JUAS 2016 – RF tutorial (solutions)

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1.) Cavities

1.1) Analyze a rectangular cavity

1. Free space wavelength: $\lambda_0 = \sqrt{2}a = 141.4\ mm$

   Resonant frequency: $f_{res} = \frac{c}{\lambda_0} = 2.122\ GHz$ → $\omega = 2\,\pi\,f_{res}$

2. Skin depth: $\delta = \sqrt{\frac{2}{\omega\sigma\mu}} = 1.435\ \mu m$ with $\mu = 4\pi \cdot 10^{-7}$ Vs/(Am)

   Quality factor: $Q_0 = \frac{1}{\delta}\frac{ab}{a+2b} = 17\,422$

3. Relation of the Q factors: $\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{EXT}}$

   with $Q_0 = Q_{EXT} = 17422$: $Q_L = Q_0/2 = 8\,711$

4. Bandwidth: $\mathrm{BW}_{3\mathrm{dB}} = f_{res}/Q_L = 243.6$ kHz

5. As the cavity is critically coupled, no RF power is reflected and all of the incident power is dissipated thermally: $P_{IN} = P_{TH} = 50$ W

6. Stored energy: $W = \frac{Q_L\,P_{IN}}{\omega} = 32.7\ \mu J$

1.2) Design a pillbox cavity (1)

1. For a pillbox cavity, the resonance wavelength λ is linked to the radius a: $\lambda_0 = 2.61\ a$

   As frequency f is related to wavelength and the velocity of light c by $\lambda f = c$

   We can write $a = \dfrac{c}{2.61 f_0} = \dfrac{3.10^8}{2.61 \times 500.10^6} = 0.23m$

2. The ratio h/2a is given to be 0.5 so that we have (with a=0.23 m) h=0.23 m

3. From the pillbox cavity chart, the first higher order mode is H_{111} (also referred to as TE_{111}).

4. For a pillbox cavity, the Q value is given by:

$$Q = \frac{0.383\frac{\lambda_0}{\delta}}{1 + 0.383\frac{\lambda_0}{h}}$$

Inserting the values for the wavelength and the skin depth δ at the resonance frequency f_0:

$$\delta = \sqrt{\frac{2}{2\pi f_0 \mu\sigma}} = \sqrt{\frac{2}{2\pi 500.10^6 4\pi 10^{-7} 58.10^6}} = 2.96\mu m$$

we obtain *Q = 38893*

5. The R/Q factor of this cavity can be approximated to: *R/Q ~ 185 h/a =185*

6. The lumped element parallel RLC circuit model of the cavity can be obtained with the resonant frequency, the Q and R/Q values:

$$R = \frac{R}{Q} Q = 7.20 M\Omega$$

$$L = \frac{R}{Q\omega_0}$$ so that $$L = \frac{185}{2\pi 500.10^6} = 58.8 nH$$

$$C = \frac{1}{\frac{R}{Q}\omega_0}$$ so that $$C = \frac{1}{185 \times 2\pi 500.10^6} = 1.72 pF$$

7. The power transferred into the cavity is $$P_{RMS} = \frac{U_{RMS}^2}{R} = \frac{U_{peak}^2}{2R}$$

so that $$U_{peak} = \sqrt{2 P_{RMS} R} = \sqrt{2 \times 300.10^3 \times 7.2.10^6} = 2.077 MV$$

8. If the cavity were in stainless steel,

$$\delta = \sqrt{\frac{2}{2\pi f_0 \mu\sigma}} = \sqrt{\frac{2}{2\pi 500.10^6 4\pi 10^{-7} 1.45.10^6}} = 19\mu m$$

$$Q = \frac{0.383\frac{\lambda_0}{\delta}}{1 + 0.383\frac{\lambda_0}{h}} = 6049$$

$$R = \frac{R}{Q} Q = 1.12 M\Omega$$

And finally

$$U_{peak} = \sqrt{2 P_{RMS} R} = \sqrt{2 \times 300.10^3 \times 1.12.10^6} = 819 \text{ kV}$$

1.3) Design of a pillbox cavity (3)

A1|

$$\lambda_0 = 2.61\, a \quad \rightarrow \quad a = \frac{\lambda_0}{2.61} = \frac{1}{2.61} = \underline{0.383\ [m]}$$

$$Q = \frac{0.383 \cdot \lambda_0}{\delta} \cdot \frac{1}{1 + 0.192 \frac{\lambda_0}{h}} = \frac{0.383 \cdot 1}{3.8E{-6}} \cdot \frac{1}{1 + 0.192 \frac{1}{0.1}} = \underline{34'517}$$

$$r/Q = 370 \cdot \frac{h}{a} = 370 \cdot \frac{0.1}{0.383} = 96.6\ [\Omega]$$

3 parameters → intrinsic cavity fully defined

A2|

$$r/Q = \omega L \quad \rightarrow \quad L = \frac{r/Q}{2\pi f} = \frac{96.6}{2\pi\, 299.9E6} = \underline{51.25\,E{-9}}$$

$$r/Q = \frac{1}{\omega C} \quad \rightarrow \quad C = \frac{1}{2\pi f\; r/Q} = \quad = \underline{5.49\,E{-12}}$$

$$r = r/Q \cdot Q \quad \rightarrow \quad r = 96.6 \cdot 34'517 = \underline{3.334E6}$$

A3|

$$BW = \frac{f_{res}}{Q} = \frac{299.98E6}{34'517} = \underline{8.69E3}$$

A4|

$$P = \frac{\hat{U}^2}{2r} = \frac{(100E3)^2}{2 \cdot 3.334E6} = 1.499E3 \sim \underline{1.5\ kW}$$

A5| At the input the impedance is $Z = 50\,\Omega$, here

$$P = \frac{\hat{U}^2}{2Z} \rightarrow \quad \hat{U} = \sqrt{2PZ} = \underline{387.26\ \hat{V}}$$

$$k = \frac{100E3}{387.26} = 258.22$$

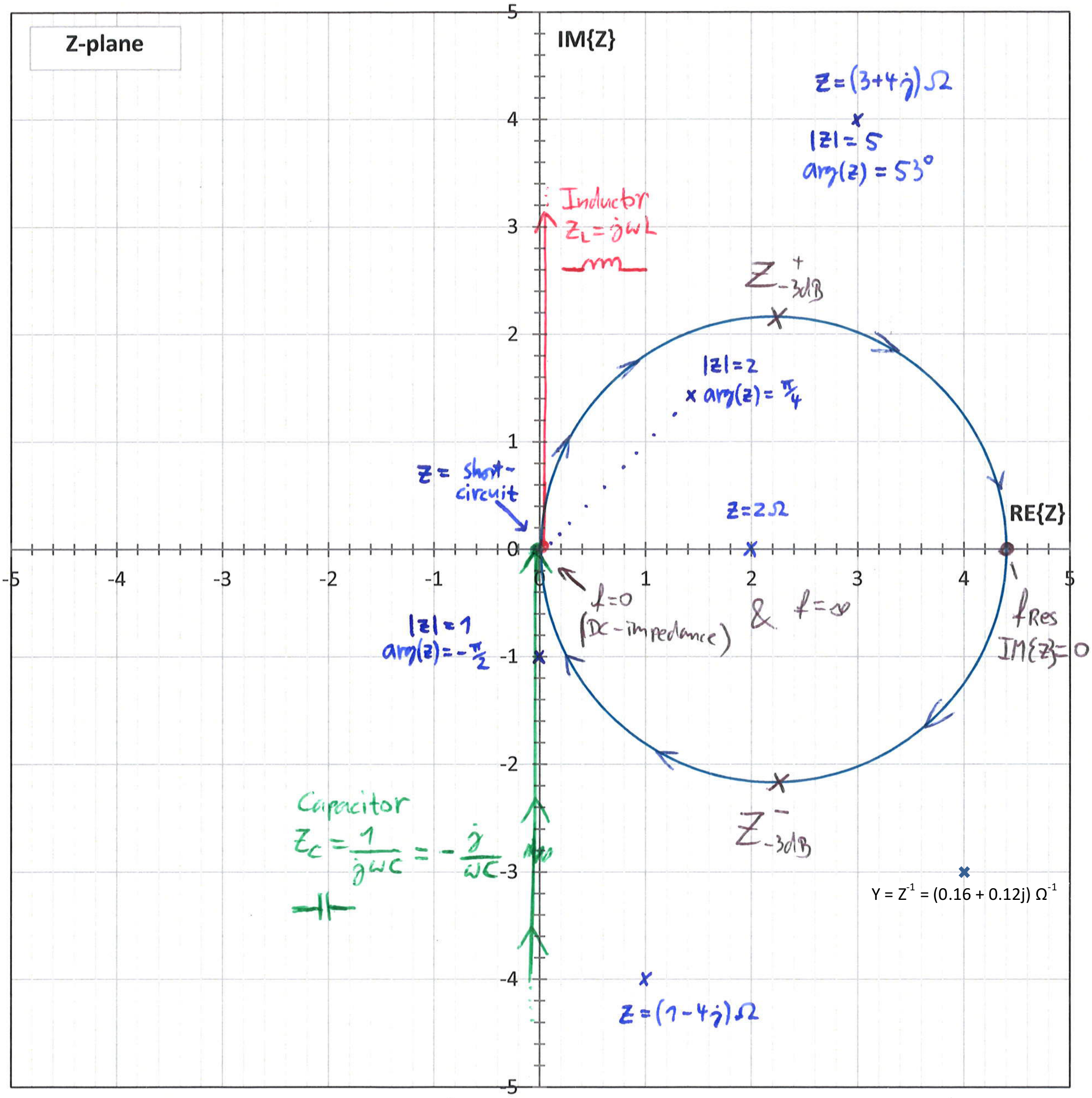
(k could also be directly evaluated by $k = \sqrt{\frac{r}{Z}}$)

2.) Multiple choice questions

1. The f_c will not change.
2. The power dissipation will not change
3. 46.7 mm
4. The mode TE_{10} or H_{10} has a cutoff frequency of 1.5 GHz.
   The electric field is orthogonal to the side with the larger dimension.
5. striplines with the top cover and the top dielectric layer taken away.
6. TE
7. TEM
8. lowers the resonance frequency
9. reduce calculation time
   rule out certain higher order modes
10. 99 percent

3.) Impedances

3.1) Impedances in the complex plane (1)



3.2) Impedances in the complex plane (1)

1. $f_{res} = f_7 = 105.2$ MHz
2. $BW = f_5 - f_2 = 105.35$ MHz – 105.05 MHz = 30 kHz
3. R-L-C parallel circuit
4. $R = 230$ kΩ (at $f_{res} = f_7 = 105.2$ MHz)
5. Straight line parallel to the Im{Y} axes, crossing Re{Y} at 1/230 mS. The 3-dB points are located on the locus as points crossing with lines from the origin under ±45°.
6. $Q = f_{res}/BW = 3506.7$, $L = R / (2\pi f_{res} Q) = 99.23$ nH, $C = 1 / [(2\pi f_{res})^2 L] = 23.066$ pF

3.5) Smith Chart (3)

1. Mark the reflection factors *Γ* of points A to F in the Smith Chart and find approximate values for the corresponding (normalized) impedances z:

Point	Reflection factor *Γ*	Normalized impedance *z*
A	1 \0°	infinity
B	1 \45°	0 + j2.4
C	1 \90°	0 + j1.0
C	1 \180°	0
E	1 \-90°	0 – j1.0
F	0.5	3.0

4.) S-Parameters

Match the following S-Matrices to the corresponding components

$$S_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad S_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad S_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \quad S_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Component	Isolator	Circulator	Transmission line, length = λ/2	3-dB attenuator
S-matrix	S_2	S_4	S_3	S_1

5.) Scaling laws

1. For the right candidate design, the parameter r/Q has to agree with the one calculated for the actual cavity. Using $r/Q = \frac{1}{\omega C}$ we find an r/Q of 50 and 200, 100, 50 for the actual cavity and the three candidate designs, respectively. Therefore, cavity C is chosen for the scaling.

2. All cavity dimension are proportional to $\lambda = 1/f$.
$D_x = D\frac{f}{f_x} = 0.236$ m

3. Using the third scaling law, $Q\frac{\delta}{\lambda} =$ const, we find $Q \propto \frac{\lambda}{\delta}$ with the skin depth δ decreasing linearly with frequency.
$\delta = \sqrt{\frac{1}{\pi f \mu \sigma}} \propto \frac{1}{\sqrt{f}} \propto \sqrt{\lambda}$.
Therefore, $Q \propto \sqrt{\lambda} \propto \frac{1}{\sqrt{f}}$. $\Rightarrow Q_x = Q\sqrt{\frac{f}{f_x}} = 6140$

6.) Questions

1. Striplines have a ground plane above and below the central conductor. Microstriplines have a groundplane only below the conductor, the other side is left open.
2. The open structure of a Microstripline resembles an antenna structure. RF power is radiated into the environment and electromagnetic interference is easily picked up. Crosstalk and mutual coupling becomes an issue if several Microstriplines are placed close to each other. The asymmetric, open structure causes a frequency-dependent characteristic impedance and a considerable dispersion.
3. While a waveguide is in general more power efficient (less attenuation), the dimensions of this structure would be impractically large for 50 MHz. The free space wavelength at 50 MHz is $\lambda = \frac{c}{f} = 6\ m$. So the waveguide needs to be at least 3 m wide. Coaxial transmission line does not have a lower cut-off frequency and would be the better choice.
4. Looking at the RLC equivalent circuit of the cavity, the ferrite increases the inductance part L and thus lowers the resonant frequency compared to air filled cavities. This allows to reduce the resonant frequency or alternatively the size of the structure.
   Placing the ferrite in a static magnetic bias field, its μ can be varied which leads to change in L and in the resonant frequency. So by changing the bias current, the cavity can be electronically tuned.
5. He is in fact fine tuning the resonant frequency of the cavity. This punch-tuning or dimple tuning procedure was used for example in the CERN Linear Injector for LEP. The structure was specially designed so it could be slightly deformed at certain points with a hammer.