Figure 27.8: The all-particle spectrum as a function of $E$ (energy-per-nucleus) from air shower measurements [88–99,101–104].

Giving a result for the all-particle spectrum between $10^{15}$ and $10^{17}$ eV that lies toward the upper range of the data shown in Fig. 27.8. In the energy range above $10^{17}$ eV, the fluorescence technique [100] is particularly useful because it can establish the primary energy in a model-independent way by observing most of the longitudinal development of each shower, from which $E_0$ is obtained by integrating the energy deposition in the atmosphere. The result, however, depends strongly on the light absorption in the atmosphere and the calculation of the detector's aperture.

Assuming the cosmic-ray spectrum below $10^{18}$ eV is of galactic origin, the knee could reflect the fact that most cosmic accelerators in the galaxy have reached their maximum energy. Some types of expanding supernova remnants, for example, are estimated not to be able to accelerate protons above energies in the range of $10^{15}$ eV. Effects of propagation and confinement in the galaxy [106] also need to be considered.

The KASCADE-Grande experiment [98] has reported observation of a second steeper rise of the spectrum near $8 \times 10^{16}$ eV, with evidence that this structure is accompanied a transition to heavy.
Galactic Cosmic Rays

- **Standard paradigm:**
  Galactic CRs accelerated in **supernova remnants**

  - sufficient power: $\sim 10^{-3} \times M_\odot$ with a rate of $\sim 3$ SNe per century
    - [Baade & Zwicky’34]

- galactic CRs via diffusive shock acceleration?

  $$n_{CR} \propto E^{-\gamma}$$  (at source)

- energy-dependent **diffusion** through Galaxy

  $$n_{CR} \propto E^{-\gamma-\delta}$$  (observed)

- arrival direction **mostly isotropic**
→ Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies.

$E_{CR} \simeq 10 \text{ TeV}, N_{CR} \sim 3.2 \times 10^{11}$ [IceCube (IC59-IC86-IV)'16 (Frank McNally's talk)]
Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies.

\[ E_{\text{CR}} \simeq 1 \, \text{TeV}, \; N_{\text{CR}} \simeq 4.9 \times 10^{10} \; \text{[HAWC'14 (HAWC-111) (Dan Fiorino's talk)]} \]
Dipole Anisotropy

- spherical harmonic expansion of **relative CR intensity**:

\[
I(\alpha, \delta) \simeq 1 + \delta \cdot n(\alpha, \delta) + \mathcal{O}(\{a_{\ell m}\}_{\ell \geq 2})
\]

- observable **dipole vector** \(\delta\) depends on:
  1. reconstruction methods (!) [e.g., Sciasio & Iuppa’14; Ahlers et al.’16]
  2. rigidity dependence of diffusion in turbulent magnetic fields
  3. relative velocity of the diffusion medium [Compton & Getting’35]
  4. (local) source distribution [Erlykin & Wolfendale’06]
  5. (local) ordered magnetic field \(B\) [e.g. Schwadron et al.’14; Mertsch & Funk’14]
• ground-based detectors can be calibrated by CR data
• true CR dipole defined by amplitude $A_1$, and orientation $(\alpha_1, \delta_1)$
  × observable: projected dipole with amplitude $A_1 \cos \delta_1$ and orientation $(\alpha_1, 0)$
• ground-based detectors can be calibrated by CR data
• true CR dipole defined by amplitude $A_1$, and orientation $(\alpha_1, \delta_1)$
  ❌ observable: projected dipole with amplitude $A_1 \cos \delta_1$ and orientation $(\alpha_1, 0)$
• ground-based detectors can be calibrated by CR data
• true CR dipole defined by amplitude $A_1$, and orientation (RA,DEC) = ($\alpha_1, \delta_1$)
• observable: projected dipole with amplitude $A_1 \cos \delta_1$ and orientation ($\alpha_1, 0$)
• ground-based detectors can be calibrated by CR data
• true CR dipole defined by amplitude $A_1$, and orientation $(\alpha_1, \delta_1)$

$x$ observable: **projected dipole** with amplitude $A_1 \cos \delta_1$ and orientation $(\alpha_1, 0)$
TeV-PeV CR Dipole Anisotropy

[Graph showing data points for different experiments (Super-K, IceCube, IceTop, K-Grande, Tibet-ASγ, EAS-TOP, ARGO-YBJ) across energy and phase, with a dashed line indicating Galactic Center. The energy axis is labeled in GeV, ranging from $10^3$ to $10^7$, and the amplitude is in $10^{-3}$.

[MA’16]
Cosmic Ray Dipole Anisotropy

- cosmic-ray (CR) arrival directions described by **phase-space distribution**

\[ f(t, r, p) = \frac{\phi(t, r, p)}{(4\pi)} + 3\hat{p}\frac{\Phi(t, r, p)}{(4\pi)} + \ldots \]

- local CR spectral density \([\text{GeV}^{-1}\text{cm}^{-3}]\)

\[ n_{\text{CR}}(p) = p^2\phi(t, r_\oplus, p) \propto p^{-\Gamma_{\text{CR}}} \propto p^{-(\Gamma_{\text{CR}}+2)} \]

- in the absence of sources, follows Liouville’s equation \((\dot{f} = 0)\)

→ **quasi-stationary dipole anisotropy** \((\partial_t \Phi \simeq 0)\):

\[ \partial_t \Phi \simeq \nabla_r (K \nabla_r \phi) \quad \text{and} \quad \Phi \simeq -K \nabla_r \phi \]

- diffusion tensor **K**:

\[ K_{ij} = \kappa_\parallel \hat{B}_i \hat{B}_j + \kappa_\perp (\delta_{ij} - \hat{B}_i \hat{B}_j) + \kappa_A \epsilon_{ijk} \hat{B}_k \]
Compton-Getting Effect

- phase-space distribution is Lorentz-invariant, \( f^*(p^*) = f(p) \)  
  - Lorentz boost to moving frame (\( \beta = v/c \), starred quantities in plasma rest-frame):
    \[
    p^* = p + p\beta + \mathcal{O}(\beta^2)
    \]

- Taylor expansion
  \[
  f(p) \simeq f^*(p) + (p^* - p)\nabla_p f^*(p) + \mathcal{O}(\beta^2) \simeq f^*(p) + p\beta\nabla_p f^*(p) + \mathcal{O}(\beta^2)
  \]

→ splitting in \( \phi \) and \( \Phi \) is not invariant:
  \[
  \phi = \phi^* \quad \Phi = \Phi^* + \frac{1}{3}\beta \frac{\partial \phi^*}{\partial \ln p}
  \]

→ energy-independent shift of dipole anisotropy:
  \[
  \delta = \delta^* + (2 + \Gamma_{\text{CR}})\beta
  \]

What is the correct plasma rest-frame? Local Standard of Rest?
Local Magnetic Field

- observed dipole:
  \[ \delta_{\text{obs}} = (\delta_{0h}, \delta_{6h}) = (A'_1 \cos \alpha_1, A'_1 \sin \alpha_1) \]

- reconstructed dipole:
  \[ \delta^* = \delta_{\text{obs}} - (2 + \Gamma_{\text{CR}}) \beta = 3K \cdot \nabla \ln n^* \]

- strong ordered magnetic fields in the local environment

- diffusion tensor reduces to projector:
  \[ K_{ij} \rightarrow \kappa_{\parallel} \hat{B}_i \hat{B}_j \]

- TeV–PeV dipole data consistent with magnetic field direction inferred by IBEX data [McComas et al.'09]
Known Local Supernova Remnants

- projection maps source gradient onto $\hat{B}$ or $-\hat{B}$

→ dipole phase $\alpha_1$ depends on orientation of magnetic hemispheres

- intersection of magnetic equator with Galactic plane defines two source groups:

  $120^\circ \lesssim l \lesssim 300^\circ \rightarrow \alpha_1 \approx 49^\circ$

  $-60^\circ \lesssim l \lesssim 120^\circ \rightarrow \alpha_1 \approx 229^\circ$
Local Magnetic Field

- 1–100 TeV phase indicates dominance of a local source within longitudes:
  
  \[ 120^\circ \lesssim l \lesssim 300^\circ \]

- plausible scenario: Vela SNR [MA’16]

  - half height: \( H \simeq 3 \) kpc
  - SNR rate: \( R_{\text{SNR}} = 1/30 \) yr\(^{-1}\)
  - (effective) isotropic diffusion:
    \[
    K_{\text{iso}} \simeq 4 \times 10^{28} (E/3\text{GeV})^{1/3} \text{cm}^2/\text{s}
    \]
  - instantaneous CR emission \( (Q^\star) \)

- or a luminous 2Myr old SNR?
  [Savchenko, Kachelrieß & Semikoz’15]

![Graph showing the distribution of CR intensity across different energies and SNRs.](image-url)
Local Magnetic Field

• 1–100 TeV phase indicates dominance of a local source within longitudes:

\[ 120^\circ \lesssim l \lesssim 300^\circ \]

• plausible scenario: Vela SNR \[\text{[MA’16]}\]
  
  • half height: \( H \approx 3 \) kpc
  
  • SNR rate: \( R_{\text{SNR}} = 1/30 \) yr\(^{-1}\)
  
  • (effective) isotropic diffusion:

\[ K_{\text{iso}} \approx 4 \times 10^{28} (E/3 \text{GeV})^{1/3} \text{cm}^2/\text{s} \]

• instantaneous CR emission (\( Q_\star \))

• or a luminous 2Myr old SNR?

[Savchenko, Kachelrieß & Semikoz’15]
Summary

- **Dipole anisotropy** can be understood in the context of standard diffusion theory:
  - Small shift by Compton-Getting effect.
  - TeV-PeV dipole phase aligns with local ordered magnetic field.
  - Amplitude variations as a result of local sources (**Vela SNR**?).

- Observed CR data shows evidence of **small-scale anisotropy**.
  - Result of local magnetic turbulence? (➔ Gwenael Giacinti’s talk)
  - ✗ Induces cross-talk with dipole anisotropy in limited field of view.

➔ Need better reconstruction & analysis methods:
  - 2D vs. 1D harmonic analysis
  - Minimize multipole cross-talk by combined FoVs (e.g. IceCube + HAWC)
Appendix
Origin of Small-Scale Anisotropy?

- CMB temperature fluctuations
- Cosmic Ray Gradient
- Local Magnetic Turbulence
- Large Scale Structure
- small scale temperature fluctuations
- small scale anisotropies
Angular Power Spectrum

- Every smooth function $g(\theta, \phi)$ on a sphere can be decomposed in terms of spherical harmonics $Y_{\ell m}$:

$$g(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \quad \leftrightarrow \quad a_{\ell m} = \int d\Omega \ (Y_{\ell m})^* (\theta, \phi) g(\theta, \phi)$$

- Angular power spectrum:

$$C_\ell = \frac{1}{2\ell + 1} \sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

- Related to the two-point auto-correlation function: (unit vectors, $n_1 \cdot n_2 = \cos \eta$)

$$\xi(\eta) = \frac{1}{8\pi^2} \int \!dn_1 \int \!dn_2 \delta(n_1 n_2 - \cos \eta) g(n_1) g(n_2) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos \eta)$$
Model Prediction

\[ \lim_{T \to \infty} \frac{\langle C_\ell \rangle (T)}{\langle C_1 \rangle (T)} \approx \frac{18}{(2\ell + 1)(\ell + 2)(\ell + 1)} \]

Appendix
Multipole Cross-Talk

- relative CR intensity (including small-scale structure):
  \[ I(\alpha, \delta) = 1 + \sum_{\ell \geq 1} \sum_{m \neq 0} a_{\ell m} Y_{\ell m}(\alpha, \pi/2 - \delta) \]

- dipole: \( a_{1-1} = (\delta_{0h} + i\delta_{6h}) \sqrt{2\pi/3} \) and \( a_{11} = -a^*_{1-1} \)

- **traditional dipole analyses** extract amplitude “\( A_1 \)” and phase “\( \alpha_1 \)” from data projected into right ascension \((s_{1/2} \equiv \sin \delta_{1/2})\)
  
  \[ A_1 e^{i\alpha_1} = \frac{1}{\pi} \int_0^{2\pi} d\alpha e^{i\alpha} \frac{1}{s_2 - s_1} \int_{s_1}^{s_2} d\sin \delta I(\alpha, \delta) \]

- the presence of high-\( \ell \) multipole moments introduces **cross-talk**

\( \rightarrow \) Can now estimate the **systematic uncertainties** of dipole measures from dipole-induced small-scale power spectrum.
Systematic Uncertainty of CR Dipole

IceTop

\((\Delta \delta/\delta^*)_{\text{IceTop}} = 0.41, \delta_1 = -90^\circ, \delta_2 = -35^\circ\)

IceCube

\((\Delta \delta/\delta^*)_{\text{IceCube}} = 0.39, \delta_1 = -90^\circ, \delta_2 = -25^\circ\)

EAS-TOP

\((\Delta \delta/\delta^*)_{\text{EAS-TOP}} = 0.31, \delta_1 = 10^\circ, \delta_2 = 58^\circ\)

ARGO-YBJ

\((\Delta \delta/\delta^*)_{\text{ARGO-YBJ}} = 0.20, \delta_1 = -10^\circ, \delta_2 = 70^\circ\)

Tibet-ASγ

\((\Delta \delta/\delta^*)_{\text{Tibet-ASγ}} = 0.12, \delta_1 = -30^\circ, \delta_2 = 90^\circ\)

HAWC

\((\Delta \delta/\delta^*)_{\text{HAWC}} = 0.07, \delta_1 = -41^\circ, \delta_2 = 79^\circ\)
Systematic Uncertainty of CR Dipole

Statistical & systematic uncertainties

IceCube (×1.54)
IceTop (×1.74)
ARGO-YBJ (×1.18)
Tibet-ASY (×1.18)
EAS-TOP (×1.38)
Powerspectrum of CR Arrival Directions

Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies

[Tibet AS-γ’05,’06; Super-Kamiokande’07; Milagro’08; ARGO-YBJ’09,’13;EAS-TOP’09]

[IceCube’10,’11; HAWC’13,’14]
Appendix

Powerspectrum of CR Arrival Directions

Cosmic ray anisotropies up to the level of one-per-mille have been observed at various energies

[Tibet AS-γ’05,’06; Super-Kamiokande’07; Milagro’08; ARGO-YBJ’09,’13;EAS-TOP’09]

[IceCube’10,’11; HAWC’13,’14]

[arXiv:1408.4805; note: low—ℓ power under-estimated]
Local Sources

\[ \delta = \frac{1}{3} \gamma + \delta = 2.67 \quad H = 1 \text{kpc} \]

SN Rate: 1/100 yr\(^{-1}\)

\[ \delta = \frac{1}{3} \gamma + \delta = 2.67 \quad H = 4 \text{kpc} \]

SN Rate: 1/100 yr\(^{-1}\)

\[ \delta = \frac{1}{3} \gamma + \delta = 2.67 \quad H = 1 \text{kpc} \]

SN Rate: 1/100 yr\(^{-1}\)

\[ \delta = \frac{1}{3} \gamma + \delta = 2.67 \quad H = 4 \text{kpc} \]

SN Rate: 1/100 yr\(^{-1}\)

[Blasi & Amato'12]

[Erlykin & Wolfendale'06; Sveshnikova et al.'13; Pohl & Eichler'13]