Exercises on Space Charge

Exercise 1

Compute the transverse space charge forces and the incoherent tune shifts for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian.

Evaluate also the tune spread (max tune shift – min tune shift) produced by the space charge forces with the same distributions.

parabolic $$\lambda(z) = \frac{3Ne}{2l_o}\left[1-\left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation $$\lambda(z) = \lambda_o + \Delta\lambda\cos(k_z z) \quad ; \quad k_z = 2\pi/\lambda_w$$

Gaussian $$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

Exercise 2

Compute the transverse space charge force and the incoherent tune shift for a cylindrical beam in a circular beam pipe, having a bi-Gaussian longitudinal and transverse distribution.

$bi-Gaussian$

$$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$\rho(r,z) = \frac{\lambda(z)}{2\pi\sigma_r^2}\exp\left(\frac{-r^2}{2\sigma_r^2}\right)$$

Exercise 3

Evaluate the dependence of the longitudinal and transverse space charge force with z at fixed r (e.g. << σ_r) for the bi-Gaussian distribution

Exercise 4

Compute the longitudinal space charge force of a transverse uniform cylindrical beam in a circular perfectly conducting beam pipe

$$E_z(r,z) = -\frac{1}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r,z)dr \quad \Longrightarrow \quad F_z(r,z) = -\frac{e}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r',z)dr'$$

Exercise 5

Compute the longitudinal space charge forces for a cylindrical beam in a circular beam pipe, having the following longitudinal distributions: parabolic, sinusoidal modulation, Gaussian

parabolic $$\lambda(z) = \frac{3Ne}{2l_o}\left[1 - \left(\frac{2z}{l_o}\right)^2\right]$$

sinusoidal modulation $$\lambda(z) = \lambda_o + \Delta\lambda \cos(k_z z) \quad ; \quad k_z = 2\pi / l_w$$

Gaussian $$\lambda(z) = \frac{Ne}{\sqrt{2\pi}\sigma_z}\exp\left(-\frac{z^2}{2\sigma_z^2}\right)$$

$$F_z(r,z) = -\frac{e}{\gamma^2}\frac{\partial}{\partial z}\int_r^b E_r(r',z)dr'$$

$$F_z(r,z) = -\frac{e}{4\pi\varepsilon_0\gamma^2}\left(1 - \frac{r^2}{a^2} + 2\ln\frac{b}{a}\right)\frac{\partial\lambda(z)}{\partial z}$$

Exercise 6

Compute the incoherent betatron tune shift of a uniform proton beam inside two perfectly conducting parallel plates