SRF Tutorial – part I

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Introduction

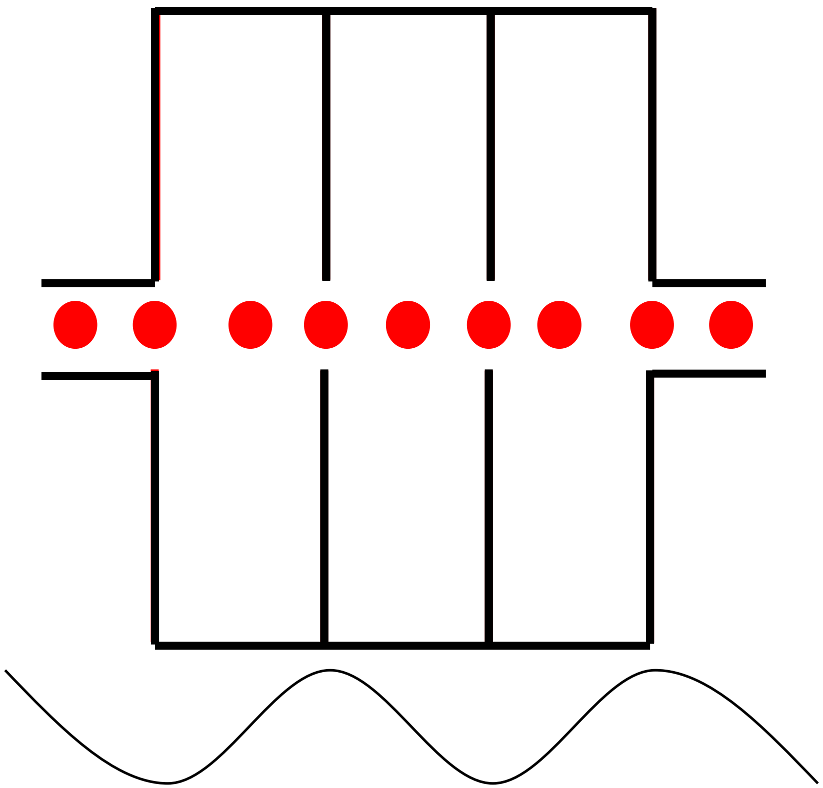
- What is a RF cavity?
  - Q factor
  - Different shapes
- Surface impedance
  - Definition
  - Normal metals
- Basics of superconductivity
  - Surface impedance of superconductors
  - Two fluids
  - BCS
  - Skin depth
- Cryogenics

Suggested readings

- Review paper:
  - J.P. Turneaure, J. Halbritter and H.A. Schwettman, J. Supercond. 4 (1991) 341
- Details:
  - J. Halbritter, Z. Physik 238 (1970) 466
  - J. Halbritter, Z. Physik 243 (1971) 201
- Books:
  - RF Superconductivity for Particle Accelerators by H. Padamsee, Vols I & II
  - HTS Thin Films at Microwave Frequencies by M. Hein (Springer, Berlin, 1999)
- CERN Accelerator Schools: Superconductivity in Particle Accelerators "Yellow reports" n. 89-04, 96-03 and 2014-05
- http://w4.lns.cornell.edu/public/CESR/SRF/SRFHome.html and links therein
- Wikipedia http://en.wikipedia.org/wiki/Superconducting_Radio_Frequency !!!

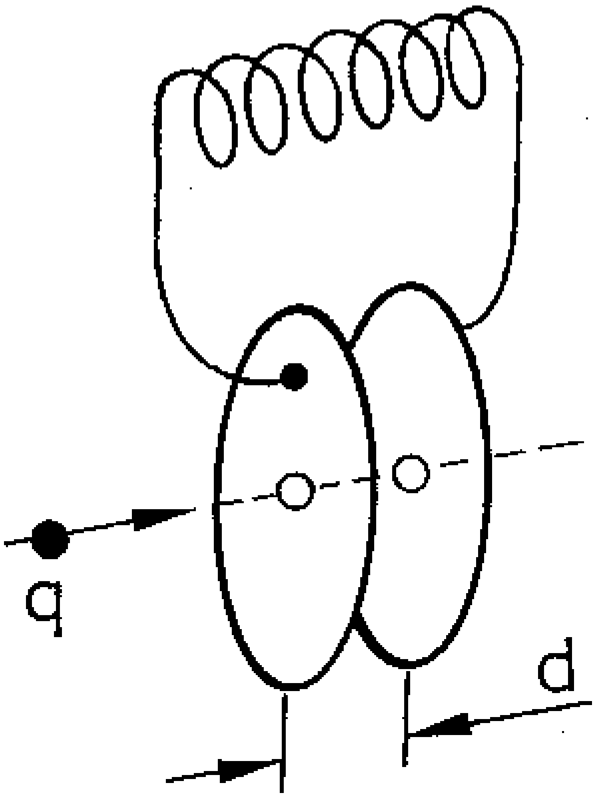
How to accelerate a particle?

- You need electric fields of the order of several MV/m
- You cannot do it in DC!
- You need a trick...



What is an RF cavity?

- An accelerating RF cavity is a resonant RLC circuit



$$\nu_0 = \frac{1}{2\pi\sqrt{LC}}$$

In order to fully exploit the accelerating effect, it is required that the transit time of the particle be ≤ than the half-period of the RF wave

$$d/v_{particle} \leq 1/(2\nu_0)$$

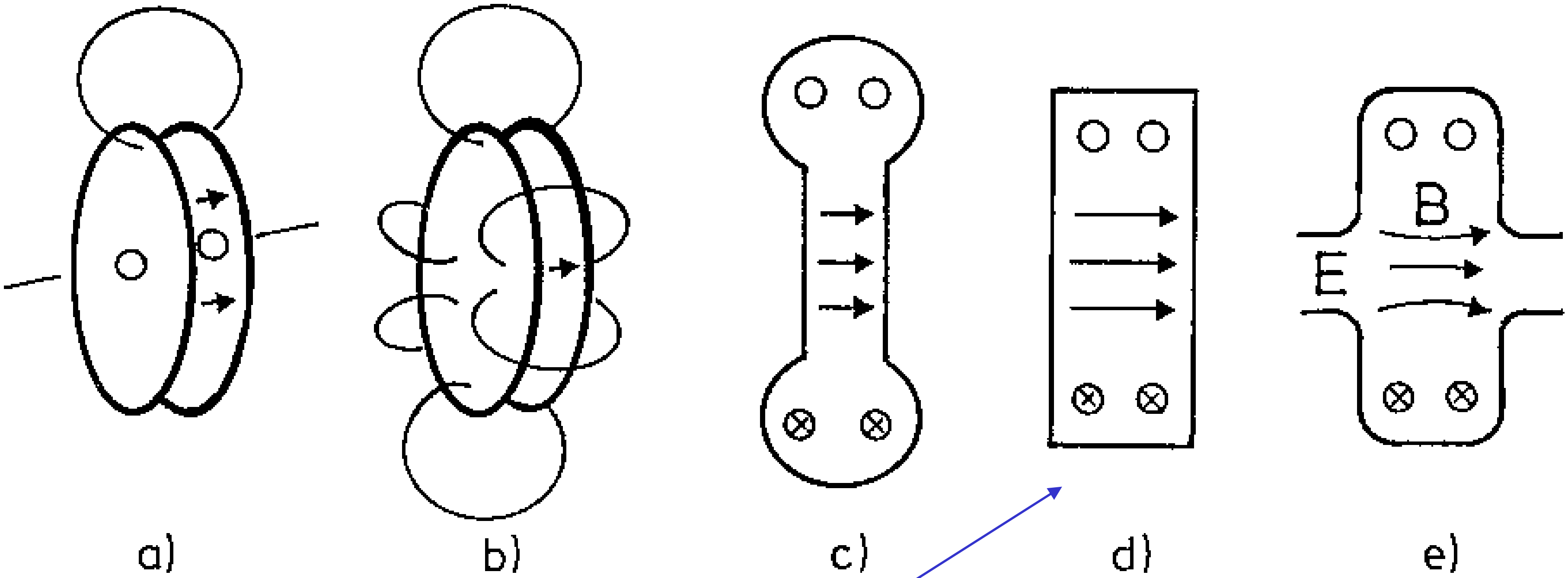
Which translates into

$$2d\nu_0 \leq \beta c$$

That is

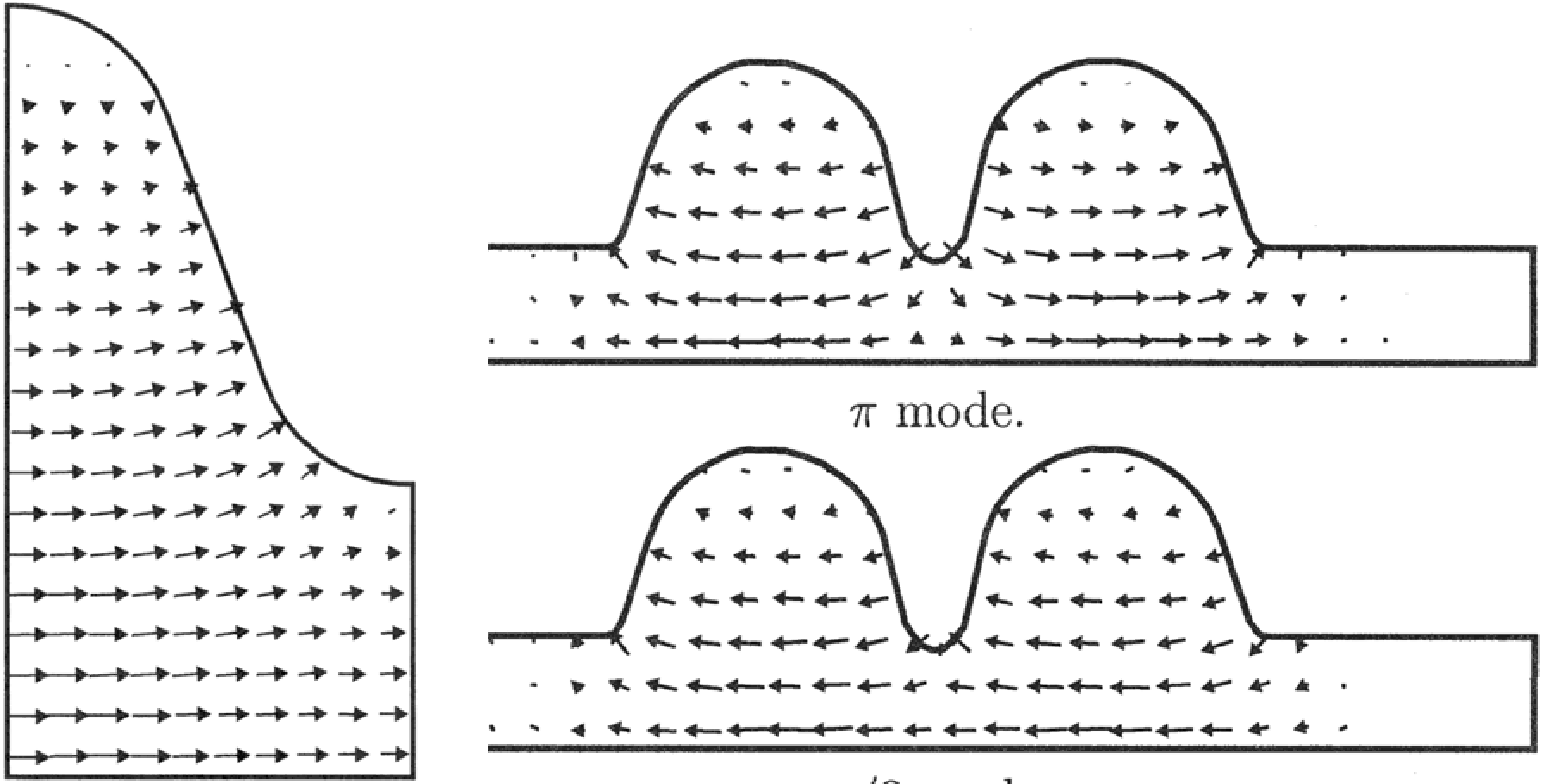
$$d \leq \beta\lambda/2$$

Metamorphosis of a RLC resonator



"Pillbox cavity" : $\nu_0 = 2.405\ c\ /(2*\pi*radius)$

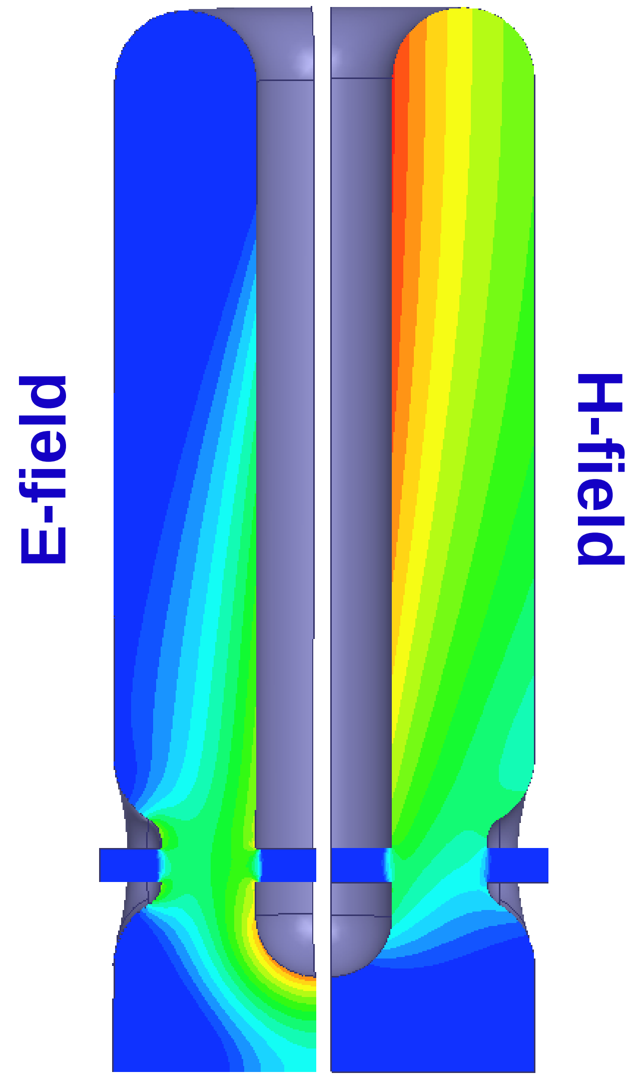
Electric field inside an elliptical cavity



π mode.

$\pi/2$ mode.

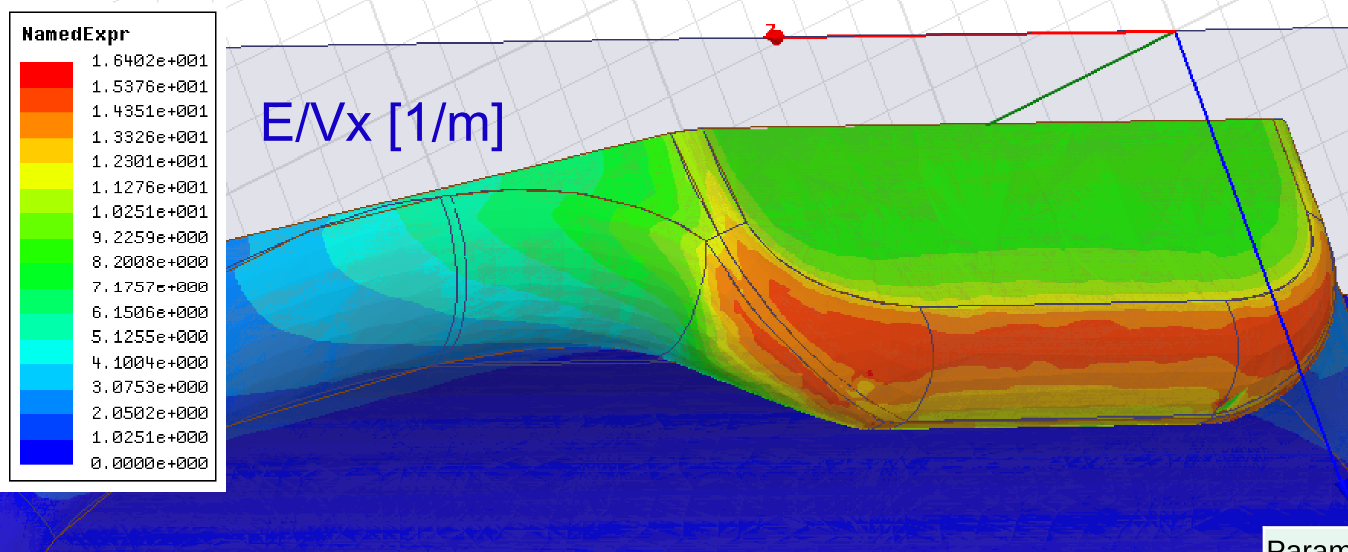
High beta QWR design (electromagnetic)

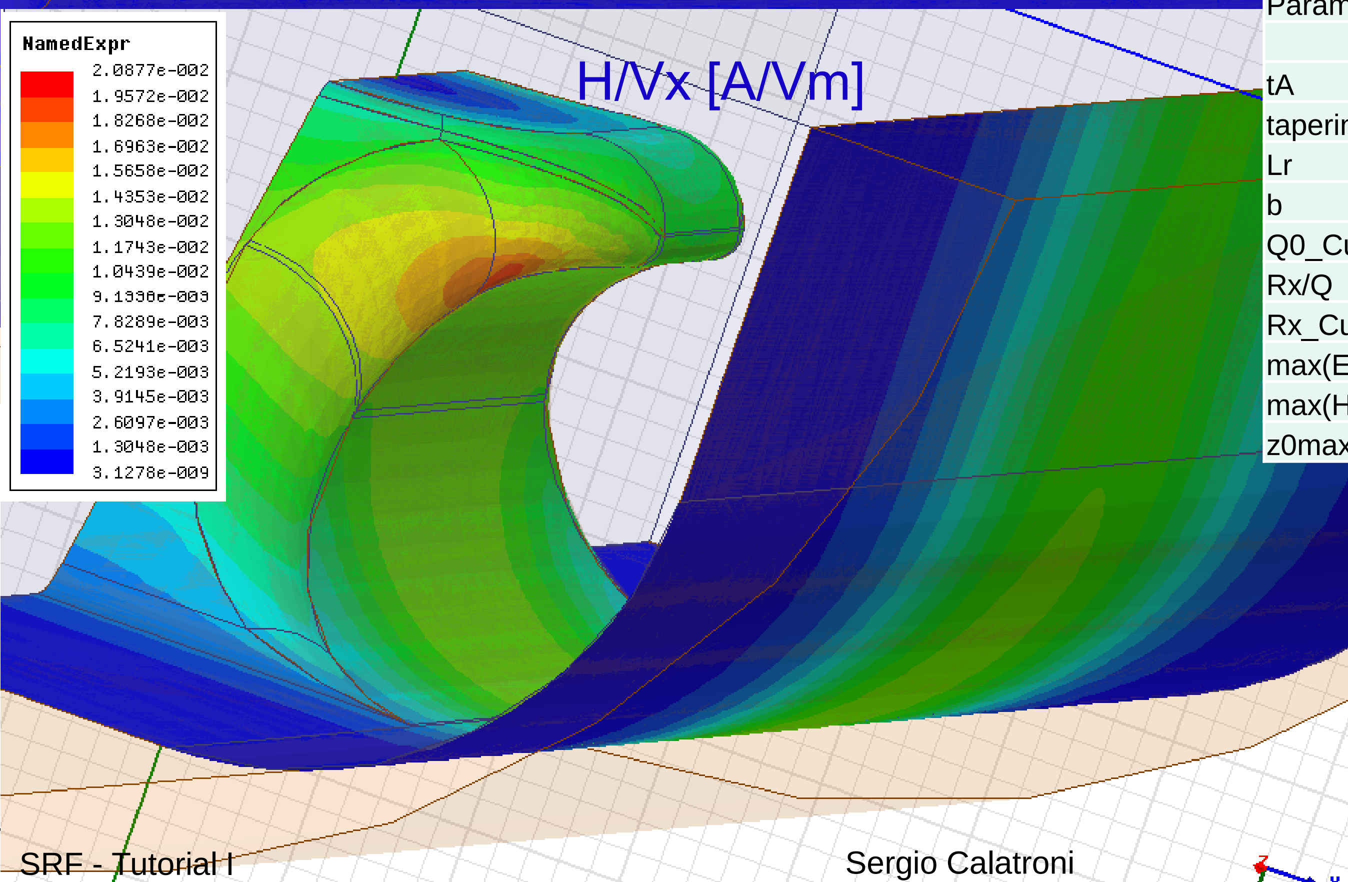


HIE ISOLDE	
f_0 at 4.5 K [MHz]	**101.28**
β_{opt} [%]	10.88
TTF at β_{opt}	0.9
R/Q [Ω] (incl. TTF)	556
E_p/E_{acc}	**5.0**
H_p/E_{acc} [G/(MV/m)]	95.3
U/E^2_{acc} [mJ/(MV/m)^2]	207
$G=R_sQ$ [Ω]	30.8
P_{diss} @ 6 MV/m [W]	**10**
P_{diss} on bottom plate [W]	**0.0018**

Ref. Proceedings of SRF2009, p. 609

Normalized field distributions

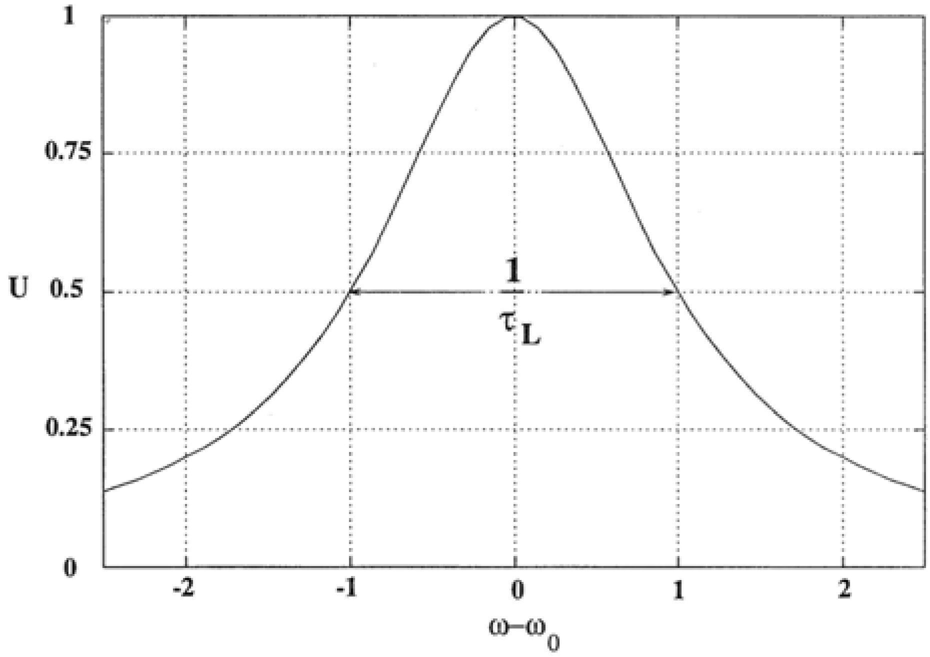




		WOW
Parameter	Unit	aR180,
		bRR63.3
tA	[deg]	VariaRR
tapering angle	[deg]	60
Lr	[mm]	30
b	[mm]	70
Q0_Cu		125.85
Rx/Q	[Ohm]	20850
Rx_Cu	[MOhm]	343.4343629
max(Esurf)/Vx	[1/m]	7.161504
max(Hsurf)/Vx	[1/Ohm*m]	15
z0max(Hsurf)/Vx	[1/Ohm*m]	0.018
		0.012

Cavities are resonators

- As we said at the beginning, an accelerating RF cavity is indeed a resonator for an electromagnetic wave. We already mentioned the idea of comparing an RF cavity to an RLC circuit.
- Two important quantities characterise a resonator (remember your first physics course..): the resonance frequency f_0 and the quality factor Q_0



$$Q_0 = \frac{f_0}{\Delta f} = \frac{\omega_0 U}{P_c}$$

$$U = U_0 \exp(-t/\tau_L)$$

- Where U is the energy stored in the cavity volume and P_c/ω_0 is the energy lost per RF period by the induced surface currents

Quality factor and resonating frequency I

- Let's expand on the latter definition:

$$Q_0 = \frac{\omega_0 U}{P_c} \qquad U = \frac{1}{2}\mu_0 \int_V |\mathbf{H}|^2 \, \mathrm{d}v \qquad P_c = \frac{1}{2} R_s \int_S |\mathbf{H}|^2 \, \mathrm{d}s$$

- Thus:

$$Q_0 = \frac{\omega_0 \mu_0}{R_s} \frac{\int_V |\mathbf{H}|^2 \, \mathrm{d}v}{\int_S |\mathbf{H}|^2 \, \mathrm{d}v} = \frac{\Gamma}{R_s} = \frac{\omega_0 L_{eff}}{R_s}$$

- Γ depends only on the shape of the cavity, and is generally calculated numerically with computer codes (HFSS...)
- For a "pillbox" cavity Γ is 257 Ω. For an accelerating cavity for relativistic particles (β=1) like those used at CERN Γ ranges in 270÷295 Ω.
- The Q_0 factor of a SC cavity is in the $10^9 \div 10^{10}$ range!
- L_{eff} is the effective inductance of the equivalent RLC circuit

Accelerating field

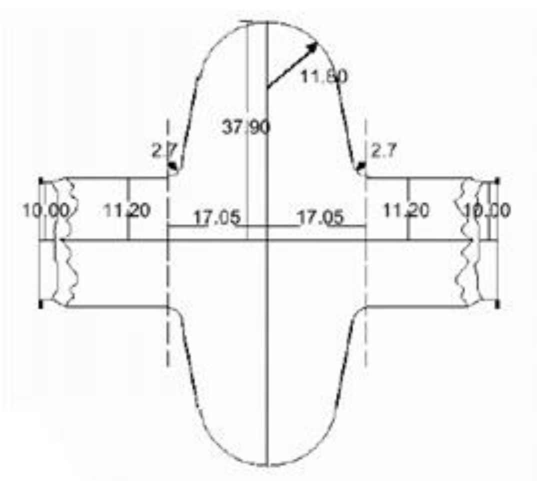
- Our cavities are meant to accelerate particles, and the accelerating electric field is of basic importance.
- However most of the considerations on the surface impedance are based on the surface magnetic field and on the induced currents.
- For a given geometry, there exists a fixed proportionality between H_0, E_0 and E_{acc}.

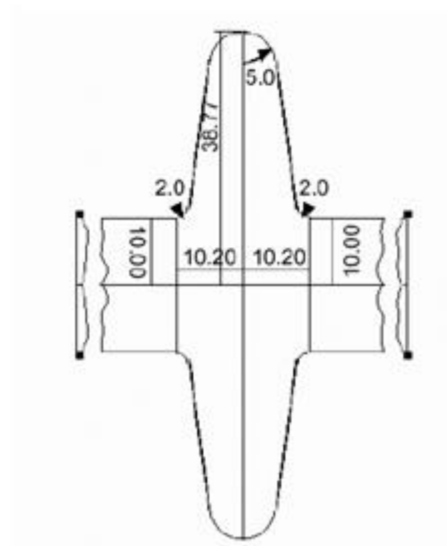
	"Pillbox"	Elliptical
H_0/E_{acc} [mT/(MV/m)]	3.33	4.55
E_0/E_{acc}	$\pi/2$	2.3
Γ [Ω]	257	295

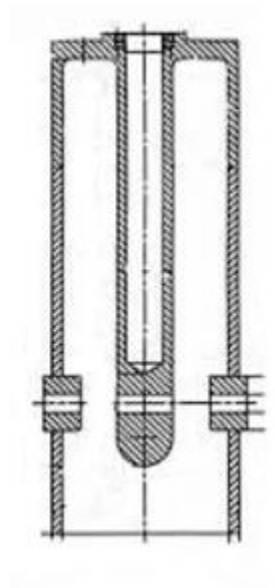
Shape vs β

Remember: $2d\nu_0 \leq \beta c$

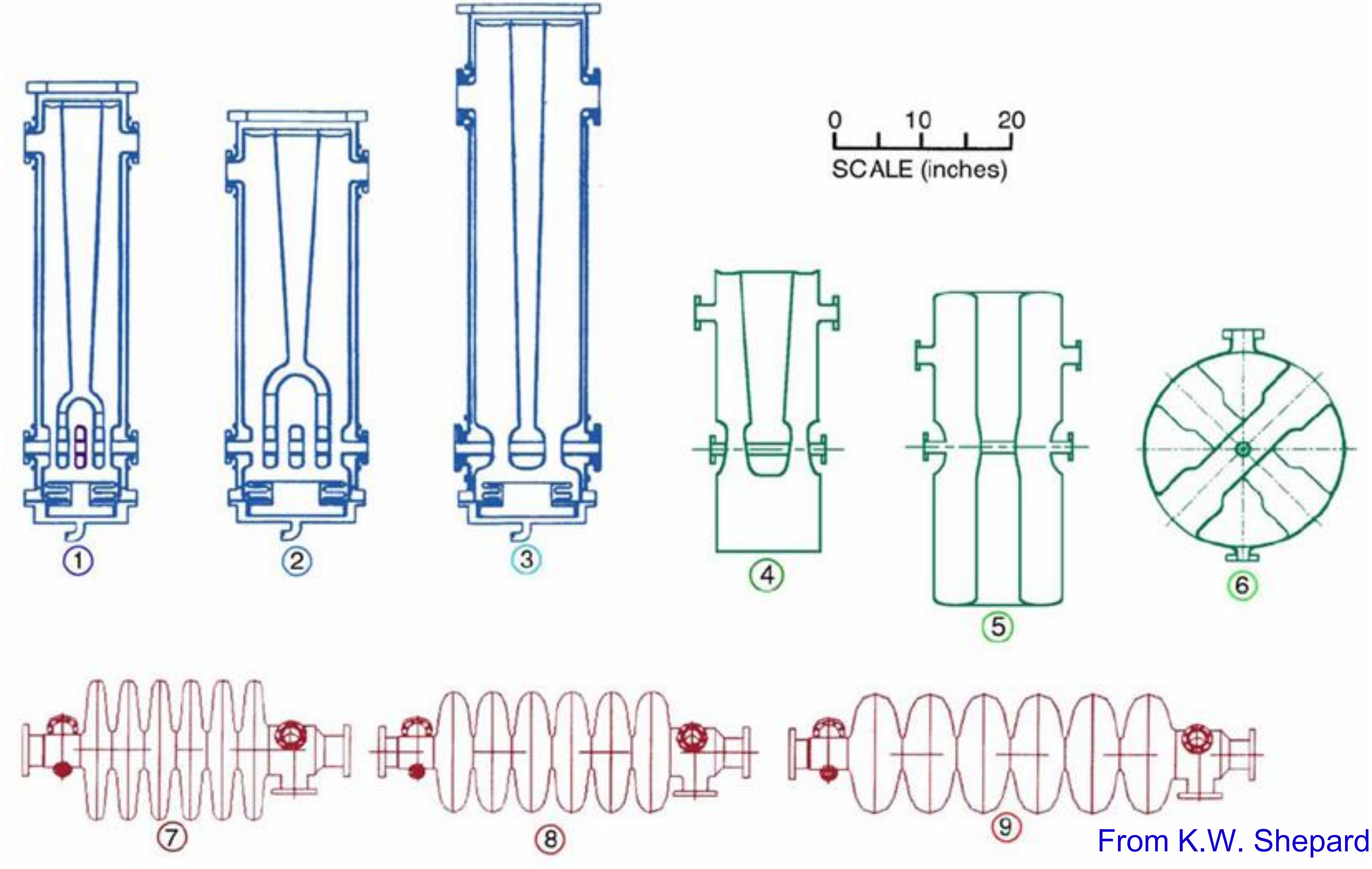
Electron accelerators	Proton accelerators	Ion accelerators
$\beta \cong 1$	$\beta \cong 0.5$	$0.05 \leq \beta \leq 0.2$
$350\ \text{MHz} \leq \nu_0 \leq 3\ \text{GHz}$	$500\ \text{MHz} \leq \nu_0 \leq 1.5\ \text{GHz}$	$50\ \text{MHz} \leq \nu_0 \leq 150\ \text{MHz}$
$\Gamma \cong 270\ \Omega$	$\Gamma \cong 170\ \Omega$	$\Gamma \cong 20\ \Omega$ (QWR)
$d \cong 35$ cm ($\nu_0 = 350$ MHz)	$d \cong 20$ cm ($\nu_0 = 350$ MHz)	$d \cong 15$ cm ($\nu_0 = 100$ MHz)







- We have already mentioned that the β of particles to be accelerated is an important parameter. The design of cavities depend strongly on this.

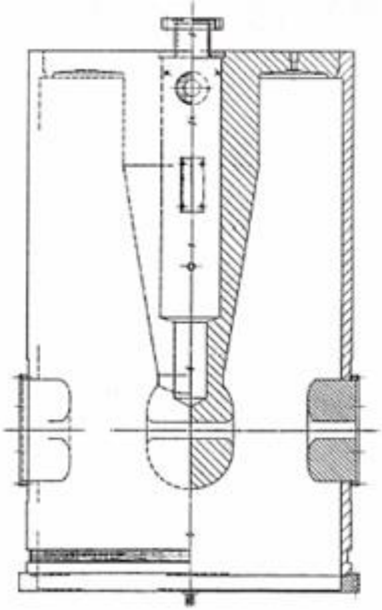


From K.W. Shepard, ANL

Split Rings (λ/4)

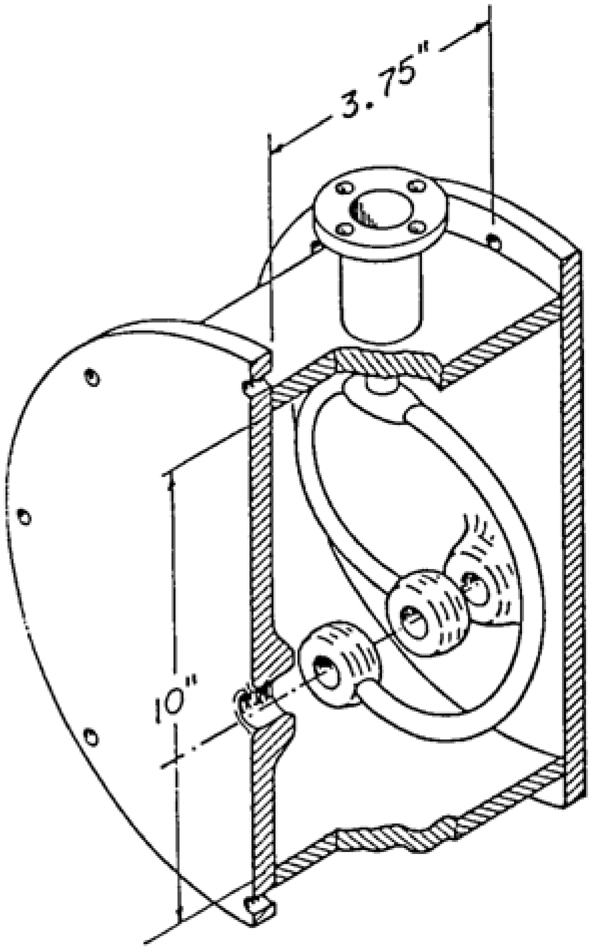


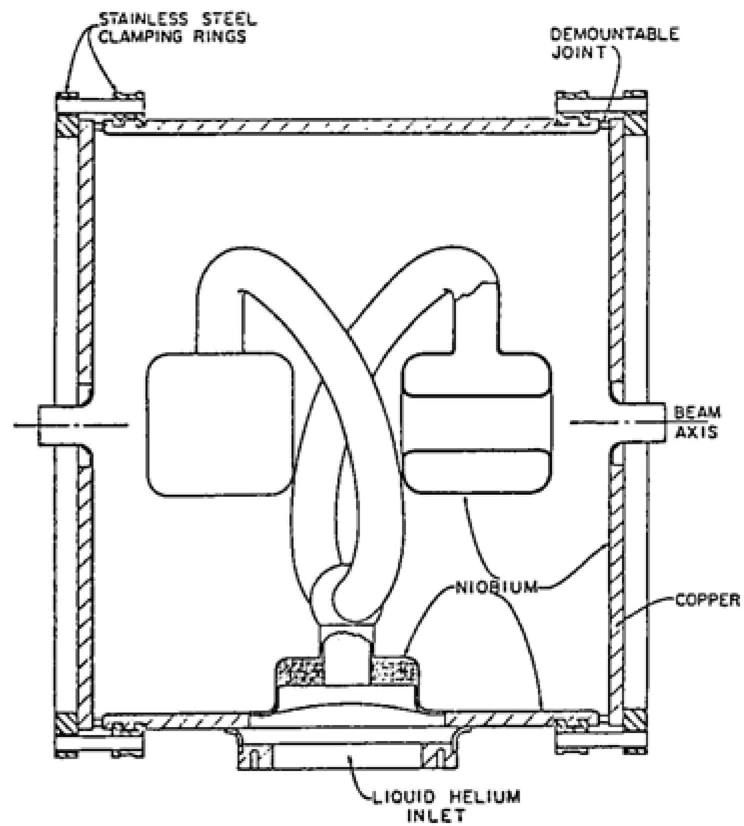


From J. Delayen, TJNAF

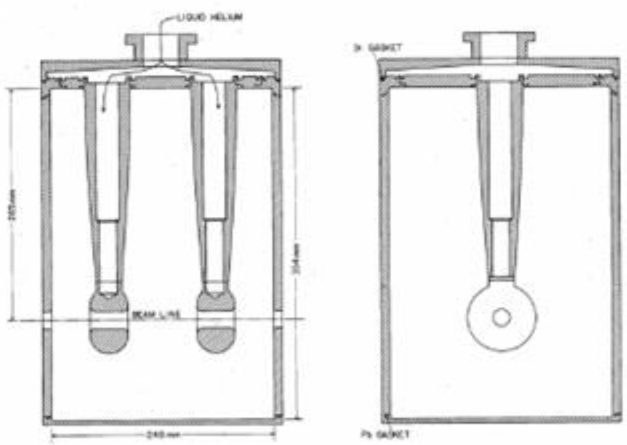
Split Rings (λ/4)

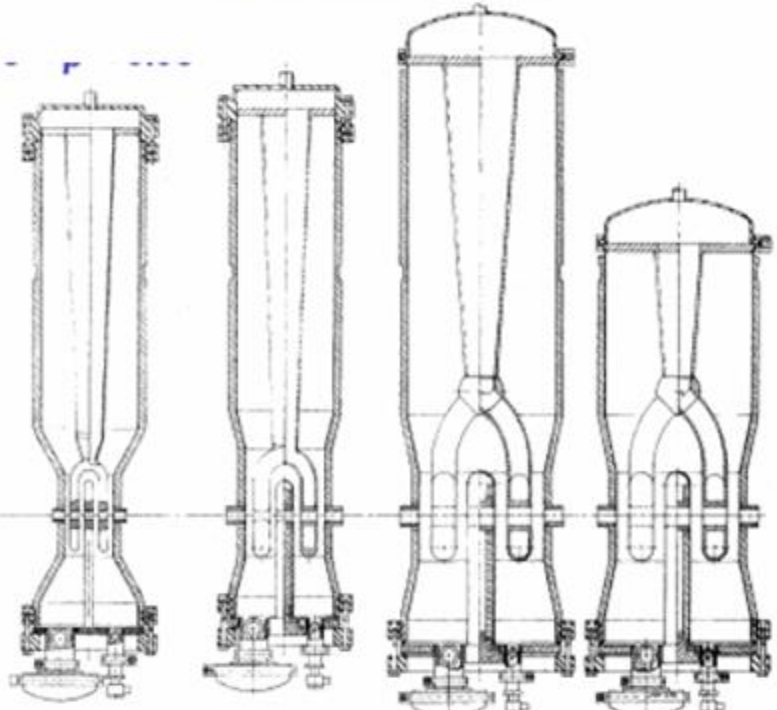




From K.W. Shepard, ANL

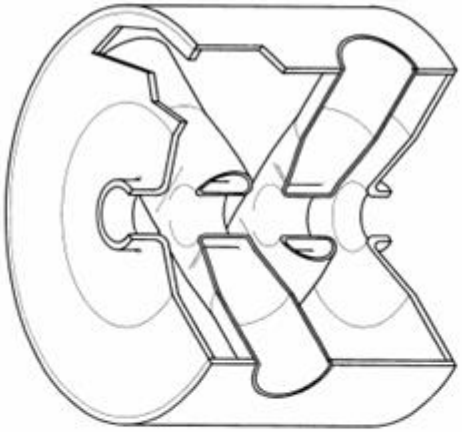
Quarter Wave (λ/4)

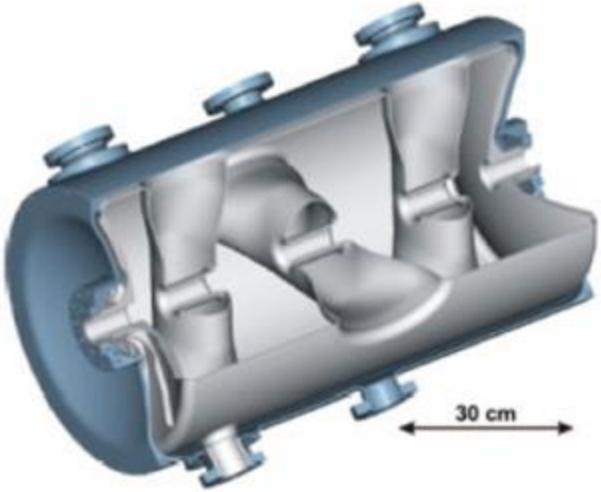


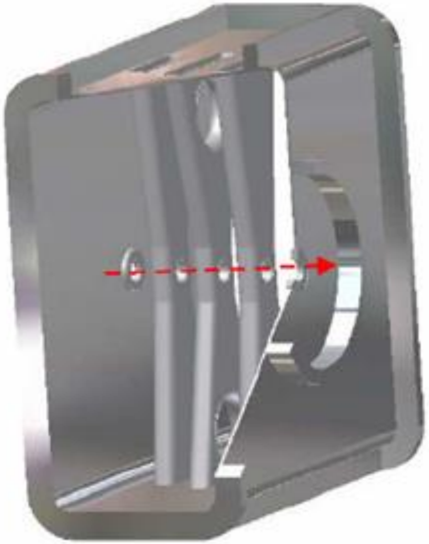


From J. Delayen, TJNAF

Spoke resonators (λ/2)







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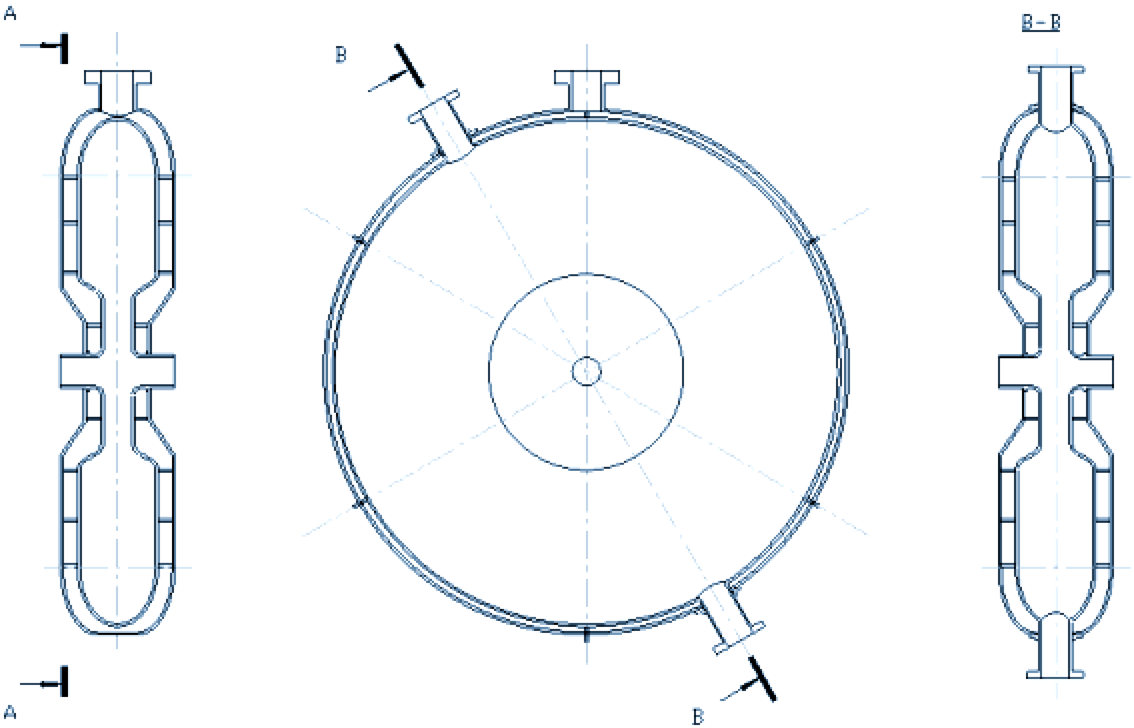
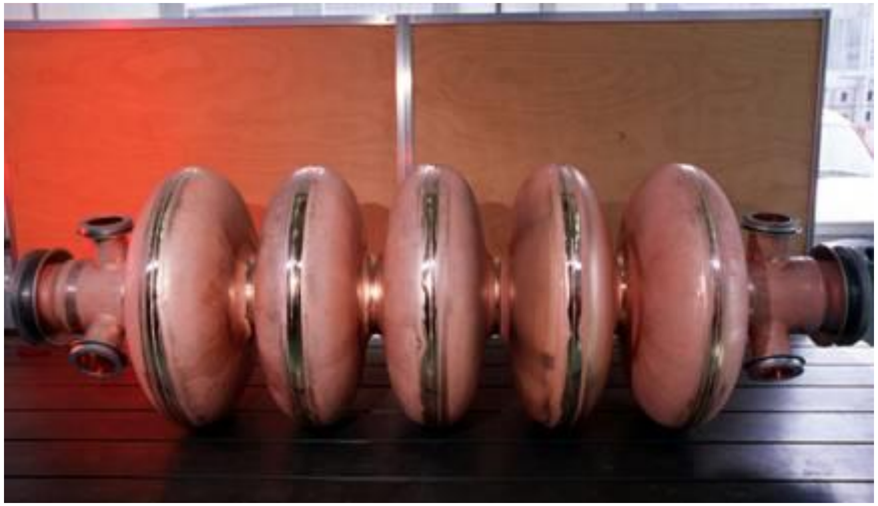
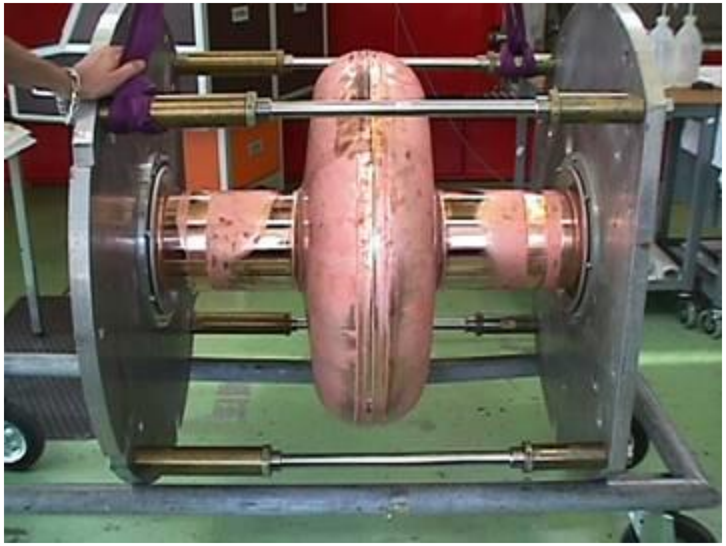


From F. Krawczyk, LANL and A. Facco, INFN-LNL

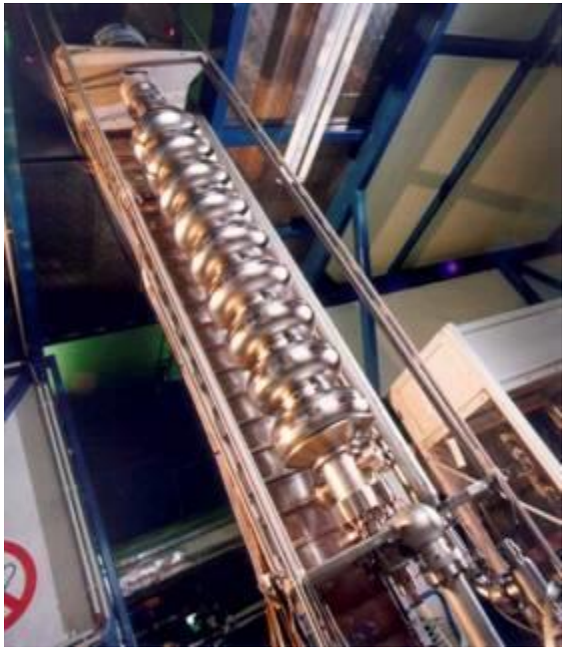
Elliptical cavities ("pizza")

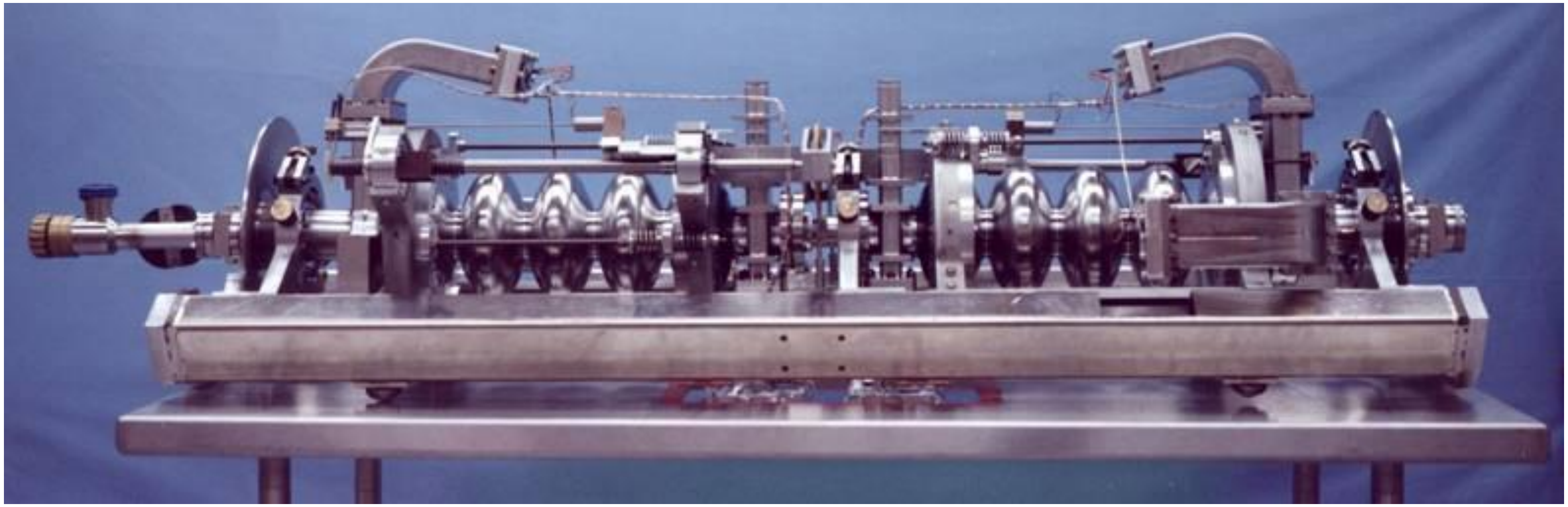




Trasco cavities (INFN) and SNS cavities (Oak Ridge)

Elliptical cavities β=1





Surface Impedance: definition

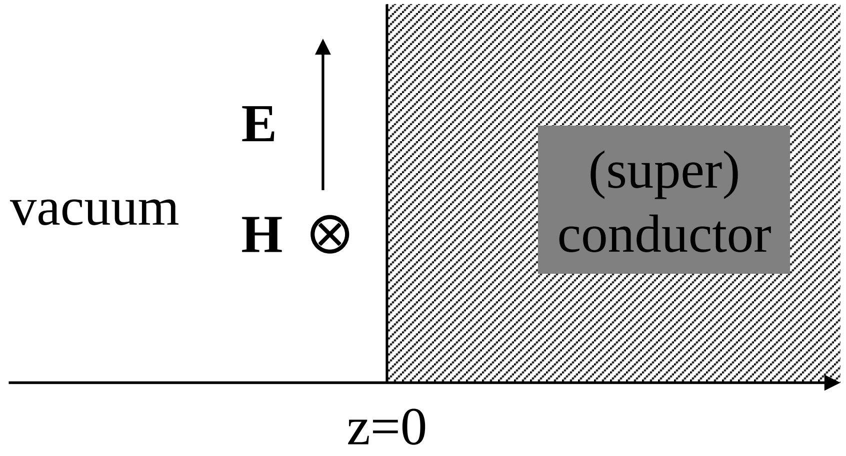
For a plane E-M wave incident on a semi-infinite flat metallic surface:

$$Z_S = R_S + iX_S = \frac{E_{\parallel}(0)}{H_{\parallel}(0)}$$

Where we use: E, H always in complex notation

E, H parallel to the surface

Our reference frame has *x, y* parallel to the surface of the metal, and the z axis directed towards its interior: *z=0* means at the surface.



Surface impedance in normal metals

- The Surface Impedance Z_s is defined at the interface between two media. It can be calculated in a similar way as for continuous media.

- You take Maxwell's equation, set the appropriate boundary conditions for the continuity of the waves (incident, reflected, transmitted), and you get:

$$Z_S = \frac{E_\parallel(0)}{H_\parallel(0)} = \sqrt{\frac{i\omega\mu_0}{\sigma}} = \sqrt{\frac{\omega\mu_0}{2\sigma}} + i\sqrt{\frac{\omega\mu_0}{2\sigma}} = R_S + iX_S$$

- Introducing appropriate numbers:

For copper ($\rho=1/\sigma=1.75\text{x}10^{-8}$ $\mu\Omega$.cm) at 350 MHz

$R_s = 5$ mΩ

$\delta = 3.5$ μm (from previous page)

Surface Impedance – Why bother? I

The surface impedance $Z_s = R_s + iX_s$ is a very useful concept, since it contains all properties of the dissipative medium. Making use of Maxwell's equations for plane waves:

- Power dissipation per unit surface (equivalent to the energy flux through the unit surface) making use of Poynting vector :

$$|S| = \mathrm{Re}[E \times H]$$

- Averaging over one period, at the surface, recalling the definition of Z_s

$$\overline{P} = \frac{1}{2}\mathrm{Re}\left[\overline{\mathbf{E}_0 \times \mathbf{H}_0}\right] = \frac{1}{2}\mathrm{Re}\left[Z_s H_0^2\right] = \frac{1}{2} R_s \mathrm{H}_0^2$$

Surface Impedance – Why bother? II

- Effective magnetic penetration depth (most general definition):

$$\lambda = \mathrm{Re}\left[\int_0^\infty H(z)\,\mathrm{d}z \Big/ H(0) \right]$$

Using Maxwell's eq.: $\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \Rightarrow \frac{\partial E}{\partial z} = -i\omega\mu_0 H$

integrating and using the fact that: $E(\infty) = 0$

$$E(\infty) - E(0) = -i\omega\mu_0 \int_0^\infty H(z)\,\mathrm{d}z \Rightarrow \int_0^\infty H(z)\,\mathrm{d}z = -\frac{i}{\omega\mu_0} E(0)$$

Substituting:

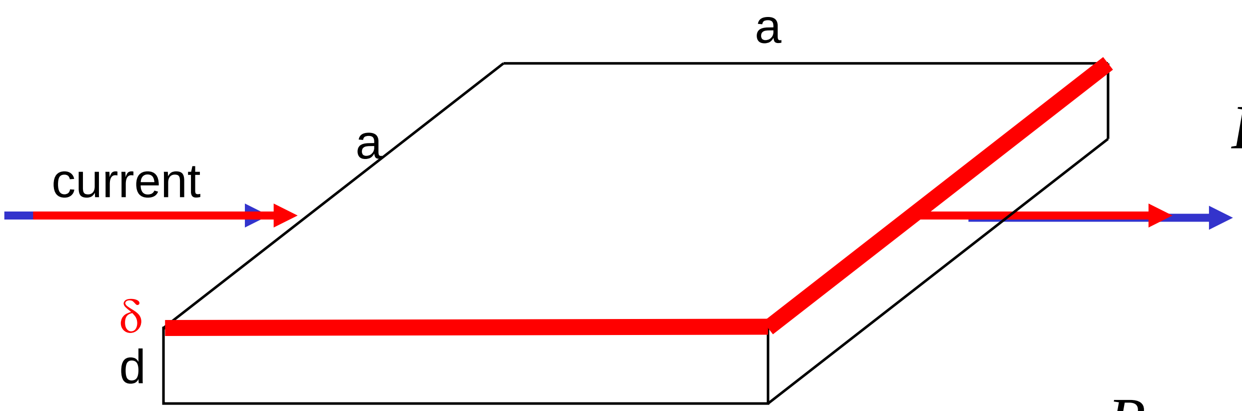
$$\lambda = \mathrm{Re}\left[-i \frac{E(0)}{\omega\mu_0 H(0)} \right] = \frac{1}{\mu_0 \omega} X_s$$

Remarkably similar to $\frac{1}{\beta} = \sqrt{\frac{\sigma\omega\mu}{2}} = \delta$ in the case of normal metals

Surface resistance: intuitive meaning

Since we will deal a lot with the surface resistance R_s in the following, here is a simple DC model that gives a rough idea of what it means:

Consider a square sheet of metal and calculate its resistance to a transverse current flow:



$$R = \frac{\rho\, a}{d\, a} = \frac{\rho}{d}$$

$$R_s = \sqrt{\frac{\mu_0 \omega}{2\sigma}} = \frac{1}{\sigma\delta} = \frac{\rho}{\delta}$$

The surface resistance R_s is the resistance that a square piece of conductor opposes to the flow of the currents induced by the RF wave, within a layer δ

More on electrical conductivity

We mentioned before the electrical conductivity σ and the law J=σE.

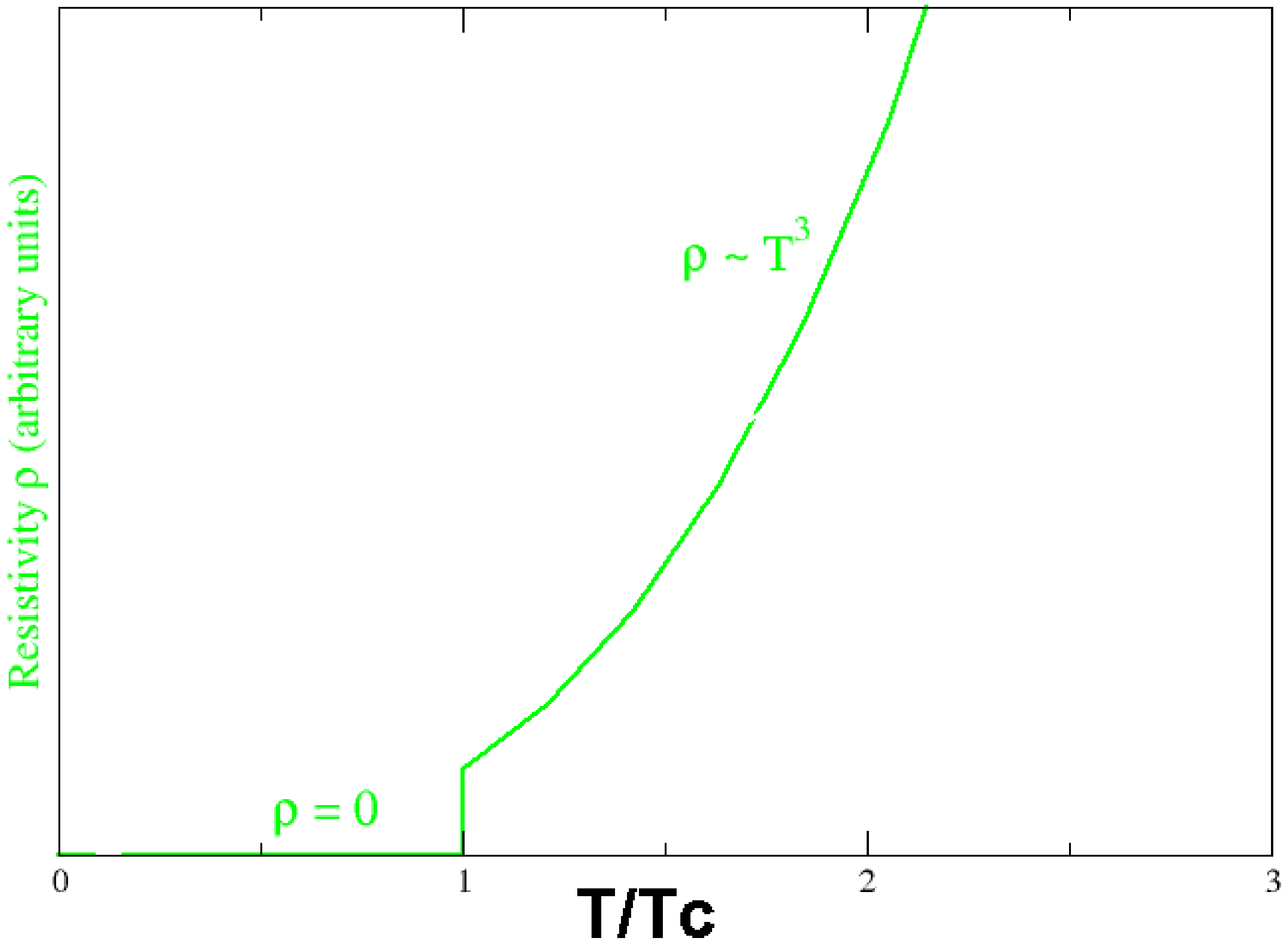
- σ depends on the frequency: $\sigma_0 = \frac{ne^2\tau}{m} = \frac{ne^2\ell}{mv_F} \to \sigma(\omega) = \frac{\sigma_0}{(1+i\omega\tau)}$

- For low temperatures τ increases and (as function of frequency), it happens $\ell \gg \delta$

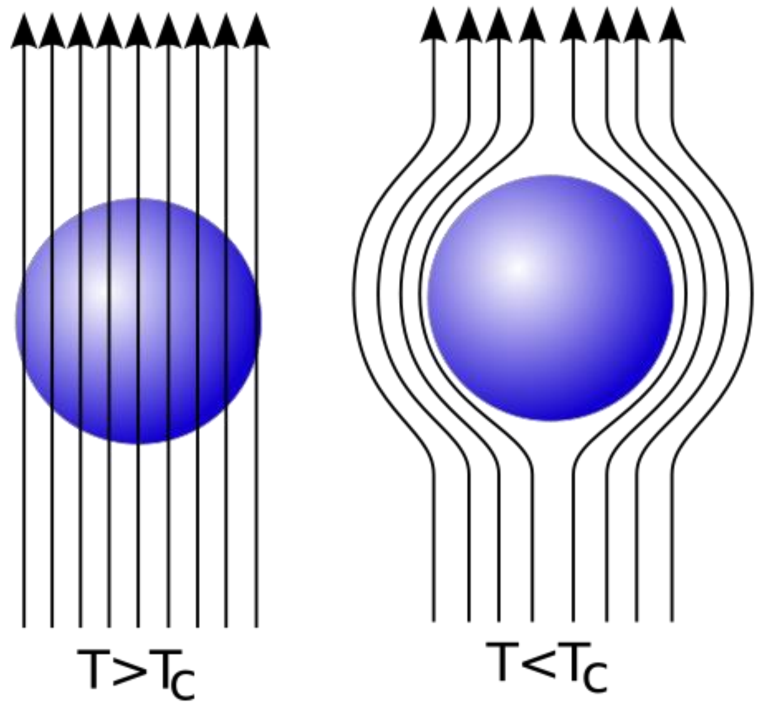
- In this case, an electron senses a variation of field during its travel between two collisions.

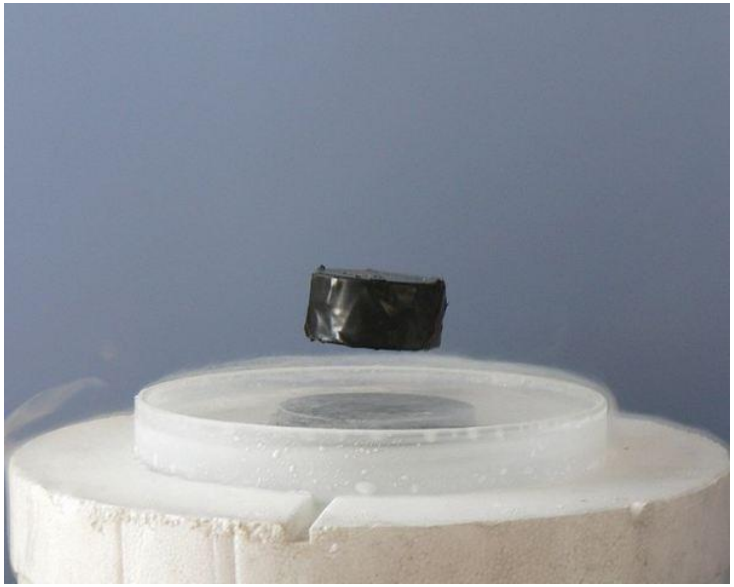
- The local relationship J=σE between current and field does not hold anymore and a new “non-local” law has to be introduced -> the Anomalous Skin Effect

Superconductivity



Meissner effect





Theories of Superconductivity

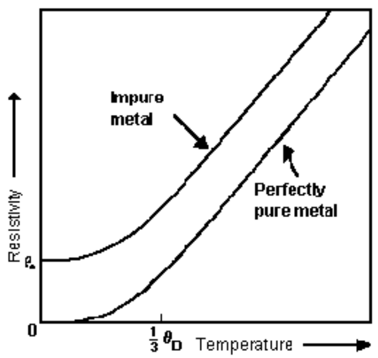
- Gorter & Casimir two fluid model
  - London Equations
  - Pippard's Coherence length ξ

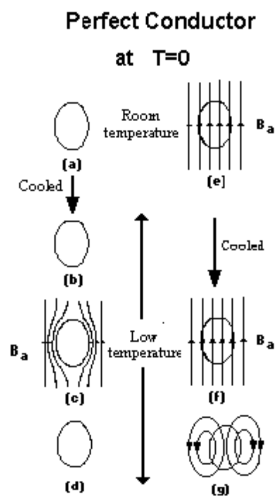
- Ginzburg-Landau
  - Second order phase transition of complex order parameter Ψ

- BCS (Bardeen-Cooper-Schrieffer)
  - Microscopic theory
  - Two Fluid Model revised

- (Strong coupling – Elihasberg)

London equations



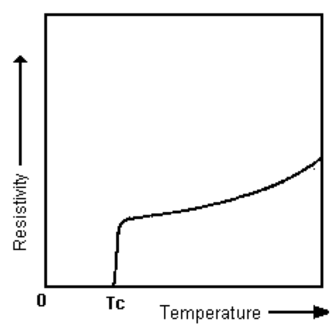


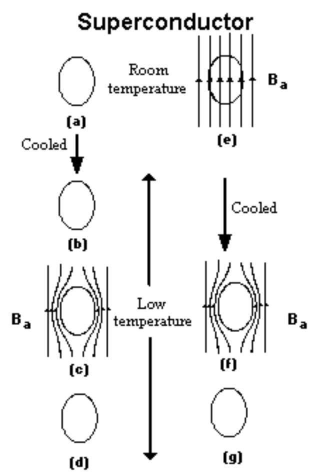
Postulated on plausibility arguments

$$\frac{\partial \mathbf{J}}{\partial t} = \frac{n_s e^2}{m}\mathbf{E} \qquad \nabla \times \mathbf{J} = -\frac{n_s e^2}{m}\mathbf{B}$$

Applying B = ∇×A

$$\mathbf{J} = -\frac{n_s e^2}{m}\mathbf{A} = -\Lambda^{-1}\mathbf{A}$$





Applying Ampere’s law
to London’s 2nd eq gives:

$$\nabla^2 \mathbf{B} = \frac{1}{\lambda_L^2}\mathbf{B} \qquad \lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

Exponential decay of B inside SC

Superconductors – Complex conductivity

- You will probably be only half surprised to hear that a superconductor at T>0 has a $R_s \neq 0$.
- This can be easily understood in the framework of the two fluids model, where a population of normal electrons of density n_n and a population of "superconducting electrons" of density $n_s = n_0(1 - T^4/T_c^4)$ coexist such as $n_n + n_s = 1$ and both give a response to the varying e.m. fields.
- Let's "invent" the conductivity of superconducting electrons:

$$\sigma(\omega) = \frac{ne^2\tau}{m_e(1+i\omega\tau)} \qquad \lim_{\tau\to\infty}\sigma(\omega) = -i\frac{ne^2}{m_e\omega}$$

Superconductors – Complex conductivity

- The conductivity of superconductors then becomes:

$$\sigma_s = \frac{n_0 e^2 \tau}{m} \left(T^4 \big/ T_c^4 \right) - i \frac{n_0 e^2}{m\omega} \left(1 - T^4 \big/ T_c^4 \right) = \sigma_1 - i\sigma_2$$

conductivity of "normal electrons"

conductivity of "superconducting electrons"

- Where $n_s = n_0(1-T^4/T_c^4)$ and $n_n = n_0(T^4/T_c^4)$ with n_0 total electron density
- Note:

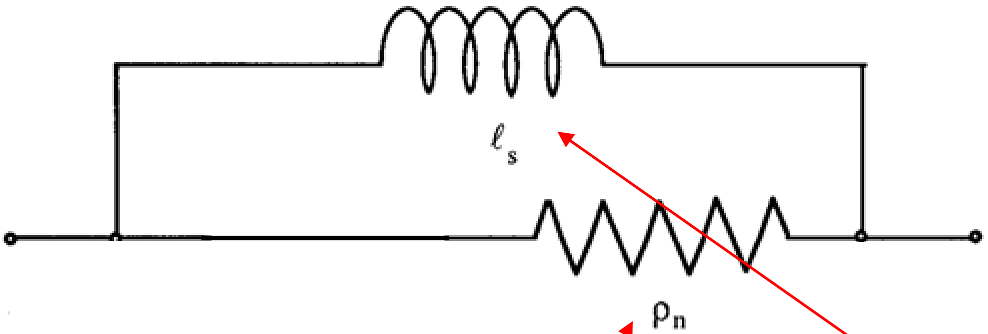
$$\sigma_2 = \frac{\sigma_0}{\omega\tau} = \frac{1}{\mu_0 \omega \lambda_L^2} = \frac{1}{\Lambda\omega} \qquad \text{with} \quad \lambda_L^2 = \frac{\Lambda}{\mu_0}$$

Superconductors – Equivalent circuit

Indeed if you take the time derivative of $\mathbf{J} = -\Lambda^{-1}\mathbf{A}$ you get the 1st London eq:

$$-\frac{\partial \mathbf{A}}{\partial t} = \mathbf{E} = \Lambda \frac{\partial \mathbf{J}}{\partial t}$$

Λ is interpreted as a specific inductance. This justifies representing the complex conductivity of a superconductor with an equivalent circuit of parallel conductors:



$$\sigma_s = \sigma_1 - i\sigma_2 = \sigma_n \left(T^4 / T_c^4 \right) - i \frac{\sigma_n}{\omega\tau} \left(1 - T^4 / T_c^4 \right)$$

Superconductors – Surface impedance I

- It is now possible, within the basic approximations we have made, to calculate the surface impedance of a superconductor.
- Take the formula for normal metals:

$$Z = (1+i)\sqrt{\frac{\mu_0 \omega}{2\sigma_n}}$$

- Perform the substitution: $\sigma_n \rightarrow \sigma_s = \sigma_1 - i\sigma_2$
- Calculate:

$$R_s = \sqrt{\frac{\mu_0 \omega}{\sigma_n}} \frac{\left[\left(\sigma_1^2 + \sigma_2^2\right)^{1/2} - \sigma_2\right]^{1/2}}{\left(\sigma_1^2 + \sigma_2^2\right)^{1/2}} \qquad X_s = \ldots\ldots$$

- In the approximation of small ℓ (small $\omega\tau \rightarrow \sigma_1 < \sigma_2$)
- and $0 < T < 0.5T_c$ ($n_n < n_s \rightarrow \sigma_1 < \sigma_2$) it gives:

$$R_s = \frac{R_N}{\sqrt{2}} \frac{\sigma_1 / \sigma_n}{\left(\sigma_2 / \sigma_n\right)^{3/2}} = \frac{1}{2} \mu_0^2 \omega^2 \sigma_1 \lambda_L^3 \qquad X_s = X_N \frac{\sqrt{2}}{\left(\sigma_2 / \sigma_n\right)^{1/2}} = \sqrt{\frac{\mu_0 \omega}{\sigma_2}} = \mu_0 \omega \lambda_L$$

- Which is a good description of the experimental data, but…

Superconductors – BCS approach I

Mattis & Bardeen, Phys. Rev. 111 (1958) 412

1st step:

$$\mathbf{j}(\mathbf{r},t)=\sum_{\omega}\frac{e^2N(0)v_0}{2\pi^2\hbar c}\times\int\frac{\mathbf{R}[\mathbf{R}\cdot\mathbf{A}_\omega(r')]I(\omega,R,T)e^{-R/l}dr'}{R^4}, \quad (3.3)$$

is

$$I(\omega,R,T)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left\{L(\omega,\epsilon,\epsilon')-\frac{f(\epsilon)-f(\epsilon')}{\epsilon'-\epsilon}\right\}\times\cos[\alpha(\epsilon-\epsilon')]d\epsilon d\epsilon', \quad (3.4)$$

2nd step:

$$\frac{\sigma_1-i\sigma_2}{\sigma_N}=\frac{I(\omega,0,T)}{-\pi i\hbar\omega}.$$

3rd step:

$$\frac{\sigma_1}{\sigma_N}=\frac{2}{\hbar\omega}\int_{\epsilon_0}^{\infty}[f(E)-f(E+\hbar\omega)]g(E)dE+\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega}^{-\epsilon_0}[1-2f(E+\hbar\omega)]g(E)dE, \quad (3.9)$$

$$\frac{\sigma_2}{\sigma_N}=\frac{1}{\hbar\omega}\int_{\epsilon_0-\hbar\omega,\,-\epsilon_0}^{\epsilon_0}\frac{[1-2f(E+\hbar\omega)](E^2+\epsilon_0^2+\hbar\omega E)}{(\epsilon_0^2-E^2)^{\frac{1}{2}}[(E+\hbar\omega)^2-\epsilon_0^2]^{\frac{1}{2}}}. \quad (3.10)$$

Superconductors – BCS approach II

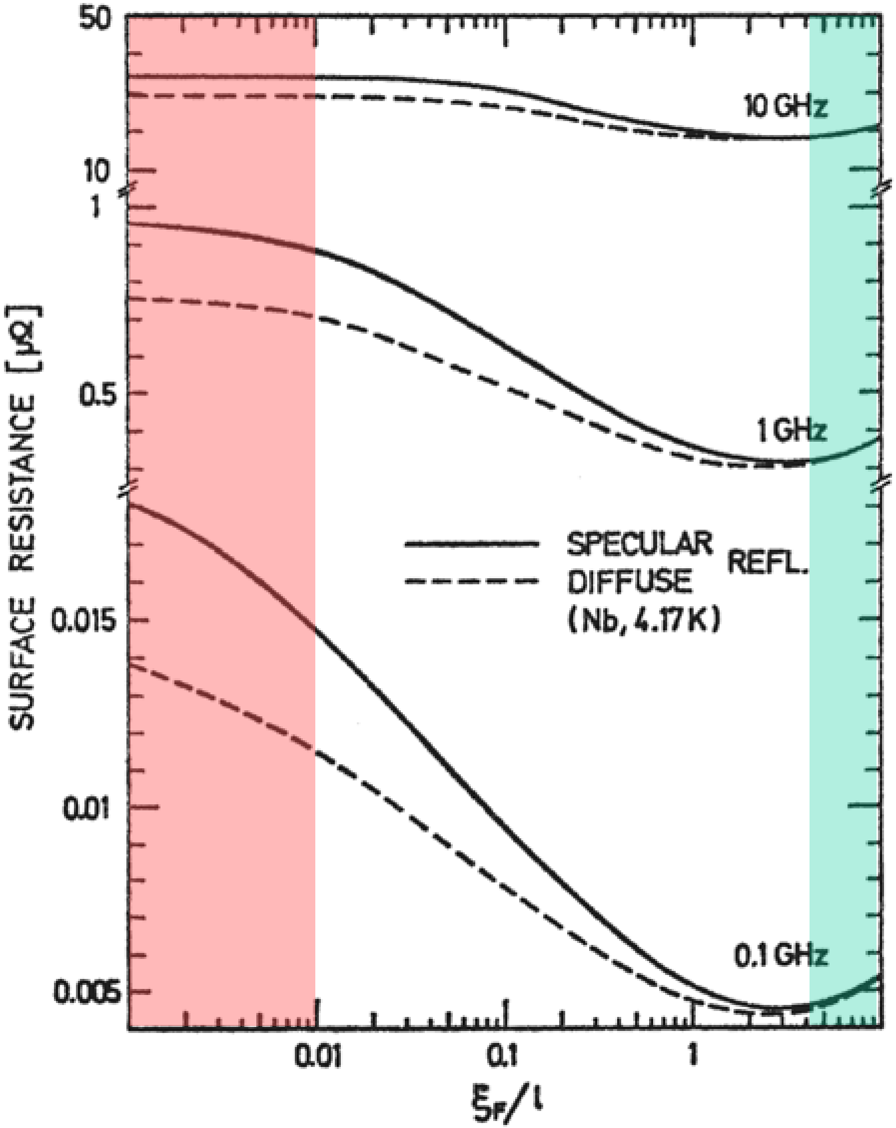
- These can be approximated for low frequencies $h\nu \ll 2\Delta$ as:

$$\frac{\sigma_1}{\sigma_n} = 2f(\Delta)\left\{1+\frac{1}{kT}\ln\left(\frac{2\Delta}{\hbar\omega}\right)[1-f(\Delta)]\right\} \qquad \frac{\sigma_2}{\sigma_n} = \frac{\pi\Delta}{\hbar\omega}\tanh\left(\frac{\Delta}{2kT}\right)$$

- The surface impedance Z can be calculated from σ_1/σ_n and σ_2/σ_n
- However the formulas seen until now are approximate valid only in a very specific limit.
- Calculations can be done having a full validity range (but only for $H_{RF} \ll H_c$)

  - Abrikosov Gorkov Khalatnikov, JETP 35 (1959) 182
  - Miller, Phys. Rev. 113 (1959) 1209
  - Nam, Phys. Rev. A 156 (1967) 470
  - Halbritter, Z. Phys. 238 (1970) 466 and KFK Ext. Bericht 3/70-6

Superconductors – Halbritter's theoretical predictions



In the green region (small ℓ):

$$\frac{Z_s}{Z_n} = \left(\frac{\sigma_1}{\sigma_n} - i\frac{\sigma_2}{\sigma_n} \right)^{1/2}$$

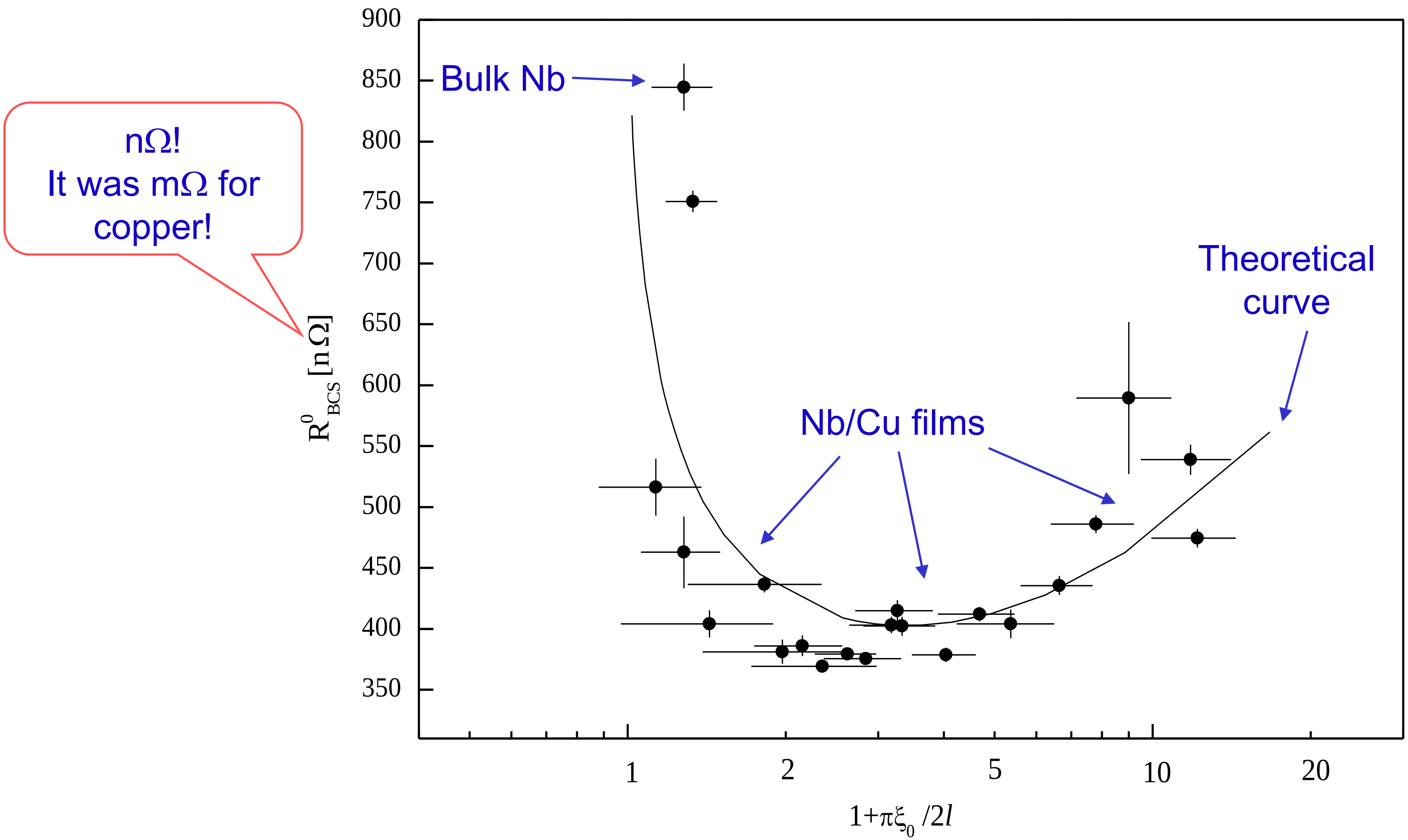
$$R_s \propto \frac{\omega^2}{T\sqrt{\sigma}} \exp\left(-\Delta/_{kT}\right)$$

In the red region (large ℓ):

$$\frac{Z_s}{Z_n} = \left(\frac{\sigma_1}{\sigma_n} - i\frac{\sigma_2}{\sigma_n} \right)^{1/3}$$

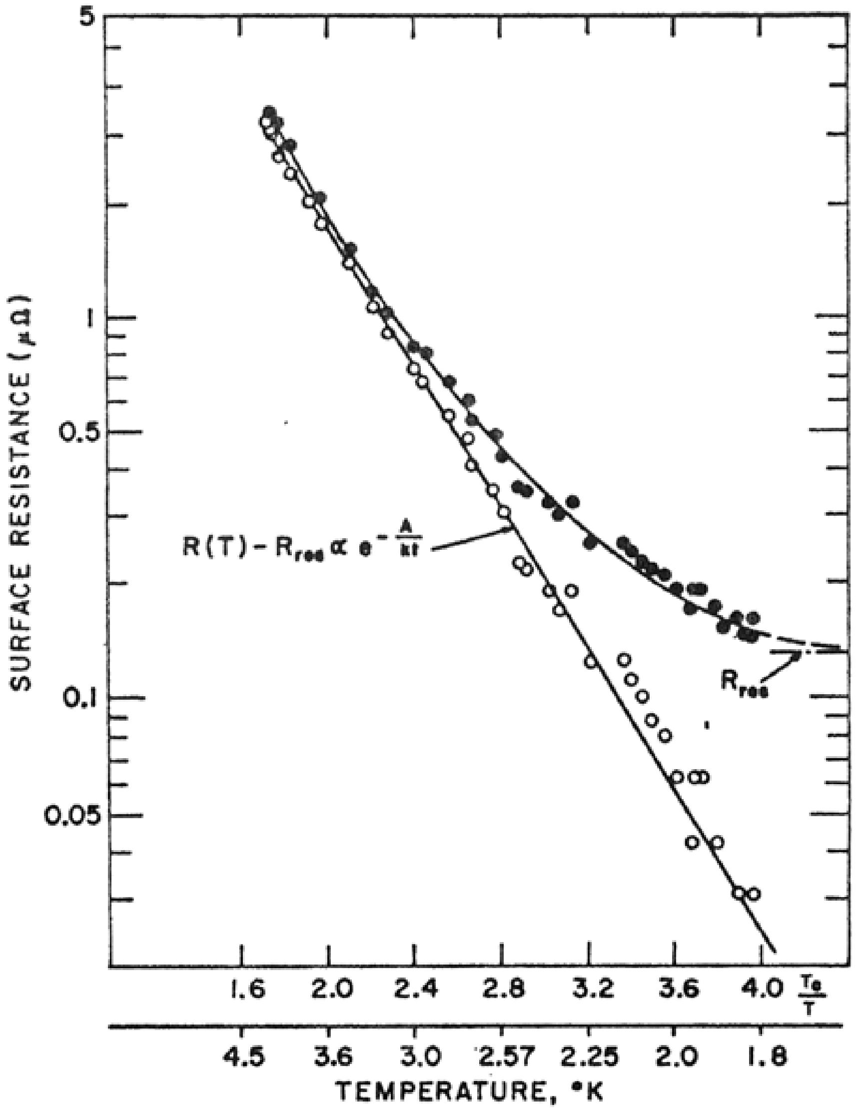
$$R_s \propto \frac{\omega^{3/2}}{T} \exp\left(-\Delta/_{kT}\right)$$

Superconductors – CERN results on films



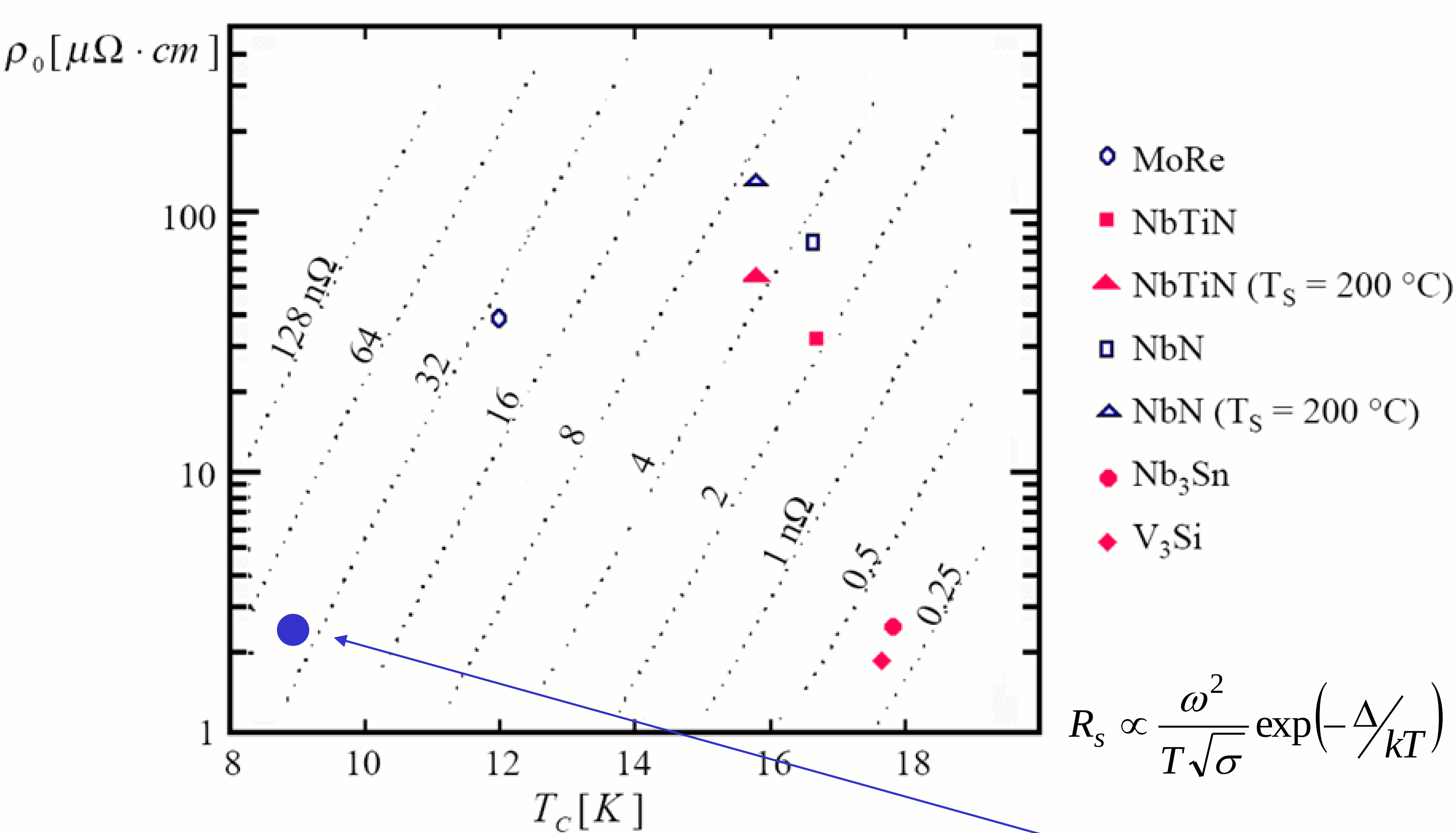
Compilation of results from several Nb/Cu and Nb bulk 1.5 GHz RF cavities

Temperature dependence of R_{BCS}



Exponential dependence limited by a temperature independent “residual” term

Other materials than niobium



$$R_s \propto \frac{\omega^2}{T\sqrt{\sigma}} \exp\left(-\Delta/_{kT}\right)$$

BCS surface resistance at 4.2 K and 500 MHz for films (Nb ~ 45 nΩ)

Again on skin depth

- London penetration depth
  - dependence on mean free path $$\lambda_L(T,\ell) = \lambda_L(T,0)\sqrt{1+\frac{\pi\xi_0}{2\ell}}$$
  - dependence on T ($T>T_c/2$) $$\lambda_L(T,\ell) = \lambda_L(0,\ell) * 1 \Big/ \sqrt{1-\left(\frac{T}{T_c}\right)^4}$$
  - define $\gamma(T,\ell) = \lambda_L(T,\ell)/\xi_F(T,\ell)$ $\quad \xi_F(0,0) = \frac{\pi}{2}\xi_0 \quad \frac{1}{\xi} = \frac{1}{\xi_0} + \frac{1}{\ell}$

- London limit $\gamma>>1$ (mfp “small”) $$\lambda_{SC}(T,\ell) = \lambda_L(T,\ell)$$
- Pippard limit $\gamma<<1$ (mfp “large”) $$\lambda_{SC}(T,\ell) = \lambda_L(T,\ell) * f_s \propto \lambda_L(T,\ell) * \gamma^{-1/3}$$
  - (non-local effects, non-exponential decay of fields, field reversal for $\gamma \le 1$)

Surface impedance and skin depth

- We are ready to meet again our friend X_s :

$$X_s = \omega \mu_0 \lambda = 2\Gamma \frac{\omega_\infty - \omega}{\omega_\infty}$$

- Where ω_∞ is the resonant frequency if the cavity walls were perfectly conducting, in which case the penetration depth would vanish.
- This formula (Slater's theorem) in its differential form is particularly useful to measure the penetration depth

- A non-zero penetration depth results in a larger resonating volume, thus a lower resonating frequency
- Changes in $\lambda(T)$ with the temperature result in changes of ω_0,
- Recall the two-fluids (London limit) formula for $\lambda(T)$:

$$\lambda_L(T,\ell) = \lambda_L(0,0)\sqrt{1+\frac{\pi\xi_0}{2\ell}}\,\frac{1}{\sqrt{1-\left(\frac{T}{T_c}\right)^4}}$$

Surface impedance and skin depth

- Expressing the frequency shift as a function of $\lambda(T)$ we can write:

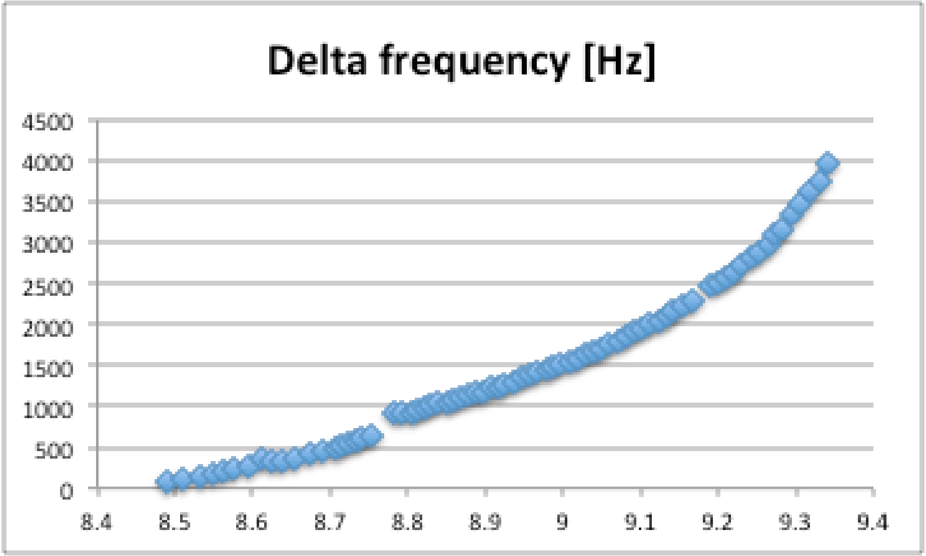
$$-\Delta\nu(t) = \frac{\pi\mu_0\nu^2}{\Gamma - \pi\mu_0\nu\Delta\lambda(T)}\Delta\lambda(T)$$

- Non-linear-fitting it to experimental data (measuring frequency as a function of temperature, approaching Tc) one can calculate the mean free path assuming reasonable values for the material (Nb) parameters:

λ_L	36 nm
ξ_0	40 nm
$\rho^*\ell$	0.28 x 10^{-15} Ωm^2
$\rho_{phononic}$(300K)	14.5 x 10^{-8} Ωm

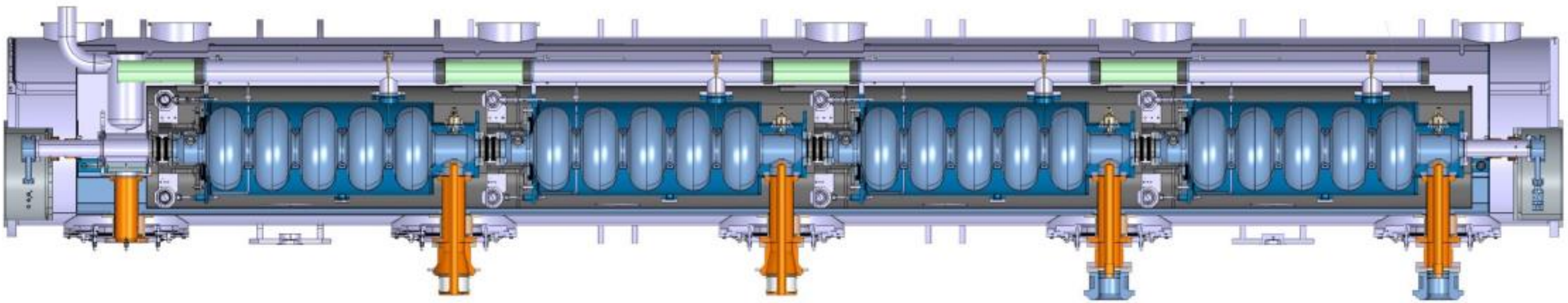
1.495 GHz cavity E8.7

- Our most studied "thin film" cavity



$\lambda(0\ K)$	7.51 nm
ℓ	19nm
ρ_{imp}	1.47 x 10^{-8} Ohm m
RRR	$(14.5\times10^{-8} + 1.47\times10^{-8})/1.47\times10^{-8} = 10.8$

SPL superconducting cryomodule



Why use superconducting cavities?

- The reason is very simple: to obtain very high accelerating fields in continuous mode, the power dissipated by normal conducting (i.e copper) cavities becomes too large. Remember that there can be a 10^5 difference in R_s, thus in dissipated power, for the same surface fields.
- This is valid also taking into account the efficiency of the cryogenic plant. The Carnot efficiency of a perfect cooler working between 300 K and T is:

$$\eta_c = \frac{T}{300 - T}$$

- For T=4.2 K, η_c=0.014. A modern cryoplant has a technical efficiency that can reach about 30% of the ideal one, thus η_{real}=0.0042. Even taking this into account the energy savings are huge.

- Moreover, SC cavities allow designs with larger beam pipes which, are beneficial for beam stability although reducing R_a. This would be unacceptable for NC cavities, but owing to the huge Q_0 the performance of SC cavities don't suffer much.

Shunt impedance

- When RF power is fed into a cavity, a voltage is developed inside it. As for any RLC circuit, we must match the impedance of the generator to that of the resonator to transfer the maximum amount of power:

$$P_c = \frac{V_c^2}{R_a}$$

- R_a is called shunt impedance [Ω / accelerating cell], and we want to maximise it in order to develop the largest voltage for a given power.
- Substituting into the definition of Q_0:

$$\frac{R_a}{Q_0} = \frac{V_c^2}{\omega_0 U}$$

- The quantity is independent of surface resistance, of cavity size and only depends on its shape. For a pillbox it is 196 Ω/cell, for LEP-type cavities it is 83 Ω/cell

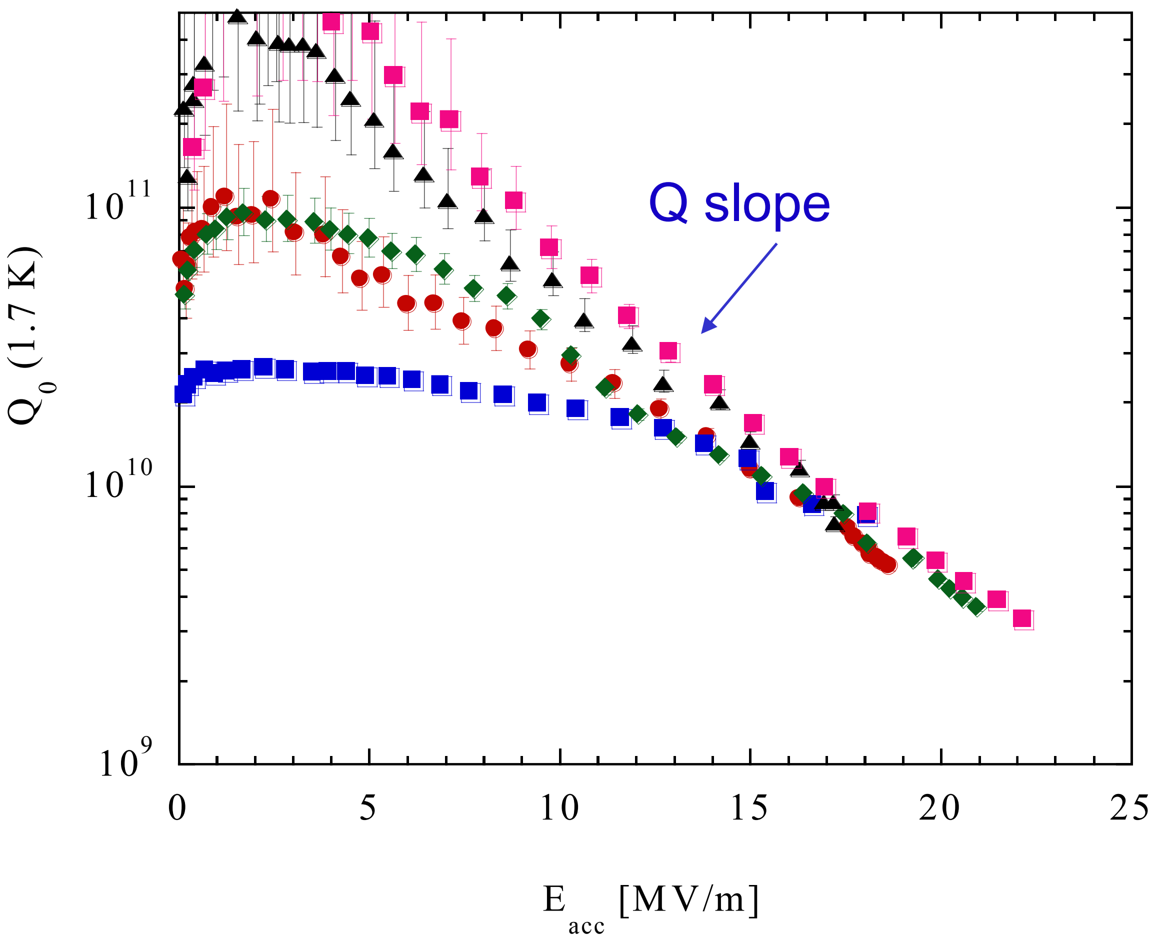
SC vs NC

- Here is a brief summary for comparing SC cavities (Nb @ 4.2 K) and NC cavities, both at 500 MHz

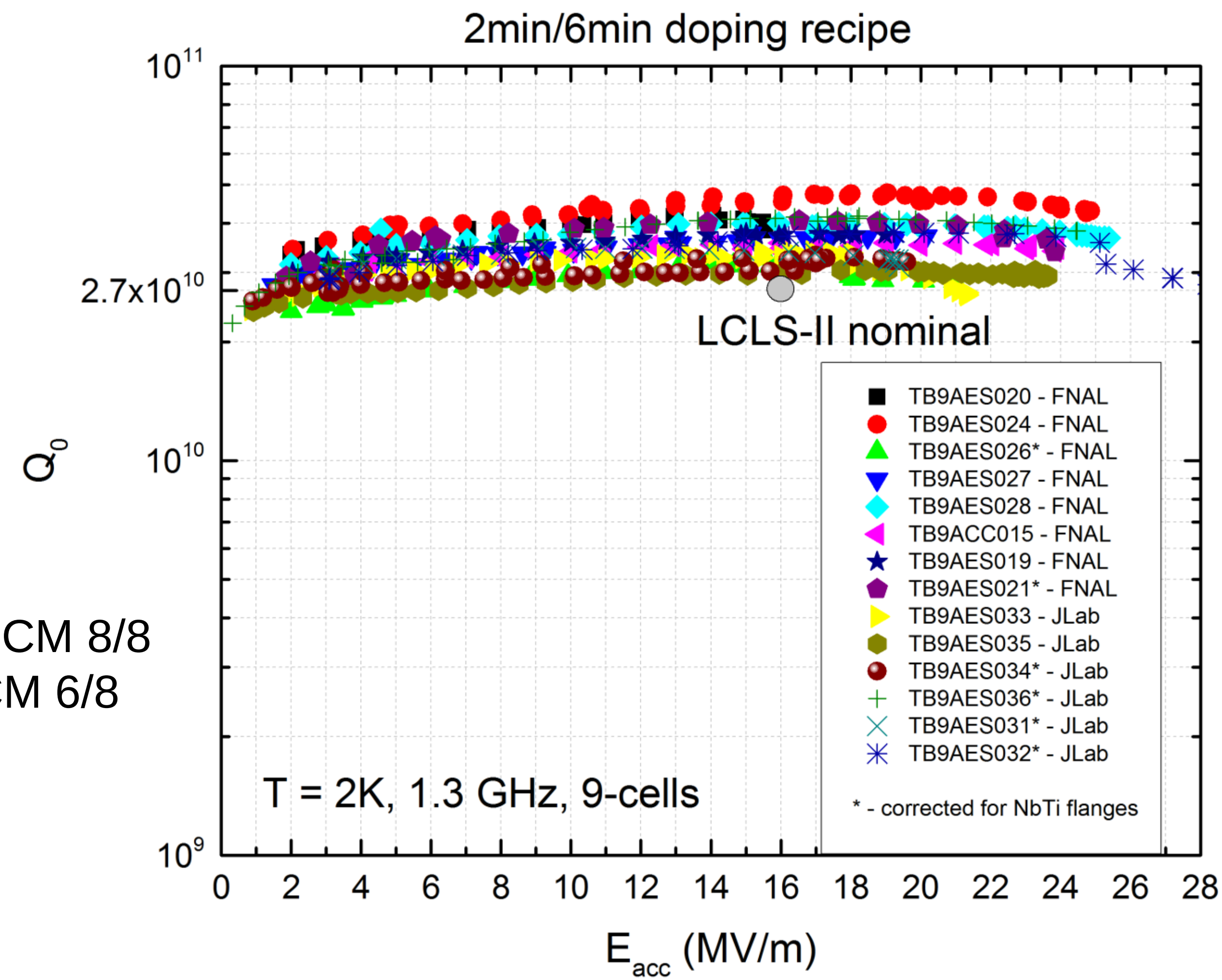
	Superconducting	Normal conducting
Q_0	$2x10^9$	$2x10^4$
R_a/Q_0 [Ω/m]	330	900
Dissipated power [kW/m] (for E_{acc}=1MV/m)	0.0015	56
Total AC power [kW/m] (for E_{acc}=1MV/m)	0.36	112

Performance of cavities II

Nb/Cu cavities 1.5 GHz at 1.7 K ($Q_0=295/R_s$)



FNAL + Jlab Nine Cell 2.0K Results, doping recipe 2/6



FNAL pCM 8/8
Jlab pCM 6/8

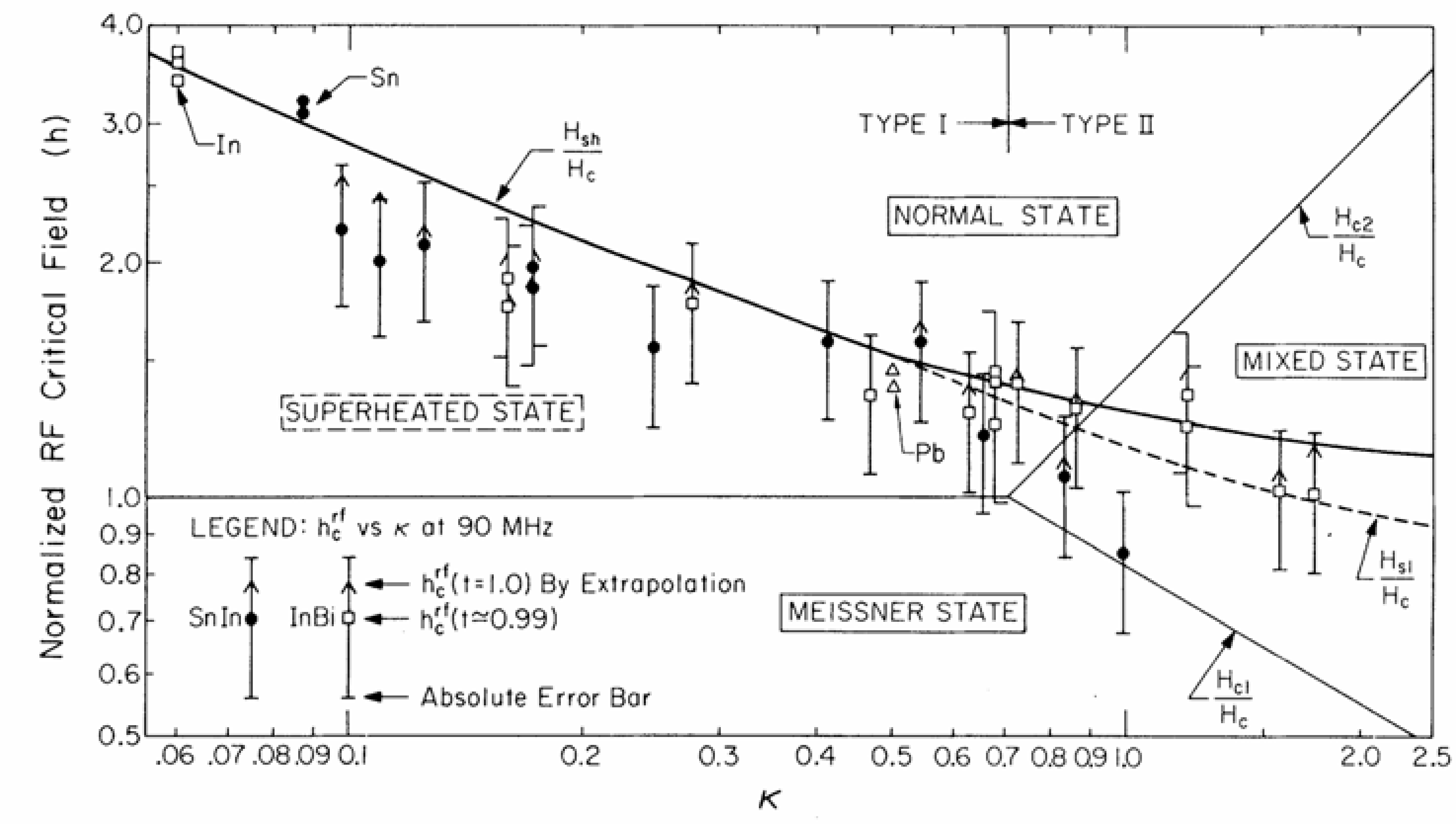
Deviations from BCS

- The first notable fact is that the surface resistance R_s is not constant with field. (The Mattis-Bardeen equations are a zero-field perturbation theory, and this should not come as a surprise)
- Second, even when lowering temperature the surface resistance does not reduce as foreseen. This is the residual surface resistance R_{res}
- Third, other phenomena take place at high fields: electron field emission, thermal breakdown (quench)
- Fourth, in the case of films, there is a stronger decrease of Q_0 factor (increase of R_s) with field compared to bulk cavities
- In fact the surface resistance is generally:

$$R_s(T, E_{rf}, H_{DC}) = R_{BCS}(T, E_{rf}, 0) + R_{res}(0, E_{rf}, 0) + R_{fl}(T, E_{rf}, H_{DC})$$

Ultimate performance

- As already said, there is in principle no intrinsic lower limit for the surface resistance ($R_s \rightarrow 0$ for $T \rightarrow 0$)
- However all superconductors are faced with the hard limit: H_c
- When H_{rf} exceeds some superheating H_{sh} value linked to H_c they loose superconductivity. This limit is around 50 MV/m for Nb at 1.7 K



Thank you!

(end of part I)

Some useful definitions

- FOR NORMAL METALS :
  - i) LOCAL OR CLASSICAL LIMIT $\ell \to 0$ or $\ell \ll \delta$
  - ii) ANOMALOUS LIMIT $\ell \to \infty$ or $\ell \gg \delta$

- FOR SUPERCONDUCTORS
  - i) LOCAL LIMIT $\ell \ll \xi, \lambda$ (INDEPENDENTLY EITHER FOR TYPE I OR II SUPERCONDUCTORS) MORE COMMONLY CALLED DIRTY LIMIT $\ell/\xi \to 0$ WHICH HAPPENS TO BE EQUIVALENT
  - { IN THE CLEAN LIMIT $\ell/\xi \to \infty$ WE HAVE TWO CASES:
  - ii) ANOMALOUS OR PIPPARD LIMIT FOR $\lambda \ll \xi, \ell$ (A CLEAN TYPE I SUPERCONDUCTOR)
  - iii) LONDON LIMIT $\xi \ll \lambda, \ell$ (A CLEAN TYPE II)

- Nb FOR $\ell \to \infty$ IS IN THE ANOMALOUS LIMIT WHEN NORMAL BUT IN LONDON LIMIT WHEN SUPERCONDUCTOR

Maxwell's equations

- Gauss' Law $$\nabla \cdot D = \rho$$
- Faraday's law of induction $$\nabla \times E = -\frac{\partial B}{\partial t}$$
- Gauss Law for magnetism $$\nabla \cdot B = 0$$
- Ampère's circuital law $$\nabla \times H = J + \frac{\partial D}{\partial t}$$
- With: $$D = \varepsilon E \qquad B = \mu H$$
- Continuity equation $$\nabla \cdot J + \frac{\partial \rho}{\partial t} = 0$$