Constraints on thermalization time and shear viscosity in heavy-ion collisions

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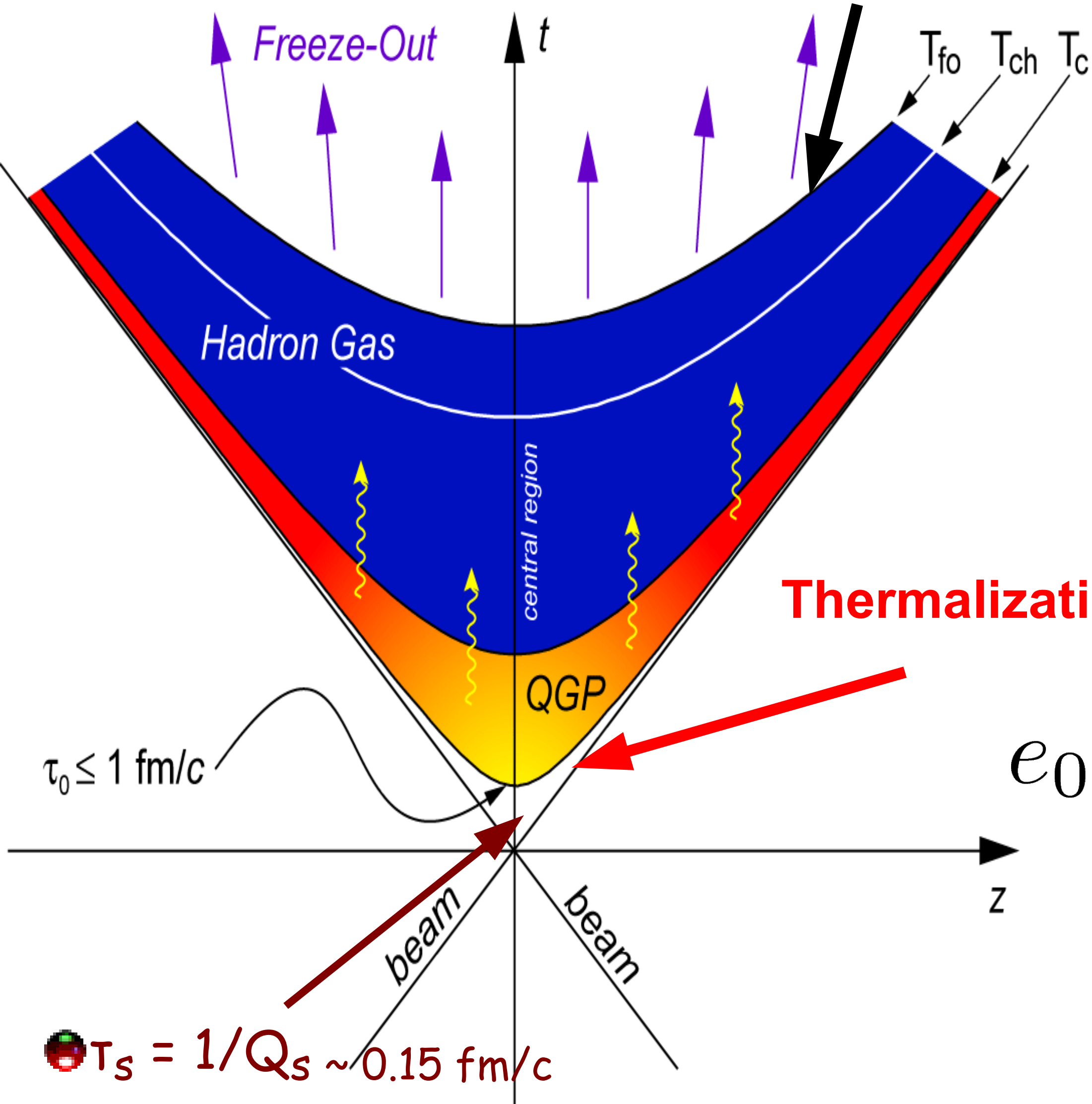
Outline

- We study the entropy production in the early stage of heavy-ion collisions.
- We employ the Israel-Stewart theory, i.e., 1+1D Bjorken model with shear viscosity.
- We show that for a given entropy production bound the initial time for hydrodynamics is correlated to the viscosity, i.e., the lower bound on η/s provides a lower limit on τ_0.

Motivation - How to apply hydrodynamics



Freeze-out hypersurface

$$\tau_f$$

Hydrodynamical evolution
+ Relaxation equations

$$\tau_f - \tau_0$$

Thermalization hypersurface

$$\tau_0$$

e_0, n_0 Initial energy and charge densities

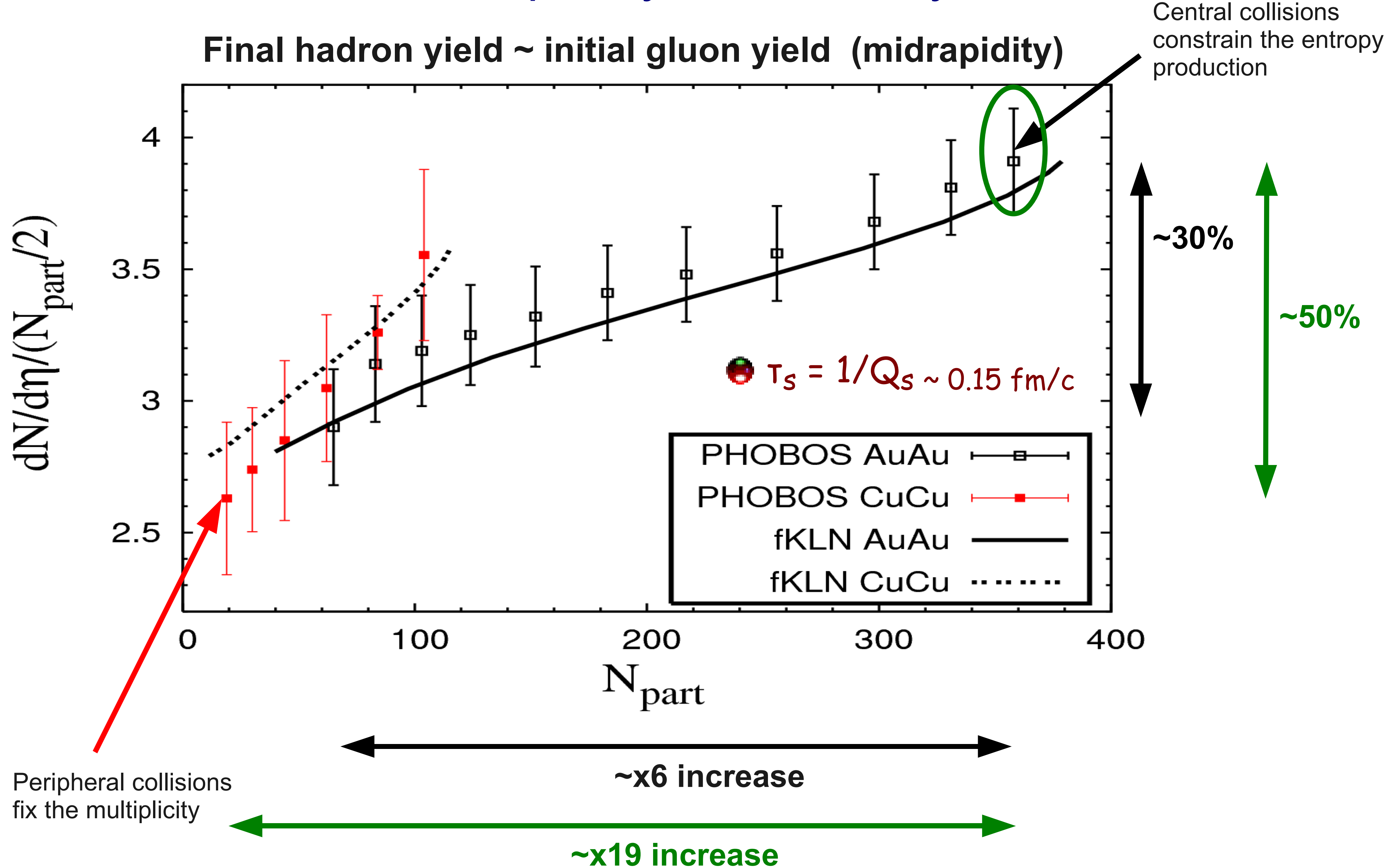
v_0 Initial flow velocity

$\pi_0^{\mu\nu}$ Initial stress tensor

Viscosity coefficient
Relaxation time

+ EOS

Motivation - The initial entropy vs. the final and the multiplictity vs. centrality



D. Kharzeev, E. Levin and M. Nardi, **Nucl. Phys. (2004); Nucl. Phys. (2005)**; A. Adil et.al., **PRC 2006;** H. J. Drescher and Y. Nara **PRC 2007**

The quantities of Dissipative Hydrodynamics

- A one component fluid is characterized locally by the conserved net-charge 4-current:

$$N^\mu = nu^\mu + V^\mu \longrightarrow V^\mu u_\mu = 0$$

the energy-momentum tensor:

$$T^{\mu\nu} = \underbrace{eu^\mu u^\nu - p\Delta^{\mu\nu}}_{reversible} + \underbrace{W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu} - \Pi\Delta^{\mu\nu}}_{irreversible}$$

$$\longrightarrow \quad W^\nu u_\nu = q^\mu u_\mu + hV^\mu u_\mu = 0,\ \pi^{\mu\nu} u_\nu = 0,\ (\pi^\mu_\mu = 0)$$

and entropy 4-current:

$$S^\mu = su^\mu + \Phi^\mu \longrightarrow \Phi^\mu u_\mu = 0$$

The Second law of thermodynamics

$$\boxed{\partial_\mu S^\mu \geq 0}$$

- Perfect fluid dynamics:

$$S^\mu = s u^\mu$$

dissipation --> 0

- First order theory; (Eckart, Landau, Lifshitz)

$$S^\mu = s u^\mu + \frac{q^\mu}{T}$$

- Second order theory; (Muller, Israel, Stewart)

$$S^\mu = s u^\mu + \frac{q^\mu}{T} - \left(\beta_0 \Pi^2 - \beta_1 q_\nu q^\nu + \beta_2 \pi_{\nu\alpha}\pi^{\nu\alpha}\right)\frac{u^\mu}{2T} - \frac{\alpha_0 \Pi q^\mu}{T} + \frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T}$$

$$\tau_\Pi = \zeta\beta_0$$

Bulk viscosity

$$\tau_q = \lambda T \beta_1$$

Thermal conductivity

$$\tau_\pi = 2\eta\beta_2$$

Shear viscosity

$$\tau_0 = \zeta\alpha_0$$

$$\tau_1 = \lambda T \alpha_1$$

Relaxation times

Coupling coefficients

Entropy production with shear viscosity

$$\boxed{S^\mu = s u^\mu}$$

$$\partial_\mu S^\mu \geq 0 \quad \xrightarrow{\text{1-st order}} \quad \frac{\pi^{\mu\nu}\pi_{\mu\nu}}{2\eta} \geq 0$$

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} = 2\eta\left[\frac{1}{2}\left(\nabla^\mu u^\nu + \nabla^\nu u^\mu\right) - \frac{1}{3}\Delta^{\mu\nu}\nabla_\lambda u^\lambda\right]$$

stress tensor **shear tensor**

$$\boxed{S^\mu = s u^\mu - \beta_2 \pi_{\nu\alpha}\pi^{\nu\alpha}\frac{u^\mu}{2T}}$$

$$\partial_\mu S^\mu \geq 0 \quad \xrightarrow{\text{2-nd order}} \quad \pi^{\mu\nu}\left[\sigma_{\mu\nu} - \beta_2 u^\lambda \partial_\lambda \pi_{\mu\nu} - T\partial_\lambda\left(\frac{\beta_2 u^\lambda}{2T}\right)\pi_{\mu\nu}\right] \geq 0$$

Relaxation equation

$$\tau_\pi u^\lambda \partial_\lambda \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \pi^{\mu\nu} - 2\eta T\partial_\lambda\left(\frac{\beta_2 u^\lambda}{2T}\right)\pi^{\mu\nu}$$

Dissipative hydrodynamics with shear viscosity

- In baryonfree matter (n = 0) using Landau's flow velocity leads:

$$W^\mu \equiv q^\mu + hV^\mu = 0 \longrightarrow V^\mu = -q^\mu \frac{\cancel{n}}{e+p}$$

- Using Bjorken's scaling flow and neglecting the bulk pressure:

$$q^\mu = 0 \qquad \Pi = 0$$

- Thus the relevant quantities become:

$$\cancel{N}^\mu = \cancel{n}u^\mu + \cancel{V}^\mu$$

$$T^{\mu\nu} = eu^\mu u^\nu - p\Delta^{\mu\nu} + \cancel{W}^\mu u^\nu + \cancel{W}^\nu u^\mu + \pi^{\mu\nu} - \cancel{\Pi}\Delta^{\mu\nu}$$

$$S^\mu = su^\mu + \cancel{\frac{q^\mu}{T}} - \left(\cancel{\beta_0\Pi^2} - \cancel{\beta_1 q_\nu q^\nu} + \beta_2 \pi_{\nu\alpha}\pi^{\nu\alpha}\right)\frac{u^\mu}{2T} - \cancel{\frac{\alpha_0 \Pi q^\mu}{T}} + \cancel{\frac{\alpha_1 \pi^{\mu\nu} q_\nu}{T}}$$

"1+1D Bjorken" dissipative hydrodynamics

$$\partial_\mu T^{\mu\nu} = 0 \longrightarrow$$

$$\partial_\mu S^\mu \geq 0 \longrightarrow$$

$$\boxed{\begin{aligned} &\frac{de}{d\tau} = -\frac{e+p}{\tau} + \frac{\Phi}{\tau} \\ &\frac{d\Phi}{d\tau} = -\frac{\Phi}{\tau_\pi} - \frac{\Phi}{2}\left[\frac{1}{\tau} + \frac{T}{\beta_2}\frac{d}{d\tau}\left(\frac{d\beta_2}{T}\right)\right] + \frac{2}{3\beta_2\tau} \\ &S^\mu u_\mu = \tilde{s} = s(e)\left(1 - \frac{3}{4}\frac{\beta_2}{Ts}\Phi^2\right) \qquad \Phi \equiv \pi^{00} - \pi^{zz} \end{aligned}}$$

A. Muronga, **PRL 2002; PRC 2004**

$$\tau_\pi = 2\eta\beta_2 = 2\eta\frac{3}{4p} = \boxed{\frac{6}{T}\ \frac{\eta}{s}}$$

Boltzmann gas – weak coupling

W. Israel and J. M. Stewart, **Ann. Phys. 1979**

$$\tau_\pi = \frac{1-\log 2}{9}\boxed{\frac{6}{T}\ \frac{\eta}{s}}$$

AdS/CFT – infinite coupling

M. P. Heller and R. A. Janik, **PRD 2007**

$$\boxed{\frac{\tau_\pi^{Boltz}}{\tau_\pi^{AdS}} \sim 30}$$

The ratio of shear viscosity to entropy density

- **Viscosity =** momentum transport

$$\frac{\eta}{s} \sim \lambda T \overline{v}$$

$$\lambda \sim 1/n\sigma$$

T - temperature
λ – mean free path
v – mean velocity
n – density
σ – cross section

Perfect = low viscosity
Fluid = large cross section
(small mean free path)

λ ~ 0.1 fm $\hbar c \sim k_B T$

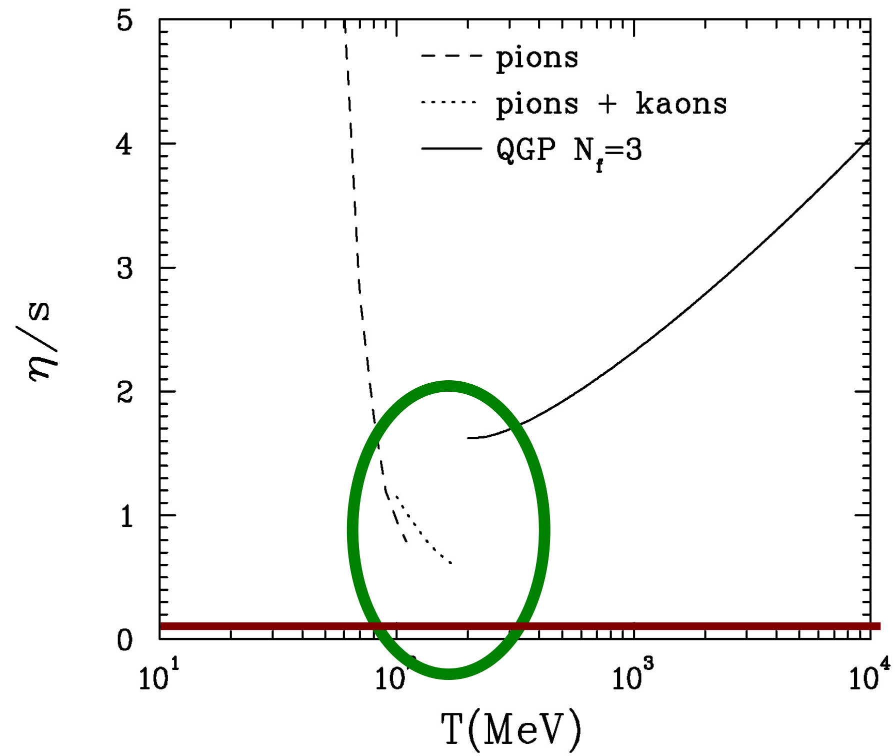
Ideal Gas = large viscosity
and small cross section
(large mean free path)

AdS/CFT correspondence universal lower bound

P. K. Kovtun, D. T. Son and A. O. Starinets, **PRL 2005**

$$\boxed{\frac{\eta}{s} \geq \frac{\hbar}{4\pi k_B}}$$

L. P. Csernai, J. I. Kapusta and L. D. McLarren, **PRL 2006**



The "R" Reynolds number

- Used to identify flow regimes (laminar or turbulent), criterion of stability

inertial force/viscous force

$$R = v\rho / \frac{\eta}{L}$$

L - characteristic length
v – flow velocity
ρ - density
η – viscosity

- If R < 100 - 1000 the flow is laminar, otherwise the flow becomes turbulent

- (Numerical) Viscosity is needed to keep the flow laminar and the solution stable!

$$R = \frac{e+p}{\Phi} \longrightarrow \frac{de}{d\tau} = (R^{-1} - 1)\frac{e+p}{\tau}$$

Effective Reynolds number

G. Baym, **Nucl. Phys. 1984;**
H. Kouno et. al., **PRD 1990;**
A. Muronga, **PRC 2004.**

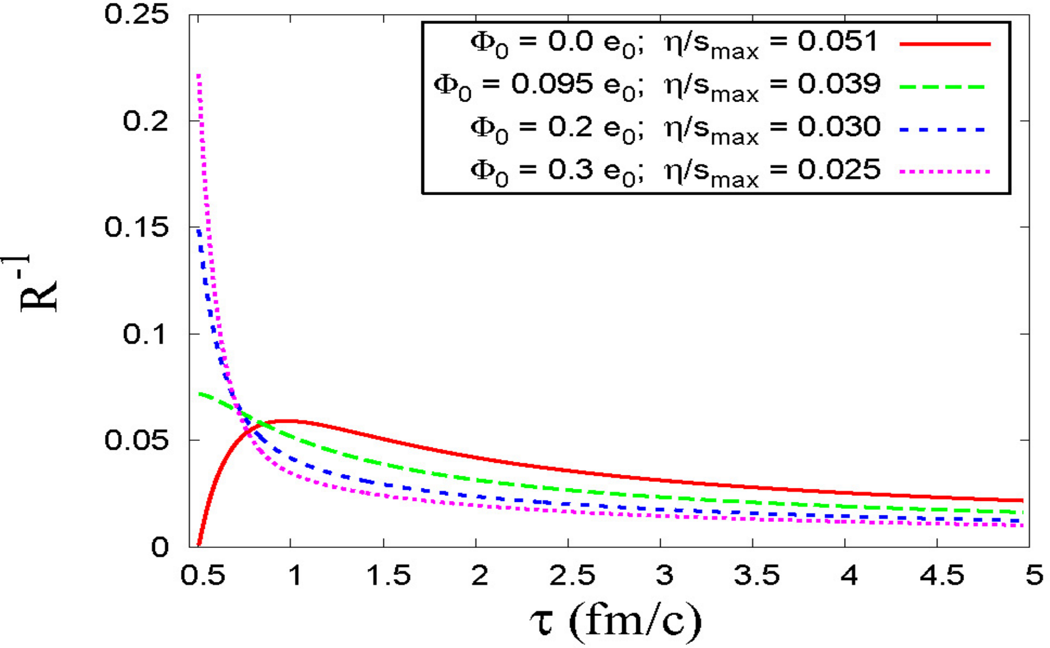
In first order theory

$$\Phi = \frac{4}{3}\frac{\eta}{\tau}$$

$$R^{-1} = \frac{4}{3T}\frac{\eta}{s}$$

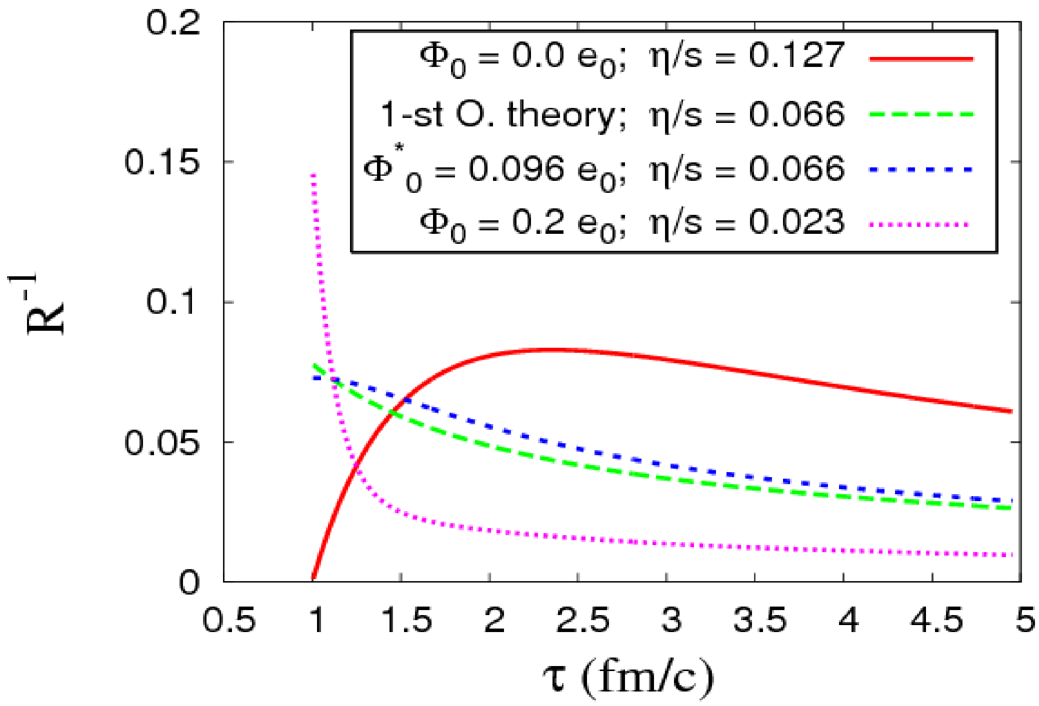
✯ Initial stress Φ_0:

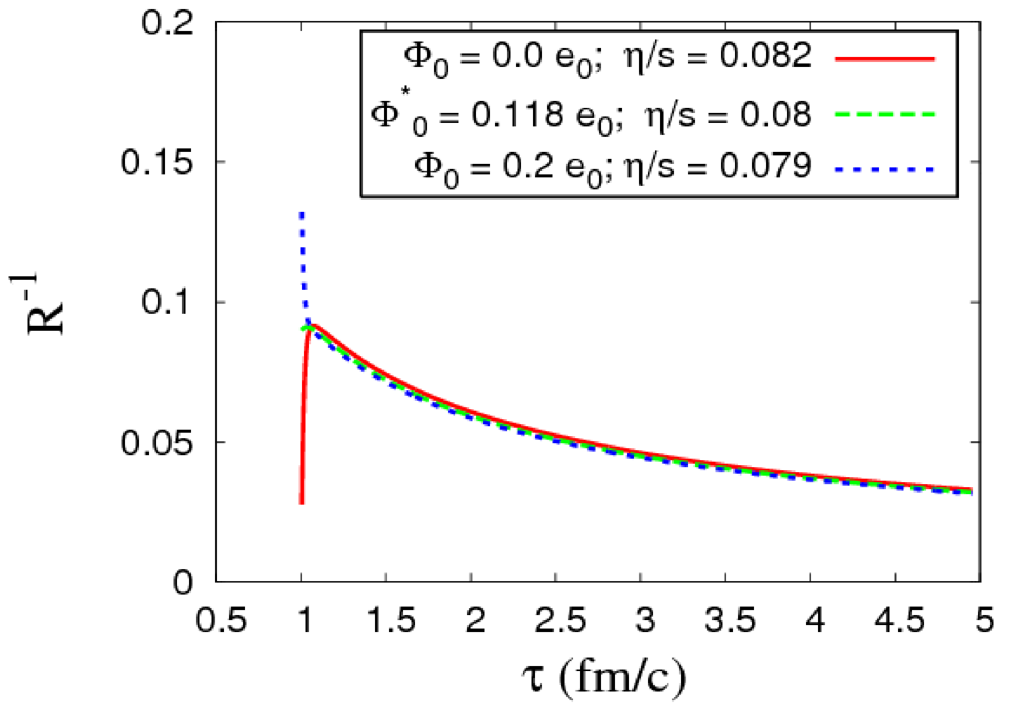
If $dR(\tau_0)/d\tau = 0$ the initial stress in the system is given by first order theory



Weak coupling - Boltzmann

Strong coupling - AdS/CFT

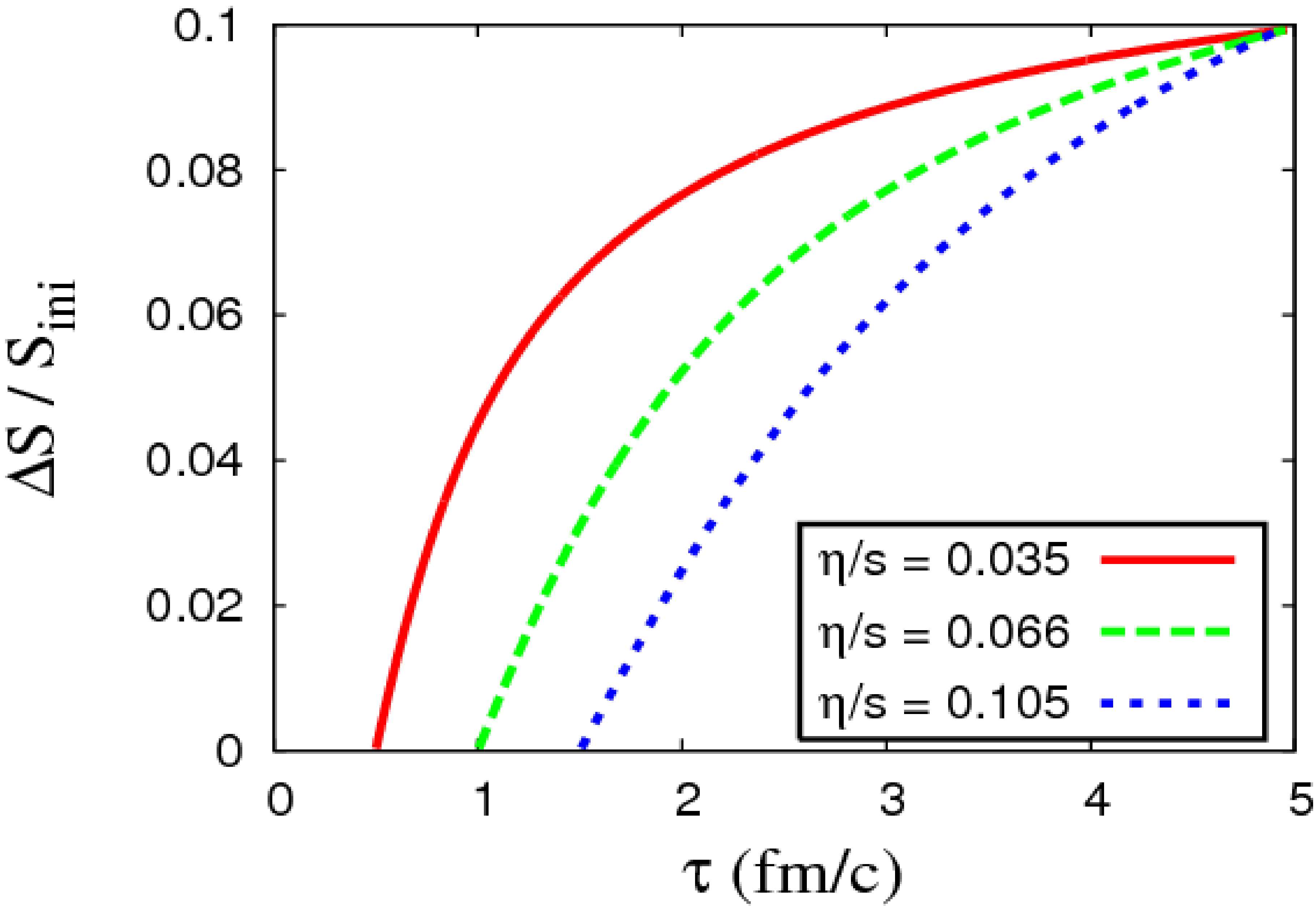




The method

- Use the KLN result and fix the initial gluon number (= entropy density).
- Run the 1+1D dissipative hydro for a given Φ and η/s.
- Allow (5%,10%,15%) entropy production and match the final multiplicity – constraint on the dissipative quantities.
- Use the Reynolds number to give a relation between Φ and η/s.

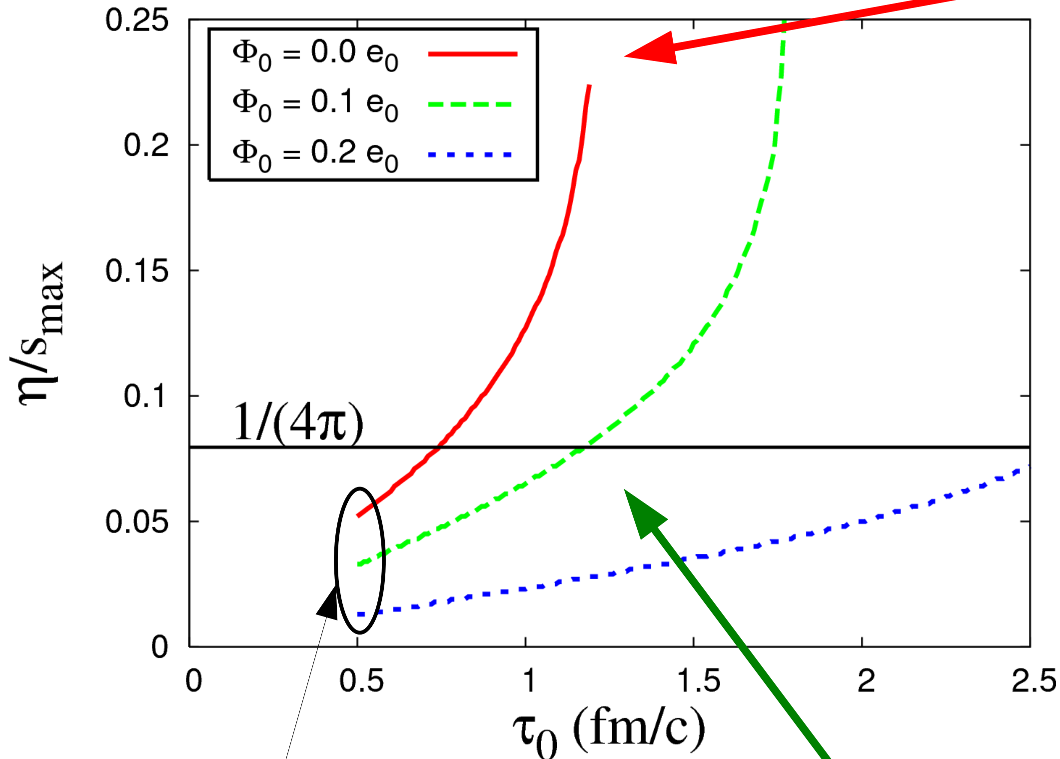
✯ Entropy production: larger τ_0 requires larger η/s



Maximum η/s vs. minimum τ_0 for 10% entropy production

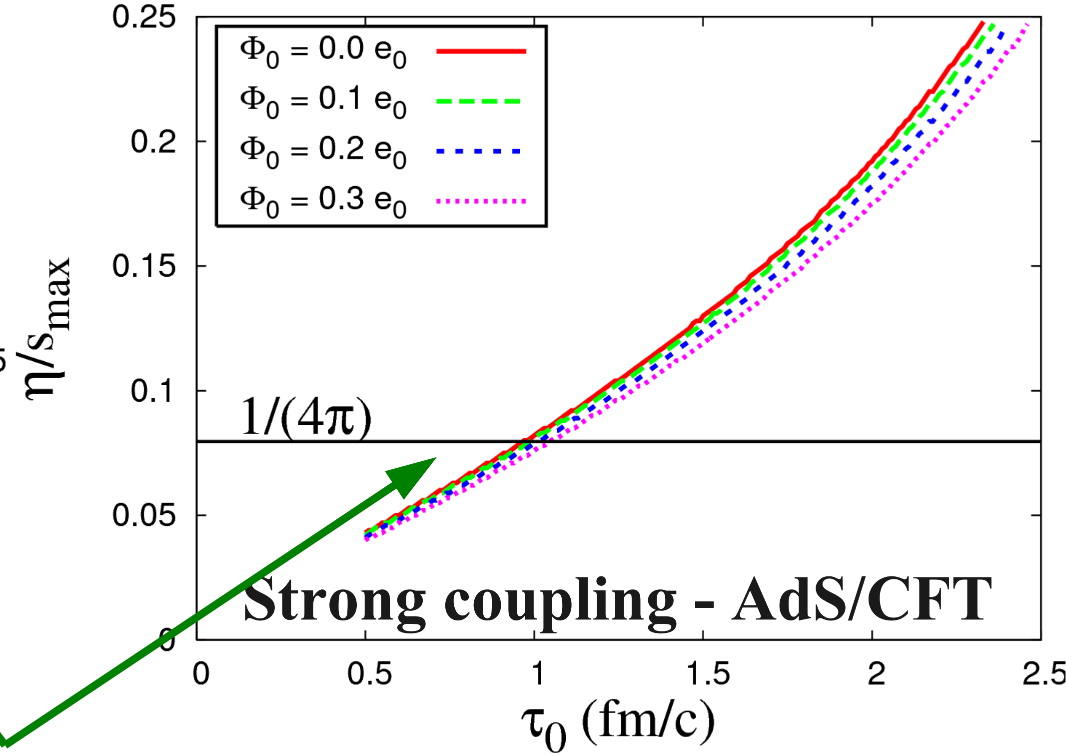
Loop over the starting time of
the hydrodynamical evolution

At large initial time, the expansion rate and entropy
production drops, and the viscosity bound disappears



Weak coupling - Boltzmann

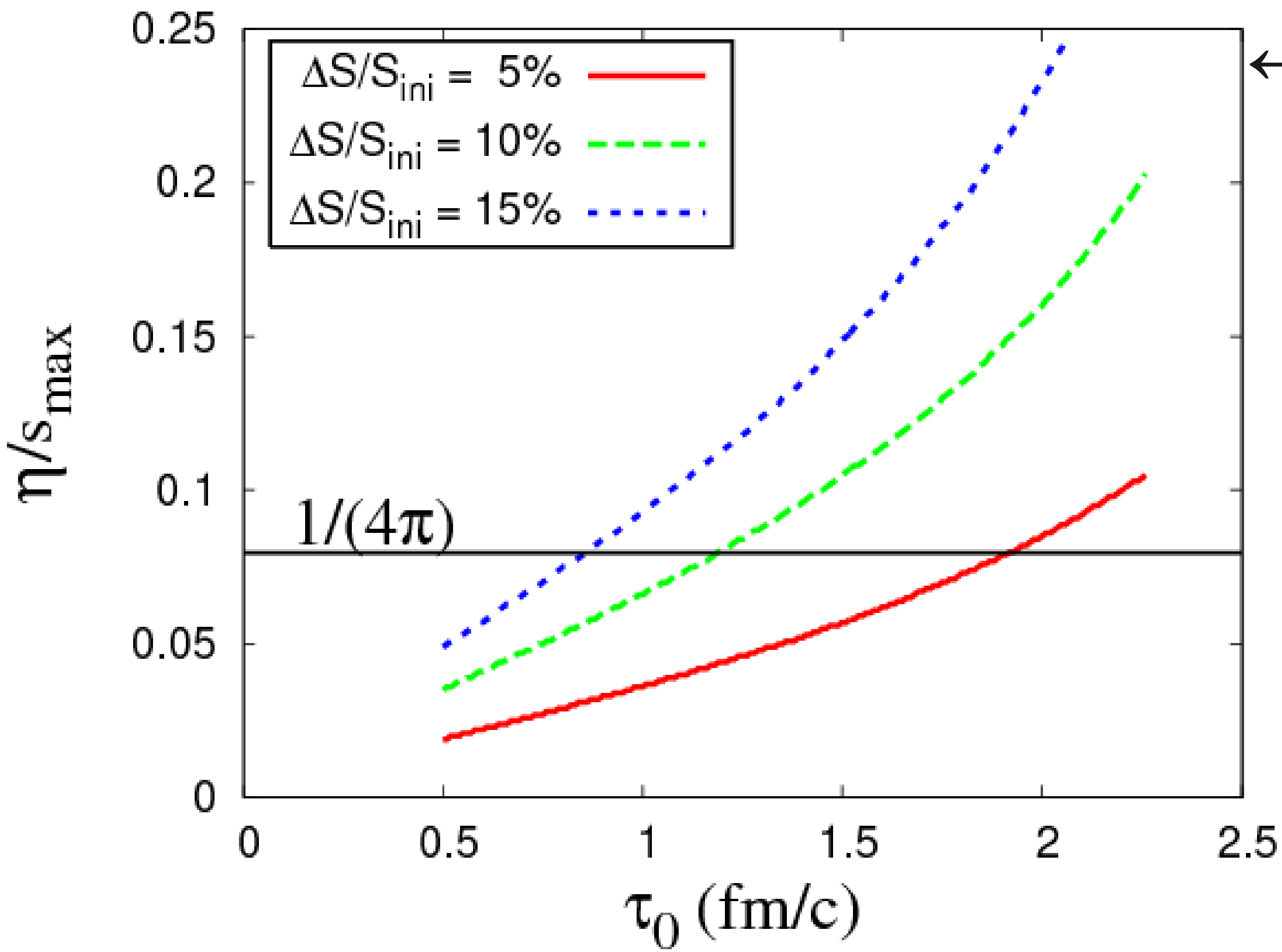
The relaxation time is 30 times smaller, and the system
is insensitive of the initial correction to equilibrium.



If the system is initially out of
equilibrium the entropy production
is larger for smaller viscosity

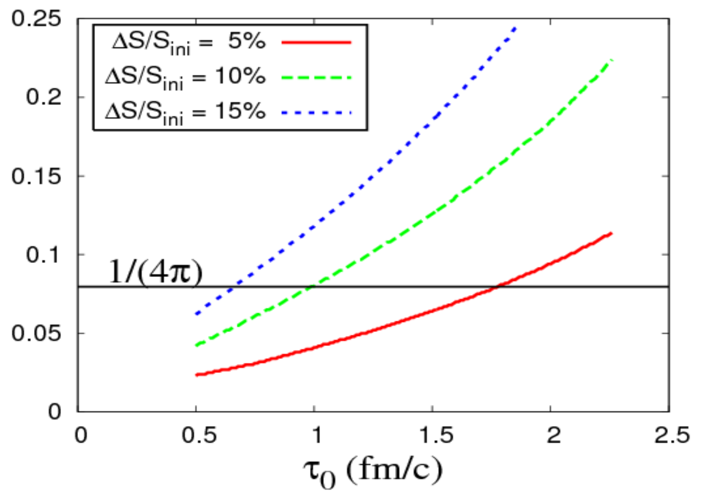
Conjectured lower-bound 1/(4π),
excludes too rapid thermalization!

$(\eta/s)_{max}$ from entropy production bound



← Boltzmann

AdS/CFT ↓

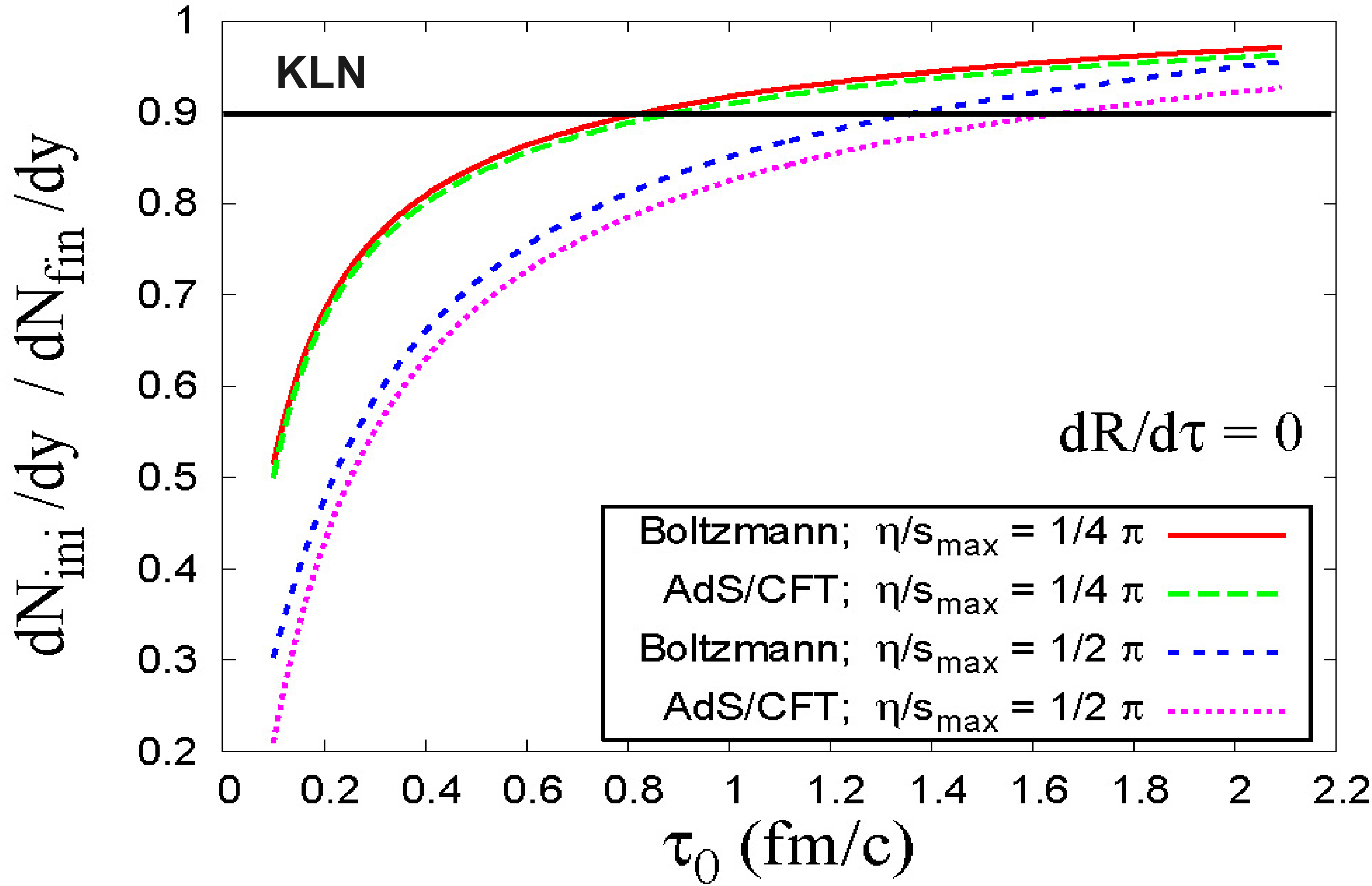


$$dR(\tau_0)/d\tau = 0$$

The initial stress is from first order theory.

Entropy production vs. the thermalization time

Fix the final multiplicity



Start the hydro such that in the end we reach the final multiplicity

Summary

- "Hydro" is a set of diff equations, and the solution depends strongly on initial condition
- Centrality dependence of dN/dy --> bound on entropy production
- Conjectured $\eta/s > 1/4\pi$ excludes too rapid thermalization
- $\tau_0 \sim 1$ fm/c