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Non-Abelian Correction to the Poisson Approximation for In Medium Multi-gluon Bremsstrahlung

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University of Cape Town, Cape Town

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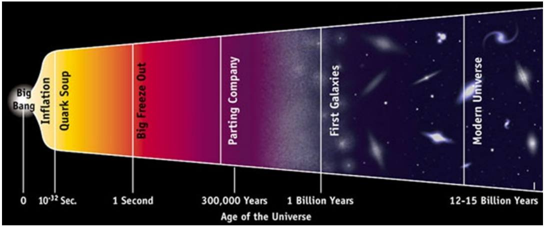
Outline

- Introduction
- Motivation and Goal
- Mathematical tools
- Computation of $J_{QCD}^{(n)}$
- Conclusion

Introduction

Early Universe

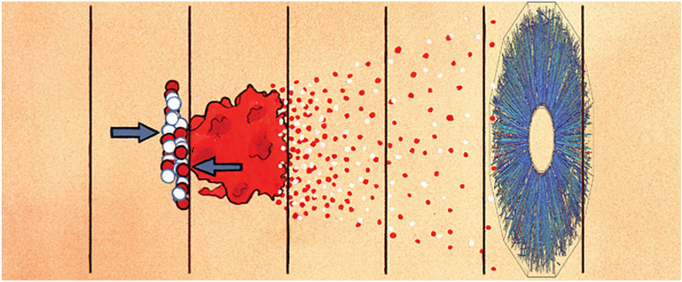
- Shortly after the big bang, the universe was filled by mixture of quarks and gluons,Quark-Gluon-Plasma.
- QGP is a phase of QCD where quarks and gluons are deconfined.
- The Universe was in the QGP state for a very short time.



Introduction

Heavy Ion Collisions

- Experiments at RHIC and at LHC,
- Recreate conditions similar to those at early universe,
- Fluid behaviour of QGP,
- Evolution of the QGP.
- Inclusive jet production events.



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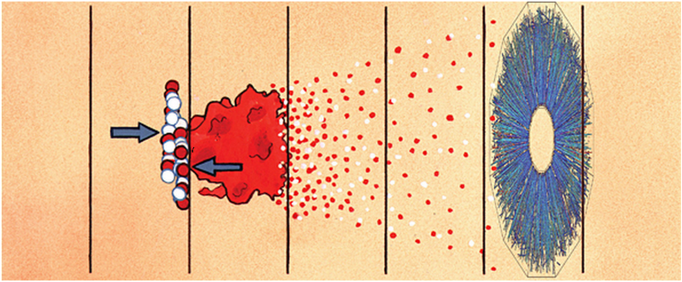

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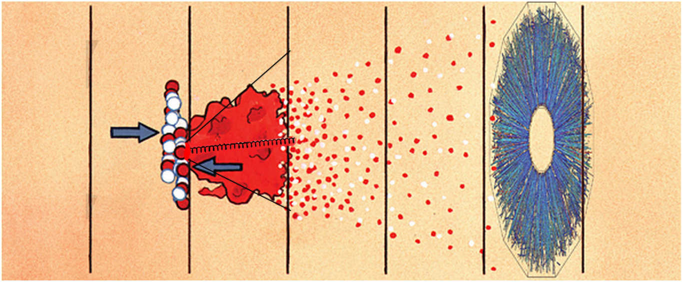

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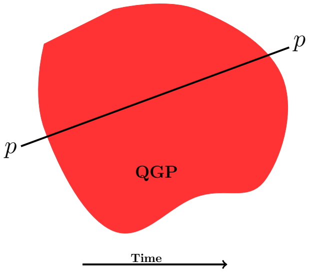
Radiative Correction

- The high p_T quark propagates in time with the medium,



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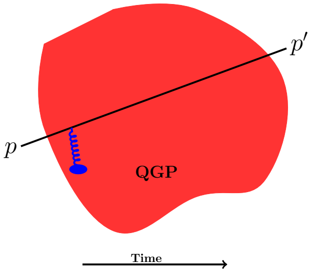

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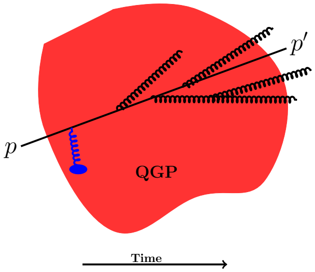

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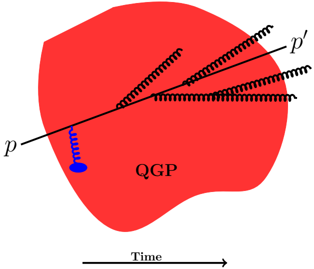


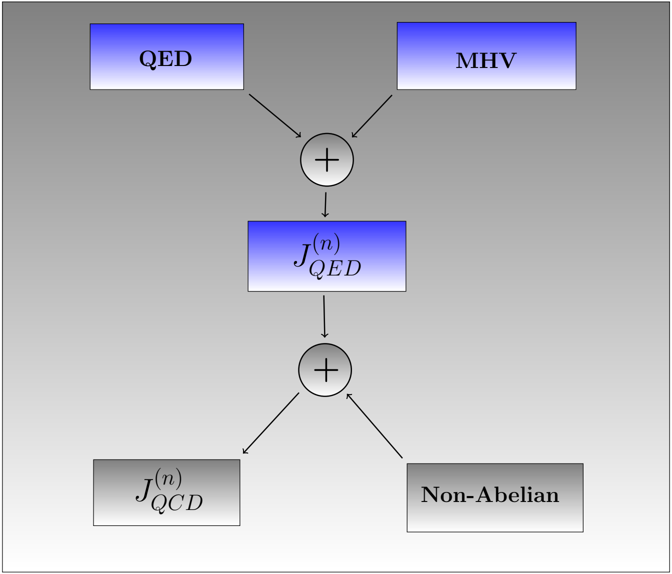
Radiative correction can be computed theoretically in the amplitude level

$$\mathcal{M}_{ng} = \mathcal{M}_0\, J^{(n)}(k_1, \ldots, k_n).$$

The resummation of those correction in the cross section level gives us the distribution of radiative gluon emitted.

How do we manage to compute those $J^{(n)}$ for gluon radiations?

Motivation



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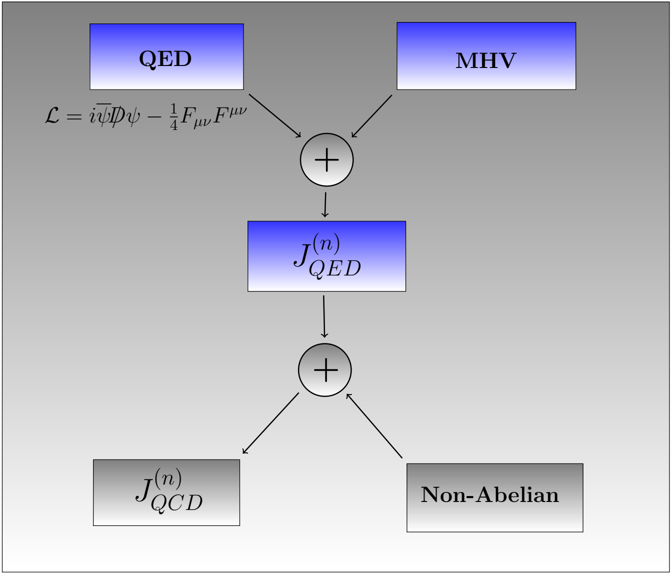
QED
MHV
$\mathcal{L} = i\bar{\psi}\not{D}\psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
+
$J^{(n)}_{QED}$
+
$J^{(n)}_{QCD}$
Non-Abelian

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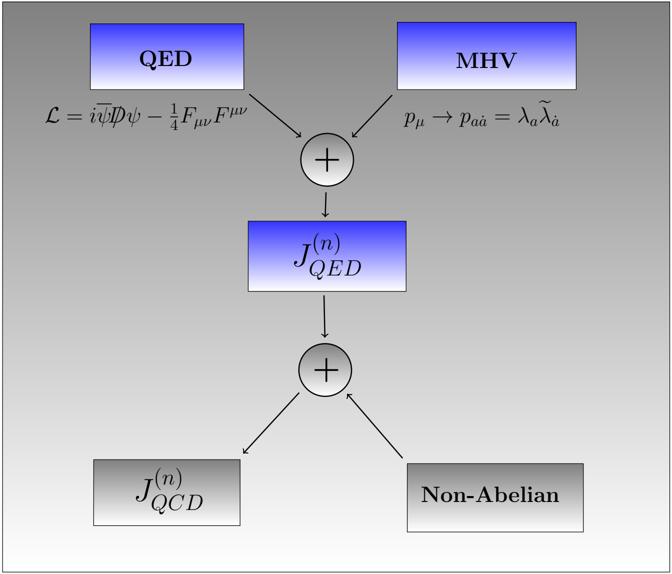
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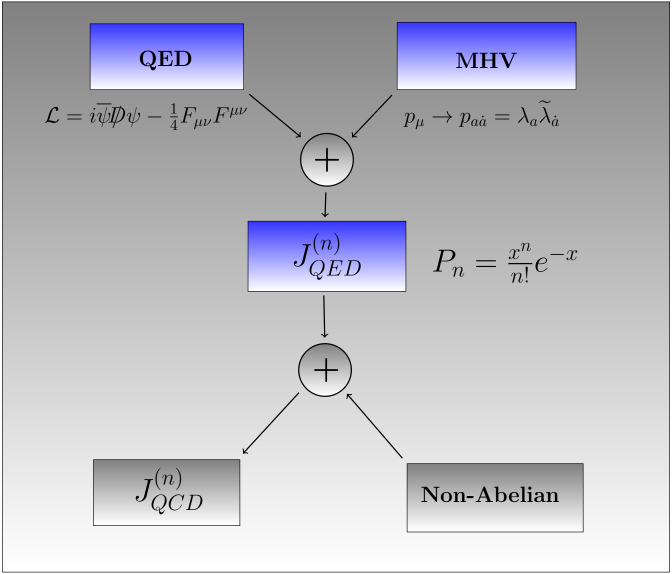
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$P_n = \frac{x^n}{n!}e^{-x}$
$J_{QCD}^{(n)}$
Non-Abelian

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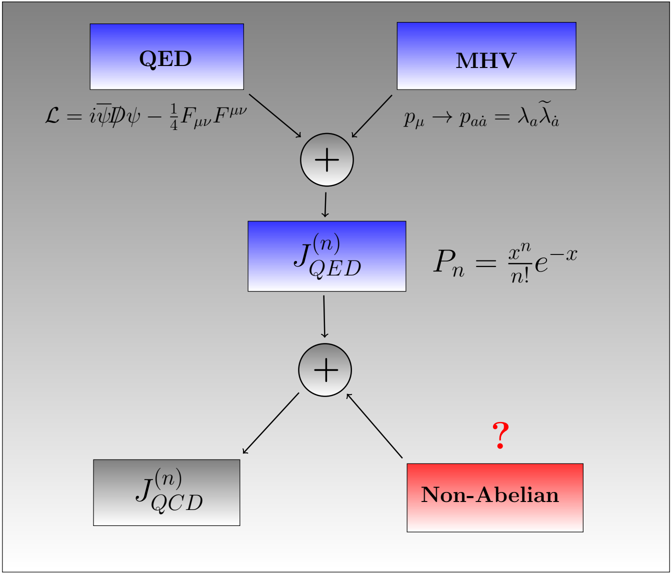
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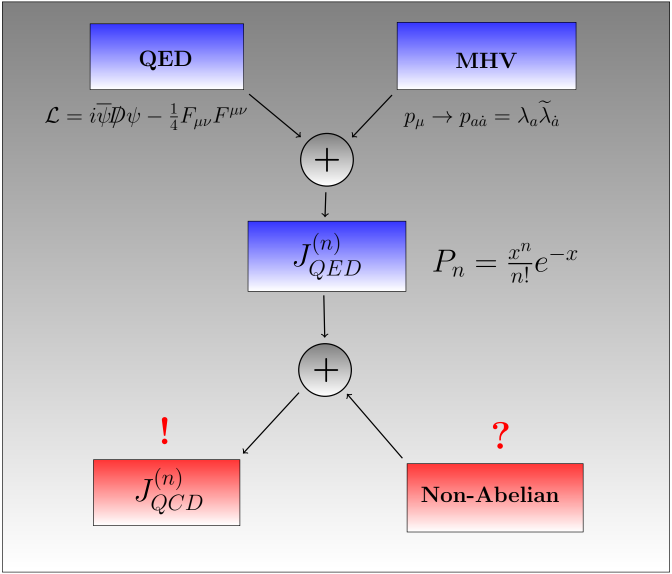
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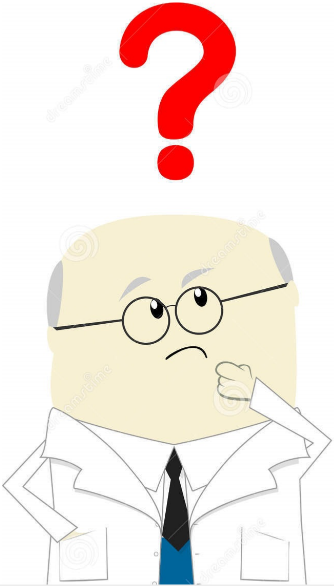


Maximal Helicity Violating (MHV)

- Spinor helicity formalism

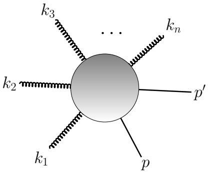
$$p_\mu \to p_{a\dot{a}} = p_\mu \sigma^\mu_{a\dot{a}} = \lambda_a \widetilde{\lambda}_{\dot{a}}$$

Invariant products

$$\langle p, q\rangle = \epsilon_{ab}\lambda^a_p \lambda^b_q \quad \text{and} \quad [p, q] = \epsilon_{\dot{a}\dot{b}}\widetilde{\lambda}^{\dot{a}}_p \widetilde{\lambda}^{\dot{b}}_q$$

- Amplitudes with helicity

$\mathcal{A}_n =$

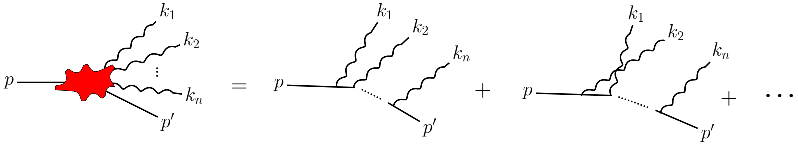


$$\Rightarrow \begin{cases} \mathcal{A}_n(p^+, p'^+, 1^+, \ldots, n^+) = 0 \\ \mathcal{A}_n(p^-, p'^+, 1^+, \ldots, n^+) = 0 \\ \mathcal{A}_n(p^-, \ldots, i^-) = \dfrac{\langle p, k_i\rangle^3 \langle p', k_i\rangle}{\langle p', p\rangle \langle p, k_1\rangle \langle k_1, k_2\rangle \cdots \langle k_n, p'\rangle} \end{cases}$$

- BCFW recursion to compute the higher number of negative helicity.

Bremsstrahlung Photon (QED)

Diagrammatic representation of amplitudes for *n* photon emissions



All permutation of photon $\Longrightarrow$ Independent photon emissions.
Soft-collinear radiation correction J for n bremsstrahlung is given by

$$J^{(n)}(k_1, k_2, \ldots, k_n) = \prod_{i=1}^{n} J^{(1)}(k_i)$$

Where in the MHV techniques

$$J^{(1)}(k) = \frac{\langle p, p' \rangle}{\langle p, k \rangle \langle k, p' \rangle}$$

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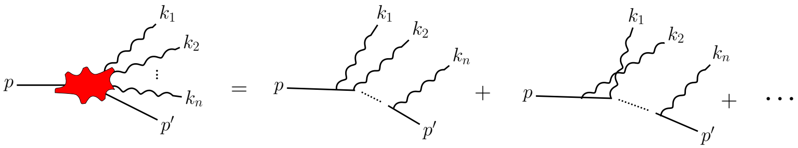

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$$J^{(1)}(k) = \frac{\langle p, p' \rangle}{\langle p, k \rangle \langle k, p' \rangle} \sim \left(\frac{p.\mathcal{E}(k)}{p.k} - \frac{p'.\mathcal{E}(k)}{p'.k} \right)$$

QED limit of *QCD*

1. QCD scattering amplitude

$$\mathcal{M}_{ng} = \sum_{\sigma \in S(n)} \left(T_{a_{\sigma_1}} \cdots T_{a_{\sigma_n}} \right)_{a_p a_{p'}} A(p, p', k_{\sigma_1}, \ldots, k_{\sigma_n})$$

2. Going from *QCD* to *QED*:
   - $A_\mu = A_\mu^a T_a$ where T_a are the generator of the $su(N)$ algebra

$$[T_a, T_b] = i f_{ab}^c T_c \quad \text{and} \quad \mathrm{tr}(T_a T_b) = \frac{1}{2}\delta_{ab}$$

   - $SU(N) \rightarrow U(N)$ by taking

$$T_a \sim \mathbb{I} \Rightarrow [T_a, T_b] = 0$$

   - Reduction from $U(N)$ to $U(1)$ by

$$N \rightarrow 1$$

3. QED scattering amplitudes

$$\mathcal{M}_{n\gamma} = \sum_{\sigma \in S(n)} A(p, p', k_{\sigma_1}, \ldots, k_{\sigma_n})$$

Symmetric Group $S(n)$

- Given n numbers of bremsstrahlung gluon emitted, we consider $S(n)$ that permutes the gluons legs.
- Introduce P_α to be the projectors into the different Young tableaux (irreducible representation)associate to $S(n)$ such that
  - $\alpha = \left\{ \square\square\cdots\square, \begin{smallmatrix}\square\square\cdots\square \\ \square\end{smallmatrix}, \ldots \right\}$
  - $\sum_\alpha P_\alpha = 1$
  - $P_\alpha P_\beta = \delta_{\alpha\beta} P_\alpha$
- Decomposition in the irreducible amplitudes

$$\mathcal{M}_{ng} = \sum_\alpha \sum_{\sigma \in S(n)} P_\alpha \left(T_{a_{\sigma_1}} \cdots T_{a_{\sigma_n}} \right) P_\alpha A(k_{\sigma_1}, \ldots, k_{\sigma_n})$$

- QED part of $\mathcal{M}_{ng}$ is the symmetrization of $A(k_1, \ldots, k_n)$

$$\mathcal{M}_{3\gamma} = P_{\square\square\square} A(k_1, k_2, k_3)$$

Procedures

1. Starting with the color kinematic scattering amplitude

$$\mathcal{M}_{ng} = \sum_{\sigma \in S(n)} C_{a_{\sigma_1} \cdots a_{\sigma n}} A(k_{\sigma_1}, \ldots, k_{\sigma_n})$$

2. Expand in the different Young symmetrizer labelled by α

$$\mathcal{M}_{ng} = C_{(a_1 \cdots a_n)} \mathcal{M}_{n\gamma} + \sum_{\sigma \in S(n)} \sum_{\alpha} C^{\alpha}_{a_{\sigma_1} \cdots a_{\sigma n}} A^{\alpha}(\sigma_1, \ldots, \sigma_n)$$

3. Factorize the parent amplitudes

$$\mathcal{M}_{ng} = \mathcal{M}_0 J^{(n)}(k_1, \ldots, k_n)$$

4. Eikonal function $J^{(n)}$

$$J^{(n)}_{QCD} = C_{(a_1 \cdots a_n)} J^{(n)}_{QED} + J^{(n)}_{\text{non-abelian}}$$

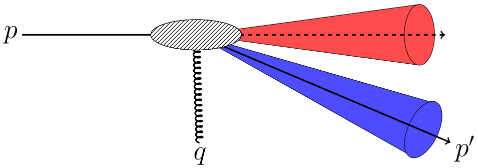
One Bremsstrahlung Gluon

One gluon

$$J^{(1)}_{QCD} = \underbrace{\frac{1}{2}\{T_{a_q}, T_{a_1}\}\frac{\langle p, p'\rangle}{\langle p, k_1\rangle\langle k_1, p'\rangle}}_{J^{(1)}_{QED}} + \underbrace{\frac{1}{2}[T_{a_q}, T_{a_1}]\frac{\langle q, p\rangle}{\langle q, k_1\rangle\langle k_1, p\rangle}}_{J^{(1)}_{\text{non-abelian}}}$$

$$\stackrel{\text{or}}{=} \frac{1}{2}\{T_{a_q}, T_{a_1}\}\frac{\langle p, p'\rangle}{\langle p, k_1\rangle\langle k_1, p'\rangle} + \frac{1}{2}[T_{a_q}, T_{a_1}]\frac{\langle q, p'\rangle}{\langle q, k_1\rangle\langle k_1, p'\rangle}$$

the gluon may be collinear along p or p'



Conclusion

Summary

- The distribution of bremsstrahlung contain the information on the Energy Loss
- Decomposition in the irreducible representation of $S(n)$ is a framework to do non-abelian corrections
- MHV techniques makes the result in a very compact form

In Progress

- Computing $J_{QCD}^{(n)}$ in the color flip
- Looking for a pattern for $J_{QCD}^{(n)}$ in order to get a resummation
- Once we obtain the distribution we wanted competing $\langle E \rangle_{\text{radiated}}$ and compare to the energy loss

Thank You!