RF Tutorial 2016

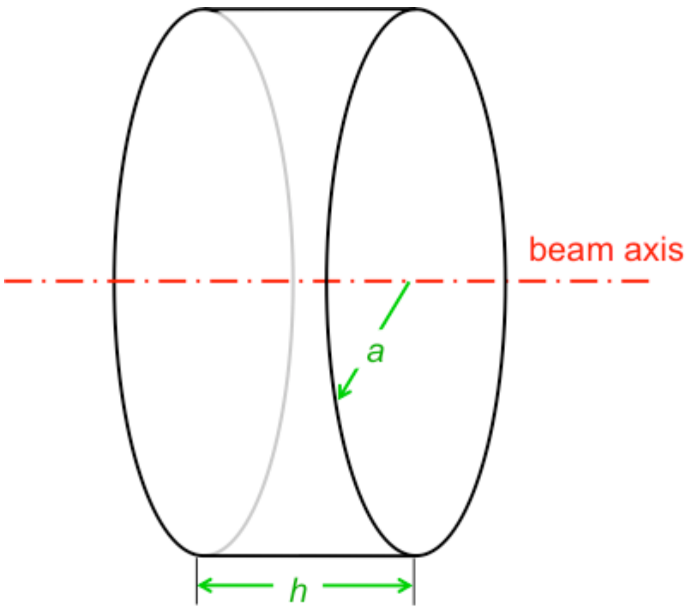
Tutorial 1

A) Design of a "pillbox" cavity

Problem: Design a simple "Pillbox" cavity with the following parameters

Frequency:	f = 299.98 MHz (λ = 1.00 m)
Wall material:	Copper (equivalent skin depth δ = 3.8 μm)
Axial length:	h = 0.2 m

For this example, we ignore beam ports, i.e. vacuum chamber stubs required for the beam passage, so that all analytical formulas describing the pillbox cavity apply.



Questions:

1. Find from the analytical formulas:
   - Cavity radius a
   - Cavity quality factor Q
   - "geometry factor", also know as "characteristic impedance" R/Q

   Is the cavity completely determined?
2. Find the equivalent circuit of the intrinsic cavity.
3. Calculate the 3-dB bandwidth of the intrinsic cavity.
4. Calculate the necessary RF power for a gap voltage of V = 100 kV
5. The cavity is fed by an amplifier, designed for a load impedance of 50 Ω. Determine:
   - The peak voltage at the cavity input.
   - The necessary transformer ratio k of the input coupler.

B) Waves of a transmission line Z = 50 Ω

Problem: Convert the circuit-based formats, voltage *V* and current *I* into the equivalent wave-based formats, forward wave *a* and backward wave *b* and vice verse using the relations:

$a = \frac{V + I\,Z}{2}$	$V = a + b$
$b = \frac{V - I\,Z}{2}$	$I\,Z = a - b$

Questions:

1. In a 50 Ω system, a directional coupler measured forward and reflected waves a and b at a certain plane as: a = 100 ∠0° and b = 60 ∠45°.
   - Calculate the corresponding voltage V and current I
   - Sketch the "phasors" of V, I Z, a and b.
2. At some plane in the 50 Ω system, a voltage of V = 100 ∠0° V and a current of I = 1.0 ∠-45° A are measured.
   - Calculate the corresponding forward and backward waves a and b.
   - Sketch the "phasors" of V, I Z, a and b.

Tutorial 2

A) "Pillbox" cavity characteristics

The following data was measured on a "pillbox" cavity":

Inductance:	L = 15.915 nH
Capacitance:	C = 1.5915 pF
3-dB bandwidth:	BW = 50 kHz

Questions:

Determine

- the frequency at resonance
- the characteristic impedance R/Q
- the quality factor Q
- the time constant τ
- the peak induced voltage immediately after the passage of a short particle bunch with charge q = 15.916e-9 As
- the remnant cavity voltage 10 μs after the passage of the bunch

B) An accelerator cavity heats up under high RF power load

A cavity is constructed from a material with"
C_2/C_1

thermal expansion coefficient:	$\Delta l/l$ = 20e-6/^{0}C (per degree Centigrade)
thermal resistivity coefficient:	$\Delta\rho/\rho$ = 4e-3/^{0}C (per degree Centigrade)

At room temperature the cavities resonance frequency is f_1 = 100 MHz, and has a 3-dB bandwidth of BW_1 = 100 kHz.
Under RF power the cavity temperature increases by 100 ^{0}C (subscripts 2 apply).

Questions:

Determine

- the ratio λ_2/λ_1
- the ratio L_2/L_1
- the ratio C_2/C_1
- the ratio Q_2/Q_1 (hint: the skin depth δ is proportional to $\sqrt{\rho/f}$
- the resonance frequency f_2 under load
- and the 3-dB bandwidth BW_2 of the resonance under load

C) Impedances in the complex plane and in the Smith chart

Plot the following impedances as points (marks)

- in a “normal” Cartesian coordinate system (complex plane)
- as reflection factors in the *Smith* chart (the reflection factor coordinates are given for convenience)

X_{rect}	X_{polar}	Γ_{rect}	Γ_{polar}
0.05	$0.05 \angle 0^0$	-0.904	$0.904 \angle 180^0$
0.5	$0.5 \angle 0^0$	-0.333	$0.333 \angle 180^0$
1	$1.0 \angle 0^0$	0	0
2	$2.0 \angle 0^0$	0.333	$0.333 \angle 0^0$
20	$20 \angle 0^0$	0.904	$0.904 \angle 0^0$
0.8	$0.8 \angle 0^0$	-0.111	$0.111 \angle 180^0$
0.8 + j0.6	$1.00 \angle 36.9^0$	0 + j0.333	$0.333 \angle 90^0$
0.8 + j1.0	$1.28 \angle 51.3^0$	0.159 + j0.472	$0.459 \angle 72.3^0$
0.8 + j1.5	$1.70 \angle 61.9^0$	0.344 + j0.546	$0.645 \angle 57.8^0$
0.8 + j2.0	$2.15 \angle 68.2^0$	0.502 + j0.552	$0.747 \angle 47.7^0$
0.8 – j0.6	$1.00 \angle -36.9^0$	0 - j0.333	$0.333 \angle -90^0$

Convince yourself with a few examples that $\Gamma(1/X) = -\Gamma(X)$

Tutorial 3

A) Gap-width optimization of a cavity

The following parameters of a 100 MHz cavity have been evaluated by a numerical simulation software as function of the gap-width g:

characteristic impedance R/Q and quality factor Q

The cavity is connected to an amplifier delivering 1 kW of RF power.
The beam has a relative velocity of $\beta = 0.15$.

Questions:

Calculate for each gap-width:

- shunt impedance R
- intrinsic cavity voltage V_{cav} for 100 kW power
- angle θ of the beam passage through the gap
- transit time factor T
- beam voltage V_{beam} maximally seen by the beam taking the transit time factor T into account

G [mm]	R/Q [Ω]	Q	R [Ω]	V_{cav} [V]	θ	T [s]	V_{beam} [V]
100	100	5000					
200	150	7000					
300	200	9000					

B) Higher-order mode of a cavity

An RF cavity has an unwanted higher-order mode (HOM) at 600 MHz, with shunt impedance $R = 6$ MΩ, a 3-dB bandwidth of $BW = 15$ kHz, and a transit time factor $T \sim 1$.

The beam consists of very short bunches, following each other at intervals of 20 μs. The circulating beam current is 0.1 A. (Reminder: current I = charge per time)

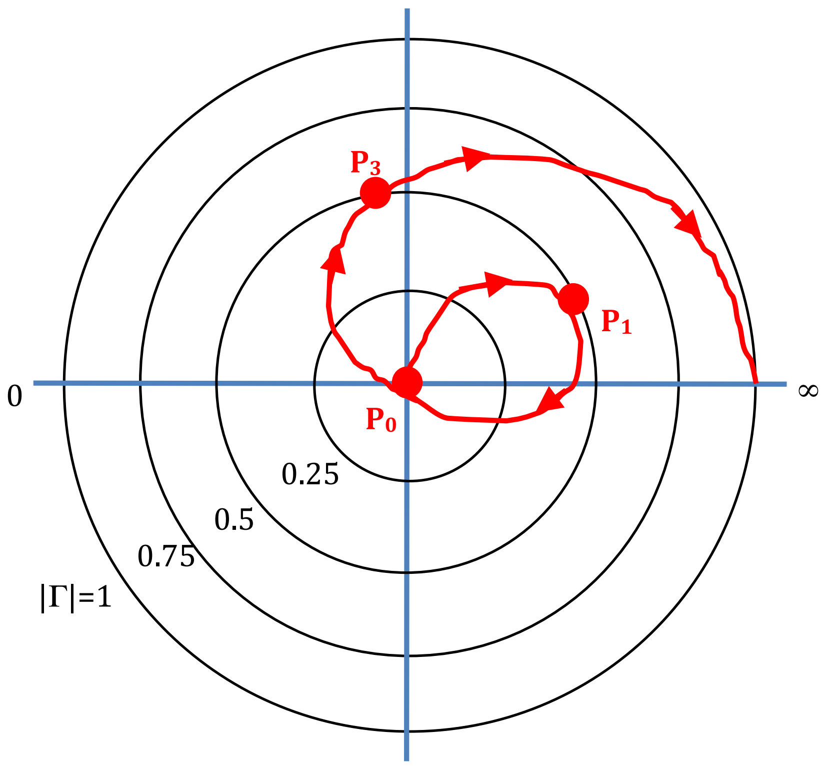
Questions:

Calculate:

- Q, R/Q, and C at the HOM frequency
- HOM voltage induced by a single bunch
- Time constant τ of the cavity
- HOM voltage at the arrival of the next bunch
- Total HOM voltage in steady state, after the passage of an infinite number of bunches, assuming the HOM resonance is an exact multiple of the beam revolution frequency and in sync with the beam.

C) Low-pass filter

A lossless low-pass filter is inserted between 50 Ω load and 50 Ω generator. The measured input impedance is shown in a very rudimentary Smith chart (only circles of constant magnitude of the reflection factor are shown). The arrows indicate the direction of increasing frequency.



Questions:

Determine:

- The relative power transmission at P_1 (normalized to the power at P_0, 100 %)
- The relative voltage transmission at P_1 (normalized as above).
- Show the point on the curve where the transmission is the same as in P_1.
- Sketch the voltage transmission (magnitude) vs. frequency.