Event reweighting with TauSpinner: spin and matrix element effects in tau lepton pair production with two high pT jets

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A: An algorithm of re-weighting, principles and examples for `TauSpinner` ($2 \to 2$)

B: Case of ($2 \to 4$) processes arXiv:1604.00964, April 4, 2016

C: Application domains, largely unexplored

TauSpinner for re-weighting τ-lepton events in Z, W, H production and decays at LHC

- τ leptons can not be observed directly, also ν_τ escapes detection. This is a nuisance making observables difficult.
- How to turn this complexity into advantage?

  a. τ is the only lepton of measurable spin state.

  b. Large mass $\to$ large coupling to H.

- TauSpinner is a tool which is devoted to manipulation of spin effects on previously generated samples of events with the help of weights which can be calculated after events are generated and stored on production files.

  c. I mean events with full detector response effects, limited acceptance etc., taken into account.

1. To calculate weight = $\frac{|matrix\ element\ new|^2}{|matrix\ element\ old|^2}$ one has to:
   (a) get phase space point at which weight is to be calculated
   (b) know $|matrix\ element\ old|^2$ with which this point was calculated.
   (c) That also means that variables used for calculation of matrix elements have to be reconstructed from information stored in event record or in production files.
2. I will review main ideas only. Basic principles how `TAUOLA interface` work and how `TauSpinner` work. More formal considerations, like for PHOTOS, would take too much time.
3. Thanks to extremely narrow width of τ lepton its production and decay can be fully separated.

http://tauolapp.web.cern.ch/tauolapp/

Formalism for $\tau^+\tau^-$, independent from production mechanism.

- Because narrow τ width approximation can be obviously used for phase space , cross section for the process $f\bar{f} \to \tau^+\tau^- Y; \tau^+ \to X^+\bar{\nu}; \tau^- \to \nu\nu$ reads:

$$d\sigma = \sum_{spin} |\mathcal{M}|^2 d\Omega = \sum_{spin} |\mathcal{M}|^2 d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

- This formalism is fine, but because of over 20 τ decay channels we have over 400 distinct processes. Also picture of production and decay are mixed.

- but (only τ spin indices are explicitly written):

$$\mathcal{M} = \sum_{\lambda_1\lambda_2=1}^{2} \mathcal{M}^{prod}_{\lambda_1\lambda_2}\ \mathcal{M}^{\tau^+}_{\lambda_1} \mathcal{M}^{\tau^-}_{\lambda_2}$$

- Cross section can be re-written into **core formula of spin algorithms**

$$d\sigma = \Big(\sum_{spin} |\mathcal{M}^{prod}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^+}|^2\Big)\Big(\sum_{spin} |\mathcal{M}^{\tau^-}|^2\Big) wt\ d\Omega_{prod}\ d\Omega_{\tau^+}\ d\Omega_{\tau^-}$$

- where

$$wt = \Big(\sum_{i,j=0,3} R_{ij} h^i h^j \Big)$$

$$R_{00} = 1, \quad < wt >= 1, \quad 0 \leq wt \leq 4.$$

R_{ij} can be calculated from $\mathcal{M}_{\lambda_1 \lambda_2}$
and h^i, h^j respectively from $\mathcal{M}^{\tau^+}$ and $\mathcal{M}^{\tau^-}$.

- Bell inequalities tell us that it is impossible to re-write wt in the following form

$$wt \neq \Big(\sum_{i,j=0,3} R^A_i h^i \Big) \Big(\sum_{i,j=0,3} R^B_j h^j \Big)$$

that means it is impossible to generate first τ^+ and τ^- in some 'quantum states' and later perform separately decays of τ^+ and τ^-

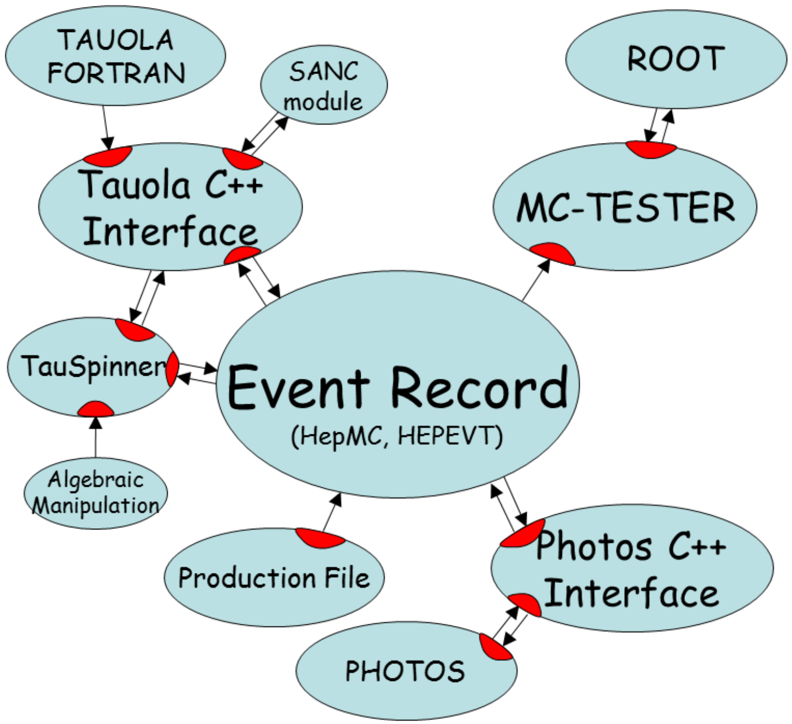
- It can be done only if approximations are used !!! (like in `TauSpinner` even though transverse spin effects are in for helicity attribution we need to approximate.

- Spin weight can be calculated after event is constructed, detector response simulated and stored in file. It can be even embedded τ event.

How TauSpinner came along...

1. TAUOLA is a generator for τ decays and in use since mid 80's, it was exstensively used by LEP experiments, also Cleo, BaBar, Belle.
2. It was communicating with the other programs through the customized interfaces.
3. Universal interface based on FORTRAN event record HEPEVT since 2001.
4. Universal interface based on C++ event record HepMC since 2010.
5. Reverted interface provided starting point for TauSpinner in 2011. It uses library of τ decay matrix elements of TAUOLA and profits from many years of experience for different difficulties, e.g. numerical stability issues.
6. No decays of τ leptons are generated. Instead complete events from the production files are read in. They may include all detector response.
7. The experience of PHOTOS Monte Carlo for bremsstrahlung in decays was also essential for avoiding numerical stability traps.

Photos, Tauola++ and TauSpinner communicate through **event record:**



- Parts:

- hard process: (Born, weak, new physics),
- parton shower,
- τ decays
- -QED bremsstrahlung

- Detector studies: acceptance, resolution lepton with or without photon.

Such organization requires:

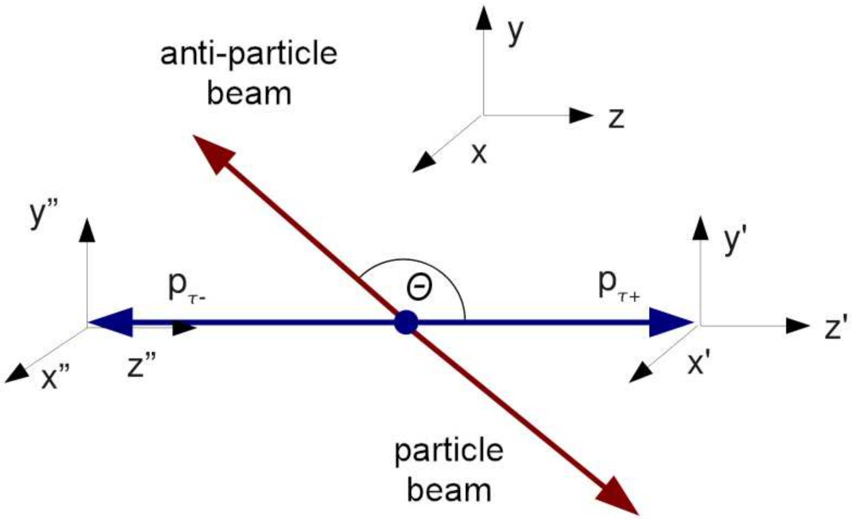
- Good control of factorization (theory)
- Good understanding of tools on user side.
- For Tauola++:

Web page **http://tauolapp.web.cern.ch/tauolapp/**

Reference **Comp.Phys.Comm. 183 (2012) 821**

What do we need to calculate spin and/or matrix element weight?

1. For matrix element of τ decay we need 4-momenta of all decay products.
2. For hard process (2$\rightarrow$2) level we need to know flavours and 4-momenta of incoming quarks/gluons .
3. Incoming quarks/gluon states can be attributed stochastically on the basis of quark level matrix elements and PDF's (`TauSpinner`) or information read from event record (`Tauola`).
4. Then weights for distinct assumptions on spin and/or hard processes can be attributed.
5. Understanding of factorization. For multi-photon (multi gluon) emissions experience of work for `KK MC, PHOTOS` was used, that is why (2 $\rightarrow$ 3) case is not so important. Second order spin amplitudes were used for design of `TauSpinner` algorithm, see later (2 $\rightarrow$ 4) level and tests.



- Configuration of hard process: flavors and 4-momenta of incoming quarks and outgoing τ's (ν_τ)
- algorithm for spin correlations has no approximation.
- However, method to calculate density matrix from that input usually will impose approximations.
- Density matrix including EW corrections is an option. This arrangement can be used to add Z’ or to play with spin correlation component by component.
- Helicity states are attributed at the end (approximation is then used). Useful for some LEP style analyses.

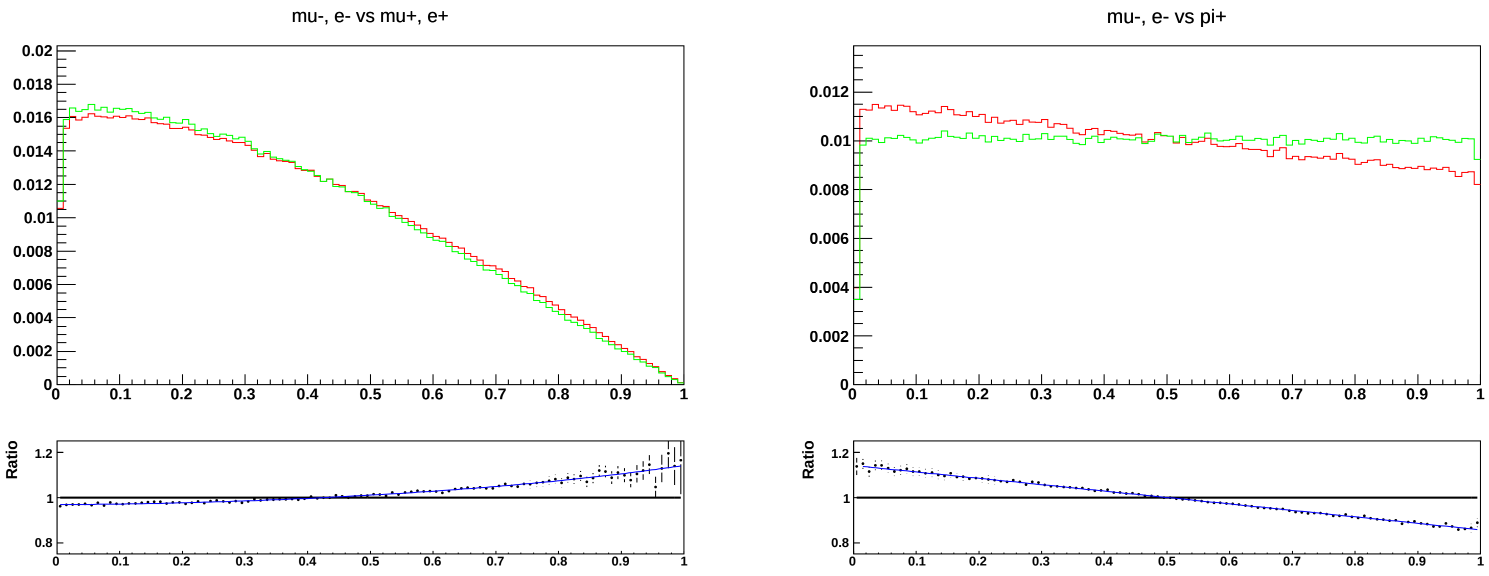
Using wt

Evaluating size of the spin effect

Left: $\tau \to l\nu_l\nu_\tau$ green line – spin effects removed with TauSpinner

Right: $\tau \to \pi\nu_\tau$

Similar plots for other τ decay channels automatically created for events stored on the production files. Also for spin correlation effects. Taken from *Application of* `TauSpinner` *for studies on* τ*-lepton polarization and spin correlations in* Z, W *and* H *decays at LHC*, A. Kaczmarska J. Piatlicki, T. Przedziński, E. Richter-Wąs and Z. Wąs, APP B45 (2014) 1921



P_{orig} = -1.89e-01 ± 3.70e-03

$P_{unweighted}$ = -3.98e-02 ± 4.20e-03

P_{orig} = -1.48e-01 ± 1.98e-03

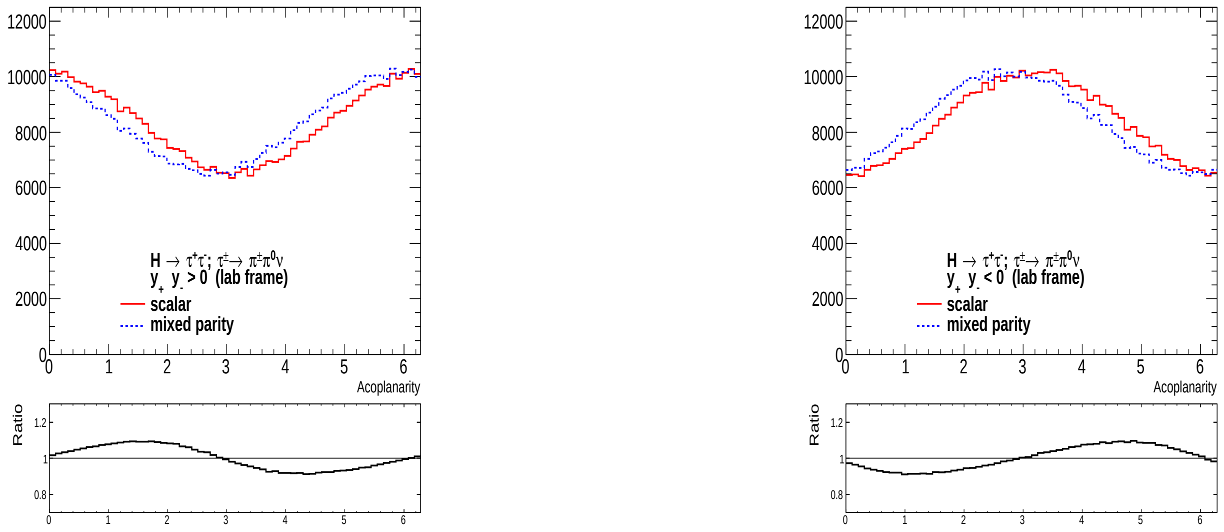
$P_{unweighted}$ = -6.20e-03 ± 2.41e-03

Using wt

Evaluating size of the parity effect

Acoplanarity distribution: $H \to \tau^+\tau^-$, $\tau^\pm \to \pi^\pm\pi^0\nu$ decays calculated in the $\pi^+\pi^0\pi^-\pi^0$ rest frame. Left plot for events with $y_- \cdot y_+ > 0$, right for $y_- \cdot y_+ < 0$, $y_\pm$ calculated in the lab. frame. Compared: scalar (red) and mixed scalar-pseudoscalar (blue dashed), with mixing angle $\theta = 0.2$, cases.

Taken from *TauSpinner: a tool for simulating CP effects in $H \to \tau\tau$ decays at LHC* ,T. Przedzinski, E. Richter-Was, Z. Was Eur.Phys.J. C74 (2014) no.11, 3177



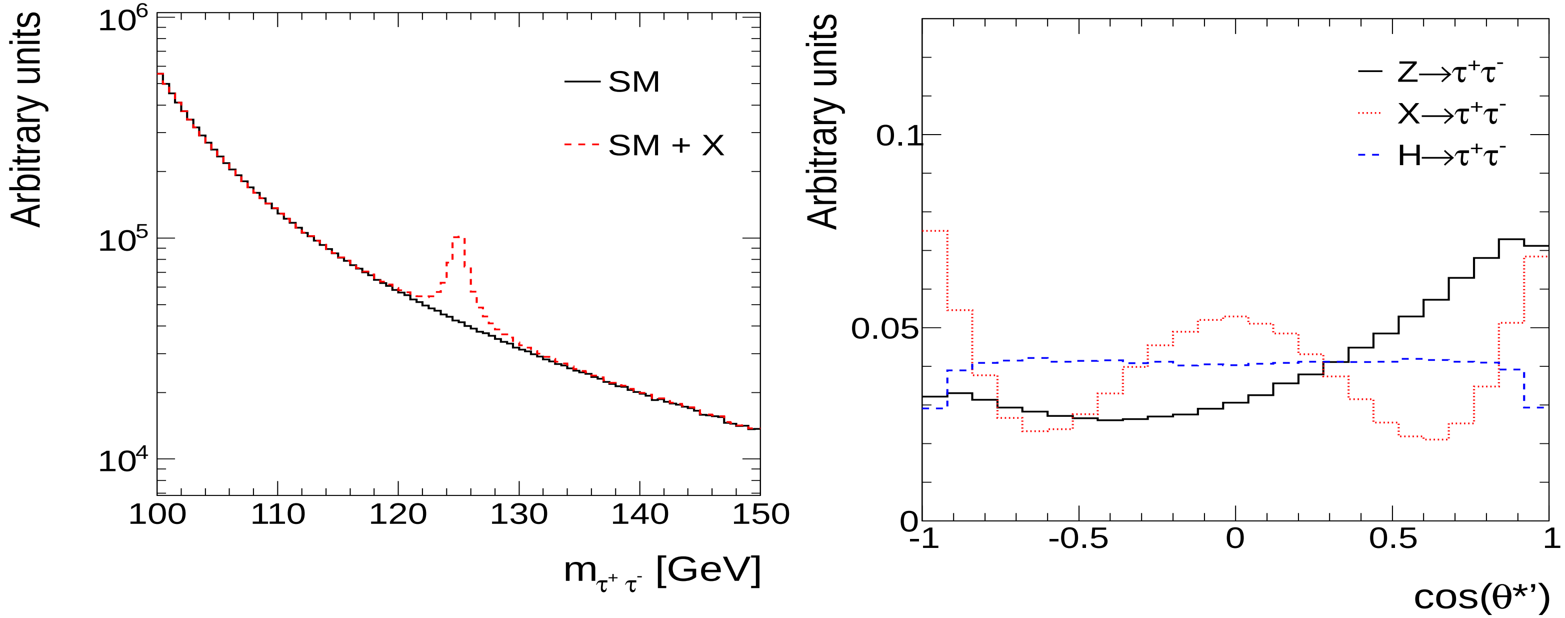
Using $\left(\sum_{spin} |\mathcal{M}^{prod}|^2\right)$

Implementing resonance with `TauSpinner` weights case of X_2

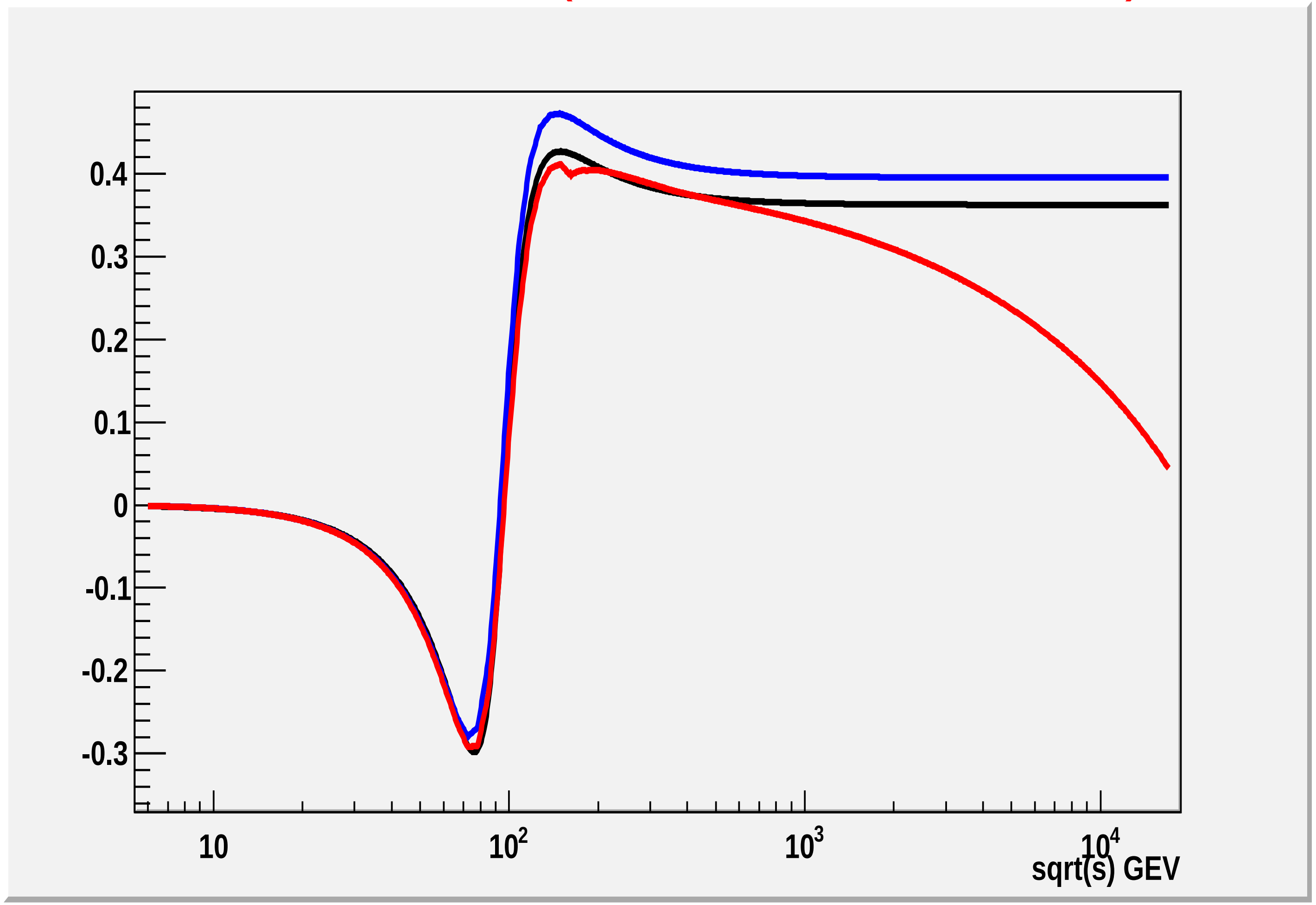
Left: invariant mass of the τ pair, SM black line, red line with effect from X.

The $\cos(\theta^\star)$ for $Z \to \tau^+\tau^-$, $X \to \tau^+\tau^-$, and $H \to \tau^+\tau^-$ events, invariant mass of $\tau^+\tau^-$ pair: 125 $\pm$ 3 GeV.

Ascertaining the spin for new resonances decaying into tau+ tau- at Hadron Colliders S. Banerjee, J. Kalinowski, W. Kotlarski, T. Przedzinski, Z. Was, Eur.Phys.J. C73 (2013) 2313



Using $\left(\sum_{spin} |\mathcal{M}^{prod}|^2\right)$



Effect of electroweak corrections on τ-polarization, up quarks. Red line includes electroweak corrections, Black is TAUOLA/TauSpinner standard and blue is Born, alpha scheme. Scattering angle $\cos\theta = -0.2$

Principles of MC methods and TauSpinner (2→4) case.

- Monte Carlo methods–part of applied mathematics. Everything can be under strict control. One can **always** parametrize the integral over the phase space:

$$G = \int_0^1 \prod_{j=1}^{n} d\hat{x}_j \;\; g(\hat{x}_1, \hat{x}_2, ..., \hat{x}_n) = \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} g(\; \hat{x}_1^{\;i},\; \hat{x}_2^{\;i}, ..., \; \hat{x}_n^{\;i}\;),$$

- If:

$$d\sigma^A = \sum_{i,j,k,l} f_i^A(x_1) f_j^A(x_2) dx_1 dx_2 \frac{1}{\Phi_{flux}} d\Omega(p_1, p_2;\; p_3, p_4, p_{\tau^+}, p_{\tau^-}) |M^A_{i,j,k,l}(p_1, p_2, p_3, p_4)|^2.$$

- Then use of weight

$$wt^{A\to B}_{prod} = \frac{\sum_{i,j,k,l} f_i^B(x_1) f_j^B(x_2) |M^B_{i,j,k,l}(p_1, p_2, p_3, p_4)|^2 \frac{1}{\Phi_{flux}} d\Omega(p_1, p_2;\; p_3.p_4, p_{\tau^+}, p_{\tau^-})}{\sum_{i,j,k,l} f_i^A(x_1) f_j^A(x_2) |M^A_{i,j,k,l}(p_1, p_2, p_3, p_4)|^2 \frac{1}{\Phi_{flux}} d\Omega(p_1, p_2;\; p_3.p_4, p_{\tau^+}, p_{\tau^-})}$$

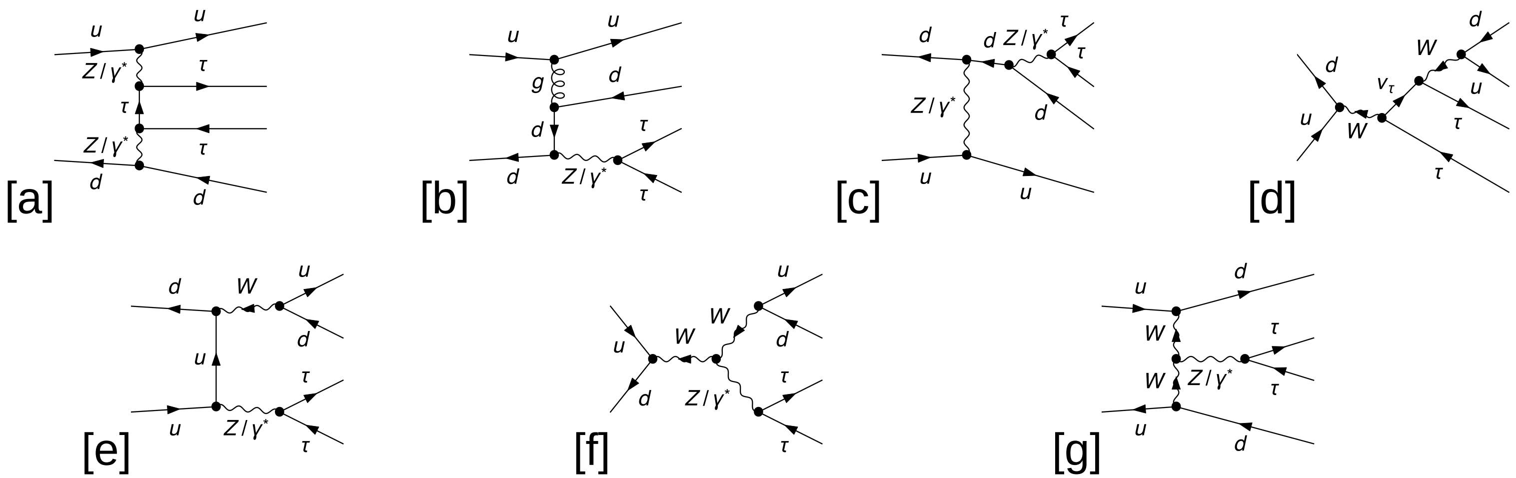
  is nothing else but change of integration variables.

- But it is usually not the case and events are not distributed accordingly to $d\sigma^A$, it is in testing environment only. Otherwise (and usually) we introduce approximations which need to be studied understood and under control.

Presentation TauSpinner $(2 \rightarrow 4)$

- *Production of τ lepton pairs with high p_T jets at the LHC and the* `TauSpinner`*, J. Kalinowski, W. Kotlarski, E. Richter-Wąs and Z. Wąs, arXiv:1604.00964*
- *Only final states of events, consisting of four momenta of two jets and τ leptons, including their subsequent decays are fed into* `TauSpinner`*.*
- Incoming parton momenta are constructed and formulae of slide 15 can be used. Similar as in the $(2 \rightarrow 2)$ case, only four momenta of outgoing τ-s and two jets are summed to hard process four momentum which is then used to reconstruct x_1, x_2 of incoming partons.
- Thus we can have at our disposal complete parton level kinematics, but sum over flavours to get contribution from all partons has to be performed.
- Let us give some details.

Typical topologies of Feynman diagrams



Diagrams contributing to the Drell-Yan–type SM process in $u\bar{d} \to \tau^+\tau^- u\bar{d}$: multi-pheripheral (a), double-t (b), t-cascade (c), s-cascade (d), double-s (e), mercedes (f) and fusion (g) type of diagrams.

To the Higgs production process $u\bar{d} \to H(\to \tau^+\tau^-)u\bar{d}$: vector boson fusion (a), Higgs-strahlung (b),

Matrix elements, grouped by incoming partons.

Table 1: For each category, files for the matrix elements (at present from MadGraph5 and in FORTRAN), are given in second column. Example processes, last column. Permutations of partons and/or CP symmetry used to reduce size of the code.

Category of Matrix Elements	Corresponding FORTRAN files	Processes
(1)	GG.f	$gg \to \sum_f q_f \bar{q}_f$
(2)	GD.f, GU.f	$gq_f(\bar{q}_f) \to gq_f(\bar{q}_f)$
(3)	DD.f, UD.f, UU.f, CC.f, CS.f, DC.f, DS.f, SS.f CD.f, CU.f, SD.f, SU.f, US.f	$q_{f_1}\, q_{f_2}\, (\bar{q}_{f_1}\, \bar{q}_{f_2}) \to q_{f_1}\, q_{f_2} (\bar{q}_{f_1}\, \bar{q}_{f_2})$
(4)	DDX.f, UDX.f, UUX.f CCX.f, CSX.f, DCX.f, DSX.f, SCX.f, SSX.f, UCX.f, USX.f, CDX.f, CUX.f, SDX.f, SUX.f	$q_{f_1}\, \bar{q}_{f_2}\, (\bar{q}_{f_1}\, \bar{q}_{f_2}) \to q_{f_1}\, \bar{q}_{f_2} (\bar{q}_{f_1}\, \bar{q}_{f_2})$ $q_{f_1}\, \bar{q}_{f_2}\, (\bar{q}_{f_1}\, \bar{q}_{f_2}) \to gg$

Typical variants of initialization (not only for MadGraph)

Table 2: Implemented EW schemes, the recommended EW scheme is EWSH=4 which gives the τ lepton polarisation on the Z-boson mass peak, in agreement with the measurement at LEP1 and physical W boson mass.

Type input:	EWSH=1 G_F, α_{QED}, m_Z	EWSH=2 $G_F, sin^2\theta_W, m_Z$	EWSH=3 G_F, m_W, m_Z	EWSH=4 $G_F, m_W, m_Z, \sin^2\theta_W^{eff}$
m_Z	91.1882 GeV	91.1882 GeV	91.1882 GeV	91.1882 GeV
m_W	80.4190	79.9407 GeV	80.4189 GeV	80.4189 GeV
$sin^2\theta_W$	0.222246	0.231470	0.222246	0.231470
$1/\alpha_{QED}$	132.5070	128.7538	132.5069	127.2272
G_F	1.1664 10^{-5} GeV^{-2}	1.1664 10^{-5} GeV^{-2}	1.1664 10^{-5} GeV^{-2}	1.1664 10^{-5} GeV^{-2}

What are numerical consequences. It can be studied with TauSpinner too...

Typical plots of tests, note numerical pseudo-problems.

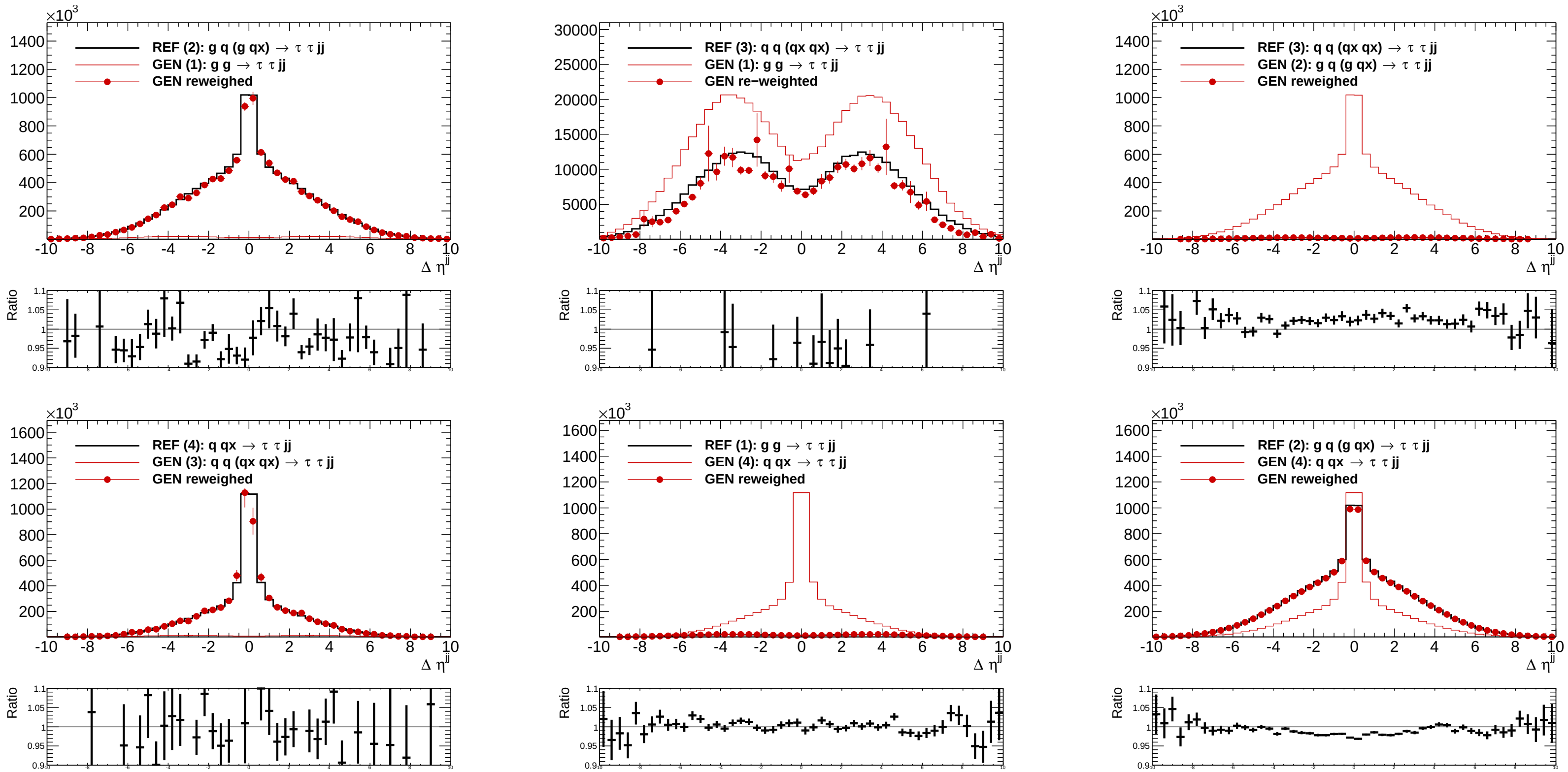


Figure 1: Distributions of the pseudorapidity gap of outgoing partons for the GEN sub-process (thin red line) and after its reweighting to the reference one (GEN reweighted, red points). Reference distr. REF is with a black line. GEN and REF sub-processes are defined in Table of slide19. The qx denote antiquark; $\bar{q}$.

Effects of ISR: parton shower or jets

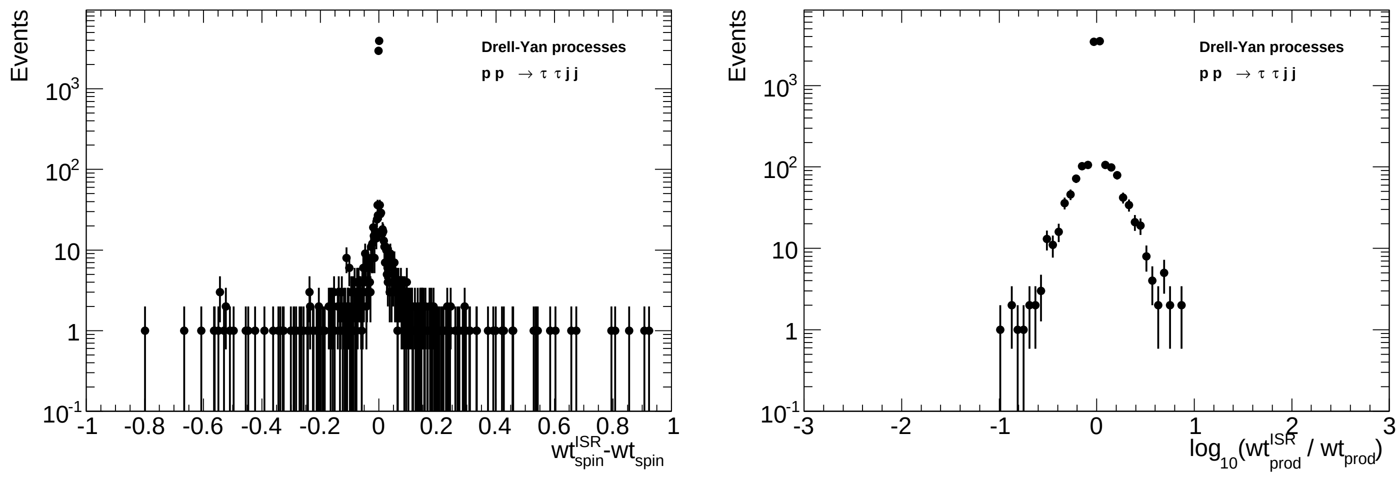


Figure 2: Impact on the matrix element calculation of parton shower smearing, as explained in the text. On the left, the difference of spin weights calculated with and without ISR parton shower kinematic smearing is shown. On the right, the ratio of matrix element weights calculated for the two cases is shown. Sample of 10000 events was used.

(2→2) or (2→4)

Table 3: τ-lepton polarisation in $\tau\tau jj$ events, calculated from weight wt_{spin} of $(2 \rightarrow 2)$ and $(2 \rightarrow 4)$ processes and G_F EW scheme: with $\sin^2\theta_W$ = 0.22222. Invariant mass of the τ pair of $m_Z \pm 10$ GeV and low threshold on outgoing partons transverse momenta, $p_T > 1$ GeV. Rows of the Table correspond to different subsets of events generated with `MadGraph5`. Last column, we restrict formula slide 15 to the parton processes used for the selected subset of events.

Process	Fraction of events	Polarisation $(2 \rightarrow 2)$ Average	Polarisation $(2 \rightarrow 4)$ Average	Polarisation $(2 \rightarrow 4)$ Process specific
All processes		-0.2142 $\pm$ 0.0003	-0.2140 $\pm$ 0.0003	-0.2135 $\pm$ 0.0003
$g\,g \rightarrow \tau\,\tau\,jj$	3.1%	-0.2085 $\pm$ 0.0018	-0.2094 $\pm$ 0.0018	-0.2122 $\pm$ 0.0018
$g\,q\,,\,g\,\bar{q} \rightarrow \tau\,\tau\,jj$	59.3%	-0.2132 $\pm$ 0.0004	-0.2133 $\pm$ 0.0004	-0.2130 $\pm$ 0.0004
$q\,q\,,\,\bar{q}\,\bar{q} \rightarrow \tau\,\tau\,jj$	1.8%	-0.2151 $\pm$ 0.0024	-0.2167 $\pm$ 0.0024	-0.2146 $\pm$ 0.0024
$q\,\bar{q} \rightarrow \tau\,\tau\,jj$	35.7%	-0.2163 $\pm$ 0.0005	-0.2156 $\pm$ 0.0005	-0.2140 $\pm$ 0.0005

EWSH choice – impact on τ polarization

Table 4: Polarisation of the τ-lepton in $\tau\tau jj$ events, calculated using `TauSpinner` weight wt_{spin} of $(2 \to 2)$ and $(2 \to 4)$ processes and different EW schemes. Required is the invariant mass of the τ pair of $m_Z \pm 10$ GeV and low threshold on gluon transverse momenta of $p_T > 1$ GeV.

EW parameter (sensitive)	EW scheme	Polarisation $(2 \to 2)$	Polarisation $(2 \to 4)$
$\sin^2\theta_W$ = 0.222246	EWSH=1	-0.2140 $\pm$ 0.0004	-0.2134 $\pm$ 0.0004
$\sin^2\theta_W$ = 0.231470	EWSH=2	-0.1488 $\pm$ 0.0008	-0.1487 $\pm$ 0.0008
$\sin^2\theta_W$ = 0.222246	EWSH=3	-0.2140 $\pm$ 0.0008	-0.2144 $\pm$ 0.0008
$\sin^2\theta_W$ = 0.231470	EWSH=4	-0.1488 $\pm$ 0.0008	-0.1486 $\pm$ 0.0008

EWSH choice – impact on $\pi^{\pm}$ energy slope

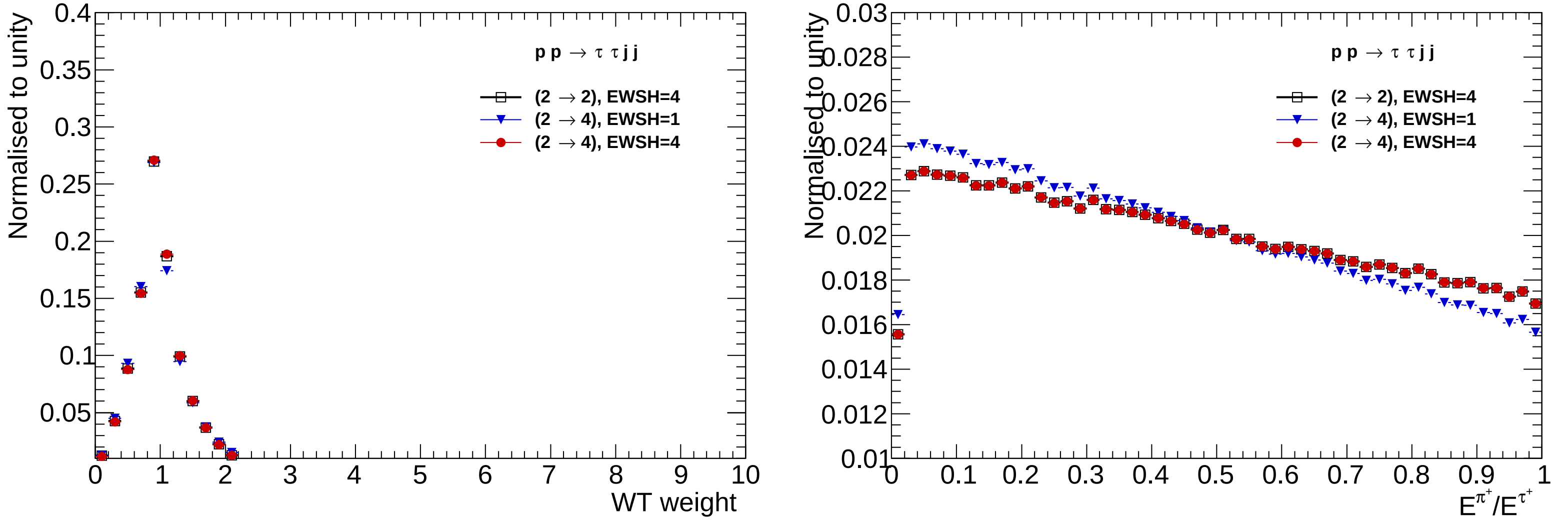


Figure 3: Spin weight (left), and the energy fraction of the τ carried by $\pi^{\pm}$ in $\tau \to \pi^{\pm}\nu$ (right), weighted with $(2 \to 2)$ and $(2 \to 4)$ matrix elements and for different EW schemes.

Summary

1. New extension of `TauSpinner` to $2 \to 4$ processes became available last week: http://tauolapp.web.cern.ch/tauolapp/ , arXiv:1604.00964.
2. Primary goal is to allow for studies on spin and alternative matrix element effects in the VBF Higgs production and Drell-Yan background processes, with the help of weights. However:
   - It seems that it can be also useful for studies of EW schemes for calculations/simulations oriented on QCD, where lowest order EW schemes are sometimes the only one available.
3. User can introduce, with the help of pointers, his own modified methods for calculation of matrix elements or $\alpha_s(Q^2)$.
4. EWSH and scales of Q^2 can be fixed with the help of input parameters.
5. Code is C++, except classical or automatically created, well isolated, routines for matrix elements, which are still in FORTRAN. `configure, make install, make, (automake)` installation, but not `cmake`.