Two-loop power corrections to DIS non-singlet structure functions and sum rules

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HiggsTools Annual Meeting, Granada

13 April 2016

Outline

Introduction

DIS structure functions and the contribution of heavy quarks

Two-loop power corrections

Structure functions

- Neutral current DIS: $g_1(x, Q^2)$, $F_1(x, Q^2)$ and $F_2(x, Q^2)$
- Charged current DIS: $F_1(x, Q^2)$, $F_2(x, Q^2)$ and $F_3(x, Q^2)$

DIS sum rules

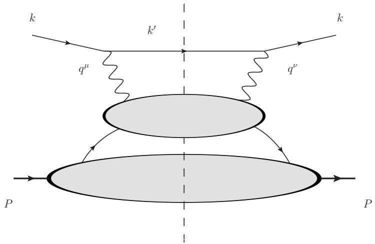
- polarized Bjorken sum rule
- unpolarized Bjorken sum rule

Conclusion

Summary and future developments

Introduction to DIS

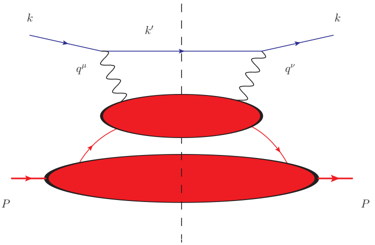
- Scattering experiments of leptons off nucleon targets provide an ideal environment to study QCD interactions
- The scattering cross section to lowest order in the electroweak interaction is factorized in its leptonic and hadronic parts



Introduction to DIS

- Scattering experiments of leptons off nucleon targets provide an ideal environment to study QCD interactions
- The scattering cross section to lowest order in the electroweak interaction is factorized in its leptonic and hadronic parts

$$\frac{d\sigma}{dxdq^2} \propto L_{\mu\nu}(k,q) \cdot W^{\mu\nu}(P,q) \quad (1)$$



The hadronic tensor is parameterized by **structure functions**: *e.g.* for unpolarized e.m. $e\,p$ scattering

$$W^{\mu,\nu}(P,q) = \left[g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right] F_L(x,Q^2) + \left[P^\mu P^\nu + \frac{q^\mu P^\nu + P^\mu q^\nu}{2x} - \frac{Q^2}{4x^2} g^{\mu\nu}\right] \frac{2x}{Q^2} F_2(x), \quad (2)$$

$$Q^2 = -q^2 \qquad x = \frac{Q^2}{2P\cdot q}$$

Heavy flavour

Define the inclusive contribution of the heavy flavour with mass m_Q to the structure function

$$F_i(x, Q^2, m_Q^2) = F_i^{\text{light}}(x, Q^2) + F_i^{\text{heavy}}(x, Q^2, m_Q^2). \quad (3)$$

$F_i^{\text{light}}(x, Q^2)$ and $F_i^{\text{heavy}}(x, Q^2, m_Q^2)$ are convolutions of non-perturbative PDFs and Wilson coefficients encoding the short-distance (perturbative) dynamics of the process.

Current status

- The massless Wilson coefficients $C_{i,j}$ are currently known up to three loops.
- The calculation of massive Wilson coefficients $\tilde{\mathcal{H}}_{i,j}$ in the asymptotic limit $Q^2 \gg m_Q^2$ at three-loop order is ongoing.

This work

- Two-loop power corrections in the ratio $\frac{m_Q^2}{Q^2}$ to the structure functions in neutral current and charged current DIS.
- Two-loop power corrections to the associated sum rules.

Neutral current

The hadronic tensor for lepton-proton e.m. scattering is decomposed $W_{\mu\nu} = W_{\mu\nu}^{(S)} + i\, W_{\mu\nu}^{(A)}$

$$W_{\mu\nu}^{(S)} = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) \frac{F_L(x,Q^2)}{2x} + \left[P_\mu P_\nu + \frac{q^\mu P^\nu + P^\mu q^\nu}{2x} - \frac{Q^2}{4x^2} g^{\mu\nu}\right] \frac{2x}{Q^2} F_2(x,Q^2),$$
$$W_{\mu\nu}^{(A)} = -\frac{M}{P\cdot q}\epsilon_{\mu\nu\rho\sigma} q^\rho \left[S^\sigma \widehat{g}_1(x,Q^2) + \left(S^\sigma - \frac{S\cdot q}{P\cdot q}P^\sigma\right)\widehat{g}_2(x,Q^2)\right], \tag{4}$$

M, P^μ and S^μ are respectively the nucleon mass, momentum and spin. Factorization for the flavour non-singlet (NS) structure functions reads

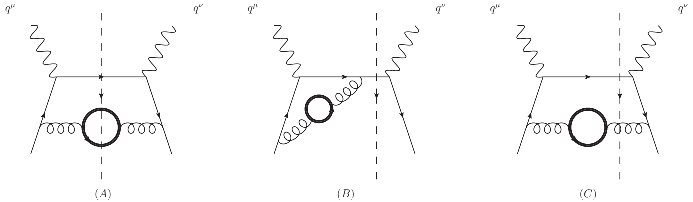
$$\widehat{g}_1(x,Q^2) = \frac{1}{2}\int_x^1 \frac{dz}{z}\left[C^{\mathrm{NS}}_{g_1,q}\left(z, \frac{Q^2}{\mu^2}\right) + L^{\mathrm{NS}}_{g_1,q}\left(z, \frac{Q^2}{m_Q^2}, \mu^2\right)\right] \cdot \tilde{\Delta}\left(\frac{x}{z}, \mu^2\right), \tag{5}$$

where $\tilde{\Delta}(x,\mu^2) = \sum_i e_i^2\ \left[\Delta f_i(x,\mu^2) + \Delta f_{\bar{i}}(x,\mu^2)\right]$.

- NS contributions are characterized by the coupling of the incoming quark and the virtual photon.
- The NS terms are selected by measuring differences of cross sections such as $\left(\frac{d\sigma}{dxdQ^2}\right)^{e\,p} - \left(\frac{d\sigma}{dxdQ^2}\right)^{e\,n}$.

$L^{\mathrm{NS}}_{g_1,q}$: two-loop power corrections

Inclusive scattering process $q + \gamma^* \longrightarrow q + X$, heavy quarks in the final state X or in loops



$$L^{\mathrm{NS}}_{g_1,q}(z, Q^2, m_Q^2) = \underbrace{\Theta\left(\frac{\xi}{\xi+4} - z\right) L^{\mathrm{NS,\,(R)}}_{g_1,q}(z,\xi)}_{(A)} + \underbrace{\delta(1-z) L^{\mathrm{NS,\,(V)}}_{g_1,q}(z,\xi)}_{(B)}$$

$$\underbrace{- \left(\frac{\alpha_s}{4\pi}\right)^2 \beta_{0,Q} \log\left(\frac{m_Q^2}{\mu^2}\right) \left[\frac{1}{2} P^{(0)}_{qq} \log\left(\frac{Q^2}{\mu^2}\right) + c^{(1)}_{g_1,q}\right]}_{(C)}, \qquad (6)$$

where $z = \dfrac{Q^2}{2p\cdot q}$, $\xi = \dfrac{Q^2}{m_Q^2}$.

Real radiation

- Re-calculation of the Compton process

$$q + \gamma^* \to q + \gamma^* + Q + \overline{Q}$$

The following variables are used

$$sq_1 = \sqrt{1 - \frac{4}{\xi}\frac{z}{1-z}},\ sq_2 = \sqrt{1 - \frac{4}{\xi}z},\ L_i = \log\left(\frac{1+sq_i}{1-sq_i}\right)\ _{(i=1,2)},\ L_3 = \log\left(\frac{sq_2+sq_1}{sq_2-sq_1}\right),$$

$$\mathrm{di}_1 = \mathrm{Li}_2\left[(1-z)\frac{1+sq_1}{1+sq_2}\right],\ \mathrm{di}_2 = \mathrm{Li}_2\left(\frac{1-sq_2}{1+sq_1}\right),\ \mathrm{di}_3 = \mathrm{Li}_2\left(\frac{1-sq_1}{1+sq_2}\right),\ \mathrm{di}_4 = \mathrm{Li}_2\left(\frac{1+sq_1}{1+sq_2}\right).$$

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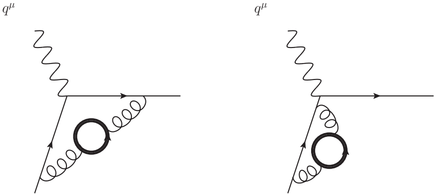
- Agreement with the literature (Buza, Matiounine, Smith, Migneron, van Neerven '96)

$$\begin{aligned} L_{g_1,q}^{\text{NS, (R)}}(z,Q^2) = \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \Bigg\{ &-\frac{8 L_3\, sq_2}{9(z-1)\xi}\Big(50z^3 - 11\xi + z(6\xi+20) - 2z^2(7\xi+12)\Big) \\ &+ \frac{2\, sq_1}{27(z-1)^2\xi}\Big(1200z^4 + 265\xi - 4z^3(109\xi+490) + 2z^2(389\xi+618) - z(607\xi+466)\Big) \\ &+ \frac{4\, L_1}{3(z-1)^3\xi^2}\Big(24z^4 - \xi^2 + 3z^2(\xi^2+6) - 2z^3(\xi^2+18)\Big) + \frac{12z^3 - \xi^2 - z^2\xi^2}{3(z-1)\xi^2} \\ &\times \left[4L_1L_2 + 8(-\text{di}_1 + \text{di}_2 + \text{di}_3 - \text{di}_4) - 4L_1 \log\left(\frac{z^2}{1-z}\right)\right]\Bigg\} \end{aligned} \quad (7)$$

Virtual corrections



- The virtual corrections are given by

$$L^{\text{NS, (V)}}_{g_1,q}(\xi) = 2\mathcal{F}_1^{(2)}\left(-\frac{Q^2}{m_Q^2}\right),$$

$\mathcal{F}_1^{(2)}$ is the two-loop Dirac form factor.

Introducing the variable $\tilde{\lambda} = \sqrt{1-\frac{4}{\xi}}$ the result is

$$\begin{aligned} L^{\text{NS, (V)}}_{g_1,q}(\xi) = 2\left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \Bigg\{ & \frac{3355}{81} - \frac{952}{9\xi} + \left(\frac{32}{\xi^2} - \frac{16}{3}\right)\zeta(3) \\ & + \left(\frac{440}{9\xi} - \frac{530}{27}\right)\log(\xi) + \tilde{\lambda}\left[\frac{184}{9\xi} - \frac{76}{9}\right]\left[\text{Li}_2\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right) - \text{Li}_2\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right)\right] \\ & + \left[\frac{8}{3} - \frac{16}{\xi^2}\right]\left[\text{Li}_3\left(\frac{\tilde{\lambda}-1}{\tilde{\lambda}+1}\right) + \text{Li}_3\left(\frac{\tilde{\lambda}+1}{\tilde{\lambda}-1}\right)\right]\Bigg\} \end{aligned} \quad (8)$$

$g_1(x, Q^2)$ structure function

Both the c quark and the b quark contributions are taken into account by summing

$$L^{\text{NS, tot}} = L^{\text{NS}}\left(z, Q^2, m_c^2\right) + L^{\text{NS}}\left(z, Q^2, m_b^2\right). \tag{9}$$

This term is compared[1] with the known asymptotic expression. Finally I show the relative contribution of massive flavours over the whole structure function.

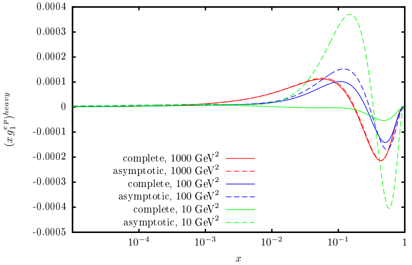


Figure: Comparison of asymptotic and exact massive contributions to $g_1(x, Q^2)$.

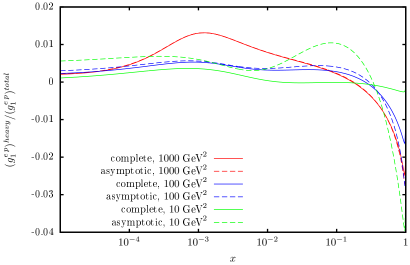


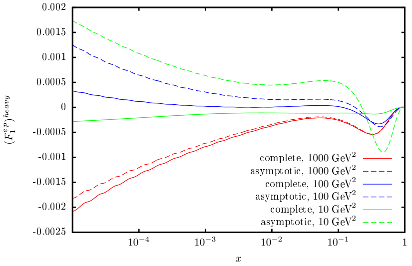
Figure: Relative contribution of the two-loop massive Wilson coefficient to $g_1(x, Q^2)$.

[1] These plots are done with BB09 POLPDF (polarized) and abm12_3_nnlo (unpolarized), $m_c = 1.59$ GeV and $m_b = 4.78$ GeV

$F_1(x, Q^2)$ structure function

Similarly, the symmetric part of the hadronic tensor, $W_{\mu\nu}^{(S)}$, is obtained from the unpolarized $q\ \gamma^*$ scattering process. Illustrations for the combination

$$F_1(x, Q^2) = \frac{1}{2x}\left[F_2(x, Q^2) - F_L(x, Q^2)\right] \tag{10}$$



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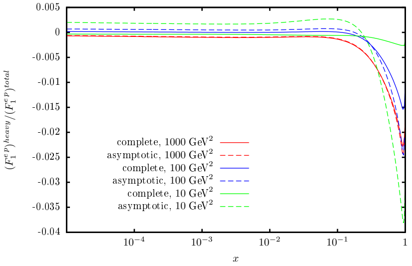

Figure: Relative contribution of the two-loop massive Wilson coefficient to $F_1(x, Q^2)$.

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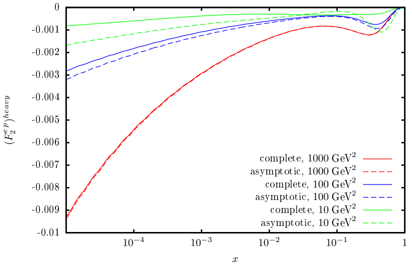
$F_2(x, Q^2)$ structure function

- The asymptotic approximation works better for the structure function $F_2(x, Q^2)$ alone. Indeed $F_1(x, Q^2)$ includes also $F_L(x, Q^2)$, which approaches the asymptotic limit at higher values of Q^2.



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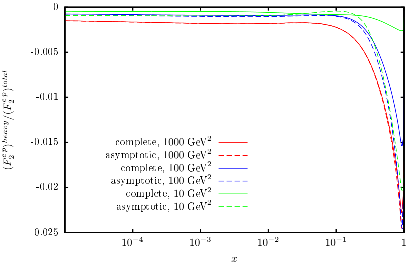

Figure: Relative contribution of the two-loop massive Wilson coefficient to $F_2(x, Q^2)$.

Charged current

The hadronic tensor in charged current DIS, $q + W^{\pm} \to X$, is characterized by

$$W_{\mu\nu}^{(A)} = i\,\epsilon_{\mu\nu\rho\sigma}\,\frac{P^{\rho} q^{\sigma}}{2P \cdot q}\,F_3^{W^{\pm}}(x, Q^2). \tag{11}$$

It is useful to consider the antisymmetric combinations under crossing:

$$F_i^{(-)}(x, Q^2) = \left[F_i^{W^+}(x, Q^2) - F_i^{W^-}(x, Q^2)\right],\ F_3^{(-)}(x, Q^2) = \left[F_3^{W^+}(x, Q^2) + F_3^{W^-}(x, Q^2)\right],$$

$i = 2, L$. Introducing the variable $\tilde{x} = x\frac{Q^2 + m_Q^2}{Q^2}$, factorization reads

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$i = 2, L$. Introducing the variable $\tilde{x} = x\frac{Q^2+m_Q^2}{Q^2}$, factorization reads

$$\begin{aligned} F_2^{(-)}(x,Q^2) = 2\Bigg\{ & x\int_x^1 \frac{dz}{z}\left[|V_{du}|^2\, d_v\left(\frac{x}{z}\right) - (|V_{du}|^2 + |V_{su}|^2)\, u_v\left(\frac{x}{z}\right)\right]\left(C_{2,q}^{\rm NS}(z) + L_{2,q}^{\rm NS}(z)\right) \\ & + \tilde{x}\int_{\tilde{x}}^1 \frac{dz}{z}|V_{dc}|^2\, d_v\left(\frac{\tilde{x}}{z}\right) H_{2,q}^{\rm NS}(z)\Bigg\} \\ F_3^{(-)}(x,Q^2) = 2\Bigg\{ & \int_x^1 \frac{dz}{z}\left[|V_{du}|^2\, d_v\left(\frac{x}{z}\right) + (|V_{du}|^2 + |V_{su}|^2)\, u_v\left(\frac{x}{z}\right)\right]\left(C_{3,q}^{\rm NS}(z) + L_{3,q}^{\rm NS}(z)\right) \\ & + \int_{\tilde{x}}^1 \frac{dz}{z}|V_{dc}|^2\, d_v\left(\frac{\tilde{x}}{z}\right) H_{3,q}^{\rm NS}(z)\Bigg\}, \end{aligned} \tag{12}$$

$q_v(x,\mu^2) = q(x,\mu^2) - \overline{q}_v(x,\mu^2)$ are the valence quark q PDFs.

Charged current

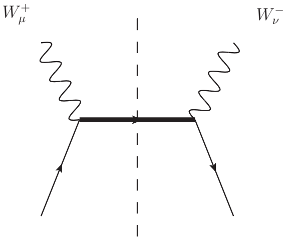
New feature

The Wilson coefficient $H^{\text{NS}}_{i,q}$ describes the coupling of the gauge boson to a heavy flavour quark.

- H^{NS}_{i} is already present at tree level.
- The Bjorken variable x is bounded by

$$\tilde{x} = x\frac{Q^2 + m_Q^2}{Q^2} \leq 1, \qquad (13)$$

  because part of the incoming energy goes in the production of the massive final state (*slow rescaling*).
- The contribution of $H^{\text{NS}}_{i,q}$ is suppressed by the CKM matrix element $|V_{dc}|^2 \simeq 0.05$ with respect to $L^{\text{NS}}_{3,q}$.



The coefficients $H^{\text{NS}}_{i,q}$ are known exactly at one loop (Gottschalk '81; Glück, Kretzer, Reya '96; Blümlein, Hasselhuhn, Kovacikova, Moch '11). CKM suppression allows to use the asymptotic two-loop formulae (Blümlein,Pfoh,Hasselhuhn '14).

Charged current plots: $F_1(x, Q^2)$

- Relying on this approximation $H_{i,q}^{\mathrm{NS}}$ are implemented up to two-loops.
- Massless Wilson coefficients at the same order are also available (Zijlstra, van Neerven '92; Moch, Vermaseren '99; Moch, Rogal, Vogt '07).
- Two-loop power corrections to $L_{i,q}^{\mathrm{NS}}$ for $i = 2, L$ are known from the neutral current calculation.
- Regarding $F_3(x, Q^2)$, $L_{3,q}^{\mathrm{NS}} = L_{g_1,q}^{\mathrm{NS}}$ at two loops.

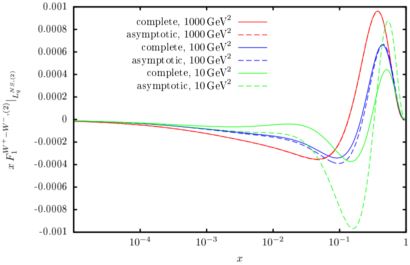


Figure: Exact and asymptotic $L_{1,q}^{\mathrm{NS}}$ contribution to the structure function.

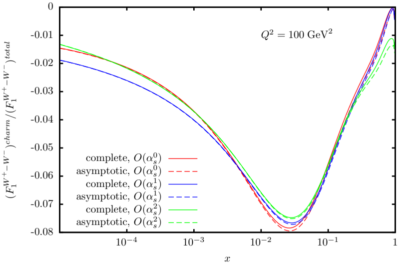
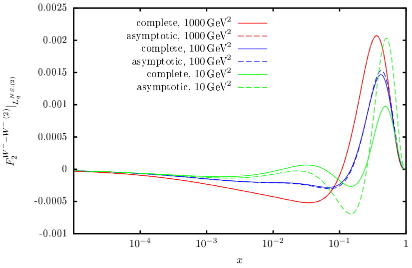
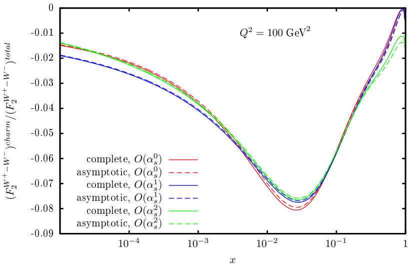
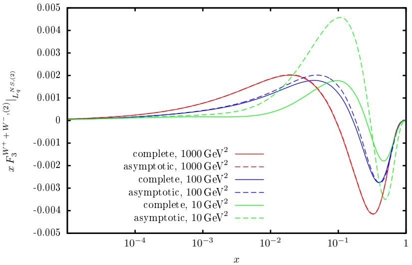


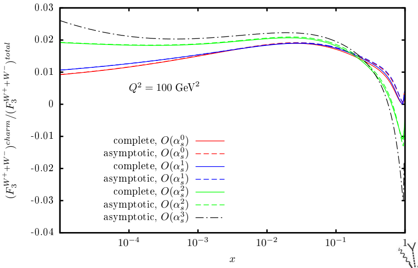
Figure: Impact of the heavy flavours (both $H_{1,q}^{\mathrm{NS}}$ and $L_{1,q}^{\mathrm{NS}}$) on the structure function.

Charged current plots: $F_2(x, Q^2)$ and $F_3(x, Q^2)$









QCD sum rules

The integrals of non-singlet structure functions give a set of sum rules

$$
\begin{aligned}
\Delta g_1(Q^2) &= \int_0^1 dx \left[g_1^{e\,p}(x,Q^2) - g_1^{e\,n}(x,Q^2) \right] = K_{g_1}(n_f)\, A^{g_1}(\alpha_s,Q^2) \quad \textit{Pol. Bjorken},\\
\Delta F_1(Q^2) &= \int_0^1 dx \left[F_1^{\overline{\nu}\,p}(x,Q^2) - F_1^{\nu\,p}(x,Q^2) \right] = K_1(n_f)\, A^{F_1}(\alpha_s,Q^2) \quad \textit{Unpol. Bjorken},\\
\Delta F_2(Q^2) &= \int_0^1 \frac{dx}{x} \left[F_2^{\overline{\nu}\,p}(x,Q^2) - F_2^{\nu\,p}(x,Q^2) \right] = K_2(n_f) \quad \textit{Adler sum rule},\\
\Delta F_3(Q^2) &= \int_0^1 dx \left[F_3^{\overline{\nu}\,p}(x,Q^2) + F_3^{\nu\,p}(x,Q^2) \right] = K_3(n_f)\, A^{F_3}(\alpha_s,Q^2) \quad \textit{Gross Llewellyn Smith}.
\end{aligned} \tag{14}
$$

- $K_i(n_f)$ is the parton model result, QCD radiative corrections are encoded in $A^i(\alpha_s,Q^2)$. Massless contributions known at 4 loops (Baikov, Chetyrkin, Kühn '10; Chetyrkin '14).

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Heavy flavour contribution

Having the analytic expression of $F_i(x, Q^2)$, we compute the power corrections to A^i. We verified the Adler sum rule as non-trivial check on the structure function F_2.

A^{g_1}: two-loop power corrections

The contribution of massive flavours to A^{g_1} is

$$\Delta g_1^{\text{massive}}(\xi) = \int_0^1 dx \int_x^1 \frac{dz}{z} L_{g_1,q}^{\text{NS}}(z) \cdot \left\{ \frac{1}{2} \left[\tilde{\Delta}_p\left(\frac{x}{z}\right) - \tilde{\Delta}_n\left(\frac{x}{z}\right) \right] \right\}$$
$$= \underbrace{\int_0^1 dy \frac{\tilde{\Delta}_p(y) - \tilde{\Delta}_n(y)}{2}}_{K_{g_1}(n_f)=\frac{1}{6}\left|\frac{G_A}{G_V}\right|} \times \underbrace{\int_0^1 dz\, L_{g_1,q}^{\text{NS}}(z)}_{C_{\text{pBj}}^{\text{Q}}(\xi)} \qquad (15)$$

By introducing $\lambda = \sqrt{1+\frac{4}{\xi}}$ and computing the integral of the Wilson coefficient[2] we get

$$C_{\text{pBj}}^{\text{Q}}(\xi) = -\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{C_F T_F}{315\xi^2} \left\{ 2100 \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 - \xi\left(6\xi^2 + 2735\xi + 11724\right) + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \right.$$
$$\left. \times\, \xi\left(3\xi^3 + 106\xi^2 + 1054\xi + 4812\right) - \xi^2\left(3\xi^2 + 112\xi + 1260\right)\log(\xi) \right\}. \qquad (16)$$

$$C_{\text{pBj}}^{\text{Q}}(\xi) \underset{\xi \gg 1}{\longrightarrow} \left(\frac{\alpha_s}{4\pi}\right)^2 C_F T_F \left[8 + \frac{1}{\xi}\left(\frac{272}{9} - \frac{64}{3}\log(\xi)\right) + O\left(\frac{\log^2(\xi)}{\xi^2}\right) \right].$$

Differently to the non-inclusive case (Blümlein, van Neerven '98), no large logarithm survives in the massless limit $Q^2 \gg m_Q^2$.

[2] Two-loop power corrections to the Gross Llewellyn Smith sum rule are identical, as $L_{g_1,q}^{\text{NS}}(z,\xi) = L_{3,q}^{\text{NS}}(z,\xi)$.

Polarized Bjorken sum rule

Features

The contribution of heavy flavours

- implements the transition to one more massless flavour $n_f \to n_f + 1$ in the limit $Q^2 \gg m_Q^2$

$$A^{g_1}(\xi) = 1 - \frac{\alpha_s}{\pi} + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-4.58333 + 0.33333\, n_f + C_{\mathrm{pBj}}^{Q,(2)}(\xi) \right] + \mathcal{O}(\alpha_s^3)\,, \qquad (17)$$

- becomes even negative at small scales $Q^2 = m_Q^2$, because of the virtual corrections.

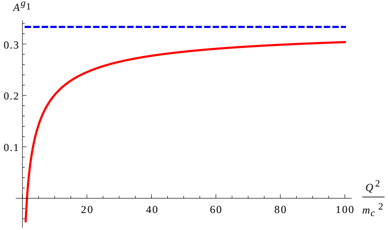


Figure: $C_{\mathrm{pBJ}}^{c,(2)}(\xi)$ (solid) compared to the transition to one more massless flavour $n_f \to n_f + 1$ (dashed).

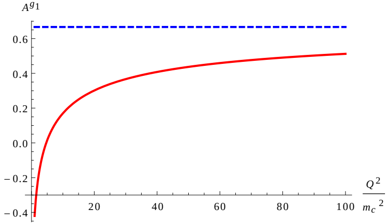


Figure: Contribution of bottom and charm quarks $C_{\mathrm{pBJ}}^{c,(2)}(\xi) + C_{\mathrm{pBJ}}^{b,(2)}(\xi)$ (solid) vs $n_f \to n_f + 2$.

Unpolarized Bjorken sum rule

$$A^{F_1} = 1 - \left(\frac{\alpha_s}{\pi}\right)\left[0.66667 + C_{\mathrm{uBj}}^{\mathrm{Q},(1)}(\xi)\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-3.83333 + 0.29630\, n_f + C_{\mathrm{uBj}}^{\mathrm{Q},(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \tag{18}$$

$C_{\mathrm{uBj}}^{\mathrm{Q},(1)}(\xi)$: first moment of the one-loop Wilson coefficient H_1 (Blümlein, van Neerven ’98).

Unpolarized Bjorken sum rule

$$A^{F_1} = 1 - \left(\frac{\alpha_s}{\pi}\right)\left[0.66667 + C_{\rm uBj}^{\rm Q,(1)}(\xi)\right] + \left(\frac{\alpha_s}{\pi}\right)^2 \left[-3.83333 + 0.29630\, n_f + C_{\rm uBj}^{\rm Q,(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \tag{18}$$

$C_{\rm uBj}^{\rm Q,(1)}(\xi)$: first moment of the one-loop Wilson coefficient H_1 (Blümlein, van Neerven '98).

$$C_{\rm uBj}^{\rm Q,(2)}(\xi) = {\rm C}_F {\rm T}_F \left\{ \frac{1129}{2520} - \frac{1}{2\xi^2} \log\left(\frac{\lambda+1}{\lambda-1}\right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda \log\left(\frac{\lambda+1}{\lambda-1}\right) \times \left(-\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840}\right) + \left(\frac{1}{6} - \frac{\xi^2}{840}\right)\log(\xi)\right\}, \tag{19}$$

$$C_{\rm uBj}^{\rm Q,(2)}(\xi) \underset{\xi\gg 1}{\longrightarrow} C_F T_F \left[\frac{4}{9} + \frac{1}{\xi}\left(\frac{20}{9} - \frac{4}{3}\log(\xi)\right)\right] + {\rm O}\left(\frac{\log^2(\xi)}{\xi^2}\right).$$

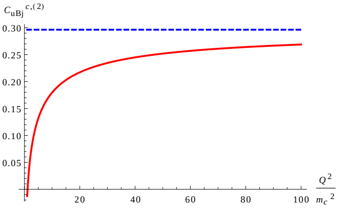
Unpolarized Bjorken sum rule

$$A^{F_1} = 1 - \left(\frac{\alpha_s}{\pi}\right)\left[0.66667 + C_{\rm uBj}^{{\rm Q},(1)}(\xi)\right] + \left(\frac{\alpha_s}{\pi}\right)^2\left[-3.83333 + 0.29630\, n_f + C_{\rm uBj}^{{\rm Q},(2)}(\xi)\right] + \mathcal{O}(\alpha_s^3), \quad (18)$$

$C_{\rm uBj}^{{\rm Q},(1)}(\xi)$: first moment of the one-loop Wilson coefficient H_1 (Blümlein, van Neerven '98).

$$C_{\rm uBj}^{{\rm Q},(2)}(\xi) = {\rm C}_F {\rm T}_F \left\{\frac{1129}{2520} - \frac{1}{2\xi^2}\log\left(\frac{\lambda+1}{\lambda-1}\right)^2 + \frac{107}{42\xi} - \frac{\xi}{420} + \lambda\log\left(\frac{\lambda+1}{\lambda-1}\right) \times\left(-\frac{67}{420} - \frac{43}{42\xi} - \frac{\xi}{420} + \frac{\xi^2}{840}\right) + \left(\frac{1}{6} - \frac{\xi^2}{840}\right)\log(\xi)\right\}, \quad (19)$$

$$C_{\rm uBj}^{{\rm Q},(2)}(\xi) \underset{\xi\gg 1}{\longrightarrow} C_F T_F\left[\frac{4}{9} + \frac{1}{\xi}\left(\frac{20}{9} - \frac{4}{3}\log(\xi)\right)\right] + {\rm O}\left(\frac{\log^2(\xi)}{\xi^2}\right).$$



- No logarithmic corrections in the mass scales involved.
- Smooth interpolation to the regime $n_f \to n_f + 1$.

Summary and outlook

Power corrections to structure functions

- We calculate the power corrections to the Wilson coefficients $L^{\mathrm{NS}}_{g_1,q}$, $L^{\mathrm{NS}}_{2,q}$, $L^{\mathrm{NS}}_{L,q}$ at two loops.
- These Wilson coefficients are used to compute all the non singlet structure functions in neutral and charged current DIS. In the latter case it is necessary to include also single-excitation processes, already known in the literature.
- A natural development of this project is an improvement of the analysis of DIS data with the complete heavy flavour effects at two loops.

Sum rules

- We compute the power corrections to the polarized and unpolarized Bjorken sum rules and to the Gross Llewellyn Smith sum rule. We verified the Adler sum rule as a check on $L^{\mathrm{NS}}_{2,q}$.
- Massive quarks provide a smooth interpolation of the sum rules from n_f to $n_f + 1$ in the limit $\frac{m_Q^2}{Q^2} \to 0$.
- The transition to $n_f + 1$ takes place at large scales; at $Q^2 \simeq m_Q^2$ the effect can be negative.

Thank you for your attention!