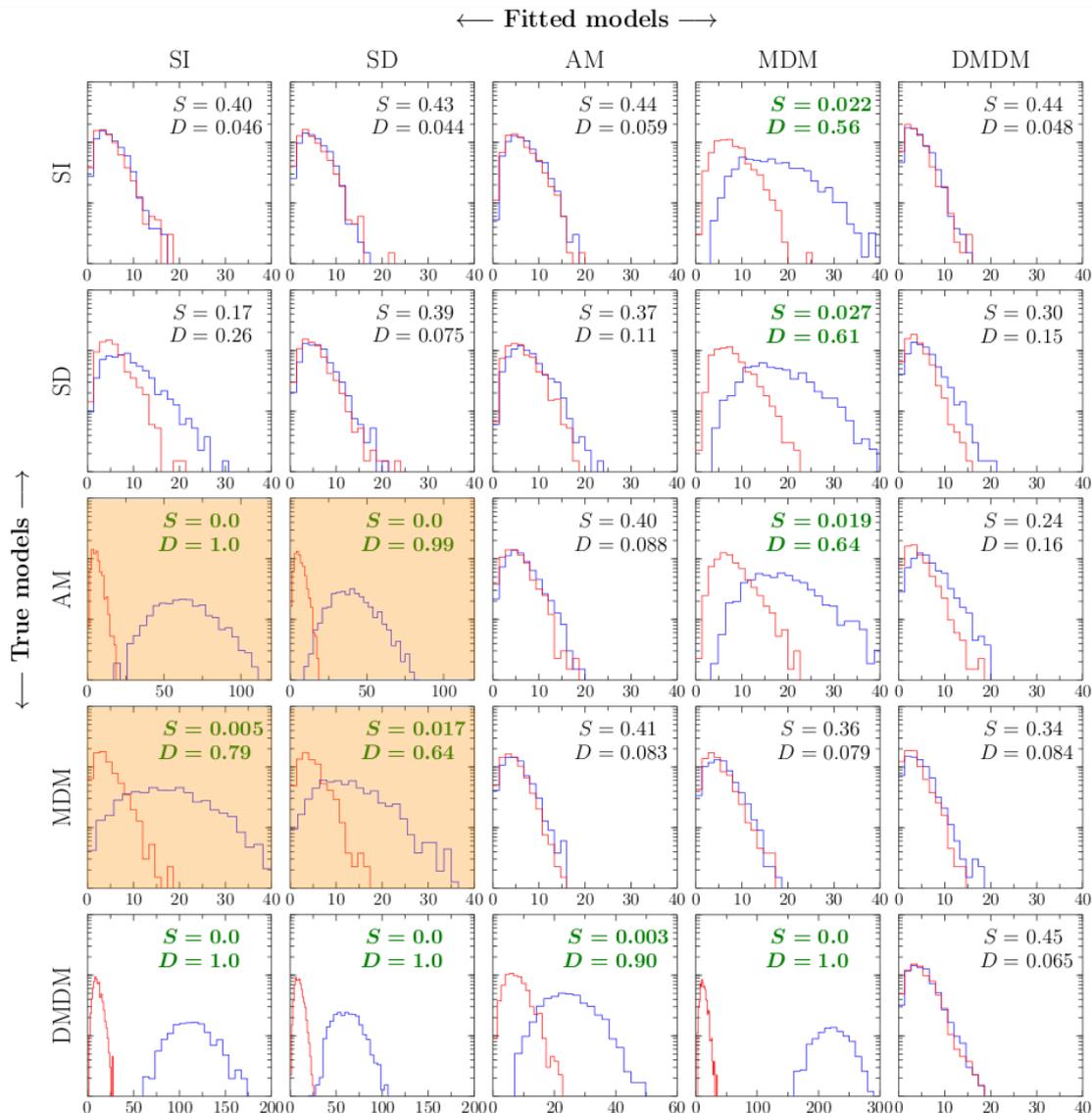
- The obvious question now is whether adding more experiments with different targets can help to distinguish certain models.
- One obvious example: Argon is completely insensitive to spin-dependent interactions, so this target allows for perfect discrimination between SI and SD.
- More non-trivial (but important) examples: iodine and germanium

Target	Exposure [kg yr]	Energy range [keV]	Number of bins	Energy resolution [keV]
Xe	2000	4–50	7	$0.6 \sqrt{E_R/1 \text{ keV}}$
Ge	200	0.3–50	9	0.06
I	100	10–100	9	$0.15 \sqrt{E_R/1 \text{ keV}}$

- We find that for all the models considered, DM interactions look very similar in xenon and germanium, so no additional discrimination power is provided.
- Iodine, on the other hand, can help quite significantly.



# Results for xenon+iodine experiments



$S$ : p-value of a typical realisation

$D$ : fraction of realisations that are excluded at 95% CL

➤ The presence of a second experiment with an iodine target makes it possible to rule out SI or SD interactions if DM has an anapole moment or a magnetic dipole moment.

➤ The reason is that iodine has a much higher sensitivity to anapole and dipole interactions.



# Anapole and dipole interactions

➤ Anapole:

$$\mathcal{L}_{\text{eff}} = 2\mathcal{A}e \sum_{N=p,n} \left( Q_N \mathcal{O}_8^{(N)} + \tilde{\mu}_N \mathcal{O}_9^{(N)} \right)$$

➤ Magnetic dipole:

$$\mathcal{L}_{\text{eff}} = \frac{2\mathcal{D}_{\text{magn}}e}{q^2} \sum_{N=p,n} \left[ Q_N \left( m_N \mathcal{O}_5^{(N)} - \frac{q^2}{4m_\chi} \mathcal{O}_1^{(N)} \right) + \tilde{\mu}_N \left( m_N \mathcal{O}_6^{(N)} - \frac{q^2}{m_N} \mathcal{O}_4^{(N)} \right) \right]$$



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Magnetic dipole moment of the nucleus

$^{129}\text{Xe}$ : -0.78 (26% abundance)

$^{131}\text{Xe}$ : -0.69 (21% abundance)

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For anapole and magnetic dipole interactions, the total rate and the shape of the recoil spectrum are rather different in xenon and iodine, independent of astrophysical uncertainties.



# Conclusions

- Astrophysical uncertainties complicate the challenge of interpreting a DM signal in future direct detection experiments.
- A promising way to extract halo-independent information is to parametrise the velocity integral  $g(v_{\min})$  as a piecewise constant function with many steps.
- For an assumed true model of DM and a model used to fit the mock data, we can define the similarity and distinguishability of the two models.
- In some cases (non-monotonic recoil spectra, long-range interactions), even a single xenon-based experiment may be sufficient to exclude the fitted model in a halo-independent way.
- Additional discrimination power (e.g. for DM with an anapole or dipole moment) is provided by combining the information from two different target materials, such as xenon and iodine.

