JUAS RF Course 2016 Tutorials

Tutorial 1

Some physical constants (in SI units, but without the units attached) :

```
c = QuantityMagnitude[UnitConvert[Quantity["SpeedOfLight"]]]
```

299 792 458

```
μ = N[QuantityMagnitude[UnitConvert[Quantity["VacuumPermeability"]]]]
```

1.25664×10^{-6}

```
ϵ = N[QuantityMagnitude[UnitConvert[Quantity["VacuumPermittivity"]]]]
```

8.85419×10^{-12}

Other resources :

$$\eta = \sqrt{\frac{\mu}{\epsilon}}$$

376.73

A) Design of a "pillbox" cavity

Problem : Design a simple cavity of the "Pillbox" type with the following parameters :

frequency :

```
λ = 1.0;
```

$$f = \frac{c}{\lambda}$$

2.99792×10^{8}

$$\omega = 2 \pi f$$

1.88365×10^{9}

```
EngineeringForm[%]
```

1.88365×10^{9}

Wall material: copper

$$\sigma_{Cu} = 5.8 \times 10^{7}$$

5.8×10^{7}

Skin depth: copper (see also page 60)

$$\delta = \sqrt{\frac{2}{\omega\, \sigma_{Cu}\, \mu}}$$

3.81677×10^{-6}

axial length:

```
h = 0.2;
```

Questions

A1 : Find from the analytical formula

cavity radius a

The fundametal mode is the TM_{010} mode, with

$\mu_r = 1;$

$\epsilon_r = 1;$

```
m = 0;
n = 1;
p = 0;
a = .
```

$$f_{mnp} = \text{FullSimplify}\left[N\left[\frac{c}{2\,\pi\,\sqrt{\mu_r\,\epsilon_r}}\sqrt{\left(\frac{\text{BesselJZero}[m,\ n]}{a}\right)^2 + \left(\frac{p\,\pi}{h}\right)^2}\,\right],\ a > 0\right];$$

with the zero of the Bessel function:

```
N[BesselJZero[m, n]]
```

2.40483

see also page 24

$f_{010} = \%\%$

$$\frac{1.14743 \times 10^{8}}{a}$$

$$\lambda_0 = \frac{c}{f_{010}}$$

2.61274 a

see also page 32

```
result = Solve[λ == %, a]
```

{{a → 0.38274}}

```
result[[1]]
```

{a → 0.38274}

```
a /. result[[1]]
```

0.38274

```
a = % λ
```

0.38274

see also page 30

cavity quality factor Q

$$Q = \frac{a}{\delta}\left(1 + \frac{a}{h}\right)^{-1}$$

34 416.2

see also page 32

"geometry factor", also known as "characteristic impedance" R/Q

```
BesselJ[1, N[BesselJZero[0, 1]]]
```

0.519147

exact result:

$$\text{RoverQ} = \frac{4\,\eta}{\text{BesselJZero}[0,1]^3\,\pi\,\text{BesselJ}[1,\text{N}[\text{BesselJZero}[0,1]]]^2}\;\frac{\text{Sin}\left[\frac{\text{BesselJZero}[0,1]}{2}\,\frac{h}{a}\right]^2}{\frac{h}{a}}$$

84.6096

approximation:

$$\text{RoverQapprox} = 185\,\frac{h}{a}$$

96.6714

see also page 33

Error due to sin(x) ≈ x approximation in %:

```
100 (1 - RoverQ / RoverQapprox)
```

12.4771

Is the cavity completely determined?

With the 3 given parameters the intrinsic cavity is fully determined!

A2 : Find the equivalent circuit of the intrinsic cavity.

```
rApprox = RoverQapprox Q
```

3.32706×10^{6}

$$\text{lApprox} = \frac{\text{RoverQapprox}}{\omega}$$

5.13213×10^{-8}

```
EngineeringForm[%]
```

51.3213×10^{-9}

$$\text{cApprox} = \frac{1}{\text{RoverQapprox}\,\omega}$$

5.49163×10^{-12}

$$\text{rExact} = \text{RoverQ}\,Q$$

2.91194×10^{6}

$$\text{lExact} = \frac{\text{RoverQ}}{\omega}$$

4.49178×10^{-8}

```
EngineeringForm[%]
```

44.9178×10^{-9}

$$\text{cExact} = \frac{1}{\text{RoverQ}\,\omega}$$

6.27451×10^{-12}

see also page 48-50

A3 : Find the 3-dB bandwidth of the intrinsic cavity.

$$\Delta f = \frac{f}{Q}$$

8710.79

see also page 56

A4 : Calculate the neccessary RF power for a gap voltage of:

$$V = 100 \times 10^{3};$$

$$P = \frac{V^2}{2\,\text{rExact}}$$

1717.07

see also page 50

A5 : The cavity shall be fed by an amplifier designed for a load impedance of

$$Z_1 = 50;$$

Determine:

the peak voltage at the cavity input.

$$\text{vPeak} = \sqrt{2\,Z_1\,P}$$

414.375

the neccessary transformer ratio k of the input coupler.

```
k = Sqrt[rExact / Z1]
```

```
241.327
```

see also page 50

B) Waves on a transmission line Z = 50 Ω

Problem : Convert the couple (volatge V, current I) into the equivalent couple (forward wave a, reflected wave b) and vise versa using the relations

$a = \frac{V + IZ}{2}$ $\qquad V = a + b$

$b = \frac{V - IZ}{2}$ $\qquad IZ = a - b$

Questions

B1 : In a 50 Ω system, a directional coupler measured the forward and reflected waves a and b at a certain plane as a = 100∠0° and b = 60∠45°

Calculate the corresponding volage V and current I.

```
a = 100;
```

```
b = N[ExpToTrig[60 Exp[I 45 Degree]]]
```

```
42.4264 + 42.4264 i
```

```
v = a + b
```

```
142.426 + 42.4264 i
```

```
{Abs[v], Arg[v] / Degree}
```

```
{148.611, 16.5879}
```

```
z = 50;
```

```
i = (a - b) / z
```

```
1.15147 - 0.848528 i
```

```
{Abs[i], Arg[i] / Degree}
```

```
{1.43035, -36.3868}
```

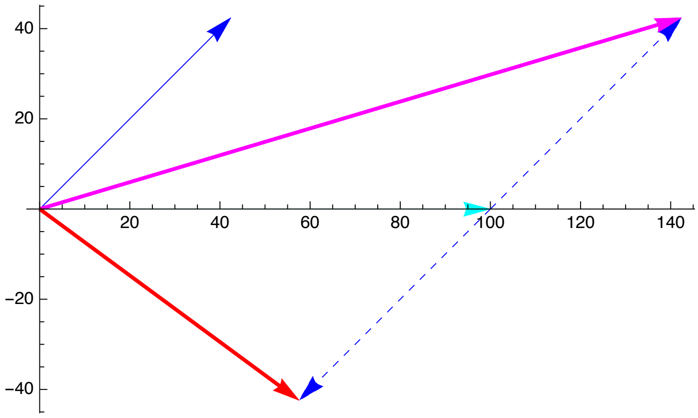
```
i z
```

```
57.5736 - 42.4264 i
```

```
{Abs[i z], Arg[i z] /Degree}
```

{71.5173, -36.3868}

```
Graphics[
 {{Cyan, Arrow[{{0, 0}, {Re[a], 0}}]}, {Blue, Arrow[{{0, 0}, {Re[b], Im[b]}}]},
  {Blue, Dashed, Arrow[{{Re[a], 0}, {Re[a + b], Im[0 + b]}}]},
  {Magenta, Thick, Arrow[{{0, 0}, {Re[a + b], Im[0 + b]}}]},
  {Blue, Dashed, Arrow[{{Re[a], 0}, {Re[a - b], Im[0 - b]}}]},
  {Red, Thick, Arrow[{{0, 0}, {Re[i z], Im[i z]}}]}},
 Axes → True, AspectRatio → Automatic]
```



B2 : At some plane of a 50 Ω system, a voltage of V = 100∠0° and a current of I = 1.0∠-45° are measured.

Calculate the corresponding foreward and backward waves a and b.

```
v = 100;
```

```
i = N[ExpToTrig[1.0 Exp[-I 45 Degree]]]
```

0.707107 - 0.707107 i

```
a = (v + i z)/2
```

67.6777 - 17.6777 i

```
{Abs[a], Arg[a] /Degree}
```

{69.9483, -14.6388}

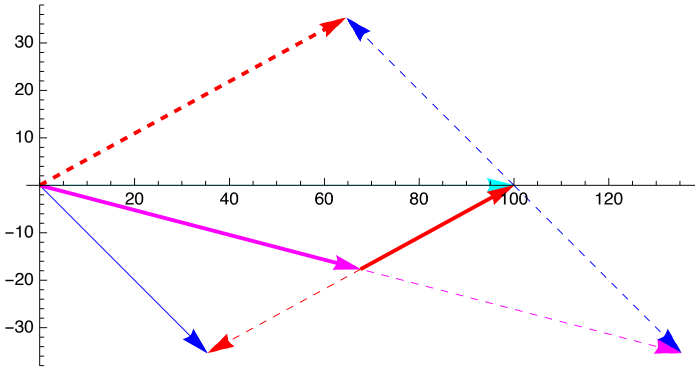
```
b = (v - i z)/2
```

32.3223 + 17.6777 i

```
{Abs[b], Arg[b] /Degree}
```

{36.8406, 28.6751}

```
Graphics[{{Cyan, Arrow[{{0, 0}, {Re[v], 0}}]},
  {Blue, Arrow[{{0, 0}, {Re[i z], Im[i z]}}]},
  {Blue, Dashed, Arrow[{{Re[v], 0}, {Re[v] + Re[i z], 0 + Im[i z]}}]},
  {Blue, Dashed, Arrow[{{Re[v], 0}, {Re[v] - Re[i z], 0 - Im[i z]}}]},
  {Magenta, Dashed, Arrow[{{0, 0}, {Re[2 a], Im[2 a]}}]},
  {Magenta, Thick, Arrow[{{0, 0}, {Re[a], Im[a]}}]},
  {Red, Thick, Arrow[{{Re[a], Im[a]}, {Re[a + b], Im[a + b]}}]},
  {Red, Dashed, Arrow[{{Re[a], Im[a]}, {Re[a - b], Im[a - b]}}]},
  {Red, Dashed, Thick, Arrow[{{0, 0}, {Re[2 b], Im[2 b]}}]}},
 Axes → True, AspectRatio → Automatic]
```



Tutorial 2

Problem #1 to be treated as "CONTINOUS ASSESMENT"

■ 1) The following data has been determined on a cavity:

Inductance:

$l = 15.915 \times 10^{-9};$

Capacitance:

$c = 1.5915 \times 10^{-12};$

3-dB bandwidth:

$\Delta f = 50 \times 10^{3};$

Questions

Determine

- the resonance frequency

$$f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{l\, c}}$$

1.00003×10^{9}

see also page 49

- the characteristic impedance R/Q

$$\text{RoverQ} = \sqrt{\frac{l}{c}}$$

100.

see also page 50

- the quality factor Q

$$Q = \frac{f_{res}}{\Delta f}$$

20 000.6

see also page 56

- the time constant τ

$$\tau = \frac{Q}{\pi f_{res}}$$

6.3662×10^{-6}

see also page 63

- the peak induced voltage immediatly after the passage of a short particle bunch with charge

$$q = 15.916 \times 10^{-9};$$

$$v_{step} = \text{Abs}\left[0 - \frac{q}{c}\right]$$

10 000.6

see also page 65

- the remaining cavity voltage 10 μs after the bunch passage

$$t = 10 \times 10^{-6};$$

$$v_{end} = v_{step}\, e^{-\frac{t}{\tau}}$$

2078.93

see also page 65

2) A cavity is constructed from material with:

thermal expansion coefficient:

$$\Delta\text{loverl} = 20 \times 10^{-6};$$

thermal resistivity coefficient:

```
Δρoverρ = 4 × 10^-3;
```

At room temperature the cavity resonates at a frequency

```
f1 = 100 × 10^6;
```

with a bandwidth of

```
Δf1 = 100 × 10^3;
```

Under RF power, the cavity temperature is increased by

```
Δt = 100;
```

(subscripts 2 apply)

Questions

Determine

- the ratio λ_2/λ_1

The wavelength scales proportional with the cavity dimension d, as λ is inverse proportional to f

$$\frac{\lambda_2}{\lambda_1} = \frac{d_2}{d_1} = \frac{d_1\,(1+\Delta l/l\,\Delta T)}{d_1} = 1 + \frac{\Delta l}{l}\,\Delta T$$

```
λ2overλ1 = N[1 + Δloverl Δt]
1.002
```

see also page 69

- the ratio L_2/L_1

The characteristic impedance R/Q stays constant, therefore:

$$\frac{L_2}{L_1} = \frac{R/Q}{\omega_2}\,\frac{\omega_1}{R/Q} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1}$$

```
l2overl1 = λ2overλ1
1.002
```

see also pages 50 and 69

- the ratio C_2/C_1

The characteristic impedance R/Q stays constant, therefore:

$$\frac{C_2}{C_1} = \frac{1}{R/Q\,\omega_2}\,\frac{R/Q\,\omega_1}{1} = \frac{\omega_1}{\omega_2} = \frac{\lambda_2}{\lambda_1}$$

```
c2overc1 = λ2overλ1
1.002
```

see also pages 50 and 69

- the ratio Q_2/Q_1

The quality factor Q $\propto \frac{\lambda_0}{\delta}$, and the skin depth follows $\delta \propto \sqrt{\frac{\rho}{f}}$, therefore:

$$\frac{Q_2}{Q_1} = \frac{\text{const}\,\lambda_2}{\delta_2}\,\frac{\delta_1}{\text{const}\,\lambda_1} = \frac{\lambda_2}{\lambda_1}\,\frac{\delta_1}{\delta_2} = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{\rho_1}{\rho_2}\,\frac{f_2}{f_1}} = \frac{\lambda_2}{\lambda_1}\sqrt{\frac{\rho_1}{\rho_2}\,\frac{\lambda_1}{\lambda_2}} = \sqrt{\frac{\rho_1}{\rho_2}\,\frac{\lambda_2}{\lambda_1}}$$

```
ρ1overρ2 = N[(1 + ΔρoverρΔt)]
1.4
```

```
q2overq1 = √(ρ1overρ2 λ2overλ1)
1.1844
```

see also pages 34 and 69

- the resonance frequency f_2

$$\frac{f_2}{f_1} = \frac{\lambda_1}{\lambda_2}$$

```
f₂ = f₁ / λ2overλ1
```

9.98004×10^{7}

```
EngineeringForm[%]
```

99.8004×10^{6}

- the bandwidth Δf_2

$$\Delta f_1 = \frac{f_1}{Q_1}\,,\ \Delta f_2 = \frac{f_2}{Q_2} \Longrightarrow \frac{\Delta f_2}{\Delta f_1} = \frac{f_2}{Q_2}\,\frac{Q_1}{f_1}$$

```
Δf₂ = (f₂/f₁) (1/q2overq1) Δf₁
```

84 262.5

```
EngineeringForm[%]
```

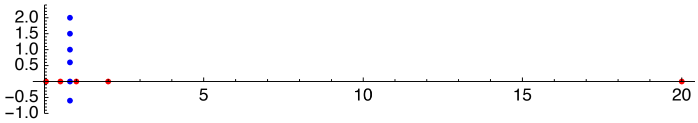
84.2625×10^{3}

3) Plot the following impedances:

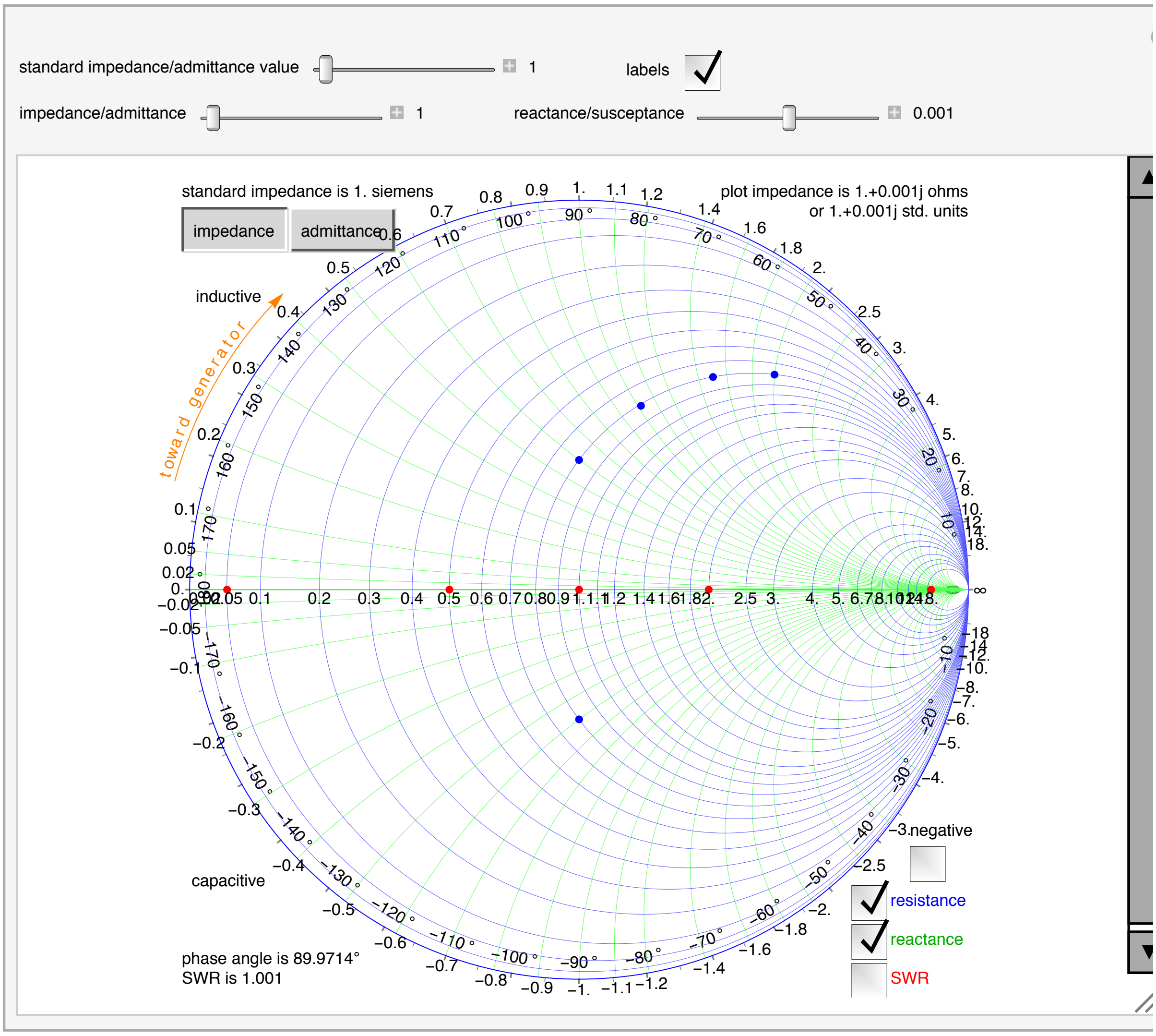
Questions

- in “normal”cartesian coordinates

```
Graphics[{{Red, Point[{0.05, 0}], Point[{0.5, 0}], Point[{1, 0}], Point[{2, 0}],
   Point[{20, 0}]}, {Blue, Point[{0.8, 0}], Point[{0.8, 0.6}],
   Point[{0.8, 1}], Point[{0.8, 1.5}], Point[{0.8, 2}], Point[{0.8, -0.6}]}},
 Axes → True, AspectRatio → Automatic]
```



- as reflection factors in the SMITH chart



Tutorial 3

▪ 1) Cavity optimization (counted for the “continous assessment”)

The following parameters of a 100-MHz cavity have benn evaluated by a cavity desing program as a function of the gap width g:
R/Q (characteristic impedance),
Q (quality factor)

```
f = 100 × 10^6;
```

The cavity is connected to an amplifier delivering a power of 1 kW

```
p = 1 × 10^3;
```

The cavity beam has a relative velocity of

```
β = 0.15;
```

Questions

Calculate for each gap width:

- shunt impedance R

```
g = {0.1, 0.2, 0.3};
RoverQ = {100, 150, 200};
q = {5000, 7000, 9000};
r = RoverQ q
{500 000, 1 050 000, 1 800 000}
```

- intrinsic cavity voltage V_{cav} for 100 kW power

```
p = 100 × 10^3;
Vcav = N[√(2 r p)]
{316 228., 458 258., 600 000.}
```

see also page 50

- phase angle Θ of the passage through the gap

Velocity v of the beam through the cavity gap g:
$v = \beta c_0 = \frac{g}{t}$
Time t through the gap:
$t = \frac{g}{\beta c_0} = \frac{g}{\beta f \lambda}$
The total phase angle acounts for $\Theta = 2\pi$,
the phase Θ is given by multipying both sides by $2\pi/f$ (remember: f t = 1):
$2\pi f t = 2\pi \frac{g f}{\beta f \lambda} \Longrightarrow 2\pi = \Theta = 2\pi \frac{g}{\beta \lambda} = \frac{2\pi f g}{\beta c_0}$

```
Θ = (2 π f g)/(β c)
{1.39723, 2.79446, 4.19169}
```

- transit time factor T

The transient time factor T is related to the phase angle Θ:
$\frac{g \omega}{2 \beta c_0} = \frac{2\pi f g}{2 \beta c_0} = \frac{\Theta}{2}$

$$T = \frac{\text{Sin}[\Theta / 2]}{(\Theta / 2)}$$

{0.920618, 0.704948, 0.412864}

see also page 79

- beam voltage V_{beam} maximally seen by the beam taking the transit time factor T into account

$V_{beam} = V_{cav}\,T$

{291 125., 323 048., 247 719.}

Which of the three versions gives the highest beam voltage?

Version 2 with a gap of g = 200 mm gives the highest beam voltage V_{beam} = 323 kV

Supposing a field enhancement factor of ~1.5 in the gap region, evaluate the danger of voltage breakdown for each design using the Kilpatrick limit as a yardstick.

$1.5\,V_{cav}$

{474 342., 687 386., 900 000.}

$V_{Kilpatrick}$ (@ 100 MHz) = {~1.15 MV, ~2 MV, ~3.3 MV}
For all gap configurations there is no danger of a voltage breakdown,
the cavity voltage is always well below the Kilpatrick limit.

see also page 94

2) A higher order-mode (HOM) in a cavity

A RF cavity has an unwanted high-order mode (HOM) at

$f_{HOM} = 600 \times 10^6$;

with a shunt impedance of

$r_{HOM} = 6 \times 10^6$;

a 3-dB bandwidth of

$\Delta f_{HOM} = 15 \times 10^3$;

and a trinsit time factor of

$T = 1$;

The beam consists of very short bunches, following eachother at intervall of

$t_{bunch} = 20 \times 10^{-6}$;

The circulating current is

$i_{beam} = 0.1$;

(Remember: I = charge per time)

Questions

Calculate:

- Q, R/Q, and C at the HOM frequency

$$Q_{HOM} = \frac{f_{HOM}}{\Delta f_{HOM}}$$

40 000

see also page 56

$$\text{RoverQ} = \frac{r_{HOM}}{Q_{HOM}}$$

150

$$C_{HOM} = N\left[\frac{1}{2\,\pi\, f_{HOM}\, \text{RoverQ}}\right]$$

1.76839×10^{-12}

see also page 50

- HOM voltage induced by a single bunch

Assuming a single bunch in the ring :

$$q_{bunch} = i_{beam}\, t_{bunch}$$

$2. \times 10^{-6}$

$$\Delta V = \frac{q_{bunch}}{C_{HOM}}$$

1.13097×10^{6}

see also page 64

- The time constant τ of the cavity

$$\tau = 2\, r_{HOM}\, C_{HOM}$$

0.0000212207

```
EngineeringForm[%]
```

21.2207×10^{-6}

see also page 64

- The HOM voltage at the arrival of the next bunch

$$V_{end}\, e^{-t/\tau} = V_{step} \implies V_{next} = \Delta V\, e^{-t/\tau}$$

$$V_{next} = \Delta V\, e^{-t_{bunch}/\tau}$$

440 696.

- The total HOM voltage in steady state, after the passage of an infinite number of bunches, supposing that the HOM resonance ilies at an exact multiple of the beam revolution frequency

$$V_{end} = \frac{q_{bunch}}{C_{HOM}\left(1 - e^{-t_{bunch}/\tau}\right)}$$

1.85303×10^{6}

3) Lossless low-pass filter

A lossless low-pass filter is inserted between 50 Ohm load and 50 Ohm generator. The measured input impedance is shown below on a very rudimentary SMITH chart (only circles of constant magnitude of the reflection factor are shown). The arrows indicate the direction of increasing frequency.

Questions

Determine:

- the relative power transmission at point P_1 (normalized to the power at P_0, 100%)

At point P_1 the magnitude of the reflection coefficient reads :

absΓ = 0.5;

In a lossless filter terminated equally at input and output, the output power equals the input power :

$$P_{out} = P_{in} = |a|^2 + |b|^2 = |a|^2\left(1 - |\Gamma|^2\right) = P_0\left(1 - |\Gamma|^2\right)$$

$$\mathbf{pOutoverp0 = 1 - abs\Gamma^2}$$

0.75

10 Log10[%]

-1.24939

see also page 176

- the relative voltage transmission at point P_1 (normalized to the power at P_0, 100%)

$$V \propto P \Longrightarrow \frac{V_{out}}{V_0} = \sqrt{\frac{P_{out}}{P_0}}$$

```
vOutoverv0 = √pOutoverp0
0.866025

20 Log10[%]
-1.24939
```