EXERCISES FOR THE COURSE ON LONGITUDINAL BEAM DYNAMICS (JUAS 2016)

1) Show that we can write $B\rho\left[\mathrm{T\,m}\right] = 3.333\, p\left[\mathrm{GeV/c}\right]$.

2) Complete the table below for some CERN machines. Which formulae do we need?

Machine	Particle	E_0 (GeV)	E_k (GeV)	E (GeV)	γ	β	p [GeV/c]	B ρ [T m]
LHC (inj.)	p^+				479.6			
LHC (coll.)	p^+			7000				
LEP (coll.)	e^+ / e^-			100				
SPS (inj.)	p^+						26	
PS (inj.)	p^+		1.4					

LHC: Large Hadron Collider
LEP: Large Electron Positron (previous collider, in the same tunnel as LHC)
SPS: Super Proton Synchrotron
PS: Proton Synchrotron

3) Derive all the equations of page 18 of the course.

4) Complete the table below for some CERN machines. Which formulae do we need?

Machine	γ_{tr}	E_{tr} (GeV)	α_p	η (inj.)	η (top energy)
LHC	55.68				
SPS			0.00192		
PS			0.027		

5) Some cosmic ray protons enter the top of the atmosphere with an energy of 1 Joule. Calculate the relativistic velocity factor β and the kinetic energy E_k.

6) No mention is made of gravity in the course as a force to be taken into account. Knowing that the earth's magnetic field is approximately 1 Gauss (i.e. 10^{-4} T), calculate the speed that a proton and an electron would have with a magnetic force equal to the gravitational one.

7) The total energy is the sum of the rest energy and kinetic energy. Establish the kinetic expression for a non-relativistic particle ($\beta << 1$) and calculate β for an accelerating voltage of 5 kV.

8) In the PS machine, antiprotons were injected in the past at 3.5 GeV/c. The magnetic and frequency variations were respectively $dB/B = 3\ 10^{-4}$ and $df/f = 10^{-4}$.

a) Calculate the momentum variation dp/p.

b) A fixed oscillator is installed and imposes df = 0. Calculate the new dp/p. What is the conclusion?

c) Calculate dR/R under these new conditions.

9) Measurement of γ_{tr}.

a) Write the expression giving γ_{tr} as a function of dB/B and df/f in the old antiproton accumulator, when the RF voltage was switched off.

b) A variation of $dB/B = 10^{-2}$ induced a variation $df/f = 1.75\ 10^{-3}$. Calculate γ_{tr}.

10) Show that the "classical" debunching time (defined as the drift time required for the beginning of overlap in azimuth of particles from adjacent bunches) is given by $t_{deb} = (\pi - \Delta\phi)/(h\,\omega_0\,|\eta|\,\Delta p/p)$, where ω_0 is the angular revolution frequency, $\Delta\phi$ the half full spread in phase and $\Delta p/p$ the half full spread in momentum. What is the numerical value for the LHC at injection energy, transition energy and collision energy?

11) See exercise of page 54 of the course.

12) The derivation of the 1st equation of page 37 (as given in the reference paper http://cdsweb.cern.ch/record/446961/files/ps-2000-008.pdf in pages 37 and 38) can be misleading. Another way to derive it is to assume that, to first order, we have $dp/p = a\, dR/R + b\, dB/B$, with a and b to be determined (remembering that the momentum compaction factor was defined for a constant magnetic field). After having found a and b, deduce that $dR/R = \alpha_p\, d\rho/\rho$.

13) See exercise of page 86 of the course.

14) General questions on the course:

1) Is this course (Longitudinal Beam Dynamics) a single-particle or multi-particle (i.e. that takes into account the effect of particles on themselves) dynamics course?

2) What are the values of the relative momentum spread $\Delta p / p_0$ and the RF phase ϕ for the synchronous particle in the absence of acceleration and deceleration (below transition energy, where $\eta > 0$)? Where is this particle located in the stationary bucket? What is the energy gain of this particle at each passage through the RF cavity? What are the values of $\Delta p / p_0$ and ϕ for the synchronous particle in the presence of acceleration (below transition energy)?

3) What does longitudinal matching mean? What happens in the longitudinal phase space if there is a longitudinal mismatch in phase or in amplitude?

4) Assuming that the longitudinal emittance is conserved, how does the bunch length vary if the peak RF voltage is increased?

5) What is the maximum value of the longitudinal emittance (give the name)? For which value of the synchronous phase is it maximum? What is the name of the limit between the stable and unstable regions, in the longitudinal phase space? Assuming that acceleration of protons is performed for a synchronous phase of 50° below transition, what do the RF system have to do above transition? Why?

6) What is the main difference between a synchrotron and a cyclotron, as concerns the acceleration?

7) In a synchrotron, a proton beam is injected and captured on a given harmonic number. What can we do if we want then to change the harmonic number without beam loss?

8) What is the main difference between electrons and protons, in the design of accelerators? Do we need a RF system in a high-energy accumulator of electrons? Why? Compare with protons.

9) What is the negative mass effect? Where does it appear (below, at or above transition energy)?

10) What does the transit time factor represent? Is it smaller or greater than 1? Is it more important for linacs or circular machines?

11) What is the difference between a circular machine and a linac, as concerns the RF system?

12) What is the main difference between a linac and a synchrotron, as concerns the longitudinal motion of particles? At transition energy in synchrotrons, the longitudinal motion of particles is similar to the longitudinal motion of particles in linacs. Why?

13) In synchrotron machines, the RF cavities are installed, in general, at location where the horizontal dispersion function is zero or close to zero. Why? How can we measure the dispersion at a beam position monitor?