NLO Correction to the Elastic Scattering Cross Section of an Electron off of a Static Scattering Center

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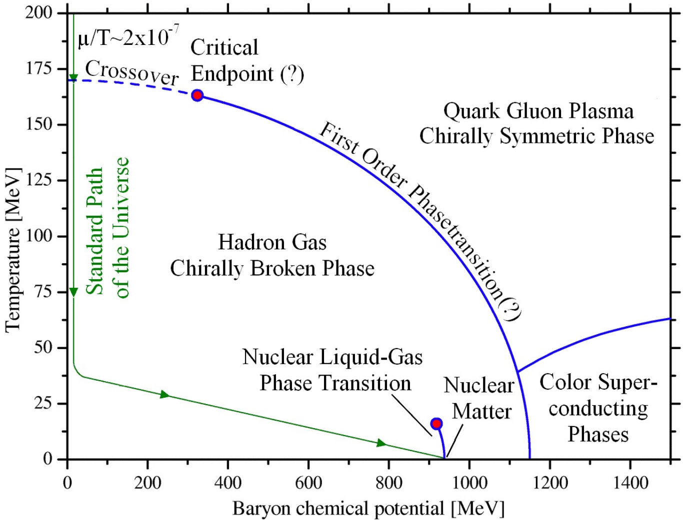
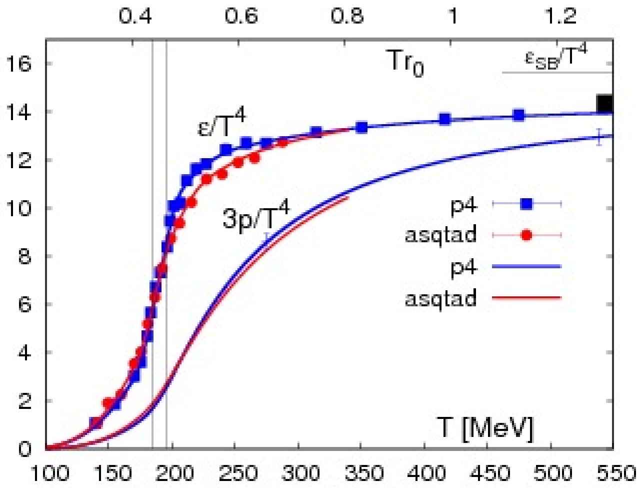
Overview

Introduction
QCD Phase Diagram



Temperature dependence of the energy density



At RHIC we study the dynamics of the QGP in two different limits:

Strongly coupled limit

- It is non-perturbative approach.
- Gives a good estimate for the dynamics of the particle at low $p_\perp$.

Weakly coupled limit

- It is perturbative approach, based on the asymptotic freedom of QCD.
- It describes the physics associated with high $p_\perp$.

Why weakly coupled limit?

QED Lagrangian with the external source

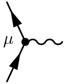
Consider the Lagrangian of an electron scattered with a fixed point charge

$$\mathscr{L} = -\frac{1}{4}\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi - e\overline{\psi}\gamma^{\mu}\psi A_{\mu} + eJ_{\mu}A^{\mu}$$

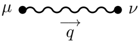
Where

$$J^{\mu} = V^{\mu}\delta(\vec{x} - \vec{v}x^{0})$$
$$V^{\mu} = (1, 0)^{\mu}$$

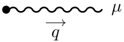
Leading Term
Feynman Rules of The Leading Term

For each vertex: 

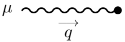
$= -ie\gamma^{\mu}$

For each internal photon: 

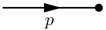
$= \frac{-ig_{\mu\nu}}{q^2+i\epsilon}$

For each incoming external photon: 

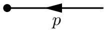
$= \varepsilon_{\mu}(q)$

For each outgoing external photon: 

$= \varepsilon^{*}_{\mu}(q)$

For each incoming external fermion: 

$= u(p)$

For each outgoing external fermion: 

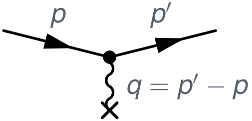
$= \bar{u}(p)$

For each external source: 
$= -ie\, V^{\mu}$

Leading Order of the Cross Section

Using feynman rules for leading term

$$i\mathcal{M}_0 =$$



$$= \frac{i\,e^2}{q^2}\,\overline{u}^{s'}(p')\,\gamma^0\,u^s(p)$$

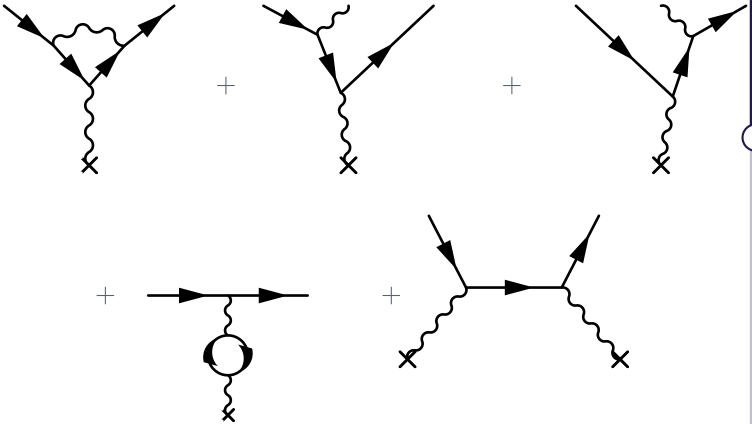
The cross section of the leading term will be

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \frac{1}{32\pi^2}\sum_{s,s'}|\mathcal{M}_0|^2$$

$$= \frac{\alpha^2}{q^4}(4E^2 - q^2)$$

Next-to-Leading Order $\mathcal{O}(\alpha^3)$

NLO Diagrams



But the first four diagrams contain divergent parts due to either the loop integrals or emitting soft photons!

We follow the renormalization steps:

1. We define the Lagrangian in terms of the bare parameters

$$\mathscr{L}_0 = -\frac{1}{4}\left(F_0^{\mu\nu}\right)^2 + \overline{\psi}_0\left(i\partial\!\!\!/ - m_0\right)\psi_0 - e_0\,\overline{\psi}\gamma^\mu\psi A_{0\mu} + e_0\,J_{0\mu}A_0^\mu$$

2. We renormalize the bare fields (ψ_0 and A_0^μ) and the bare parameters (e_0 and m_0) by defining the renormalization parameters Z_ψ, Z_A, Z_e and Z_m

$$\psi_0 = Z_\psi^{\frac{1}{2}}\psi$$
$$A_0^\mu = Z_A^{\frac{1}{2}}A^\mu$$
$$Z_\psi m_0 = Z_m m$$
$$e_0 Z_\psi Z_A^{\frac{1}{2}} = Z_e e$$

Renormalization Procedure Cont...

3. We expand the renormalization patrameters in terms of the counter terms

$$Z_\psi = 1 + \delta_\psi$$
$$Z_A = 1 + \delta_A$$
$$Z_e = 1 + \delta_e$$
$$Z_m = 1 + \delta_m$$

4. We rewrite the Lagrangian in terms of the Renormalized fields and parameters (ψ, A, m and e and the counter terms)

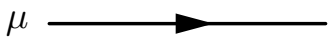
$$\mathscr{L} = -\frac{1}{4}\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\partial\!\!\!/ - m\right)\psi - e\,\overline{\psi}\gamma^\mu\psi A_\mu + e\,J_\mu A^\mu$$
$$-\frac{1}{4}\delta_A\left(F^{\mu\nu}\right)^2 + \overline{\psi}\left(i\delta_\psi\,\partial\!\!\!/ - m\,\delta_m\right)\psi - e\,\delta_e\,\overline{\psi}\gamma^\mu\psi A_\mu + e\,J_\mu A^\mu$$

Renormalization

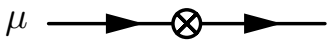
Feynman Rules of The Renormalized Lagrangian

$= \frac{-ig_{\mu\nu}}{q^2+i\epsilon} \quad \Rightarrow$

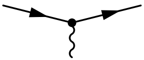
$= -i(g^{\mu\nu}q^2 - q^\mu q^\nu)\,\delta_A$



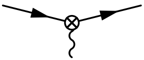
$= \frac{i}{\not{p}-m+i\epsilon} \quad \Rightarrow$



$= i(\not{p}\,\delta_\psi - m\,\delta_m)$



$=$ -$ie\gamma^\mu \quad \Rightarrow$



$= -ie\gamma^\mu\delta_e$

- Dimensional Regularization to regularize the U.V divergences, which requires Introducing the mass scale μ.

$$\int \frac{d^4 k}{(2\pi)^4} \rightarrow \int \frac{d^d k}{(2\pi)^d} \qquad \Rightarrow \qquad e \rightarrow e\,\mu^{\frac{4-d}{2}}$$

- Mass Regularization (m_γ) to regularize the IR divergences.

$$\frac{-ig_{\mu\nu}}{k^2} \rightarrow \frac{-ig_{\mu\nu}}{k^2 + m_\gamma^2}$$

- $\overline{MS}$ Renormalization Scheme.

Why did we use $\overline{MS}$?

On-shell renormalization scheme:

- We just use the renormalization conditions to get ride of the divergence parts.
- The physical quantities are the renormalized ones.
- The differential cross section is divergent as we send the mass of the electron (m_e) to be zero.

$\overline{MS}$ renormalization scheme:

- We just choose the counter terms such that it removes the $(\frac{1}{\epsilon} + \log(4\pi) - \gamma_E)$ terms.
- The renormalized parameters aren't necessarily the physical ones and the value of the residue is no longer one.
- The differential cross section is finite as we send the electron mass (m_e) to be zero.

The Fourier transform of the two point correlation function of the electron self energy is given by

$$\int d^4x \, \langle\Omega|\, T(\psi(x)\bar{\psi}(0))\, |\Omega\rangle \, e^{ip\cdot x} = \frac{i}{\not{p}-m} + \frac{i}{\not{p}-m}\left(\frac{\Sigma(\not{p})}{\not{p}-m}\right) + \frac{i}{\not{p}-m}\left(\frac{\Sigma(\not{p})}{\not{p}-m}\right)^2 + \dots = \frac{i}{\not{p}-m-\Sigma(\not{p})}\,.$$

This means that the pole is shifted by $\Sigma(\not{p})$, so the renormalized mass is not the physical mass and the residue of this pole is no longer one.

The Physical Mass

The physical mass can be given by the position of the pole

$$\left(\not{p} - m - \Sigma(\not{p})\right)\Big|_{\not{p}=m_e} = 0$$

Which implies

$$\begin{aligned} m_e &= m + \Sigma(m_e) \\ &= m + \Sigma(m) + O(\alpha^2) \\ &= m\left(1 + \frac{\alpha}{4\pi}\left(4 + 3\log\left(\frac{\mu^2}{m^2}\right)\right) + O(\alpha^2)\right) \end{aligned}$$

Correction to the Residue

The inverse of the residue is given by

$$\begin{aligned} R^{-1} &= \frac{d}{d\not{p}}(\not{p} - m - \Sigma(\not{p}))\Big|_{\not{p}=m_e} \\ &= 1 - \Sigma'(m_e) \\ &= 1 - \Sigma'(m) + O(\alpha^2) \\ &\approx 1 - \frac{\alpha}{4\pi}\left(2\log\left(\frac{m^2}{m_\gamma^2}\right) - \log\left(\frac{\mu^2}{m^2}\right) - 4\right) + \mathcal{O}(\alpha^2) \end{aligned}$$

We should multiply the amplitude by $R^{-1/2}$ for each external leg, which means that we should multiply the differential cross section by R^{-2}

The NLO Cross Section

The final formula will be

$$\left(\frac{d\sigma}{d\Omega}\right) = \frac{1}{32\pi^2}\sum_{s,s'}\left(R^{-2}|\mathcal{M}_0|^2 + \mathcal{M}_0^*\mathcal{M}_V + \mathcal{M}_V^*\mathcal{M}_0 + \mathcal{M}_0^*\mathcal{M}_P\right.$$
$$\left. + \mathcal{M}_p^*\mathcal{M}_0 + \mathcal{M}_0^*\mathcal{M}_{BO} + \mathcal{M}_{BO}^*\mathcal{M}_0 + |\mathcal{M}_B|^2\right)$$
$$= \left(\frac{d\sigma}{d\Omega}\right)_L + \left(\frac{d\sigma}{d\Omega}\right)_{VL} + \left(\frac{d\sigma}{d\Omega}\right)_{PL} + \left(\frac{d\sigma}{d\Omega}\right)_{BOL} + \left(\frac{d\sigma}{d\Omega}\right)_B$$
$$= \left(\frac{d\sigma}{d\Omega}\right)_0 \left[1 + \frac{\alpha}{\pi}\left(\frac{3}{2}\log\left(\frac{m^2}{-q^2}\right) - \frac{2}{3}\log\left(\frac{\mu^2}{-q^2}\right) - \frac{47}{18}\right)\right]$$
$$+ \frac{\pi\alpha^3 E}{|q|\, q^2}\left(\frac{|q|}{p} - 1\right) + \mathcal{O}(\alpha^4)$$

What happened to the infinities?

Dependence on μ

The Physics (the physical parameters and the cross section) shouldn't depend on μ, so the renormalized parameters depend on μ and we find

$$\beta(\alpha) = \mu \frac{d\alpha}{d\mu} = \frac{2\alpha^2}{3\pi}$$

$$\gamma_m(\alpha) = \frac{\mu}{m}\frac{dm}{d\mu} = \frac{-3\alpha}{2\pi}$$

Then, it is straightforward to check that

$$\frac{dm_e}{d\mu} = 0 \quad , \quad \frac{d}{d\mu}\left(\frac{d\sigma}{d\Omega}\right) = 0$$

Conclusion

- All U.V and I.R divergences has been cancelled.
- We included all diagrams that contribute to the $\mathcal{O}(\alpha^3)$ in the cross section.
- Our final result for the differential cross section is finite and valid up to arbitrary large momentum exchange.
- We have used a very simple and powerful renormalization scheme which can be used for the QCD calculations as we deal with the light quarks with zero mass.
- Something missing with our calculations, where we expect not to have $\log(m^2)$ term in the final formula.

Thank you!