New Lattice Calculations for Quantities in Flavor Physics

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Why is Lattice QCD important for the LHC?

The QCD coupling α_s runs with distance scale. At long distances, the theory is strongly coupled.

We need a reliable nonperturbative tool to calculate all the low energy phenomena of QCD, from the hadron spectrum to quark masses to weak matrix elements.

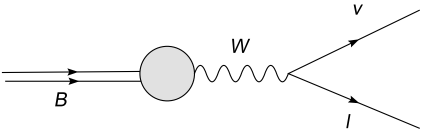
Precision determination of Standard Model hadronic parameters is required to make best use of experimental data from Belle II. Most indirect physics searches require nonperturbative input.

Lattice QCD in the LHC era

If a “particle zoo" is discovered, ATLAS and CMS will measure the spectrum. Precision flavor measurements still important as part of studies to learn the underlying structure of the theory.

If new physics is beyond the reach of direct production at LHC, indirect searches using high precision low energy quantities may be our best bet to discover new physics.

Nonperturbative input needed



$$\Gamma = (\text{known factor})(\text{CKM factor})(\text{QCD factor}) \tag{1}$$

$$\mathscr{B}(B \to \tau \overline{\nu}_\tau) = \frac{G_F^2 m_B m_\tau^2}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 f_B^2 |V_{ub}|^2 \tau_B \tag{2}$$

Lattice QCD Calculations

Calculate expectation values on an ensemble of gauge fields $[\mathscr{U}]$ with an exponential weight

$$\langle \mathscr{O} \rangle = \frac{1}{Z} \int D\mathscr{U}\, D\psi_{\mathrm{sea}} D\overline{\psi}_{\mathrm{sea}} e^{-S_{\mathrm{QCD}}[\mathscr{U},\psi_{\mathrm{sea}},\overline{\psi}_{\mathrm{sea}}]} \mathscr{O}[\mathscr{U},\psi_{\mathrm{val}},\overline{\psi}_{\mathrm{val}}], \tag{3}$$

$$\langle \mathscr{O} \rangle = \frac{1}{Z} \int D\mathscr{U} \prod_{f=1}^{n_f} \det(\not{D} + m_f) e^{-S_{\mathrm{QCD}}[\mathscr{U}]} \mathscr{O}[\mathscr{U},\psi_{\mathrm{val}},\overline{\psi}_{\mathrm{val}}], \tag{4}$$

The action is discretized, so that derivatives become finite differences. Integral is still too large to do directly ($N_s^3 \times N_t \times 4 \times N_f \times N_c$), so we use Monte Carlo importance sampling.

Computing

Cluster at Fermilab and BlueGene/Q at Argonne

Types of Errors

Because QCD with physical quark masses is a nonlinear multiscale problem ($\Lambda_{QCD} \approx 100 - 200$ MeV, $m_{u,d} \approx 2 - 6$ MeV, $m_b \approx 4.3$ GeV), it is very expensive to simulate at the physical quark masses.

1.) Statistics and fitting

2.) Tuning lattice spacing, a, and quark masses

3.) Matching lattice gauge theory to continuum QCD

4.) Extrapolation to continuum

5.) Chiral extrapolation to physical up, down quark masses

6.) Quenching. Uncontrolled!

7.) Neglecting QED and/or isospin effects

Quenched Approximation

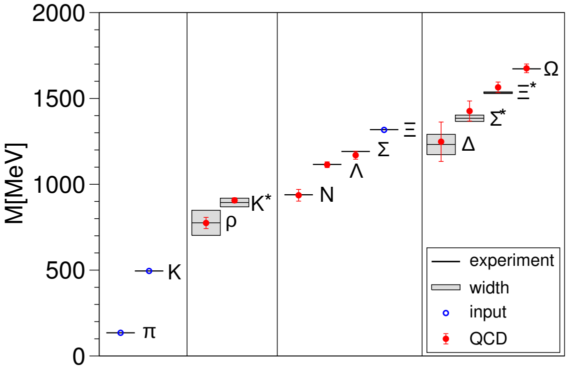
Configurations are generated with a weighting given by the gauge field and fermion determinant. Including the fermion determinant in this weighting is the most computationally demanding step in lattice QCD.

The quenched approximation ignores fermion-antifermion vacuum bubbles. This is an uncontrolled systematic error.

“Unquenched" calculations, where the fermion determinant is included, are now the norm.

Some calculations quench the strange quark (2 flavor) and most quench the charm (2+1 flavors). Only the most recent calculations include charm sea quark effects. Such calculations are called in the lingo "2+1+1 flavor".

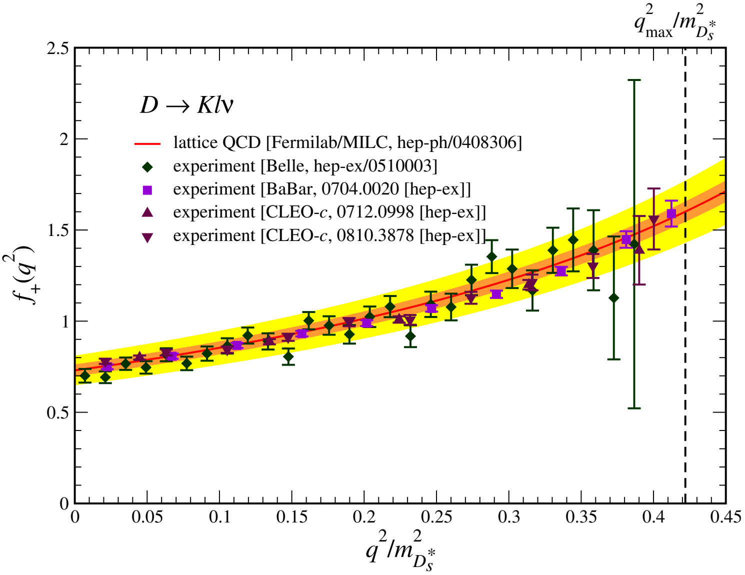
Hadron Spectrum from Lattice QCD



BMW Collaboration, Science 322:1224-1227, 2008.

- *Good agreement between theory and experiment!*

Prediction of form factor



Lattice Averages

Many lattice quantities have reached the mature stage of having controlled systematic errors and results from several groups using different methods.

FLAG II has produced a large number of averages for many quantities of interest to flavor physics.
`http://itpwiki.unibe.ch/flag/index.php/Review_of_lattice_results_concerning_low_energy_particle_physics`

This includes many weak-matrix elements of heavy- and light-quark quantities.

What the lattice can do

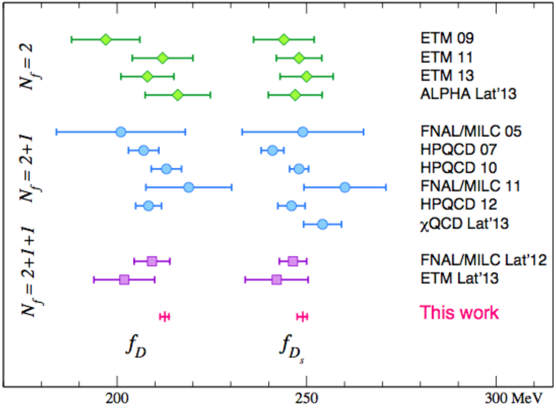
Lattice calculations are definitely doable when there is at most one hadron (stable under QCD) in the initial and final states. Baryons are more challenging than mesons. Unstable particles are very challenging.

Around twenty weak matrix elements have mature calculations, with existing results and expected improvements: f_K, f_π, $K \to \pi\ell\nu$, $D \to K\ell\nu$, $B \to \pi\ell\nu$, ...

Another class of problems where methods exist, but are more challenging because of disconnected diagrams, more than one meson in the final state, or both. (A few years?): $K \to \pi\pi$ with (results now exist), ΔM_K, $K \to \pi\ell^+\ell^-$,...

Yet another class of problems (when initial state mass is greater than the inelastic threshold of final states) where new ideas are needed, and probably a lot more computing: $D \to \pi\pi$, $D \to KK$, $B \to \pi\pi$, D-mixing and CP violation...

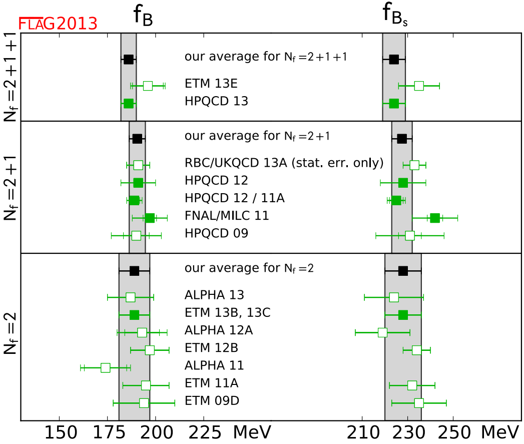
f_{D^+}, f_{D_s}



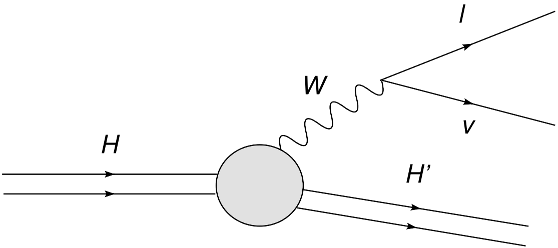
New MILC/Fermilab results (arXiv:1407.3772):

$f_{D_s} = 249.0^{+1.1}_{-1.5}$ MeV, $\quad f_{D^+} = 212.6(^{+1.1}_{-1.3})$ MeV.

f_B, f_{B_s}, FLAG II averages



Heavy-light semileptonic decays



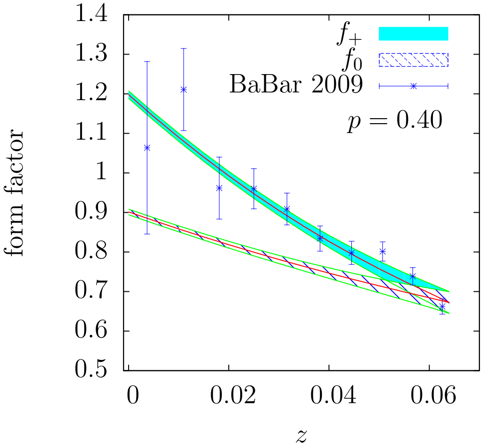
Vertex proportional to $|V_{qq'}|$. In order to extract it, a nonperturbative determination of the form factors is needed.

Obtaining V_{cb} from $\overline{B} \to D^* l \overline{\nu}_l$

$$\frac{d\Gamma}{dw} = \frac{G_F^2}{4\pi^3} m_{D^*}^3 (m_B - m_{D^*})^2 \sqrt{w^2 - 1} \times |V_{cb}|^2 \mathscr{G}(w) |\mathscr{F}_{B\to D^*}(w)|^2 \quad (5)$$

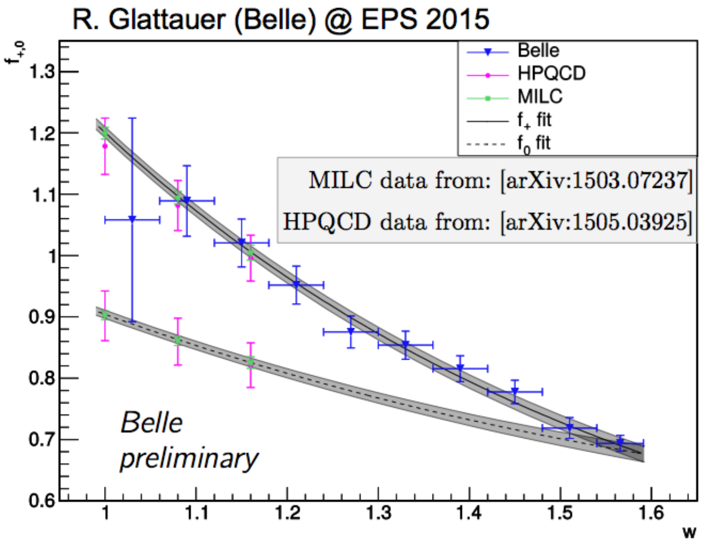
where $\mathscr{G}(w)|\mathscr{F}_{B\to D^*}|^2$ contains a combination of form-factors which must be computed non-perturbatively. $w = v' \cdot v$ is the velocity transfer from initial (v) to final state (v').

$B \to D\ell\nu$ at non-zero recoil

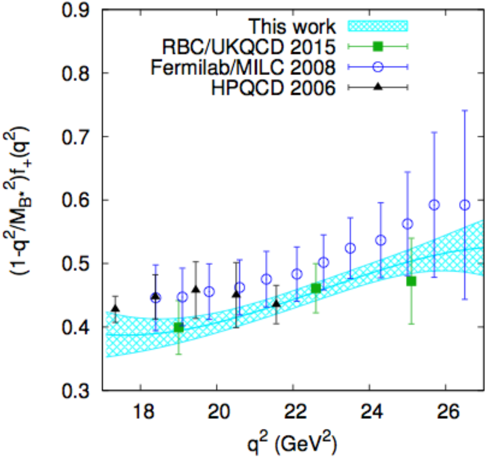


A fit to the form factor shape using lattice calculations (FNAL/MILC) and the z expansion to BaBar data. This fit yields a value of $|V_{cb}| = (39.6 \pm 1.7) \times 10^{-3}$.

$B \to D\ell\nu$ at non-zero recoil

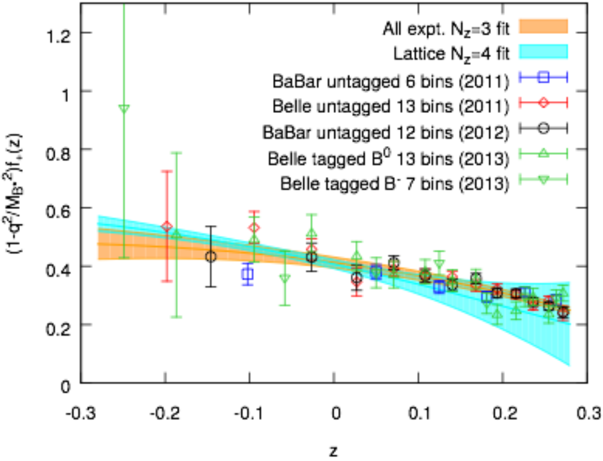


$B \to \pi \ell \nu$



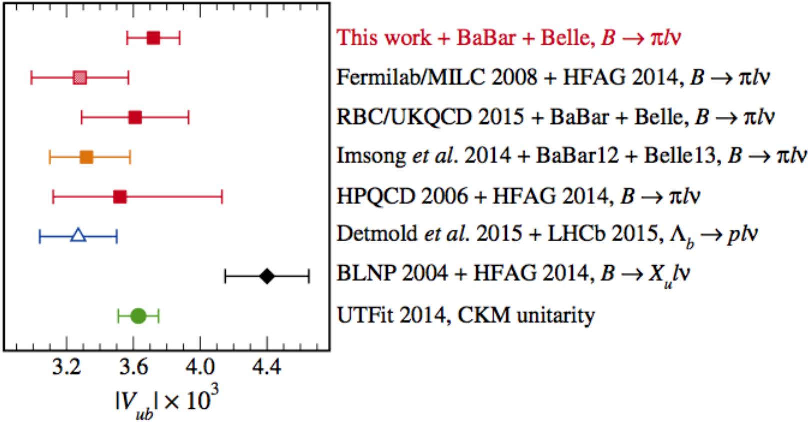
“This work" is FNAL/MILC 2015 (arXiv:1503.07839).

$B \to \pi \ell \nu$



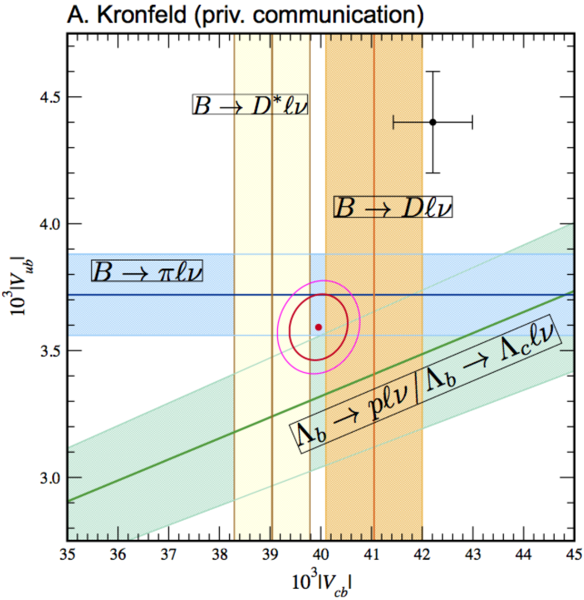
“Lattice" is FNAL/MILC 2015 (arXiv:1503.07839).

$|V_{ub}|$

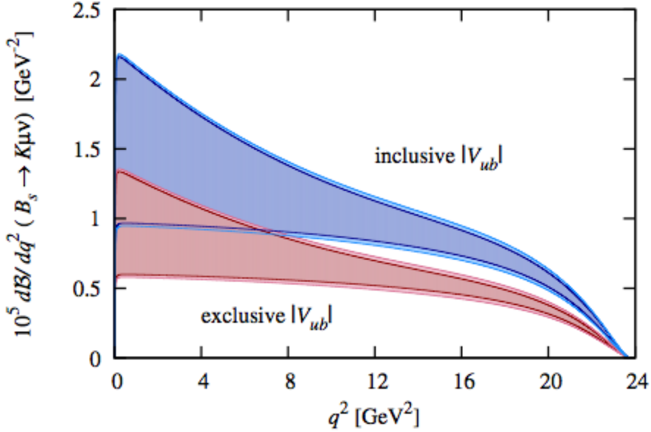


“This work" is FNAL/MILC 2015 (arXiv:1503.07839).
$|V_{ub}| = (3.72 \pm 0.16) \times 10^{-3}$ (See talk by Daping Du.)

$|V_{cb}|$ and $|V_{ub}|$

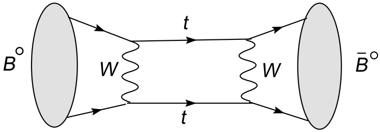

A. Kronfeld (priv. communication)
$B \to D^* \ell\nu$
$B \to D\ell\nu$
$B \to \pi\ell\nu$
$\Lambda_b \to p\ell\nu \,|\, \Lambda_b \to \Lambda_c\ell\nu$
4.5
4.0
3.5
3.0
35
36
37
38
39
40
41
42
43
44
45
$10^3|V_{ub}|$
$10^3|V_{cb}|$

$B_s \to K\ell\nu$



From Bouchard, et al., HPQCD (arXiv:1406.2279).

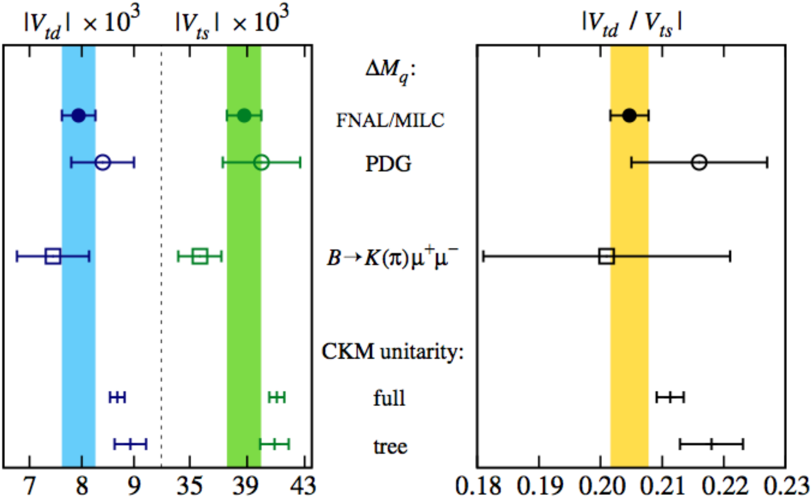
B-$\overline{B}$ Mixing



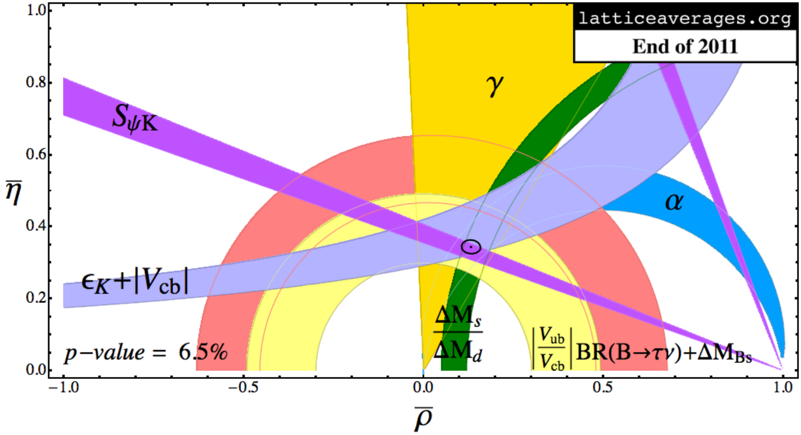
$$\langle \overline{B}^0 | (\overline{b}d)_{V-A} (\overline{b}d)_{V-A} | B^0 \rangle \equiv \frac{8}{3} m_B^2 f_B^2 B_B, \tag{6}$$

$$\Delta M_s = \frac{G_F^2 M_W^2}{6\pi^2} |V_{ts}^* V_{tb}|^2 \eta_2^B S_0(x_t) M_{B_s} f_{B_s}^2 \hat{B}_{B_s} \tag{7}$$

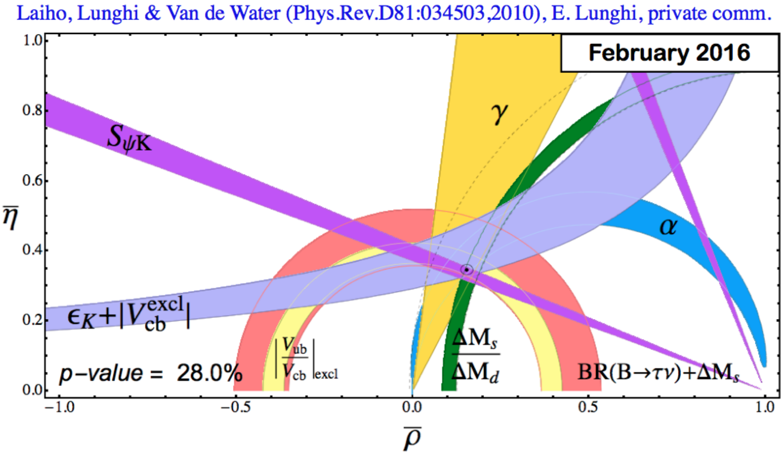
Implications from B-mixing



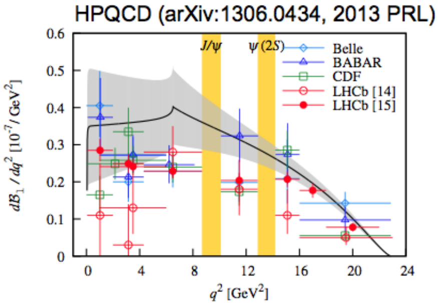
UT triangle 2011

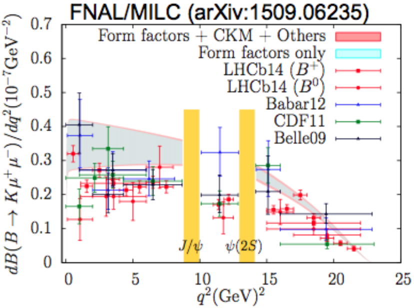


UT triangle 2016

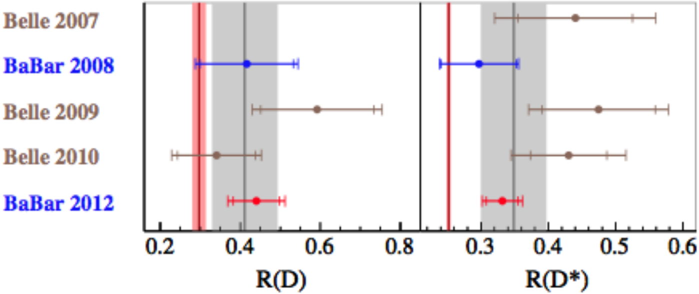


Rare decays: $B \to K\ell^+\ell^-$



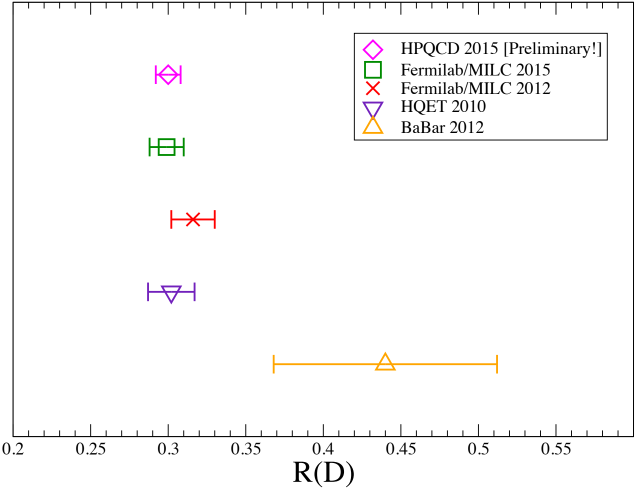


Rare Decays: R(D)

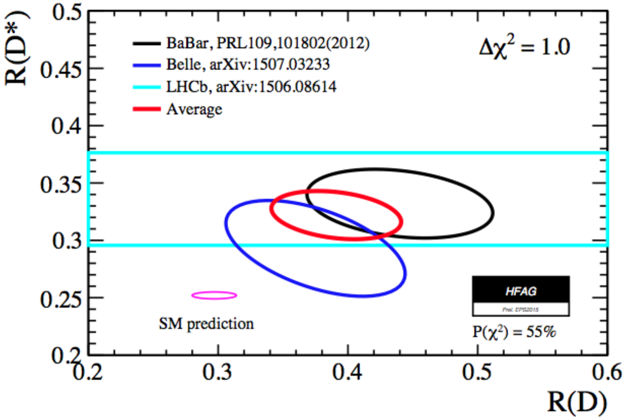


Gray band is the average from 2007-2010 experiments. The pink band is the Standard Model (non-lattice) prediction.

Comparison for R(D)



R(D^*)



BaBar, PRL109,101802(2012)
Belle, arXiv:1507.03233
LHCb, arXiv:1506.08614
Average
$\Delta\chi^2 = 1.0$
HFAG
SM prediction
$P(\chi^2) = 55\%$
R(D*)
R(D)
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0.6

Forcast

Error forcast

Quantity	CKM element	Present expt. error	Present lat error	2014 lat error	2018 lat error
f_K/f_π	$\lvert V_{us}\rvert$	0.2%	0.5%	0.3%	0.15%
$f_+^{K\pi}(0)$	$\lvert V_{us}\rvert$	0.2%	0.5%	0.35%	0.2%
f_D	$\lvert V_{cd}\rvert$	4.3%	2%	1%	$< 1\%$
f_{D_s}	$\lvert V_{cs}\rvert$	2.1%	2%	1%	$< 1\%$
$D \to \pi\ell\nu$	$\lvert V_{cd}\rvert$	2.6%	4.4%	3%	2%
$D \to K\ell\nu$	$\lvert V_{cs}\rvert$	1.1%	2.5%	2%	1%
$B \to D^*\ell\nu$	$\lvert V_{cb}\rvert$	1.3%	1.8%	1.5%	$< 1\%$
$B \to \pi\ell\nu$	$\lvert V_{ub}\rvert$	4.1%	8.7%	4%	2%
f_B	$\lvert V_{ub}\rvert$	9%	2.5%	1.5%	$< 1\%$
ξ	$\lvert V_{ts}\rvert/\lvert V_{td}\rvert$	0.4%	2.5%	1.5%	$< 1\%$
ΔM_s	$\lvert V_{ts}V_{tb}\rvert^2$	0.24%	11%	8%	5%
B_K	$\mathrm{Im}(V_{td}^2)$	0.5%	1.3%	1%	$< 1\%$

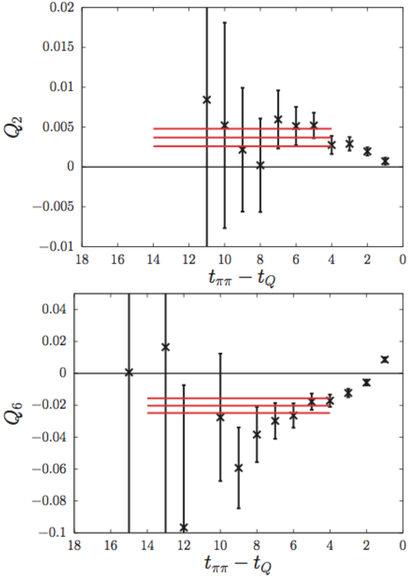
From the USQCD 2013 white paper

Approaches to $K \to \pi\pi$ matrix elements

Maiani-Testa no-go theorem tells us that we cannot extract physical matrix elements from Euclidean correlation functions with multi-hadron states.

Difficulties simulating at physical kinematics for $K \to \pi\pi$ matrix elements avoided by using Lellouch-Lüscher finite volume method. This is still costly. Most straightforward implementation requires a large (6 fm) box, momentum insertion, and physical light quark masses. The $\Delta I = 1/2$ channel also requires power subtractions and evaluation of disconnected diagrams (very noisy).

$K \to \pi\pi$



From RBC/UKQCD (PRL 115, 212001 (2015))

$Re(\varepsilon'/\varepsilon) = 1.38(5.15)_{\rm stat}(4.59)_{\rm sys}$

Prospects

Simple quantities (semileptonic form factors, decay constants, mixing parameters) from many different groups using different methods can be calculated with controlled systematic errors.

Many quantities are now precision calculations, and results are in good agreement. Prospects for improvement are excellent! Given our extensive experience with these quantities, it is straightforward to predict how the errors will scale into the future with existing methods assuming Moore's Law for computing. Improvements in methodology usually mean that these predictions are conservative.

There is a catch... A challenge for the future is to incorporate QED effects. This is difficult because, e.g. it turns decay constants into form factors. New methods for dealing with this have been introduced (see arXiv:1502.00257).

Backup slides

Some types of lattice fermions

Lots of ways to solve the lattice fermion doubling problem:

- Staggered Fermions: Identifies some of the extra fermions with the different spin components of a single fermion. There are still 4 extra species of fermions, and these are eliminated by taking the 4th root of the determinant. Some open theoretical issues with this, though theoretical progress has been made on this front. Very cheap.

- Wilson Fermions: Introduces an additional “irrelevant" term to the action. Improved variants, i.e. “clover" or "twisted-mass" used in practice. Fairly cheap.

- Domain Wall Fermions: Solves chiral symmetry problem by using Wilson type quarks in five dimensions. More costly because of the extra dimension. There is a small chiral symmetry breaking due to the finiteness of the fifth dimension. Expensive.

- Overlap Fermions: Exact lattice chiral symmetry. Very expensive.

Heavy quarks on the lattice

The lattice cut-off is smaller than the heavy quark masses for realistic lattices.
The solution(s): heavy quark effective theory(HQET) or nonrelativistic QCD

Fermilab Method:

Continuum QCD $\rightarrow$ Lattice gauge theory
(using HQET)

nonrelativistic QCD method:

Continuum QCD $\rightarrow$ Nonrelativistic QCD $\rightarrow$ Lattice gauge theory

- Both methods require tuning parameters of the lattice action
- The currents and 4-quark operators must be matched as well. Typically this is done in lattice perturbation theory.