Joint University Accelerator School

Examination of Transverse Beam Dynamics

JUAS 2015

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1 Exercise:

Consider a quadrupole magnet (in thin-lens approximation) focusing electrons with $E = 1$ GeV. Its aperture (diameter) is 20 mm, and its length is 40 mm. The magnetic field at the radius is 0.1 T.

(i) What is the field gradient?

(ii) Calculate the normalised quadrupole field strength, k. What are the units of k?

(iii) What is the maximum angular deviation that this quadrupole can impart to a particle?

2 Exercise:

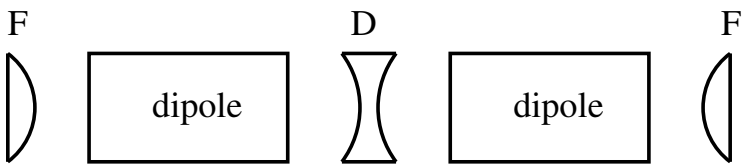
The position x of particles can easily be measured, for example using phosphor screens. It is more difficult to measure the particle's angular divergence x'.

(i) Assume a drift section with length L and two phosphor screens located at its ends. Using the transfer matrix formalism, show how the initial angle x'_0 can be derived from the measurements of the displacements x_0 and x at the two screens.

(ii) Now suppose that there is no initial screen, but instead a focusing quadrupole with focal length f. (Therefore the initial position x_0 cannot be measured anymore). How can the quadrupole be set to measure initial angles x'_0 without knowing x_0?

3 Exercise:

A Muon Collider is one of the options being considered for a forthcoming Higgs factory. The Muon Collider design is conceived as a ring with a circumference 6 km long where muons and anti-muons circulate for about 1000 turns before they decay. Let us assume that the optics of this machine is made of identical FODO cells, each cell with the following structure:



Assuming for the calculations the thin lens approximation, and considering the following parameters:

Parameter	Value
Beam energy	1.5 TeV
Total FODO cell length	80 m
Quadrupole gradient G	20 T/m
Quadrupole length	4 m
Dipole length	21.3 m

answer the following questions:

(i) Calculate: (1) for each dipole the bending angle and the necessary magnetic field; (2) the value of the focal length of each quadrupole

(ii) Is this lattice stable?

(iii) What are the vertical and horizontal tunes for this machine?

(iv) Compute the maximum and the minimum value for the betatron functions, and the maximum and minimum value of the dispersion

(v) Calculate the natural chromaticities for this machine

(vi) Let's imagine that all focusing quadrupoles of our ring are connected to faulty power supplies that coherently provide +10% more current than required. What it the tune shift induced? (neglect beta beat effects)

(vii) If instead the errors are randomly distributed, that is, each quadrupole is powered with a random current, Gaussian distributed within 10% RMS from the nominal value, what is the tune shift induced? (neglect beta beat effects)

4 Exercise:

Let's now assume that in such a Muon Collider there is an Interaction Point (IP). Preliminary designs foresee an IP with $\beta_x^* = \beta_y^* = 0.3$ cm. The peak luminosity available at a $\mu^+\mu^-$ collider can be written as:

$$L = \frac{N_\mathrm{b} N_{\mu^-} N_{\mu^+} f_\mathrm{rev}}{4\pi\sigma_x^*\sigma_y^*} \ [\mathrm{cm}^{-2}\mathrm{s}^{-1}]$$

where $N_\mathrm{b} = 4$ is the number of bunches per beam (we assume the same number of bunches for both the μ^- and the μ^+ beams), $N_{\mu^-} = N_{\mu^+} = 2 \times 10^{12}$ is the number of particles per bunch (we assume the same number for both μ^- and μ^+ bunches), and f_rev is the revolution frequency. The horizontal and vertical normalised emittances are: $\epsilon_{x,N} = \epsilon_{y,N} = 50$ μm. [Remember that the mass of the muon is $m_{\mu^\pm} \simeq 105.6$ MeV]

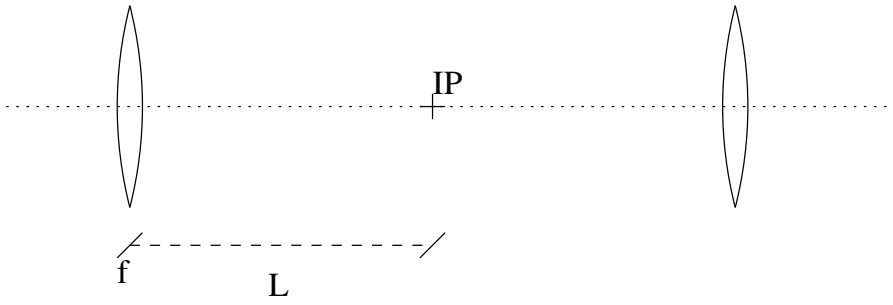
(i) Compute the revolution frequency f_rev, knowing that the circumference is 6 km and that the beam moves nearly at the speed of light

(ii) Calculate the beam transverse beam sizes σ_x^* and σ_y^* at the IP, and the luminosity L, at beam energy 1.5 TeV

(iii) What is the value of the betatron function at position $s = 0.5$ m from the IP?

5 Exercise:

Consider the following low-beta insertion around an interaction point (IP). The quadrupoles are placed with mirror-symmetry with respect to the IP:



The beam enters the quadrupole with twiss parameters $\beta_0 = 20$ m and $\alpha_0 = 0$. The drift space has length $L = 10$ m.

(i) Determine the focal length of the quadrupole in order to locate the waist at the IP.

(ii) What is the value of $\beta^\star$?

(iii) What is the phase advance between the quadrupole and the IP?