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Guided study and tutorial on Cyclotrons

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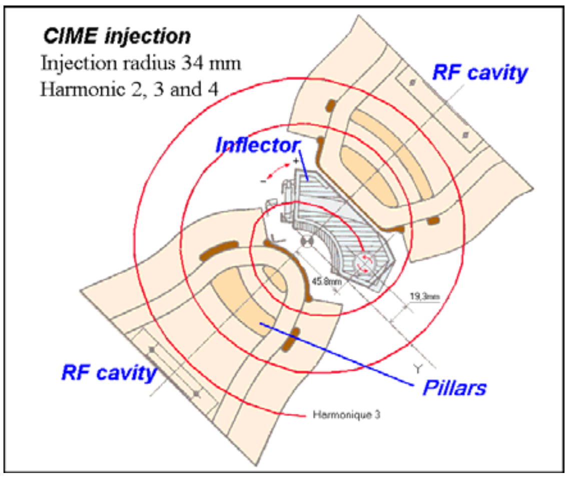
PROBLEM 1

In a variable energy isochronous cyclotron with an axial injection system and a fixed geometry of the central region (posts, pillars, etc…) all beams must follow the same path at least on the first turns.

1. It is also convenient to carry this condition up to extraction, so that the position of the electrostatic deflector remains constant.



If r_0 is the first and unique radius of curvature of the orbit at the exit of the inflector, for an ion of charge Q and mass m, find a relation linking the mean magnetic field B_0, the RF frequency F_{RF} , h the harmonic number and the input voltage V_{inj}: $\frac{1}{\pi \cdot r^2{}_0} = \frac{F_{RF} B_0}{h V_{inj}}$

V_{inj} is this extraction potential of the ion source.

Answer:

$$\begin{cases} W = \frac{1}{2}mv^2 = QV_{inj} \\ \omega = \frac{QB}{m} \Rightarrow \omega R = \frac{QBR}{m} \\ then \\ \frac{1}{2}mv^2 = \frac{1}{2}m(\omega R)^2 = \frac{1}{2}m(\frac{QBR}{m})^2 = \frac{1}{2}\frac{(QBR)^2}{m} = QV_{inj} \\ \frac{1}{2m}Q(BR)^2 = V_{inj} \Rightarrow \frac{1}{2}(\frac{QB}{m})BR^2 = V_{inj} \\ with \\ \omega = 2\pi F_{rev} = 2\pi\frac{F_{RF}}{h} = \frac{QB_0}{m} \\ then \\ 1/2(\frac{QB}{m})BR^2 = \frac{1}{2}(2\pi\frac{F_{RF}}{h})BR^2 = \frac{F_{RF}}{h}B\pi R^2 = V_{inj} \\ \frac{F_{RF}B}{hV_{inj}} = \frac{1}{\pi R^2} \end{cases}$$

$$\begin{cases} W = 1/2mv^2 = \frac{(QB_0R)^2}{2m} = QV_{inj} \\ \omega = 2\pi F_{rev} = 2\pi\frac{F_{RF}}{h} = \frac{QB_0}{m} \end{cases}$$

$$\frac{(QB_0)^2}{2mQV_{inj}} = \frac{1}{r_0^2}$$

$$\frac{QB_0 \times 2\pi m F_{RF}/h}{2mQV_{inj}} = \frac{1}{r_0^2}$$

$$\boxed{\frac{B_0 \times F_{RF}/h}{V_{RF}} = \frac{1}{\pi r_0^2} = \text{constante}}$$

2. **if $R_{injection}$=0.04 m, $R_{extraction}$= 2 m and W = 10 MeV/A**

   a. **Find the revolution frequency**

   Answer: The formulaire gives the expression of γ as a function of the energy per nucleon (A).

$$\gamma = \frac{931.478 + W}{931.478} = \frac{931.478 + 10}{931.478} = 1.010736$$

One can derive the expression of β:

$$\beta = \sqrt{\frac{\gamma^2 - 1}{\gamma^2}} = 0.146141$$

The frequency is given by:

$$\beta = 2.0958.10^{-2} \frac{F_{RF} R_{extraction}}{h} = 0.146141$$

Leading to

$$\boxed{\frac{F_{RF}}{h} = F_{rev} = 3.487\text{MHz}}$$

b. Find the magnetic field at the injection

$$\omega_{rev} = 2\pi F_{rev} = 2\pi \frac{F_{RF}}{h} = \frac{qB}{m} = \frac{QeB}{m} \quad \text{with m = Am}_p$$

$$B = 2\pi F_{rev} \times Am_p / Q = 2\pi m_P / e(A/Q) F_{rev} 10^6$$

Then

$$B = 6.5121 \times 10^{-2} (A/Q) F_{rev}$$

With F_{rev}=3.48 MHz

$$\boxed{B = 6.5121 \times 10^{-2} (A/Q) \times 3.48}$$

c. For an ion of carbon 12C6+ (A=12 and Q=6) find the voltage, V_{inj}, to inject the ion on the good radius

$$\boxed{B = 6.5121 \times 10^{-2} (12/6) \times 3.48 = 0.4542T}$$

Therefore with:

$$\frac{B_0 \times F_{RF} / h}{V_{inj}} = \frac{1}{\pi \cdot r_0^2} = \text{constante}$$

$$\boxed{V_{HT} = 7.96kV}$$

PROBLEM 2

From the lecture, it has been stated that to reach high energy with cyclotron, the magnetic field requires satisfying the relation below :

$$\overline{B}_z(r) = \gamma(r) \overline{B}_z(0)$$

1. What sign should be the field index $n = -\frac{r}{B(r)}\frac{dB(r)}{dr}$ in order to satisfy the isochronism condition to reach high energies:

Answer: $\gamma(r)$ grows with the radius and the energy of the ion.

Therefore $\overline{B}_z(r) = \gamma(r)\overline{B}_z(0) \Rightarrow \frac{\partial B_z}{\partial r} > 0$ then $n < 0$

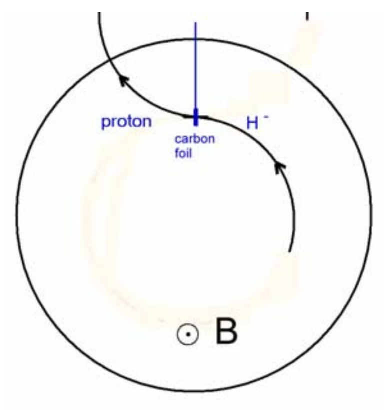
PROBLEM 3

Stripping Problem

To extract protons out of a cyclotron it is easier to accelerate H^- (constituted by a proton and 2 electrons) and in the cyclotron to insert a thin carbon foil to strip the ion the last two electrons.

1. Draw the particle trajectory before and after stripping

Answer:

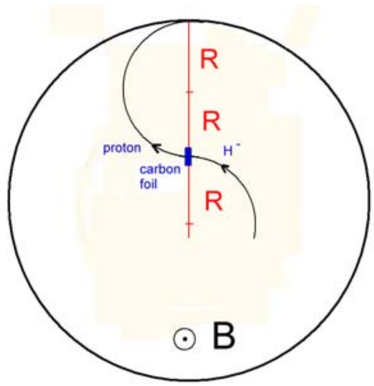


2. What is the minimum energy of the proton beam as a function of R_{max} and W_{max} that can be produced?

Answer: The ejection of the proton beam can be achieved until that the trajectory exit the magnetic field at R_{max} The limit condition is reached when $3R=R_{max}$.

The energy of the beam is equivalent to R^2 then $W \alpha R^2$ therefore $W_{min} \alpha R^2$ and $W_{max} \alpha R^2_{max}$

By substituting the R term with Rmax one get : $W_{min} \; \alpha \; (R_{max})^2/9 = W_{max}/9$



PROBLEM 4

The kinetic energy W (in MeV/nucleon) reached by an ion of mass A and charge Q accelerated to a mean radius R in a mean magnetic field $B(r)=\gamma(r)B_0$ can be expressed by:

$$W[\text{MeV/n}] = 96.488\left(\frac{Q\overline{B}\overline{R}}{A}\right)^2 \frac{1}{\gamma+1}$$

1. Show that this energy W can also be simply expressed in terms of γ, R, the RF frequency F_{RF} and the harmonic number h. The parameters B, Q and A should not appear in the formula.

ANSWER:

$$W = 0.40924\frac{f_{RF}^2\overline{R}^{\,2}}{h^2(\gamma+1)}$$

2. Check that the two formulae give the same numerical result in the following case:

A=12, Q=6, R=1 m, B=1.5 Tesla, F_{RF}=34.56 MHz and h=3

ANSWER:

W=26.75 MeV/nucleon