Gauge-Higgs Unification in 5 dimensions for an SU(3) gauge group

Mohammed Omer Khojali

National Institute for Theoretical Physics;
School of Physics and Mandelstam Institute for Theoretical Physics
University of the Witwatersrand

E-mail: khogali11@gmail.com

February 9, 2016

Motivations

- A gauge theory defined in more than four dimensions has many attractive features:
  - The interactions at low energies may be truely unified.
  - Some of the distinct fields in four dimensions can be integrated as a single multiplet in higher dimensions, like in gauge-Higgs models.
- The simplest theories of this type have problems:
  - In reproducing the low energy observables, such as the Weinberg angle.
  - The SM fermion content and Yukawa coupling are different from the gauge couplings.

The basic construction

- Our goal is to discuss the gauge couplings evolution for a model which contains a bulk field, gauge fields and one pair of fermions ψ_a and $\tilde{\psi}_a$.
- The matter field can be introduced either as a bulk field in the representations of the unified group $G = SU(3)$ or as a boundary field localised at the fixed point where this is broken to a subgroup H .

The basic construction

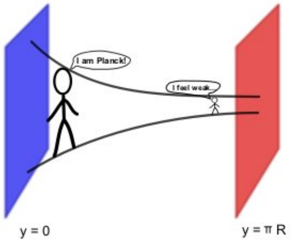
- The gauge bosons arise from the 4-dimensional components of the 5-dimensional gauge fields, whilst the Higgs field arises from the internal components of the gauge group $G = SU(3)$ compactified on an S^1/Z_2 orbifold; the orbifold boundary condition can be written in the following way

$$P = e^{i\pi\lambda_3} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

- The orbifold projection P breaking the gauge group G to the subgroup $H = SU(2) \times U(1)$.

The Matter Fields Lagrangian

- The brane fields of the model we going to focus on it consist of a left-handed fermion doublet $Q_L = (u_L, d_L)$ and two right-handed fermions singlets u_R and d_R.
- We are going to assume that the doublet and the two singlets fields are located respectively at position y_1 and y_2, which equal to either 0 or πR.



The Matter Fields Lagrangian

- The matter fields lagrangian for the bulk field, gauge field and the pair of fermions is given by the following lagrangian

$$\mathcal{L}_{\text{matter}} = \sum_a \Big[i\bar{\psi}_a(x,y) \not{D}_5 \psi_a(x,y) + i\bar{\tilde{\psi}}_a(x,y) \not{D}_5 \tilde{\psi}_a(x,y)$$

$$+ \bar{\psi}_a(x,y) M_a \tilde{\psi}_a(x,y) + \bar{\tilde{\psi}}_a(x,y) M_a \psi_a(x,y) \Big]$$

$$+ \delta(y - y_1) \Big[i\tilde{Q}_L(x,y) \not{D}_\mu Q_L(x,y) \Big]$$

$$+ \delta(y - y_2) \Big[i\bar{d}_R(x,y) \not{D}_\mu d_R(x,y) + i\bar{u}_R(x,y) \not{D}_\mu u_R(x,y) \Big]$$

The Matter Fields Lagrangian

- where $\not{D}_\mu$ and $\not{D}_5$ are the 4-dimension and 5-dimension covariant derivatives respectively, and they related by the following equality

$$\not{D}_5 = \not{D}_\mu + i\gamma_5 D_5,$$

and

$$\not{D}_M = \gamma^M \partial_M - i\gamma^M g_M A^a_M T^a,$$

where M $= (\mu \equiv 0, 1, 2, 3$ and $5)$, $\gamma_5 = i\gamma_\mu$, T^a is the generators of lie algebra of the gauge group G, A^a_μ is the 4-dimensional gauge bosons and the scalar fields A^a_5 are identified with the component of the Higgs field H.

Extra Dimension

- The 5D KK expansions of the fields are (where the corresponding coupling constants among the KK modes are simply equal to the SM couplings up to normalization):

$$\psi_(x,y) = \frac{1}{\sqrt{2\pi R}}\psi_R^0(x) + \frac{1}{\sqrt{2\pi R}}\sum_{n=1}^{\infty}\left[\sin(\frac{ny}{R})\psi_L^n(x) + \cos(\frac{ny}{R})\psi_R^n(x)\right],$$

$$Q_L(x,y) = \frac{1}{\sqrt{2\pi R}}Q_L^0(x) + \frac{1}{2\sqrt{\pi R}}\sum_{n=1}^{\infty}\left[\cos(\frac{ny}{R})Q_L^n(x) + \sin(\frac{ny}{R})Q_R^n(x)\right],$$

$$u_R(x,y) = \frac{1}{\sqrt{2\pi R}}u_R^0(x) + \frac{1}{2\sqrt{\pi R}}\sum_{n=1}^{\infty}\left[\cos(\frac{ny}{R})u_R^n(x) + \sin(\frac{ny}{R})u_L^n(x)\right],$$

Extra Dimension

- The zero modes are the 4D SM fields and there is a left- and a right-handed KK mode for each SM chiral fermion.
- After integrating out the compactified dimension, the 4D effective Lagrangian has interactions involving the zero mode and the KK modes.
- However, these KK modes cannot affect EW processes at tree level, and only contribute to higher order EW processes.

Renormalization Group Equation

- β-functions, is the crucial object to determine the evolution of a coupling constant. In general, in a theory with n couplings g_i , we have to solve a set of coupled differential equations of the kind

$$\beta_i(\mu, m) = \mu \frac{dg_i}{d\mu} = \frac{dg_i}{dt},$$

  where $t = \ln[\frac{\mu}{M_Z}]$, generally β-functions depend on all the couplings and masses of the theory.

- Now let us write the general term for the gauge interaction of the fermions and the gauge bosons as follows

$$g\bar{\psi}\gamma^{\mu}\psi A_{\mu},$$

Renormalization Group Equation

- Now going to rewrite the previous equation in term of renormalizable quantities by rescaling

$$\bar{\psi} = Z_{\bar{\psi}}^{1/2} \bar{\psi}^R,$$

$$\psi = Z_{\psi}^{1/2} \psi^R,$$

$$A_\mu = Z_{A_\mu}^{1/2} A_\mu^R,$$

where Z_ψ, $Z_{\bar{\psi}}$ and Z_{A_μ} are the wave function renormalization constant related to the fermions and the gauge boson.

Renormalization Group Equation

- Then we can rewrite the gauge interaction of the fermions and the gauge bosons in terms of the renormalizable quantities as follows

$$gZ_{\bar{\psi}}^{1/2}Z_{\psi}^{1/2}Z_{A_\mu}^{1/2}\bar{\psi}^R\gamma^\mu\psi^R A_\mu^R = Z_g^{1/2}g^R\bar{\psi}^R\gamma^\mu\psi^R A_\mu^R,$$

- From the above equation one can see

$$gZ_{\bar{\psi}}^{1/2}Z_{\psi}^{1/2}Z_{A_\mu}^{1/2} = Z_g^{1/2}g^R,$$

- We have, therefore,

$$\frac{d\ln g^R}{dt} = \frac{1}{2}\frac{d\ln Z_{\bar{\psi}_R}}{dt} + \frac{1}{2}\frac{d\ln Z_{\psi_R}}{dt} + \frac{1}{2}\frac{d\ln Z_{A_\mu}}{dt} - \frac{1}{2}\frac{d\ln Z_g}{dt}.$$

One-loop

- The one loop β-functions for the gauge couplings is given by:

$$16\pi^2 \frac{dg_i}{dt} = b_i^{\text{SM}} g_i^3 + (b_i + S(t)\tilde{b_i}) g_i^3,$$

where $S(t) = e^t M_Z R$ for $M_Z < \mu < \ln[\frac{1}{M_Z R}]$.

- The one-loop β-functions for the gauge couplings in 0-mode are given by:

$$16\pi^2 g_i^{-3} \frac{dg_i}{dt} = b_i,$$

One-loop

- The one-loop β-functions for the gauge couplings in n-mode are given by:

$$16\pi^2 g_i^{-3}\frac{dg_i}{dt} = S(t)\tilde{b}_i,$$

- The numerical coefficients given by:

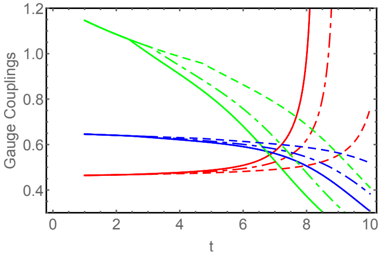
$$b_i^{\rm SM} = \left[\frac{41}{10}, -\frac{19}{6}, -7\right],$$

$$b_i = \left[\frac{10}{3}, -\frac{51}{16}, -\frac{20}{3}, \frac{3}{8}\right]$$

and

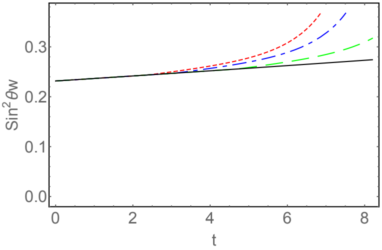
$$\tilde{b}_i = \left[\frac{45}{16\pi}, -\frac{15}{16\pi}, -\frac{5}{\pi}, 0\right].$$

Result and discussion



The evolution of the gauge couplings g_1 (red), g_2 (blue) and g_3 (green); for three different values of the compactification scales (1 TeV (solid line), 2 TeV (dot-dashed line), 10 TeV (dashed line)) as a function of the scale parameter t.

Result and discussion



The evolution of Weinberg angle $\sin^2\theta_W$ with the bulk fermions, also with the doublet located at position y_1 and two singlets located at position y_2, for three different values of compactification scales $R^{-1} = 1$TeV (red), $R^{-1} = 2$TeV (blue) and $R^{-1} = 10$TeV (green) as a functiono of the scale parameter t.

Conclusions

- I test in a simplified 5-dimensional model with SU(3) gauge symmetry, the evolution equations of the gauge coupling and Weinberg angle of the model containing bulk fields, gauge fields and one pair of fermions.
- This is not a complete and realistic model for the electroweak interactions, it is just toy model.
- I wish to extend this to studies different types of models and different types of the unified gauge groups.

Thank-you