The Hebrew University of Jerusalem האוניברסיטה העברית בירושלים
מכון רקח לפיסיקה
The Racah Institute of Physics

Non-linear diffusion of cosmic rays escaping from supernova remnants

Nava, Gabici, Marcowith, Morlino, Ptuskin
MNRAS, 2016, 461, 3552

Lara Nava
The Hebrew University of Jerusalem

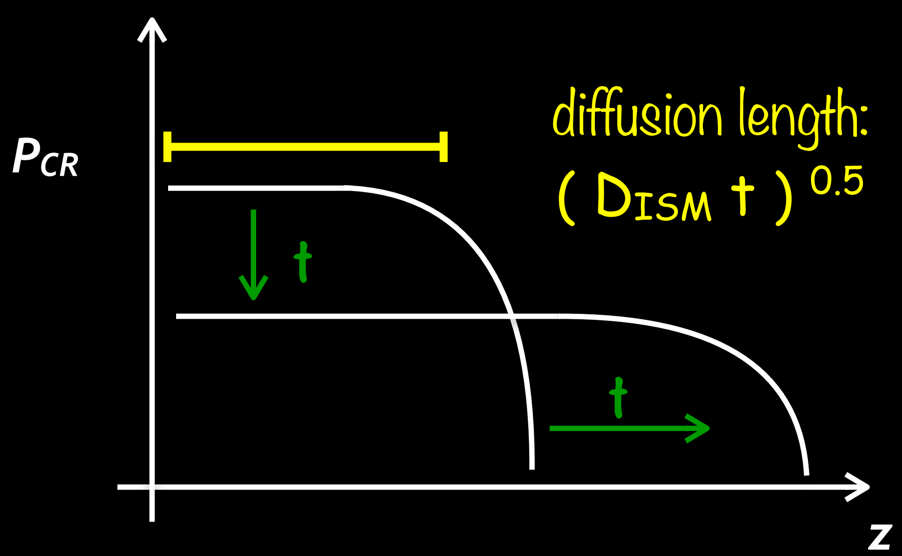
Since 1925

Test Particle (TP) solution

for a constant diffusion coefficient....

$$\frac{\partial P_{CR}}{\partial t} + V_A \frac{\partial P_{CR}}{\partial z} = \frac{\partial}{\partial z}\left(\frac{D_B}{I}\frac{\partial P_{CR}}{\partial z}\right)$$

$$P_{CR} \propto (D_{ISM} t)^{-1/2} \exp(-z^2/D_{ISM}\, t)$$



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CR pressure $P_{CR}(p, z, t)$

wave energy density $I(k, z, t)$

$$\frac{\partial P_{\rm CR}}{\partial t} + V_{\rm A}\frac{\partial P_{\rm CR}}{\partial z} = \frac{\partial}{\partial z}\left(\frac{D_{\rm B}}{I}\frac{\partial P_{\rm CR}}{\partial z}\right),$$

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Alfvén wave energy density I(k)

$$\frac{\delta B^2}{8\pi} = \frac{B_0^2}{8\pi}\int I(k)\,{\rm d}\ln k$$

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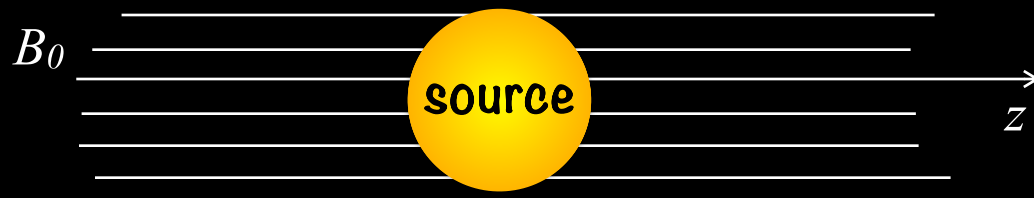

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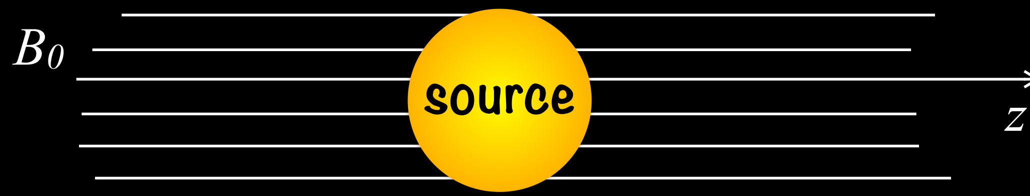

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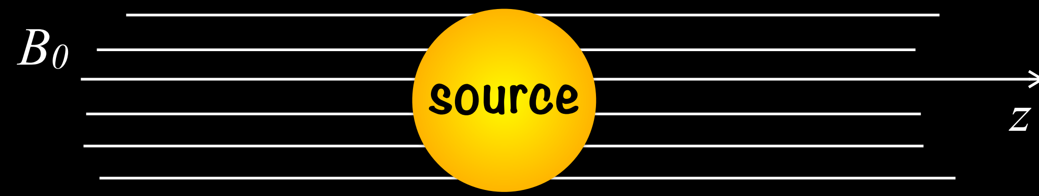

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$$D = \frac{4\, c\, r_L(E)}{3\pi\, I(k)} = \frac{D_B(E)}{I(k)}$$

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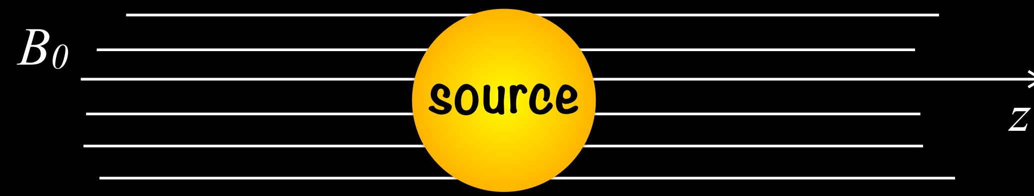

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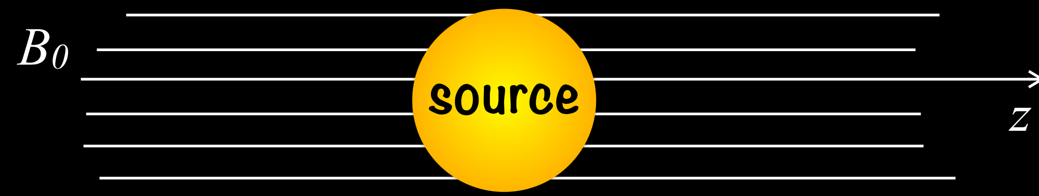

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damping rate
next slide...

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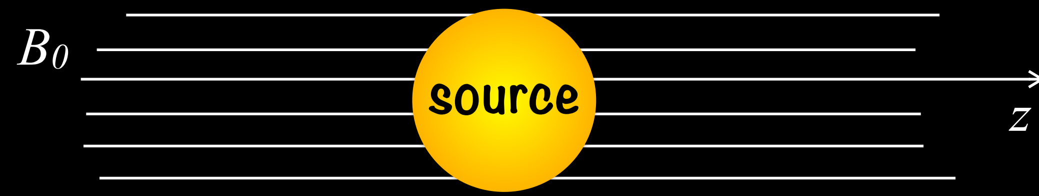

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injection term
$Q=2\Gamma_d I_0$

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ISM phases: WIM, WNM

	WIM	WNM
T (K)	8×10^3	8×10^3
B_0 (μG)	5	5
n_{tot} (cm^{-3})	0.35	0.35
f_{ion}	0.9	0.02

[Jean et al. 2009]

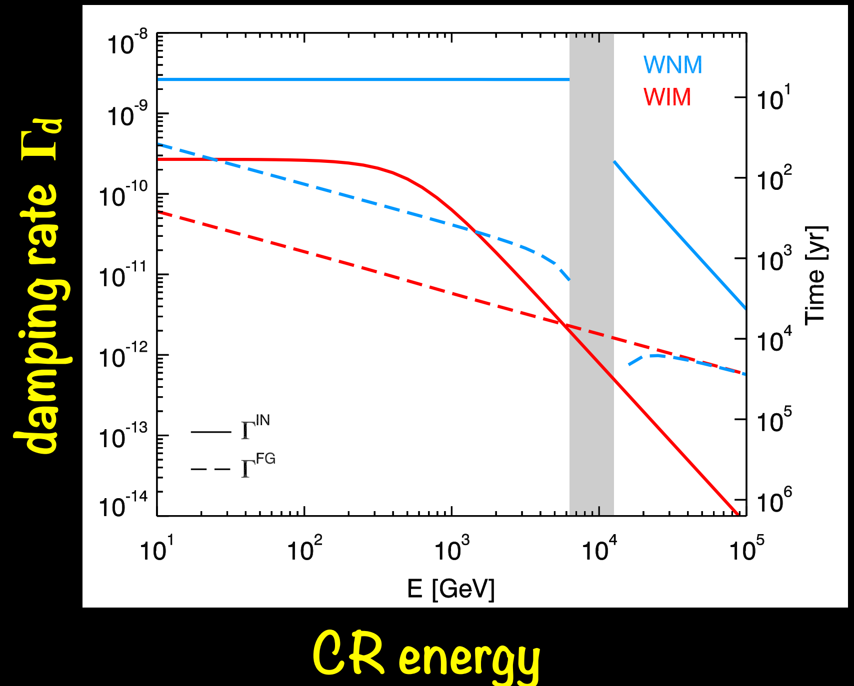
Two main damping mechanisms:

Ion-neutral collisions (Γ_{IN}): momentum-exchanging collisions between ions and neutral particles

$$\Gamma_{IN} = -\frac{\omega^2}{2\nu_c} \quad \text{for} \quad \omega \ll \nu_c$$

$$\Gamma_{IN} = -\frac{\nu_c}{2} \quad \text{for} \quad \omega \gg \nu_c$$

[Kulsrud & Pierce 1969; Zweibel & Shull 1982]



Farmer & Goldreich (Γ_{FG}): wave damping by background MHD turbulence. MHD turbulence acts as a damping mechanism for CR-generated waves

$$\Gamma_{FG} = \frac{V_A}{\sqrt{L_{MHD} r_L}}$$

[Yan & Lazarian 2002; Farmer & Goldreich 2004]

Conditions for growth of waves

Timescales

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$$\tau_{\rm g} \approx I_0/(V_{\rm A}\partial P_{\rm CR}/\partial z) \approx a I_0/V_{\rm A} P^0_{\rm CR}$$ WAVE GROWTH

$$\tau_{\rm diff} \approx a^2/D \approx a^2 I_0/D_{\rm B}$$ CR DIFFUSION

$$\tau_{\rm damp} = 1/2\Gamma_{\rm d}$$ WAVE DAMPING

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$$\tau_g < \min(\tau_{diff}, \tau_{damp})$$

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Malkov et al 2013

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$$\Pi \approx 3 \times 10^4 \; W_{CR,50} \; R_1^{-2} \; n_{i,-1}^{-1/2} \; E_1^{-1.2},$$

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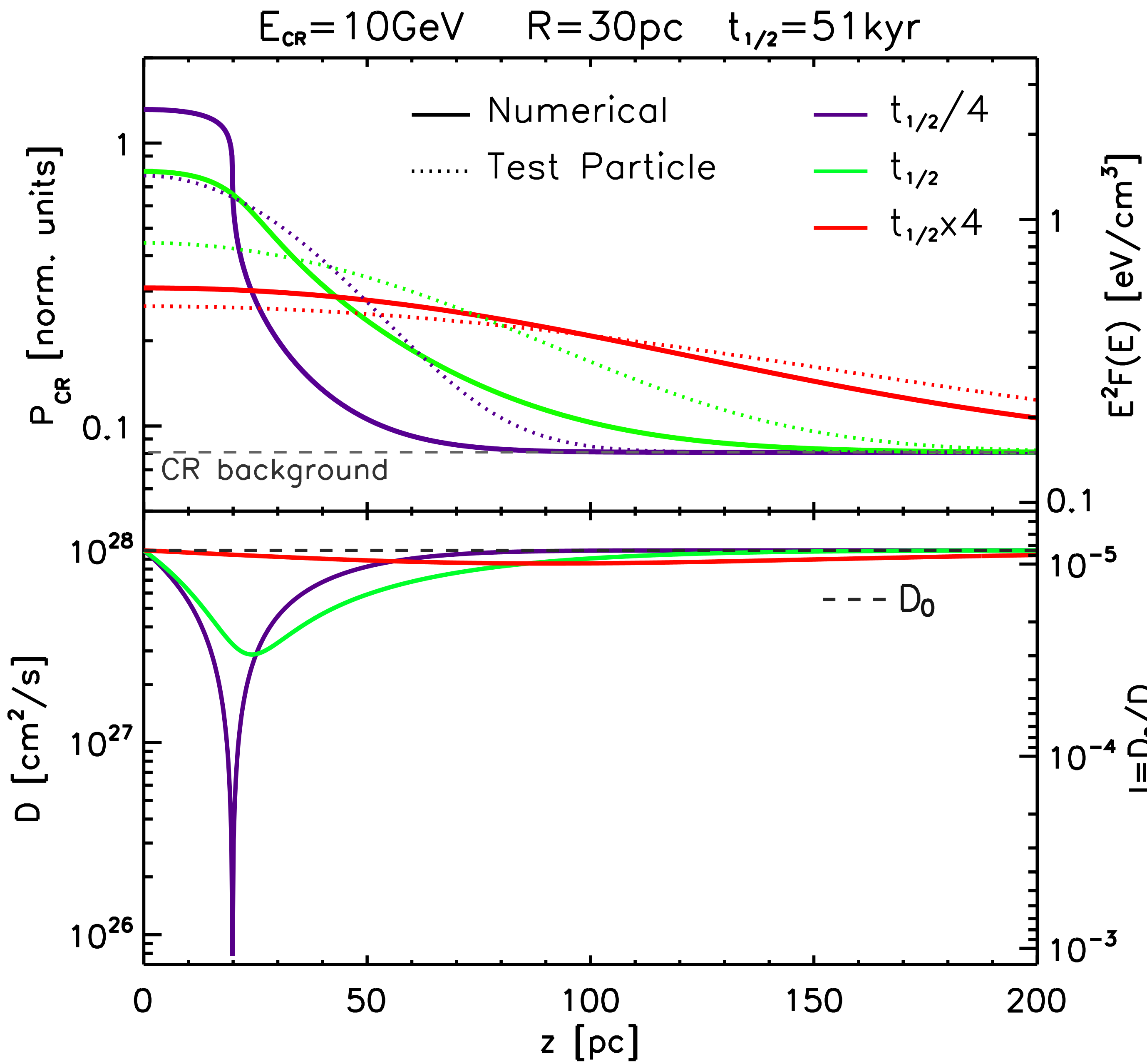
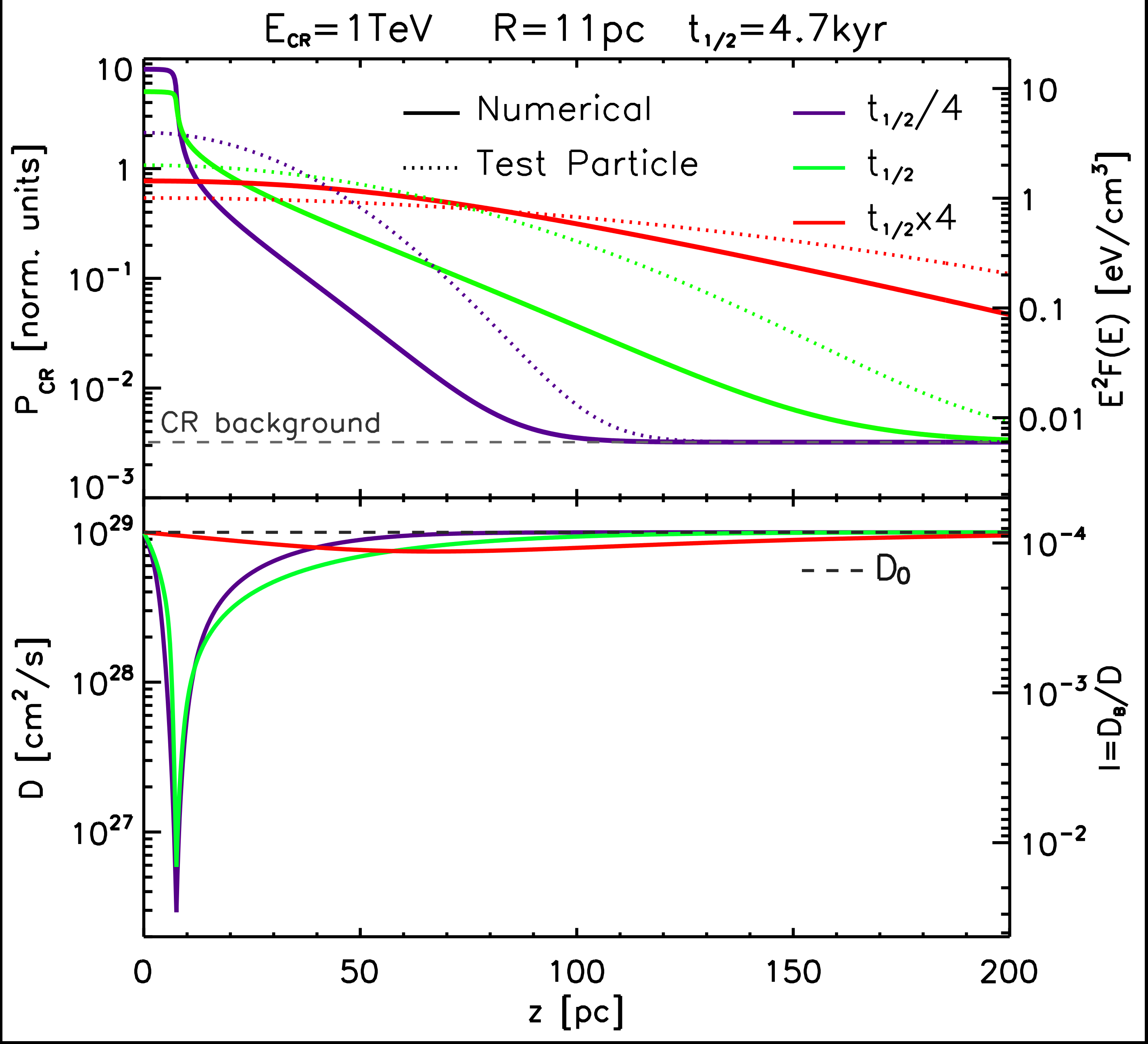
dependence on the initial radius...



E_CR=10GeV R=30pc t_1/2=51kyr
Numerical
Test Particle
t_1/2/4
t_1/2
t_1/2x4
CR background
P_CR [norm. units]
E^2F(E) [eV/cm^3]
1
0.1
D_0
D [cm^2/s]
10^28
10^27
10^26
10^-5
10^-4
10^-3
I=D_B/D
0
50
100
150
200
z [pc]



E_CR=1TeV R=11pc t_1/2=4.7kyr
Numerical
Test Particle
t_1/2/4
t_1/2
t_1/2x4
CR background
P_CR [norm. units]
E^2F(E) [eV/cm^3]
D [cm^2/s]
I=D_B/D
D_0
z [pc]
0
50
100
150
200
10
1
10^-1
10^-2
10^-3
0.1
0.01
10^29
10^28
10^27
10^-4
10^-3
10^-2

Summary

How/when CRs are released?

How do CRs propagate?

What is the impact of ISM on propagation?

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LARGE IMPACT
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Upcoming Observations