Absorption probabilities associated to spin-3/2 particles near N dimensional Schwarzschild black holes

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9th February 2016

Gravitino fields in N dimensional Schwarzschild black hole spacetimes

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Black hole absorption

- Field theory allows for particle scattering off a black hole.
- This scattering implies "Grey body", not pure "Black body", behaviour.
- Pioneered by Unruh in 1976.
- Unruh ideas describe Hawking radiation.

Rarita-Schwinger equation

- The relativistic field equation of spin-3/2 particles.

Rarita-Schwinger equation

$$\gamma^{\mu\nu\alpha}\nabla_\nu \Psi_\alpha = 0$$

where $\gamma^{\mu\nu\alpha} = \gamma^\mu\gamma^\nu\gamma^\alpha - \gamma^\mu g^{\nu\alpha} + \gamma^\nu g^{\nu\alpha} - \gamma^\alpha g^{\mu\nu}$.

- Gravitino is predicted to have a spin of 3/2.
- Lightest supersymmetic particle.

Schwarzschild metric

N dimensional Schwarzschild metric

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2 d\Omega_{N-2}^2,$$

with $f(r) = 1 - \left(\frac{2M}{r}\right)^{N-3}$.

N dimensional spherical metric

$$d\Omega_N^2 = d\theta^2 + \sin^2(\theta) d\Omega_{N-1}^2,$$

with $d\Omega_2^2 = d\theta^2 + \sin^2(\theta) d\phi^2$.

Radial equation

Radial equation

$$-\frac{d^2}{dr_*^2}\bar{\phi}_1 + V_1\bar{\phi}_1 = \omega^2\bar{\phi}_1; \quad -\frac{d^2}{dr_*^2}\bar{\phi}_2 + V_2\bar{\phi}_2 = \omega^2\bar{\phi}_2,$$

where $\frac{d}{dr_*} = f(r)\frac{d}{dr}$

Potential function

N dimensional potential function

$$V_{1,2} = \pm f \frac{dW}{dr} + W^2,$$

with,

$$W = \frac{\left(j + \frac{N-3}{2}\right)\sqrt{f}}{r} \left(\frac{\left(\frac{2}{N-2}\right)^2 \left(j - \frac{1}{2}\right)\left(j + \frac{2N-5}{2}\right) - \frac{N-4}{N-2}\left(\frac{2M}{r}\right)^{N-3}}{\left(\frac{2}{N-2}\right)^2 \left(j - \frac{1}{2}\right)\left(j + \frac{2N-5}{2}\right) + \left(\frac{2M}{r}\right)^{N-3}} \right)$$

Unruh method

Consider three regions:

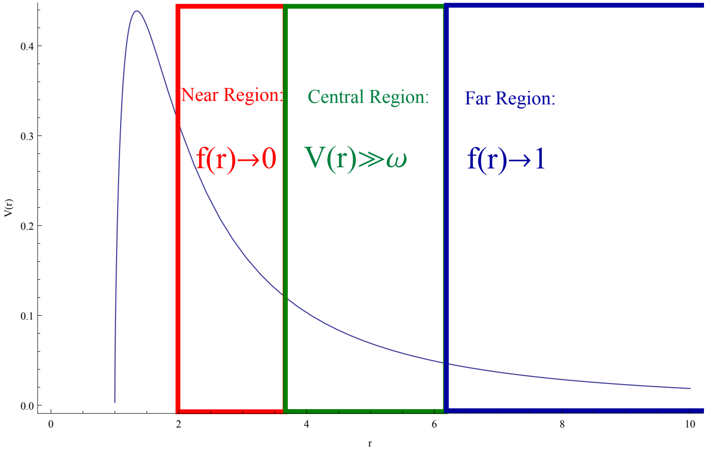

Figure: Potential for the 4 dimensional Schwarzschild black hole

Unruh method

The absorption probability is given as,

Unruh absorption probability

$$|A_j(\omega)|^2 = 4\pi C^2\omega^{2j+1}\left(1+\pi C^2\omega^{2j+1}\right)^{-2} \approx 4\pi C^2\omega^{2j+1},$$

where,

$$C = \frac{1}{2^{2j+1}\Gamma(j+1)}\frac{j+\frac{3}{2}}{j-\frac{1}{2}},$$

where Γ is the gamma function and $\omega < 1$.

Potential near black hole

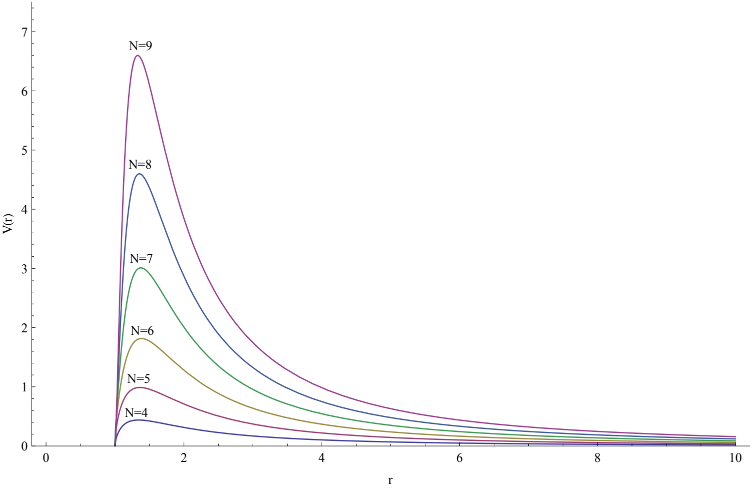

Figure: Potential function for spin-3/2 particles in 4 and higher dimensional Schwarzschild backgrounds with angular momentum j=3/2.

Potential near black hole

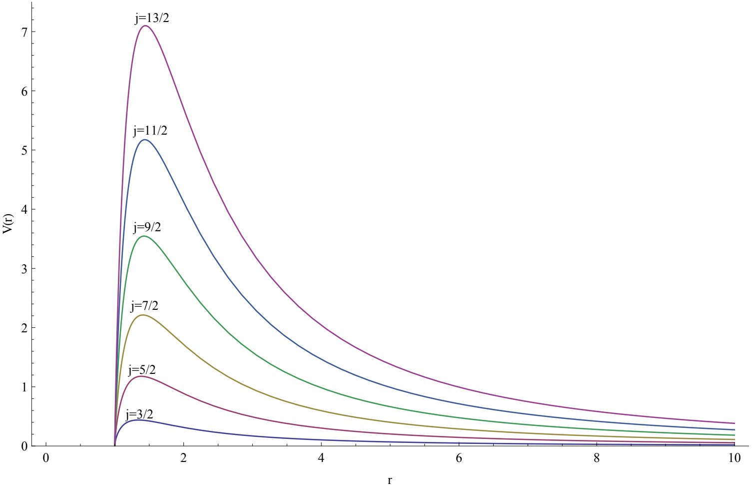

Figure: Absorption probability for spin-3/2 particles in 4 dimensional Schwarzschild background with angular momentum j=3/2 to 13/2.

WKB method

WKB absorption probability

$$|A_j(\omega)|^2 = \frac{1}{1+e^{2S(\omega)}},$$

with,

$$\begin{aligned} S(\omega) &= \pi k^{1/2}\left[\frac{1}{2}z_0^2 + \left(\frac{15}{64}b_3^2 - \frac{3}{16}b_4\right)z_0^4\right] \\ &+ \pi k^{1/2}\left[\frac{1155}{2048}b_3^4 - \frac{315}{256}b_3^2 b_4 + \frac{35}{128}b_4^2 + \frac{35}{64}b_3 b_5 - \frac{5}{32}b_6\right]z_0^6 \\ &+ \pi k^{-1/2}\left[\frac{3}{16}b_4 - \frac{7}{64}b_3^2\right] \\ &- \pi k^{-1/2}\left[\frac{1365}{2048}b_3^4 - \frac{525}{256}b_3^2 b_4 + \frac{85}{128}b_4^2 + \frac{95}{64}b_3 b_5 - \frac{25}{32}b_6\right]z_0^2, \end{aligned}$$

Results

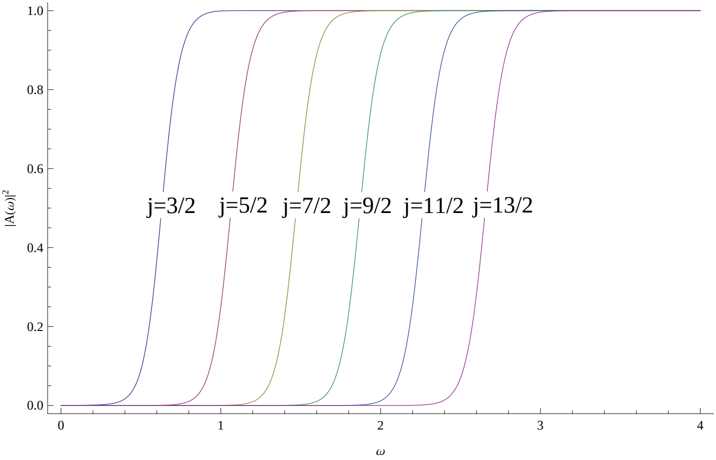

Figure: Absorption probability for spin-3/2 particles in 4 dimensional Schwarzschild backgrounds for angular momentum j=3/2 to 13/2.

Results

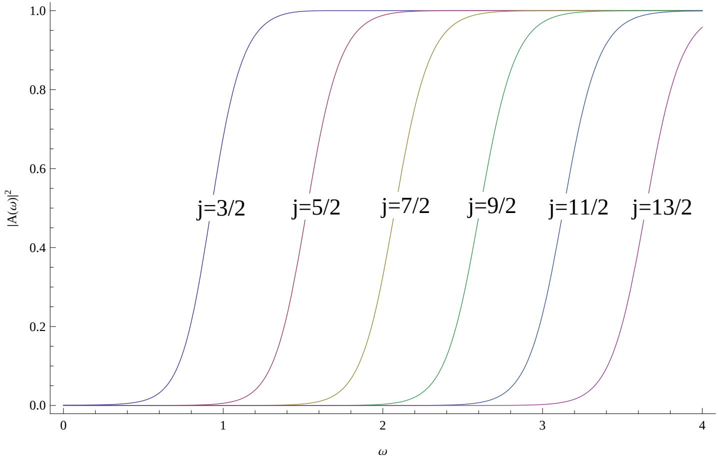

Figure: Absorption probability for spin-3/2 particles in 5 dimensional Schwarzschild backgrounds with angular momentum j=3/2 to 13/2.

Results

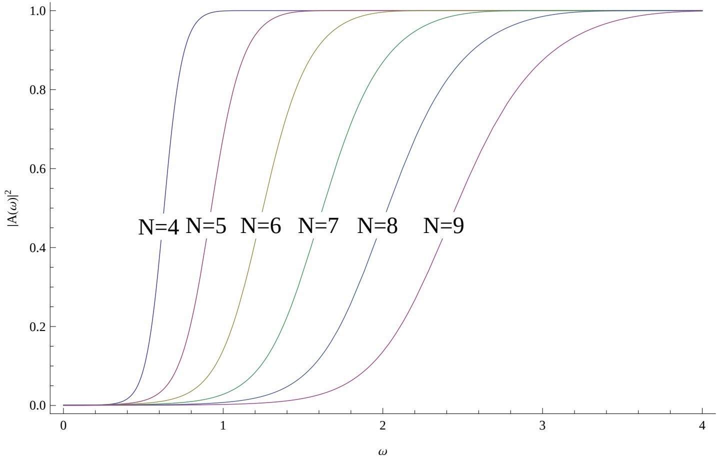

Figure: Absorption probabilities for spin-3/2 particles with an angular momentum of j=3/2 in 4 and higher dimensional Schwarzschild backgrounds.

Results

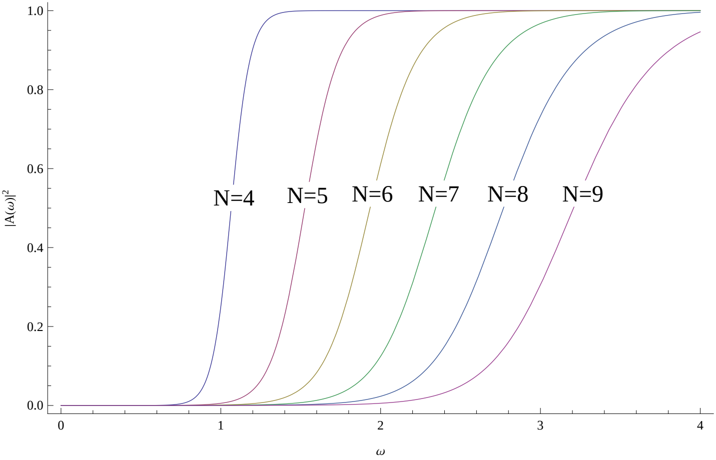

Figure: Absorption probabilities for spin-3/2 particles with an angular momentum of j=5/2 in 4 and higher dimensional Schwarzschild backgrounds.

Concluding remarks

- An increase in quantum number j results in an increase in the required particle energy for absorption.
- Increase in space time dimensions results in an increase in the required particle energy for absorption.
- Higher dimensional black holes exhibit a slower changes from total reflection to total absorption

Acknowledgements to: