Exercises for RF Tutorial

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1 Cavities

Question 1

Design a pillbox cavity which has a resonance frequency f_{res} of 150 MHz for the TM_{010} resonance.

1. Determine the free space wavelength λ for $150\,$MHz.
2. What is the cavity diameter?
3. The R/Q is $300\,\Omega$. Calculate the cavity height.
4. Determine the Q value for this cavity when made from copper and and when made from stainless steel. (conductivities: $\sigma_{\text{copper}} = 58 \cdot 10^6$ S/m, $\sigma_{\text{StSt}} = 1.4 \cdot 10^6$ S/m)
5. Determine the parameters R, L, C of the equivalent circuit for this cavity.

Question 2

A rectangular $TE_{101} = H_{101}$ mode cavity has the dimensions $a = c = 100\,$mm $b = 50\,$mm.

1. Determine its resonant frequency f_{res}.
2. Determine the unloaded Q-factor for copper walls. ($\sigma_{\text{copper}} = 58 \cdot 10^6$ S/m)
3. The coupler, connecting the cavity to the outside world is adjusted for critical coupling. Then $Q_0 = Q_{ext}$ which allows maximum power transfer into or out of the resonator. What is the loaded Q value (Q_L) of the cavity?
4. What is the $3\,$dB bandwidth of the loaded cavity for the TE_{101} mode?
5. The critically coupled cavity is driven by $P_{in} = 50\,$W of input power on its resonant frequency. How much power is thermally dissipated in the cavity?
6. How much energy is stored in the cavity?

Question 3

A stainless steel pillbox cavity with a radius $a = 160\,$mm and a Q value of 4000 is given. ($\sigma_{\text{StSt}} = 1.4 \cdot 10^6$ S/m)

1. What is the resonance frequency of the fundamental mode?
2. Determine the height of the cavity.
3. What is a suitable tuning range for this cavity with no external impedance matching? (3dB bandwidth?)
4. Determine the R/Q value for this cavity. Will it change if the cavity was made of copper? ($\sigma_{\text{copper}} = 58 \cdot 10^6$ S/m)
5. What are the parameters R, L, C of the equivalent circuit for this cavity?
6. The cavity will be powered by a $10\,$W amplifier via critical coupling. What is the peak voltage for this configuration? The amplifier is designed for a load impedance of $50\,\Omega$. Determine the necessary transformer ratio k of the input coupler.
7. What would be the Q factor and the gap voltage if the cavity would be made out of copper?

2 Decibel

Question 4

Fill in the missing fields in the tables below:

Voltage ratio	Power ratio	dB
3.1623		
	100	
		40

Question 5

Fill in the missing fields in the tables below:

dBm (50 Ω)	RMS Voltage (50 Ω)	milli Watt
0		
+ 30		
– 60		
		100

3 Multiple choice

Question 6

1. Which mode is the fundamental mode (lowest cut-off frequency) in a cylindrical waveguide of circular cross-section without inner conductor? (check 1)

   ○ TE
   ○ TEM
   ○ TM

2. Which mode is the fundamental mode in a cylindrical waveguide with inner conductor (coaxial line)? (check 1)

   ○ TE
   ○ TEM
   ○ TM

3. Adding capacitive loading to a cavity (check 1)

   ○ lowers the resonance frequency
   ○ does not affect the resonance frequency
   ○ increases the resonance frequency

4. Advantages of a nose cone cavity compared to an ordinary pill box cavity of same dimension (check 1)

   ○ Smaller skin depth
   ○ Higher R/Q
   ○ Higher Q

5. Superconducting cavities usually do not have nose cones because (check 2)

- ○ Superconductors are expensive, so don't waste them for nose cones
- ○ Nose cones are sensitive to multipactoring, which causes excessive heating and must therefore be avoided
- ○ The shunt impedance is so high that it can't be increased any more by changing the geometry
- ○ Superconductors are sensitive to high electric field around the nose cones

6. When doing numerical simulations, geometrical symmetries are exploited in order to (check 2)

   - ○ ensure convergence of the simulation algorithms for resonant structures
   - ○ reduce calculation time
   - ○ account for the transit time factor
   - ○ rule out certain higher order modes

7. The GSM standard specifies a minimum sensitivity requirement of about -100 dBm, while the maximum output power is in the order of 1 W. This corresponds to how many orders of magnitude in power? (Exact values: -102 dBm minimum sensitivity, 1 to 5 W maximum output power) (check 1)

   - ○ 5
   - ○ 8
   - ○ 13

8. When you cover then antenna of your mobile with your hand while using it, the attenuation caused is in the order of 20 dB. Human tissue is a rather good absorber, so you can neglect reflections for this calculation. How many percent of the mobile's output power stay in the head and hand? (check 1)

   - ○ 9
   - ○ 99
   - ○ 99.99

4 Resonant circuits and impedance plane

Question 7

A resonant circuit consists of an inductance with L = 100 nH, a capacitance with C = 10 pF and a resistor with R = 1 kΩ at resonance.

1. Calculate the resonance frequency, the Q value, Δf and the R/Q.
2. Sketch the locus of impedance of the circuit in the admittance plane.
3. Sketch the locus of impedance of the circuit in the impedance plane.

Question 8

Design a RLC parallel resonance circuit with a resonance frequency f_{res} of 15.92 MHz.

1. For Q = 20 at resonance this circuit has an impedance R of 4 kΩ. What are the values for L and C?
2. Sketch the locus of impedance of the resonant circuit in the complex impedance plane.

5 Transmission lines and striplines

Question 9

A TEM transmission line has a characteristic impedance of $Z_C = 75\,\Omega$ ZC and a velocity of propagation of v$= 0.5$c (c = $3 \cdot 108$ m/s, $\mu_r = 1$)

1. What is the capacitance C′ and inductance L′ per unit length?
2. What is the relative permittivity ϵ_r of the dielectric?
3. The transmission line with the above parameters is realized as a coaxial structure. The outer radius is R= $10\,\text{mm}$. What is the inner radius r?

Question 10

The impedance of a stripline is given via:

$$Z = \frac{60\Omega}{\sqrt{\epsilon_r}} \cdot \ln\left[\frac{1.9b}{0.8w + t}\right]$$

1. Determine the Impedance for a stripline with dimensions $b = 15\,\text{mm}$, $t = 0.02\,\text{mm}$ and $w = 3.1\,\text{mm}$. The stripline is filled with PVC which has a dielectric constant of $\epsilon_r = 4$.
2. What is the capacitance C′ per unit length and what is the inductance L′ per unit length?
3. What is the phase velocity of the line?

Question 11

A stripline has an impedance Z= $50\,\Omega$. It is filled with Teflon as a dielectric ($\epsilon_r = 2.1$).

1. What is the capacitance C′ per unit length and what is the inductance L′ per unit length?
2. What is the phase velocity of the line?

6 S-parameters

Question 12

Match the S-matrices to the corresponding components

$$S_1 = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad S_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad S_3 = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad S_4 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Component	Isolator	Circulator	Transmission line, length $\lambda/2$	3 dB attenuator
S-matrix				

Question 13

Denote the S-Matix of a transmission line of length $\lambda/4$ and an ideal $10\,\text{dB}$ amplifier.

7 Impedances and Smith chart

Question 14

1. Sketch the locus of impedance of a resistor in the impedance and addmittance plane.
2. Sketch the locus of impedance of an inductance in the impedance and addmittance plane.
3. Sketch the locus of impedance of a capacitance in the impedance and addmittance plane.

You may use figure 1

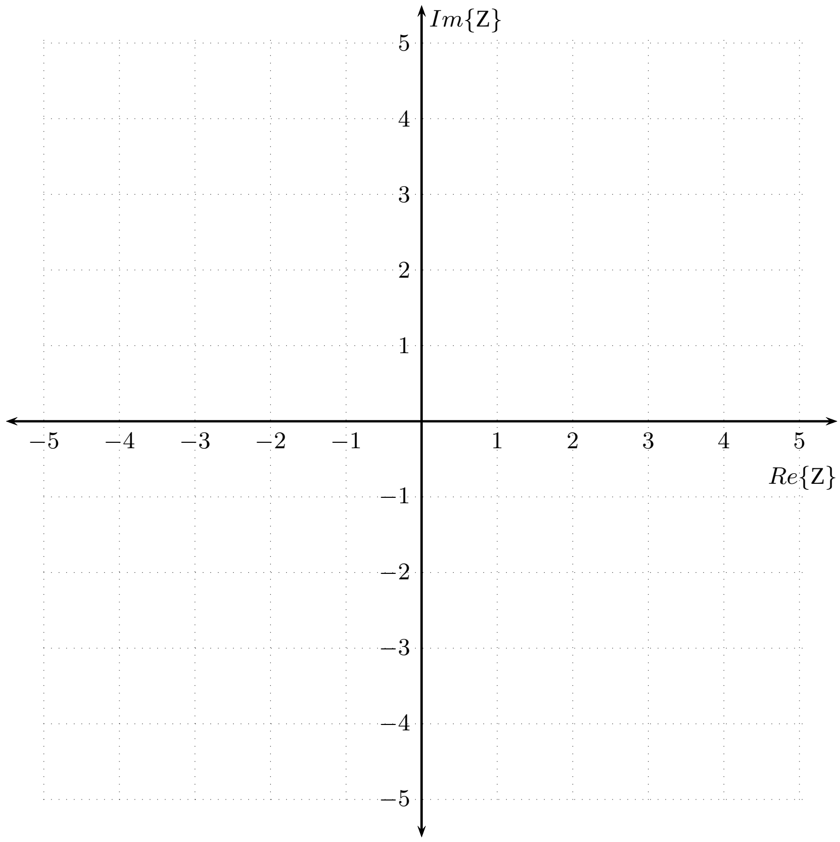


Fig. 1: The impedance plane. (Question 14 and 15)

Question 15

Mark the following impedances in the Z-plane:

Z=$(3+4\text{j})\,\Omega$	$\lvert Z\rvert = 2$, arg(Z)= $\frac{\pi}{4}$	Z = short circuit
Z= $2\,\Omega$	$\lvert Z\rvert = 1$, arg(Z)= $-\frac{\pi}{2}$	Y = Z^{-1} = $(0.16+0.12\text{j})\,\Omega^{-1}$
Z=$(1-4\text{j})\,\Omega$	$\lvert Z\rvert = 5$, arg(Z)= $53°$	

You may use figure 1

Question 16

The input impedance of an RLC circuit has been plotted in the Z-plane (blue circle — see figure 2 below). Mark the points in the diagram describing:

1. Impedance at resonance frequency
2. DC impedance
3. 3 dB bandwidth
4. Impedance at f$\rightarrow\infty$

For a capacitance of 80 pF and an inductance 150 nH, what is the resonance frequency?

Question 17

The impedance of a resonant circuit is a function of frequency. For a given resonator, the impedance has been measured at 7 different frequencies (f_1 to f_7 — figure 3). The result is plotted in the complex Z-plane.

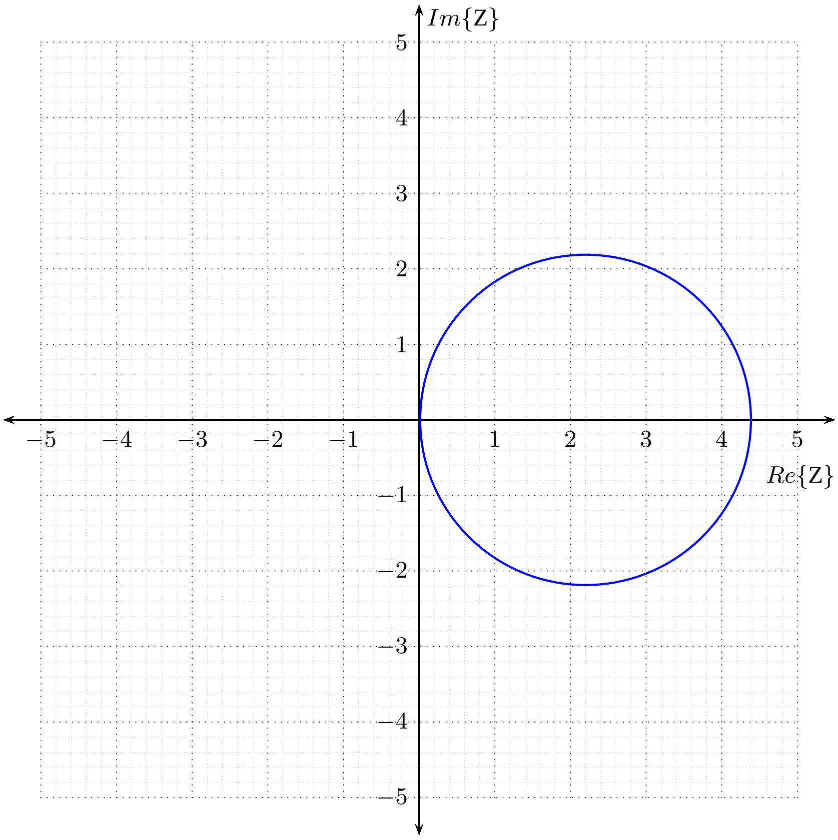


Fig. 2: Reonant RLC circuit in the impedance plane. (Question 16)

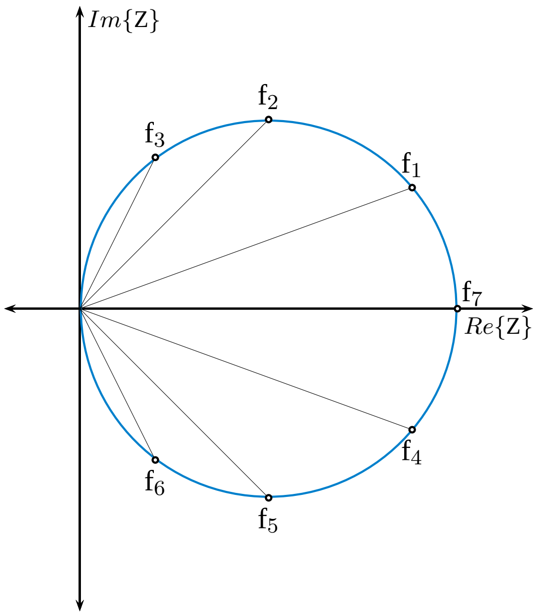


Fig. 3: Different measured frequency points of a resonant circuit in the impedance plane. (Question 17)

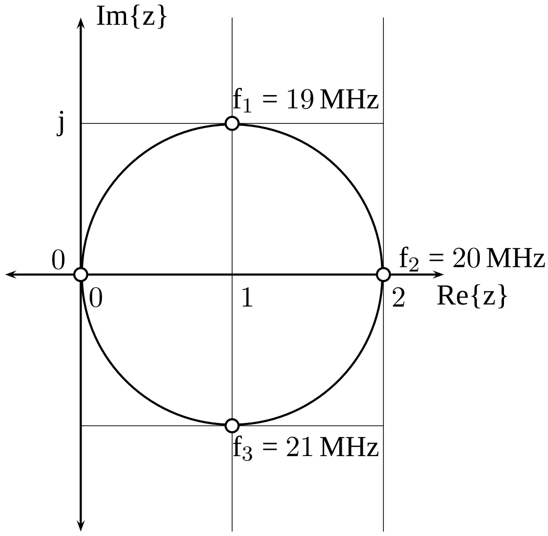
	f [MHz]	Z [kΩ]
f_1	105.11	$200.0\ e^{j30°}$
f_2	105.05	$162.6\ e^{j45°}$
f_3	104.94	$115.0\ e^{j60°}$
f_4	105.29	$200.0\ e^{-j30°}$
f_5	105.35	$162.6\ e^{-j45°}$
f_6	105.46	$115.0\ e^{-j60°}$
f_7	105.20	$230.0\ e^{j0°}$

1. Determine the resonance frequency.
2. Determine the 3 dB bandwidth of this resonator.
3. Draw the equivalent circuit for the measured resonator.
4. Determine R.
5. Draw the locus of admittance of this circuit in the Y-plane and indicate the lower and upper 3 dB point.
6. Determine the Q value as well as L and C for this circuit.

Question 18

The locus of inpedance of a parallel RLC resonant circuit is given in the complex z-plane (z-plane = normalized Z-plane, normalization to 50 Ohm; $z = Z/50$ Ohm).

1. Transform this locus of impedance into the Smith Chart.
2. Mark the resonance frequency both in the z-plane and in the Smith Chart.
3. Mark the -3 dB points (for unloaded Q) both in the z-plane and in the Smith Chart.



Question 19

1. Plot the following **normalized** impedances z in the Smith chart:

Point	z
A	0.6 + j 0
B	0.6 – j 0.6
C	0.6 – j 0.8
D	0.6 – j 1.0

2. Plot the following impedances Z in the Smith chart:

Point	Z
E	50 + j 0
F	20 – j 15
G	10 + j 25
H	0 – j 150

Question 20

1. Mark the reflection factors Γ of points A to F in the Smith chart and find approximative values for the corresponding (normalized) impedances z.

Point	Reflection factor Γ	(Normalized) Impedance z
A	$1\angle 0°$	
B	$1\angle 45°$	
C	$1\angle 90°$	
D	$1\angle 180°$	
E	$1\angle -90°$	
F	0.5	

2. Mark the following (normalized) impedances z in the chart:

Point	(Normalized) Impedance z
G	0 + j 2
H	1.00 + j 1.73
I	1.41 + j 1.41
J	1.73 + j 1.00
K	2.00 + j 0
L	0.30 + j 0.40

3. Sketch in the chart the locus of Re(z) = 0.5.
4. Sketch in the chart the locus of Im(z) = 2.