Joint Universities Accelerator School

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# Normal-conducting accelerator magnets


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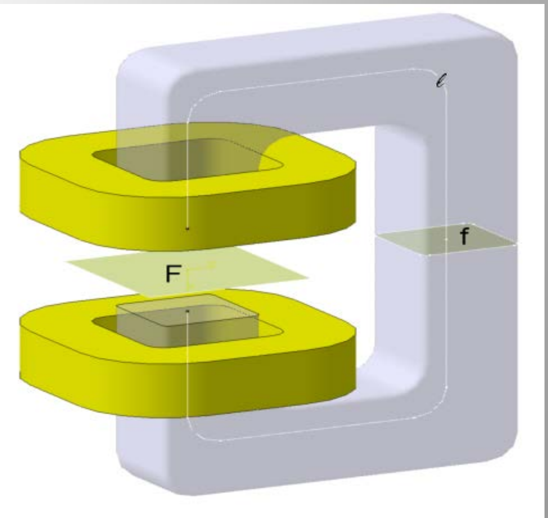
# Lecture 2: Analytical design

- Goals in magnet design and coherence
- What do we need to know before starting?
- Defining the requirements & constraints
- Deriving the magnet main parameters
- Coil design and cooling

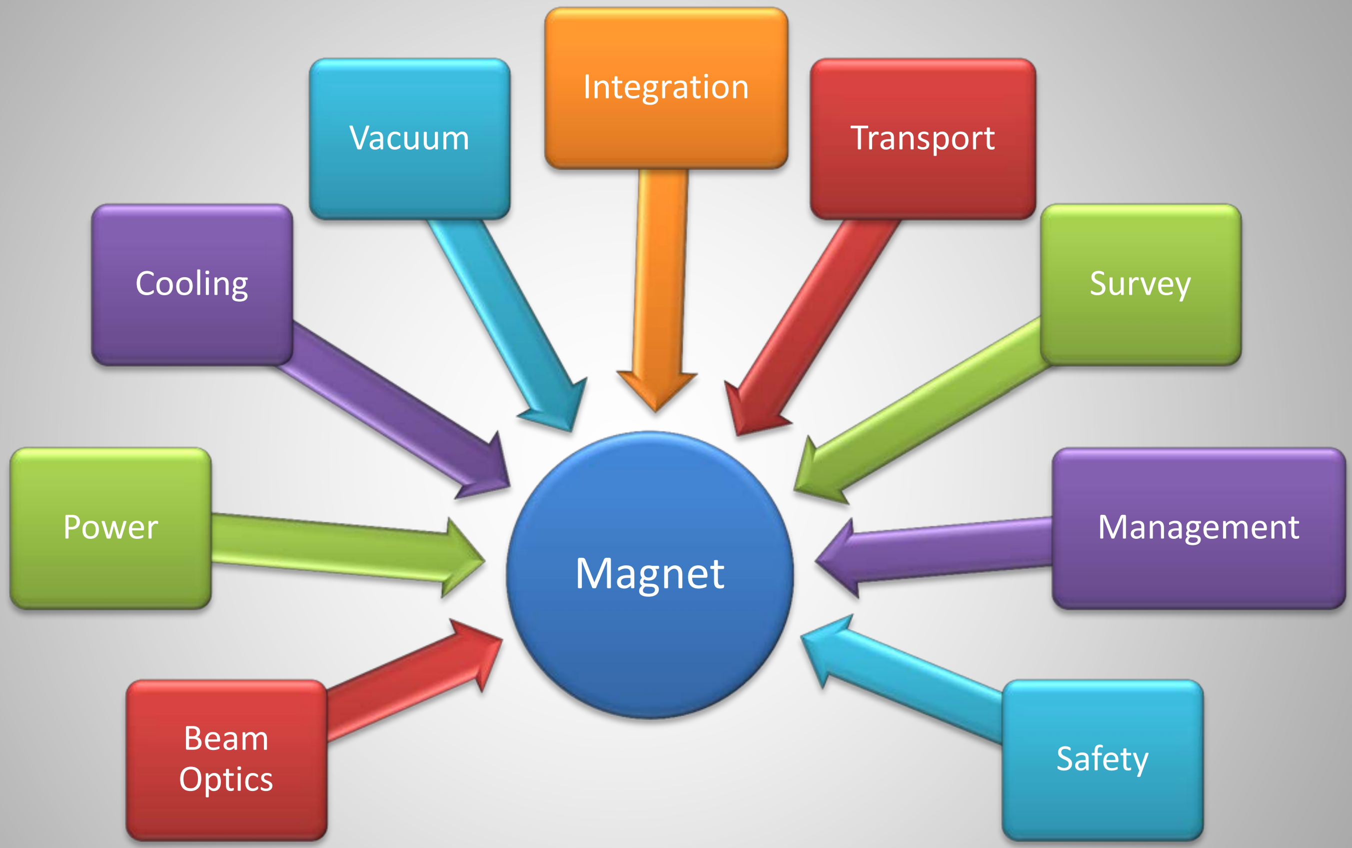


# Goals in magnet design

The goal is to produce a product just good enough to perform reliably with a sufficient safety factor at the lowest cost and on time.

- Good enough:
  - Obvious parameters are clearly specified, but tolerance difficult to define
  - Tight tolerances lead to increased costs
- Reliability:
  - Get MTBF and MTTR reasonably low
  - Reliability is usually unknown for new design
  - Requires experience to search for a compromise between extreme caution and extreme risk (expert review)
- Safety factor:
  - Allows operating a device under more demanding condition as initially foreseen
  - To be negotiated between the project engineer and the management
  - Avoid inserting safety factors a multiple levels (costs!)

# Magnet interfaces



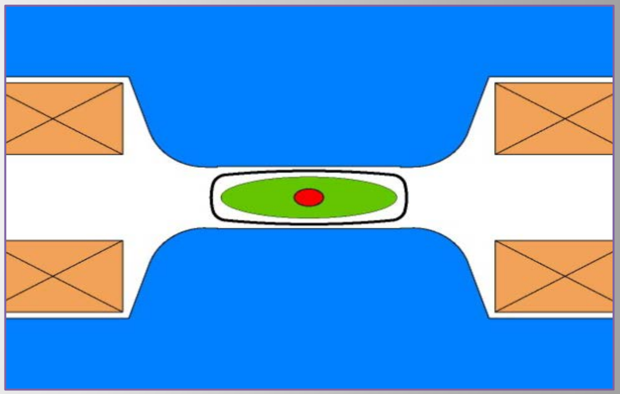

A magnet is not a stand-alone device!

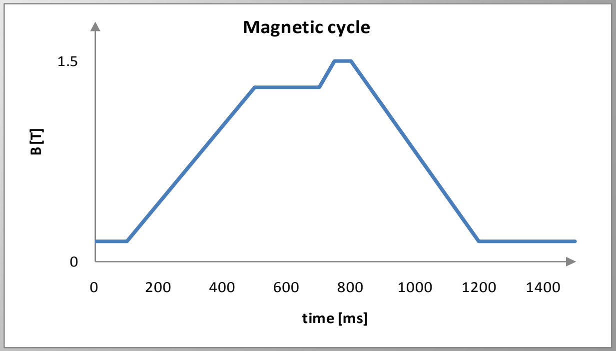
# Design process

## Electro-magnetic design is an iterative process:

Collect input data → Analytical design → Numerical 2D/3D simulations → Mechanical design → Drawings & specifications

- Field strength (gradient) and magnetic length
- Integrated field strength (gradient)
- Aperture and ‚good field region'
- Field quality:
  - field homogeneity
  - maximum allowed multi-pole errors
  - settling time (time constant)
- Operation mode: continous, cycled
- Electrical parameters
- Mechanical dimensions
- Cooling
- Environmental aspects





# General requirements

| | |
|---|---|
| Magnet type and purpose | • Dipole: bending, steering, extraction<br>• Quadrupole, sextupole, octupole<br>• Combined function, solenoid, special magnet |
| Installation | • Storage ring, synchrotron light source, collider<br>• Accelerator<br>• Beam transport lines |
| Quantity | • Installed units<br>• Spare units (~10 %) |

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# Performance requirements

**Beam parameters**

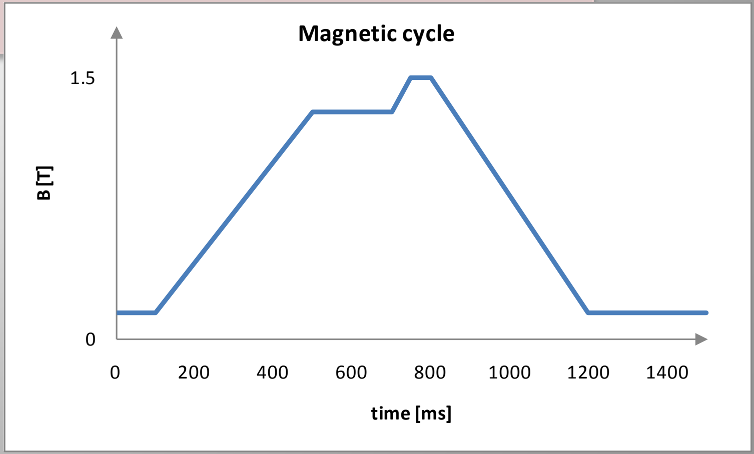
- Type of beam, energy range, deflection angle
- Integrated field (gradient)
- Local field (gradient) and magnetic length

**Aperture**

- Physical aperture
- 'Good field region'

**Operation mode**

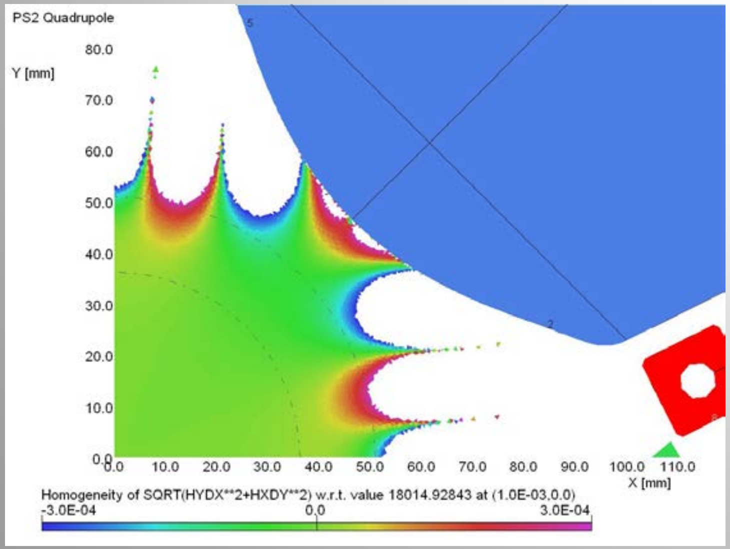
- Continuous
- Pulse-to-pulse modulation (ppm)
- Ramp rate (T/s)

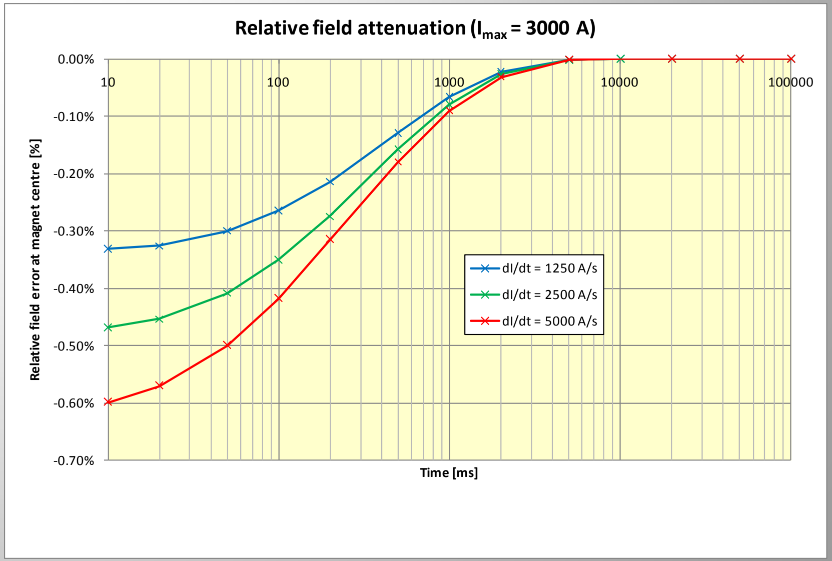


# Performance requirements

## Field quality

- Homogeneity (uniformity)
- Maximum allowed multipole errors
- Stability & reproducibility
- Settling time (time constant)
- Allowed residual field)



# Physical requirements

**Geometric boundaries**
- Available space
- Transport limitations
- Weight limitations

**Accessibility**
- Crane
- Connections (electrical, hydraulic)
- Alignment targets


# Interfaces

Equipment linked to the magnet is defining the boundaries and constraints

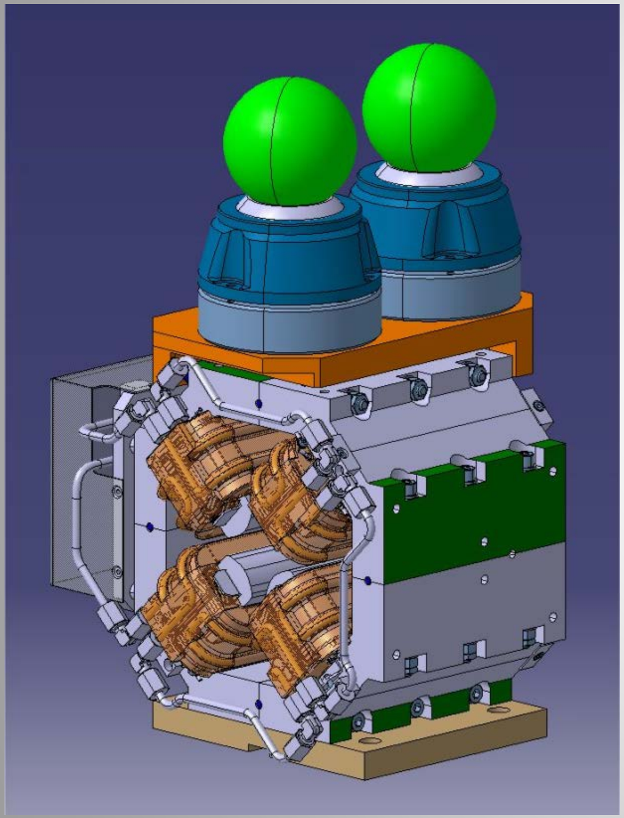
| | |
|---|---|
| Power converter | • Max. current (peak, RMS)<br>• Max. voltage<br>• Pulsed/dc |
| Cooling | • Max. flow rate and pressure drop<br>• Water quality (aluminium/copper circuit)<br>• Inlet temperature<br>• Available cooling power |
| Vacuum | • Size and material of vacuum chamber<br>• Space for pumping ports, bake-out<br>• Captive vacuum chamber |

# Environmental aspects

Other aspects, which can have an influence on the magnet design

| | |
|---|---|
| Environment temperature | • Risk of condensation<br>• Heat dissipation into the tunnel |
| Ionizing radiation | • High radiation levels require radiation hard materials<br>• Special devices to allow fast repair/replacement |
| Electro-magnetic compatibility | • Magnetic fringe fields disturbing other equipment (beam diagnostics)<br>• Surrounding equipment perturbing field quality |
| Safety | • Electrical safety<br>• Interlocks |

# Magnet Components



Alignment targets

Yoke

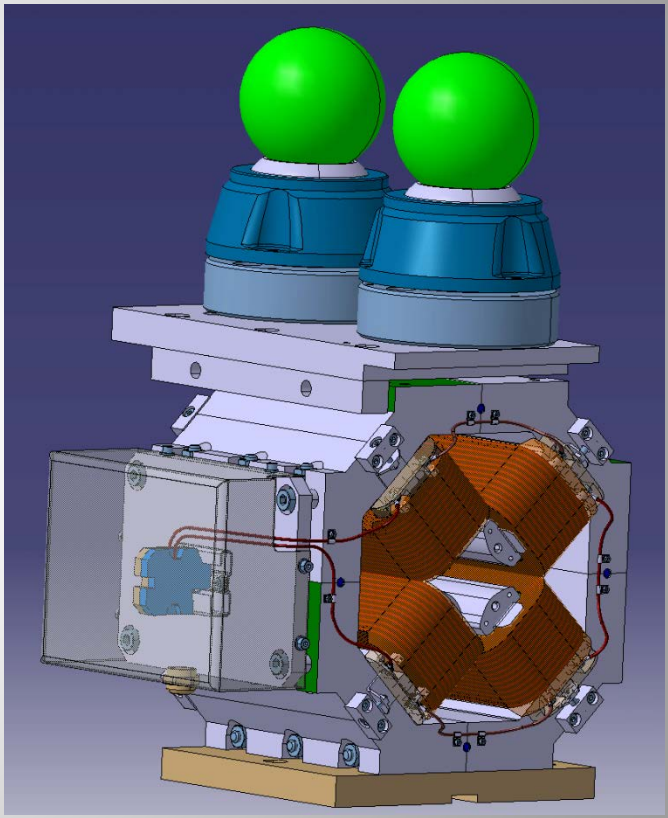
Coils

Sensors

Cooling circuit

Connections

Support



# Magnetic design

Translate the beam optic requirements into a magnetic design

- Magnetic induction / field quality
- Aperture size
- Magnet excitations (Amp-turns)
- Yoke cross-section
- Yoke material

# Beam rigidity

Beam rigidity $(B\rho)$ [Tm]: $$(B\rho) = \frac{p}{q} = \frac{1}{qc}\sqrt{T^2 + 2T\,E_0}$$

$p$: particle momentum [kg m/s]

$q$: particle charge number [Coulombs]

$c$: speed of light [m/s]

$T$: kinetic beam energy [eV]

$E_0$: particle rest mass energy [eV]
( 0.51 MeV for electrons, 938 MeV for protons)

*" ...resistance of the particle beam against a change of direction when applying a bending force..."*

# Magnetic induction

Dipole bending field $B$ [T]:

$$B = \frac{(B\rho)}{r_M}$$

$B$: Flux density or magnetic induction (vector) [T]

$r_M$: magnet bending radius [m]

Quadrupole field gradient $B'$ [T/m]:

$$B' = (B\rho)k$$

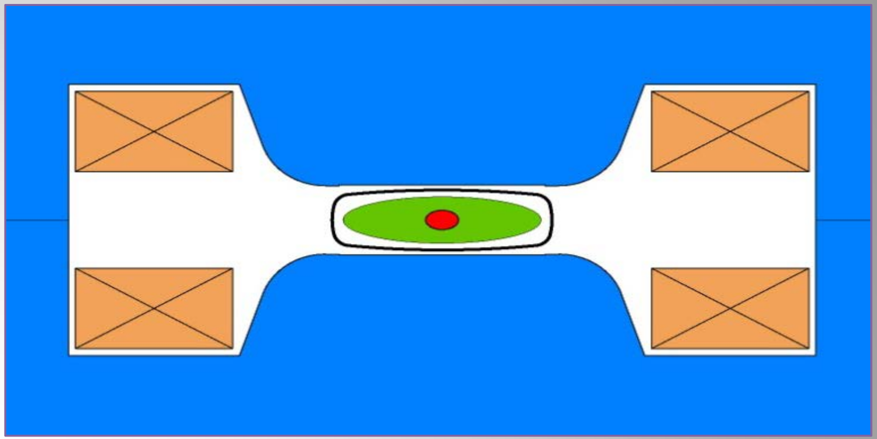
$k$: quadrupole strength [m$^{-2}$]

Sextupole differential gradient $B''$ [T/m$^2$]:

$$B'' = (B\rho)m$$

$m$: sextupole strength [m$^{-3}$]

# Aperture size



## Aperture =

Good field region

Maximum beam size

- Lattice functions: beta functions and dispersion
- Geometrical transverse emittancies (energy depended)
- Momentum spread
- Envelope (typical 3-sigma)
- Largest beam size usually at injection

$$\sigma = \sqrt{\varepsilon \beta + \left( D \frac{\Delta p}{p} \right)^2}$$

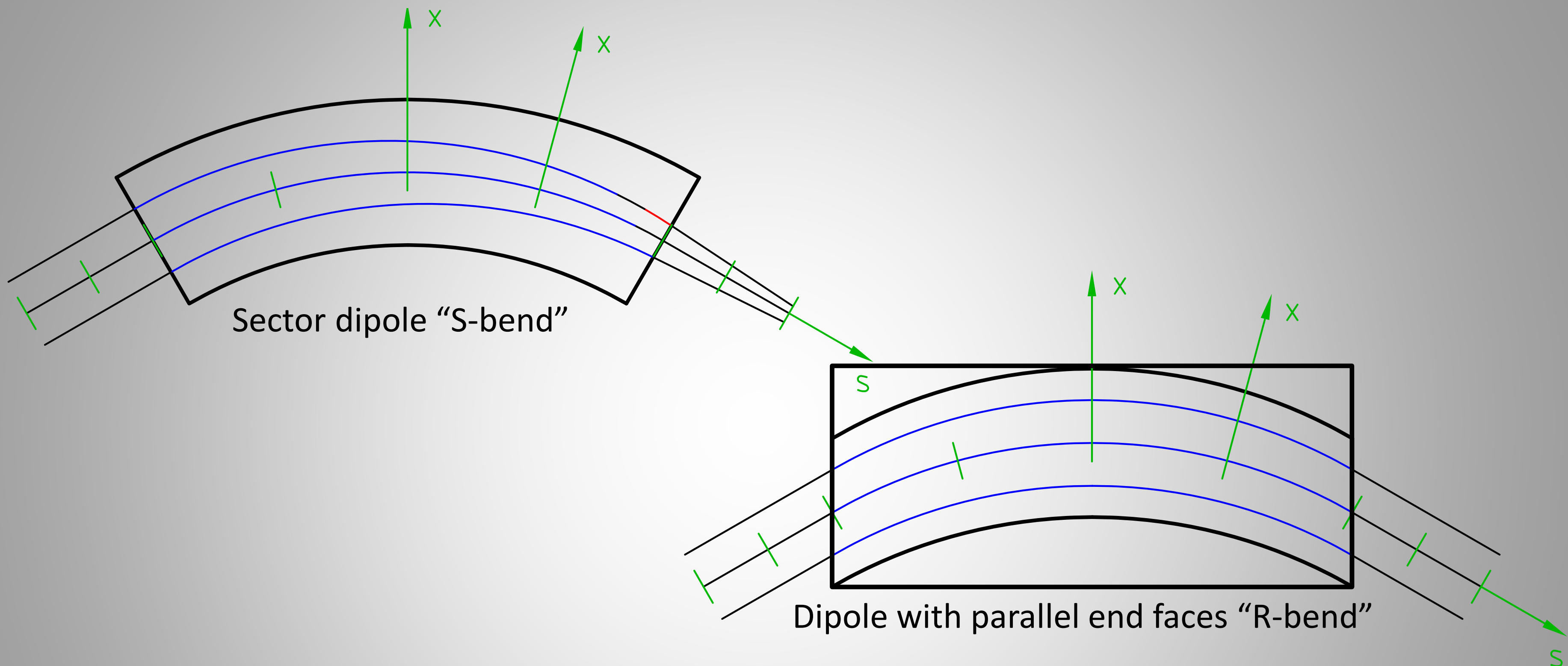
+ Closed orbit distortions (few mm)

+ Vacuum chamber thickness (0.5 – 5 mm)

+ Installation and alignment margin (0 – 10 mm)

*"...good field region: region where the field quality has to be within certain tolerances..."*

# S-bend vs. R-bend




The two types are slightly different in terms of focusing:

- S-bend: focuses horizontally
- R-bend: no horizontal focusing, but small vertical defocusing at the edges

Note: the curvature has no effect, it is just for saving material, otherwise the pole would have to be wider ("*sagitta*").

# Excitation current in a dipole

Ampere's law $\oint \vec{H} \cdot d\vec{l} = NI$ and $\vec{B} = \mu \vec{H}$ with $\mu = \mu_0 \mu_r$

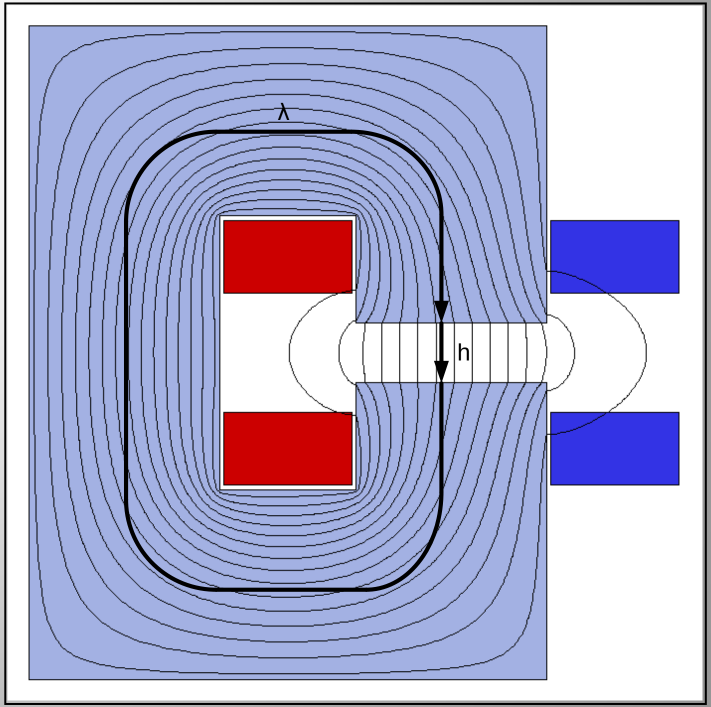
leads to $$NI = \oint \frac{\vec{B}}{\mu} \cdot d\vec{l} = \int_{gap} \frac{\vec{B}}{\mu_{air}} \cdot d\vec{l} + \int_{yoke} \frac{\vec{B}}{\mu_{iron}} \cdot d\vec{l} = \frac{Bh}{\mu_{air}} + \frac{B\lambda}{\mu_{iron}}$$

assuming, that B is constant along the path

If the iron is not saturated: $$\frac{h}{\mu_{air}} >> \frac{\lambda}{\mu_{iron}}$$

then: $$\boxed{NI_{(per\ pole)} \approx \frac{Bh}{2\eta\mu_0}}$$

$h$: gap height [m]
$\eta$: efficiency (typically 95% - 99 %)



# Reluctance and efficiency

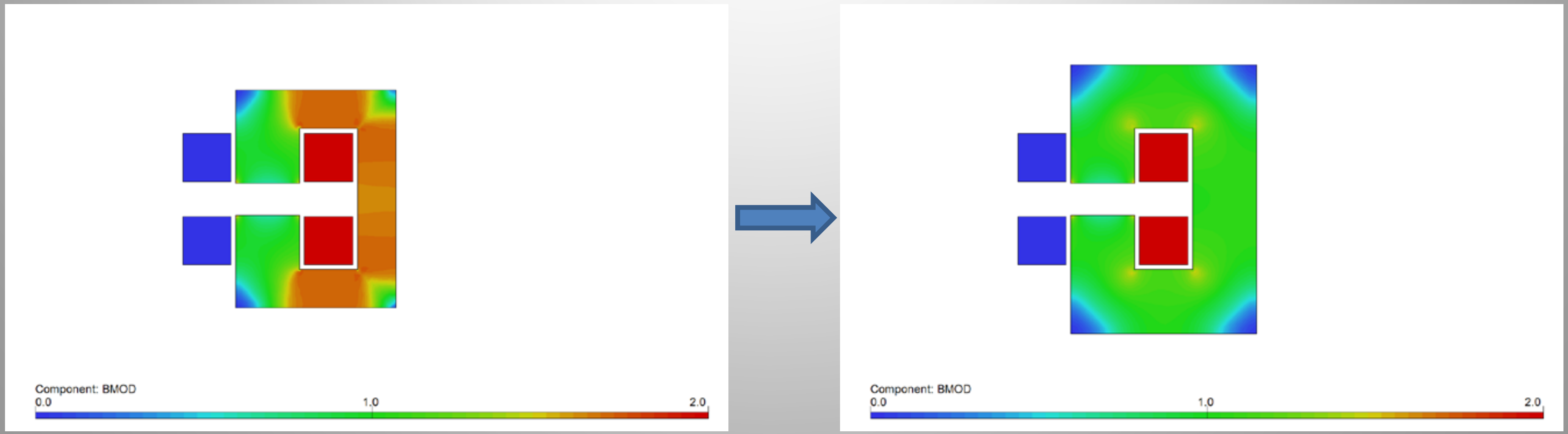


Reluctance: $R_M = \frac{NI}{\Phi} = \frac{l_M}{A_M \mu_r \mu_0}$

Term ( $\frac{\lambda}{\mu_{iron}}$ ) in previous slide is called 'normalized reluctance' of the yoke

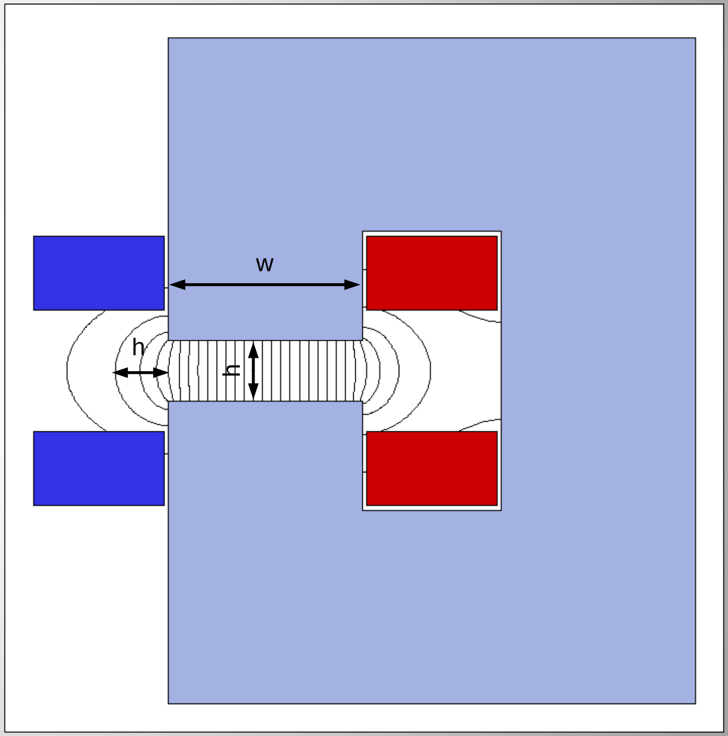
Keep iron yoke reluctance less than a few % of air reluctance ( $\frac{h}{\mu_0}$ ) by providing sufficient iron cross section ($B_{iron} < 1.5$ T)

Efficiency: $\eta = \frac{R_{M,gap}}{R_{M,gap} + R_{M,yoke}} \approx 99\,\%$



# Magnetic flux

Flux in the yoke includes the gap flux <u>and</u> stray flux




Total flux in the return yoke:

$$\Phi = \int_A B \cdot dA \approx B_{gap}(w+2h)l_{mag}$$

$$B_{leg} \cong B_{gap}\frac{w+2h}{w_{leg}}$$

# Pole design

It is easy to derive perfect mathematical pole configurations for a specific field configuration

In practice poles are not ideal: finite width and end effects result in multi-pole errors disturbing the main field

The uniform field region is limited to a small fraction of the pole width

Estimate the size of the poles and calculate the resulting fields

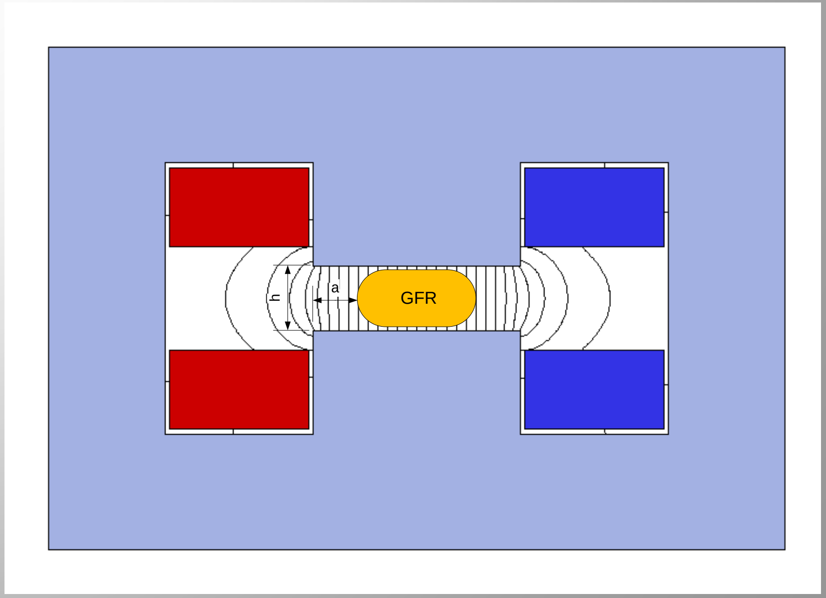
Better approach: calculate the necessary pole overhang using:

$$x_{unoptimized} = 2\frac{a}{h} = -0.36 \ln \frac{\Delta B}{B_0} - 0.90$$

$x$: pole overhang normalized to the gap

$a$: pole overhang: excess pole beyond the edge of the good field region to reach the required field uniformity

$h$: magnet gap

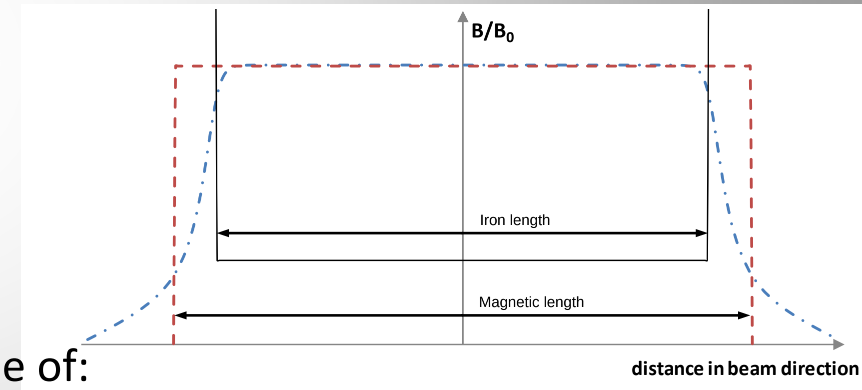


# Magnetic length

Coming from ∞, B increases towards the magnet center (stray flux)

Magnetic length: $$l_{mag} = \frac{\int_{-\infty}^{\infty} B(z) \cdot dz}{B_0}$$

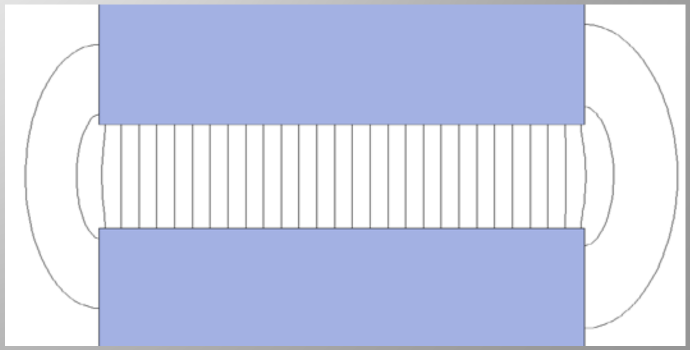
'Magnetic' length > iron length




Approximation for a dipole: $l_{mag} = l_{iron} + 2hk$

Geometry specific constant $k$ gets smaller in case of:

- pole length < gap height
- saturation
- precise determination only by measurements or numerical calculations



# Excitation current in a Quadrupole



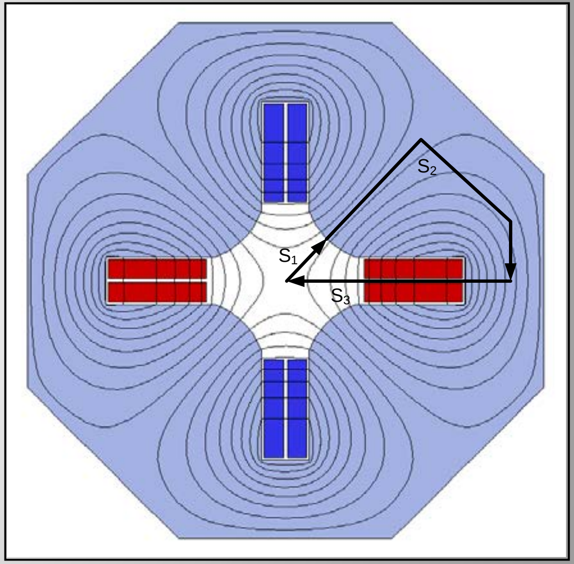
Choosing the shown integration path gives:

$$NI = \oint \vec{H} \cdot \vec{dl} = \int_{s1} \vec{H}_1 \cdot \vec{dl} + \int_{s2} \vec{H}_2 \cdot \vec{dl} + \int_{s3} \vec{H}_3 \cdot \vec{dl}$$

For a quadrupole, the gradient $B' = \frac{dB}{dr}$ is constant

and $B_y = B'x \qquad B_x = B'y$

Field modulus along $s_1$: $H(r) = \frac{B'}{\mu_0}\sqrt{x^2 + y^2} = \frac{B'}{\mu_0} r$




Neglecting $H$ in $s_2$ because: $R_{M,s2} = \frac{s_2}{\mu_{iron}} << \frac{s_1}{\mu_{air}}$

and along $s_3$: $\int_{s3} \vec{H}_3 \cdot \vec{dl} = 0$

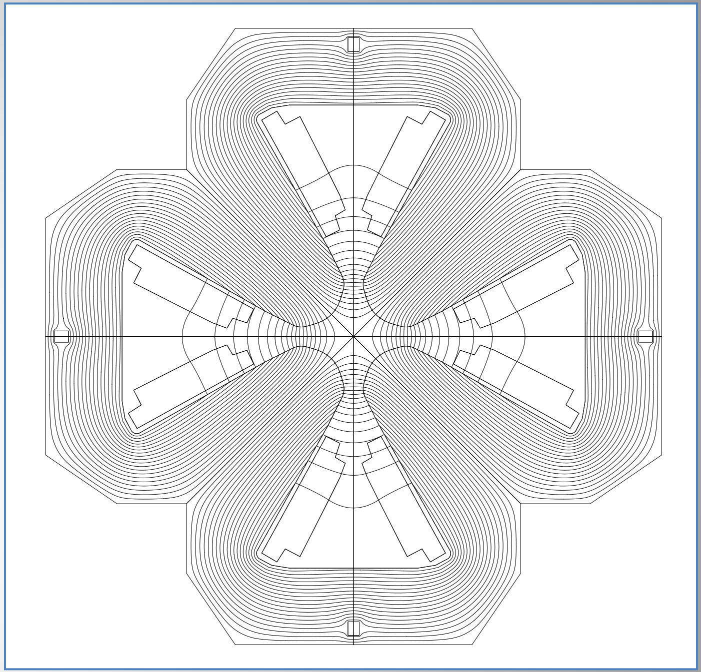
Leads to: $NI \approx \int_0^R H(r)\, dr = \frac{B'}{\mu_0} \int_0^R r\, dr$

$$\boxed{NI_{(per\ pole)} = \frac{B' r^2}{2\eta\mu_0}}$$

# Magnetic length

Magnetic length for a quadrupole:

$$l_{mag} = l_{iron} + 2r\,k$$



$NI$ increases with the square of the quadrupole aperture:

$$NI \propto r^2 \qquad P \propto r^4$$

More difficult to accommodate the necessary Ampere-turns (= coil cross section)

$\rightarrow$ truncating the hyperbola leads to a decrease in field quality

# Coil design

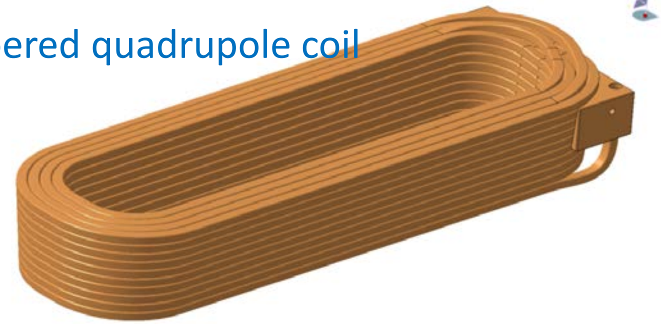
Ampere-turns $NI$ are determined, but the current density $j$, the number of turns $N$ and the coil cross section need to be defined

- Coil type selection
- Power requirements
- Cooling circuit computation
- Conductor selection
- Optimization

Bedstead or saddle coil

Racetrack coil

Tapered quadrupole coil



# Power requirements

Assuming the magnet cross-section and the yoke length are known, one can estimate the total dissipated power per magnet:

$$\boxed{P_{dipole} = \rho \frac{Bh}{\eta \mu_0} j l_{avg} 10^6}$$

$$\boxed{P_{quadrupole} = 2\rho \frac{B' r^2}{\eta \mu_0} j l_{avg} 10^6}$$

$j$: current density [A/mm²]: $j = \frac{NI}{f_c A} = \frac{I}{a_{cond}}$

$\rho$: resistivity [Ωm] of coil conductor

$l_{avg}$: average turn length [m]; approximation: 2.5 $l_{iron}$ < $l_{avg}$ < 3 $l_{iron}$ for racetrack coils

$a_{cond}$: conductor cross section [mm²]

$A$: coil cross section [mm²]

$f_c$: filling factor = $\frac{\text{net conductor area}}{\text{coil cross section}}$

(includes geometric filling factor, insulation, cooling duct, edge rounding)

Note: for a constant geometry, the power loss $P$ is proportional to the current density $j$.

Note: If the magnet is not operated in dc, the rms power has to be considered.

# Number of turns

The determined power can be divided into voltage and current: $P = UI$

Basic relations: $P_{magnet} \propto j$ $V_{magnet} \propto Nj$ $R_{magnet} \propto N^2 j$

**Large N = low current = high voltage**

- Small terminals
- Small conductor cross-section
- Thick insulation for coils and cables
- Less good filling factor in the coils
- Large coil volume
- Low power transmission loss

**Small N = high current = low voltage**

- Large terminals
- Large conductor cross-section
- Thin insulation in coils and cables
- Good filling factor in the coils
- Small coil volume
- High power transmission loss

The number of turns $N$ are chosen to match the impedances of the power converter and connections

Attention when ramping the magnet: $V_{tot} = RI + L\dfrac{dI}{dt}$

# Practical example

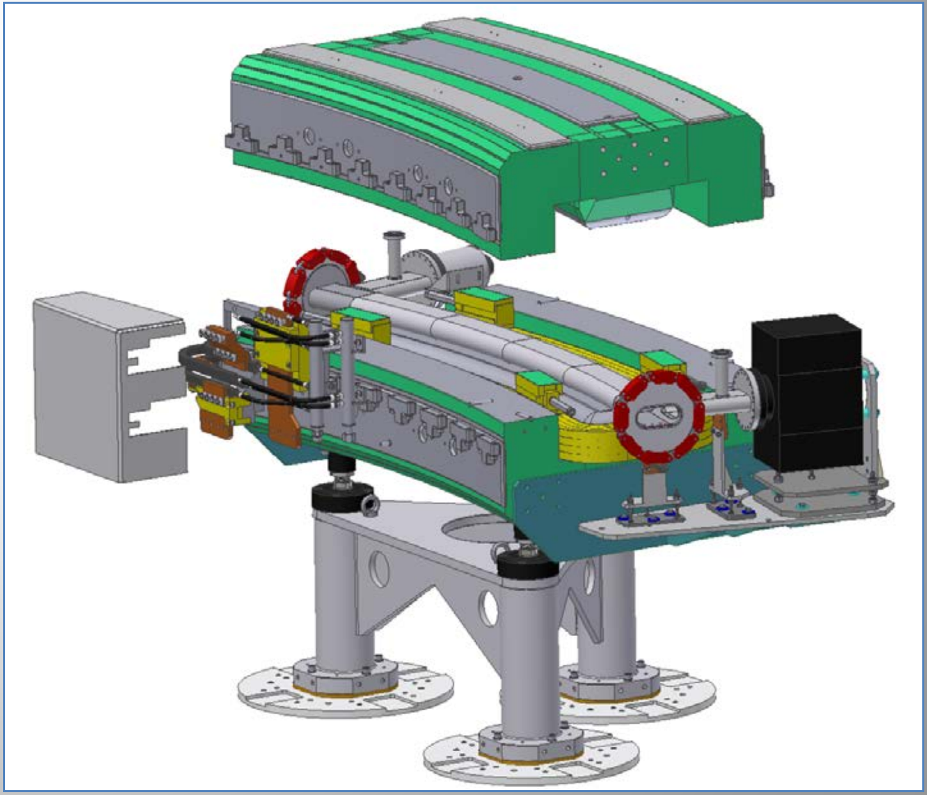
ebg *Med*Austron

MedAustron: ion therapy facility under construction near Vienna/Austria

Providing beam energies from 120 to 400 MeV/u for carbon ions and from 60 to 220 MeV for protons

16 synchrotron bending magnets:

- Bending angle: 22.5°
- Bending radius: 4.231 m
- Field ramp rate: 3.75 T/s
- Max. current: 3000 A
- Overall length: < 2 m
- Good field region: 120 x 56 mm
- Field quality: $\frac{\Delta \int B \cdot dl}{\int B \cdot dl} = 2 \cdot 10^{-4}$



# Coil examples

## MedAustron Synchrotron

### Dipole

- # of turns $N_{\text{(per pole)}}$: 16
- Current: 3000 A
- Voltage: 100 V

### Quadrupole

- # of turns $N_{\text{(per pole)}}$: 20
- Current: 650 A
- Voltage: 12 V

### Sextupole

- # of turns $N_{\text{(per pole)}}$: 14
- Current: 650 A
- Voltage: 16 V

### Corrector

- # of turns $N_{\text{(per pole)}}$:240/96
- Current: 15/30 A
- Voltage: 7/3 V

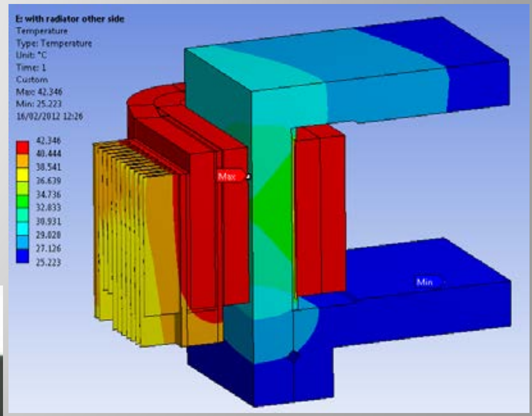
# Coil cooling



## Air cooling by natural convection:

- Current density
  - $j \leq 2$ A/mm$^2$ for small, thin coils
- Cooling enhancement:
  - Heat sink with enlarged radiation surface
  - Forced air flow (cooling fan)
- Only for magnets with limited strength (e.g. correctors)

## Direct water cooling:




- Typical current density $j \leq 10$ A/mm$^2$
- Requires demineralized water (low conductivity) and hollow conductor profiles

## Indirect water cooling:

- Current density $j \leq 3$ A/mm$^2$
- Tap water can be used

# Direct water cooling

Practical recommendations and canonical values:

- Water cooling: 2 A/mm² ≤ $j$ ≤ 10 A/mm²
- Pressure drop: 1 ≤ $\Delta p$ ≤ 10 bar (possible up to 20 bar)
- Low pressure drop might lead to more complex and expensive coil design
- Flow velocity should be high enough so flow is turbulent
- Flow velocity $u_{avg} \leq$ 4 m/s to avoid erosion and vibrations
- Acceptable temperature rise: $\Delta T \leq$ 30°C
- For advanced stability: $\Delta T \leq$ 15°C

Assuming:

- Long, straight and smooth pipes without perturbations
- Turbulent flow = high Reynolds number ($Re$ > 4000)
- Good heat transfer from conductor to cooling medium
- Temperature of inner conductor surface equal to coolant temperature
- Isothermal conductor cross section

Note: practical (non-SI) units are used in the following slides for convenience

# Direct water cooling

Useful simplified formulas using water as cooling fluid:

Water flow $Q$ [litre/min] necessary to remove power $P$: $Q_{water} = 14.3\frac{P}{\Delta T}10^{-3}$

$P$: dissipated power [W]

$\Delta T$: temperature increase [°C]

Average water velocity $u_{avg}$ [m/s] in a round tube: $u_{avg} = \frac{Q}{A} = 66.67\frac{Q}{\pi d^2}$

$A = \frac{\pi d^2}{4}$ : tube section [mm²]

$d$: hydraulic diameter [mm]

Pressure drop $\Delta p$ [bar] : $\Delta p \approx 60\, l\, \frac{Q^{1.75}}{d^{4.75}}$ (from Blasius' law)

$l$: cooling circuit length [m]

Reynolds number $Re$ []: $Re = d\frac{u_{avg}}{\nu}10^{-3}$

*Re:* dimensionless quantity used to help predict similar flow patterns in different fluid flow situations

$\nu$: kinematic viscosity of coolant is temperature depending, for simplification it is assumed to be constant ($6.58 \cdot 10^{-7}$ m²/s @ 40°C for water)

# Direct water cooling

Number of cooling circuits per coil: $\Delta p \propto \frac{1}{K_W^3}$

$\rightarrow$ Doubling the number of cooling circuits reduces the pressure drop by a factor of eight for a constant flow

Diameter of cooling channel: $\Delta p \propto \frac{1}{d^5}$

$\rightarrow$ Increasing the cooling channel by a small factor can reduce the required pressure drop significantly

# Cooling circuit design

Already determined: current density $j$, power $P$, current $I$, number of turns $N$

1. Select number of layers $m$ and number of turns per layer $n$
2. Round up $N$ if necessary to get reasonable (integer) numbers for $n$ and $m$
3. Define coil height $c$ and coil width $b$: $A = bc = \frac{NI}{j\, f_c}$ (Aspect ratio $c : b$ between 1 : 1 and 1 : 2 and $0.6 \le f_c \le 0.8$)
4. Calculate average turn length $l_{avg} = pole\ perimeter + 4b$
5. The total length of cooling circuit $l = \frac{K_c N l_{avg}}{K_w}$ (start with single cooling circuit per coil)
6. Select $\Delta T$, $\Delta p$ and calculate cooling hole diameter $d = 0.5 \left( \frac{P}{\Delta T\, K_w} \right)^{0.368} \left( \frac{l}{\Delta p} \right)^{0.21}$
7. Change $\Delta p$ or number of cooling circuits, if necessary
8. Determine conductor area $a = \frac{I_{nom}}{j} + \frac{d^2 \pi}{4} + r_{edge}(4 - \pi)$
9. Select conductor dimensions and insulation thickness
10. Verify if resulting coil dimensions, $N$, $I$, $V$, $\Delta T$ are still compatible with the initial requirements (if not, start new iteration)
11. Compute coolant velocity and coolant flow
12. Verify if Reynolds number is inside turbulent range ($Re > 4000$)

$K_c$: Number of coils

$K_w$: Number of cooling circuits per coil

# Cooling water properties

- For the cooling of hollow conductor coils demineralised water is used (exception: indirect cooled coils)
- Water quality essential for the performance and the reliability of the coil (corrosion, erosion, short circuits)
- Resistivity > $0.1 \times 10^6\ \Omega m$
- pH between 6 and 6.5 (= neutral)
- Dissolved oxygen below 0.1 ppm
- Filters to remove particles and loose deposits to avoid cooling duct obstruction

# Summary

- Magnetic design means translating beam optic requirements
- Before starting the design, all input parameters, requirements, constraints and interfaces have to be known and well understood
- Establishing the coherence between beam physics requirements, magnet design & manufacture, and measurements is indispensable for the success of a project
- Analytical design is necessary to derive the main parameters of the future magnet before entering into a detailed design using numerical methods
- Magnet design is an iterative process often requiring a high level of experience