Lecture 3: Magnets training and stability
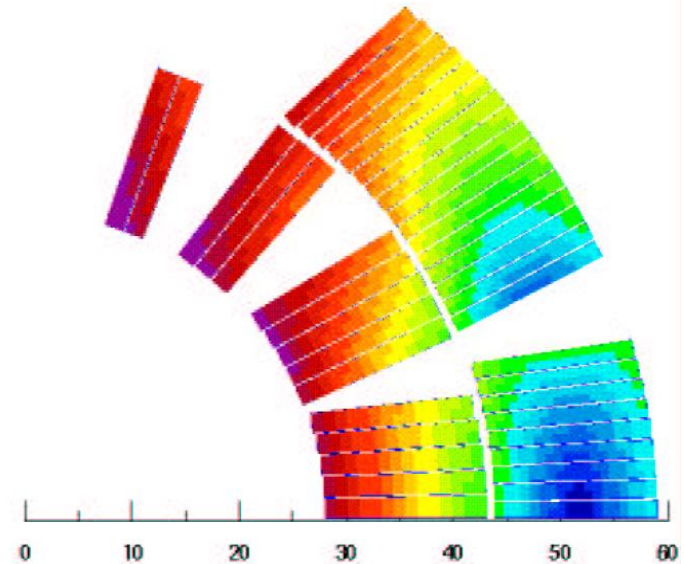
Magnets

- magnetic fields above 2 Tesla

- coil shapes for solenoids, dipoles and quadrupoles

- engineering current density

- load lines

Degradation & Training

- causes of training - release of energy within the magnet

- reducing training - stability and minimum quench energy MQE
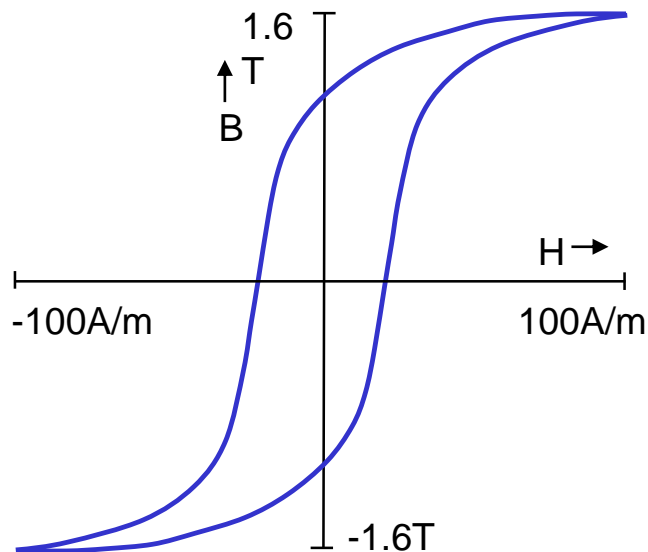
- need copper and fine filaments for low MQE

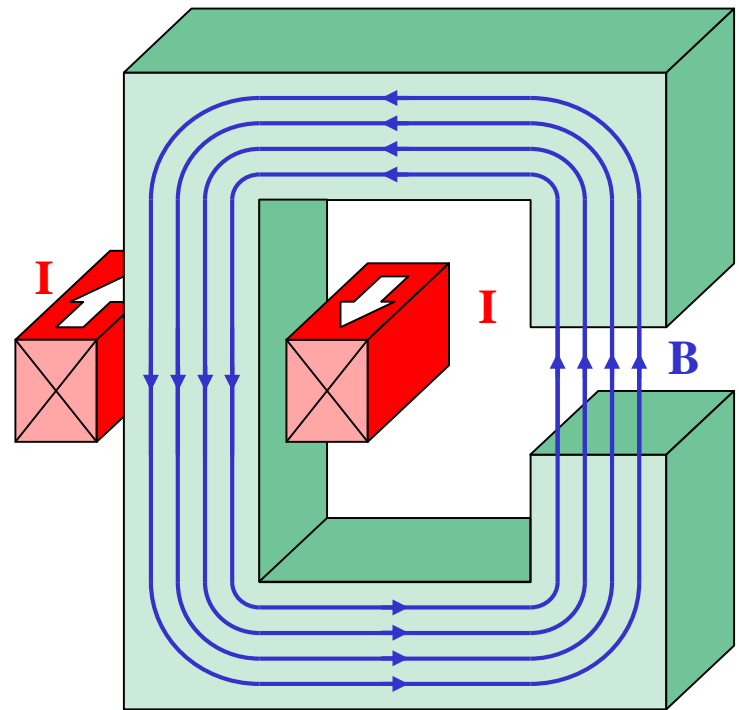## Flux Jumping

- need for fine filaments

Fields and ways to create them: conventional

- conventional electromagnets have an iron yoke
  - reduces magnetic reluctance
  - reduces ampere turns required
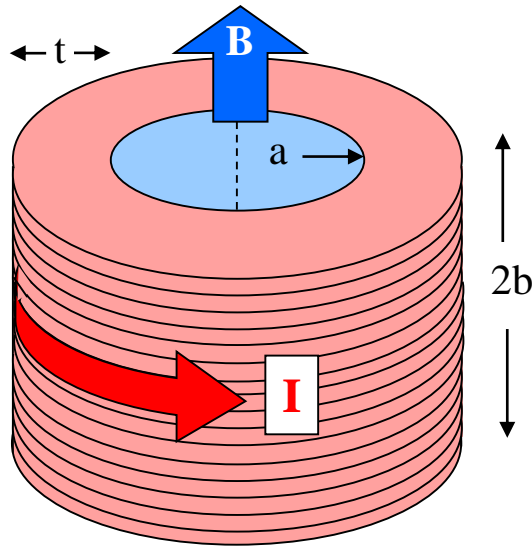 - reduces power consumption
- iron guides and shapes the field

**Iron electromagnet
– for accelerators, motors,
transformers, generators etc**

BUT iron saturates at ~ 2T

for higher fields we cannot rely on iron
field must be created and shaped by the winding

Solenoids

- no iron - field shape depends only on the winding

- azimuthal current flow, eg wire wound on bobbin, axial field

- the field produced by an infinitely long solenoid is

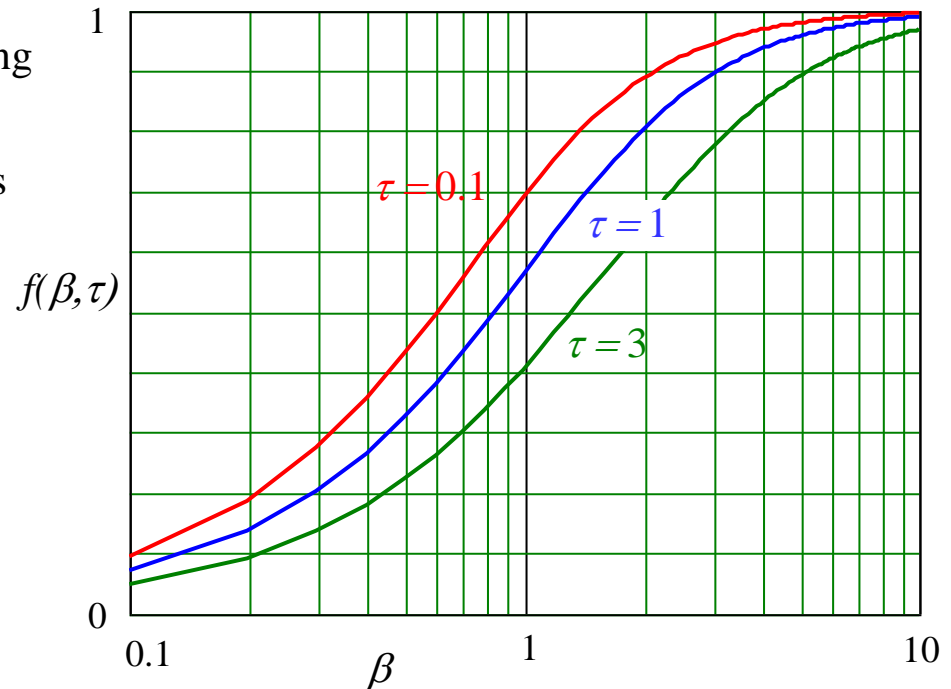$$B = \mu_o \, NI = \mu_o \, J_e \, t$$

where N = number of turns/unit length, I = current , J_e = engineering current density

- so high $J_e \Rightarrow$ thin compact economical winding

- in solenoids of finite length the central field is

$$B = \mu_o J_e \, t \, f(\beta, \tau)$$

where $\beta = b/a$ $\tau = t/a$

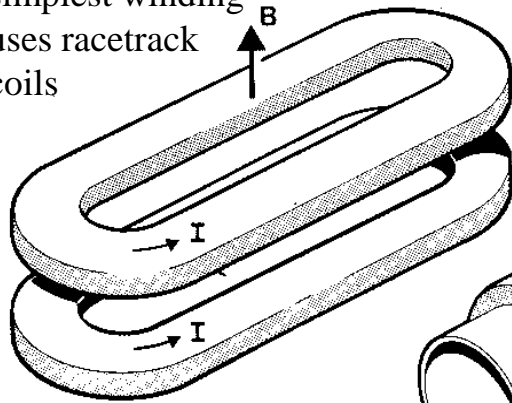- field uniformity and the ratio of peak field to central field get worse in short fat solenoids

Superconducting solenoids

Delphi solenoid for HEP experiments at CERN
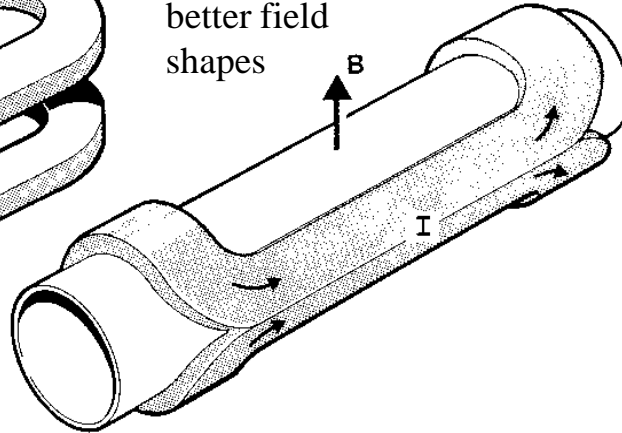1.2T 5.5m dia 6.8m long 110MJ

Superconducting solenoid for research

Accelerators need transverse fields

simplest winding
uses racetrack
coils

'saddle' coils make
better field
shapes

- some iron - but field shape is set mainly by the winding
- used when the long dimension is transverse to the field, eg accelerator magnets
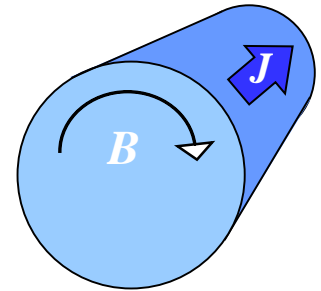- known as *dipole* magnets (because the iron version has 2 poles)

special winding cross
sections for good
uniformity
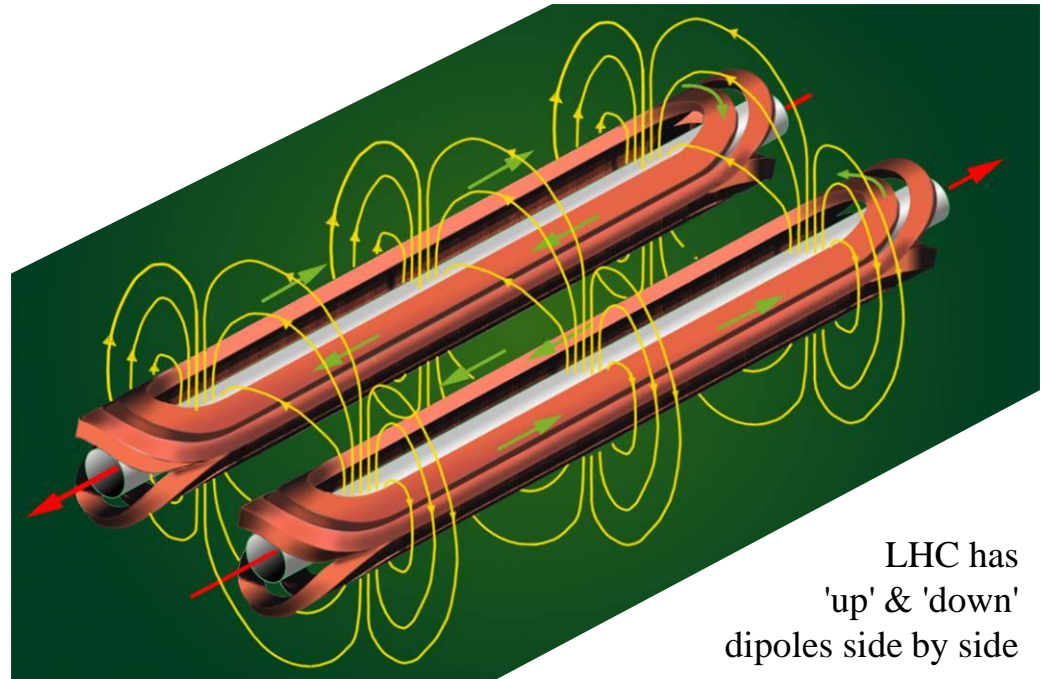
LHC has
'up' & 'down'
dipoles side by side

Dipole field from overlapping cylinders

Ampere's law for the field inside a cylinder carrying uniform current density

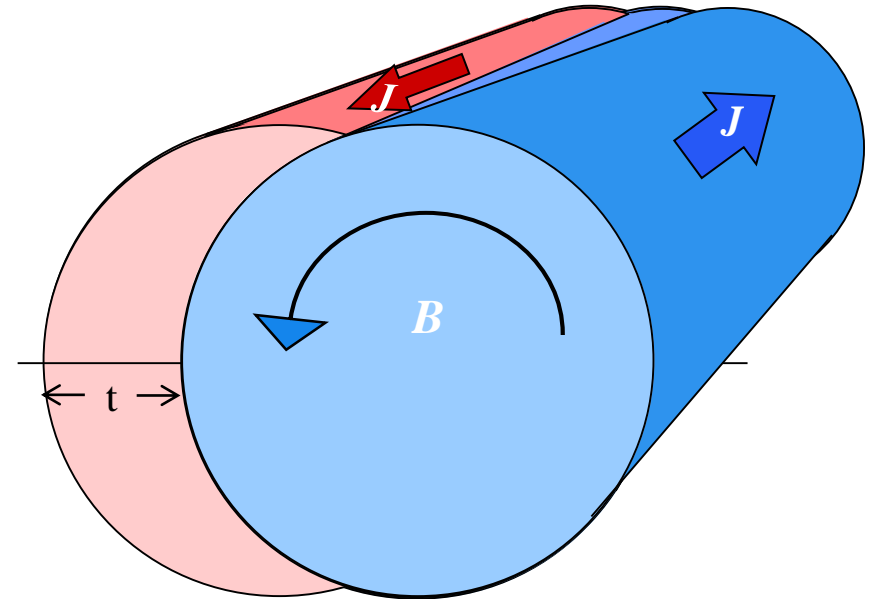$$\oint B.ds = 2\pi r B = \mu_o I = \mu_o \pi r^2 J \qquad\qquad B = \frac{\mu_o J r}{2}$$

- two cylinders with opposite currents

- push them together

- currents cancel where they overlap $\Rightarrow$ aperture

- fields in the aperture:

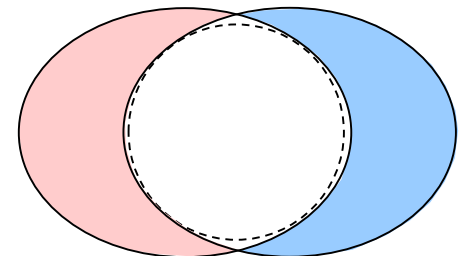$$B_y = \frac{\mu_o J}{2}\left(-r_1 cos\theta_1 + r_2 cos\theta_2\right) = \frac{-\mu_o J t}{2}$$

$$B_x = \frac{\mu_o J}{2}\left(-r_1 sin\theta_1 + r_2 sin\theta_2\right) = 0$$

- thus the overlapping cylinders give a perfect dipole field

$$B_y = \frac{-\mu_o J_e t}{2}$$

- same trick with ellipses

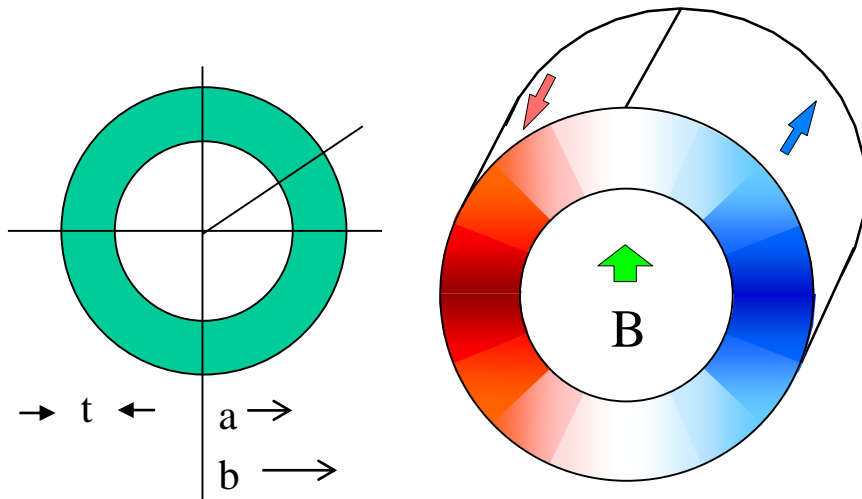- circular aperture

Windings of distributed current density

Analyse thin current sheets flowing on the surface of a cylinder using complex algebra. Let the *linear* current density *(Amps per m of circumference)* be $g_n(\theta) = g_o \cos(n\theta)$ *(Am⁻¹)*

For $n = 1$ we find a pure dipole field inside the cylinder, $n = 2$ gives a quadrupole etc.

Now superpose many cylinders of increasing radius to get a thick walled cylinder carrying an (area) current density *(Am⁻²)* $J_n = J_o \cos(n\theta)$
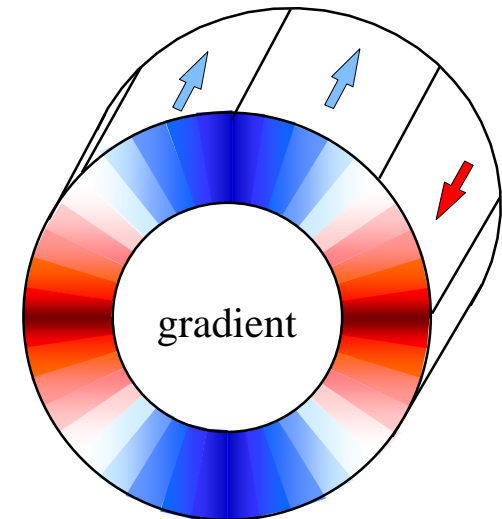
n=1 $J_1(\theta) = J_0 \cos\theta$

$B_x = 0$ $B_y = -\mu_o J_0 (b-a)/2 = -\mu_o J_0 t/2$
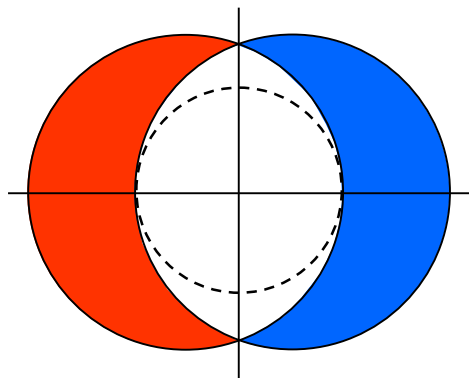
n=2 $J_2(\theta) = J_0 \cos 2\theta$

$B_x = \dfrac{\mu_o J_0}{2} y \ln\left(\dfrac{b}{a}\right)$ $B_y = \dfrac{\mu_o J_0}{2} x \ln\left(\dfrac{b}{a}\right)$

$\rightarrow$ t $\leftarrow$ a $\rightarrow$
b $\longrightarrow$

B

gradient

Summary of simplified dipole windings

Overlapping circles

Overlapping ellipses

Cos (θ)

60° sector

$$B = \frac{\mu_o}{2} J_o t$$

$$B = \mu_o J_o t \alpha$$

$$B = \frac{\mu_o}{2} J_o t$$

$$B = \frac{2\mu_o}{\pi} J_o t \, Sin(60^o)$$

t = winding thickness J_o = *engineering current density*
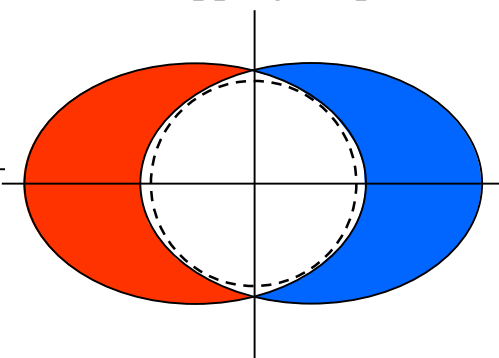
where $\alpha = c/(b+c)$
c = height of ellipse
b = width
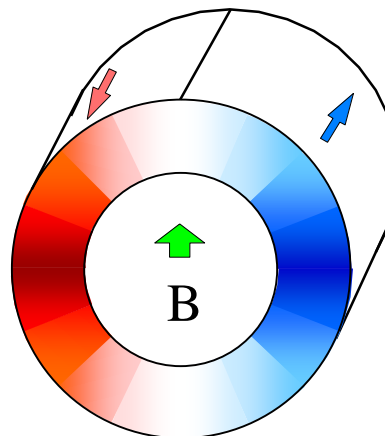
best estimate of forces
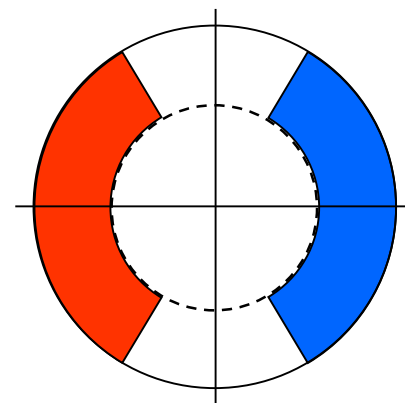
$$\frac{2}{\pi} Sin(60^o) = 0.55$$

best estimate of peak field
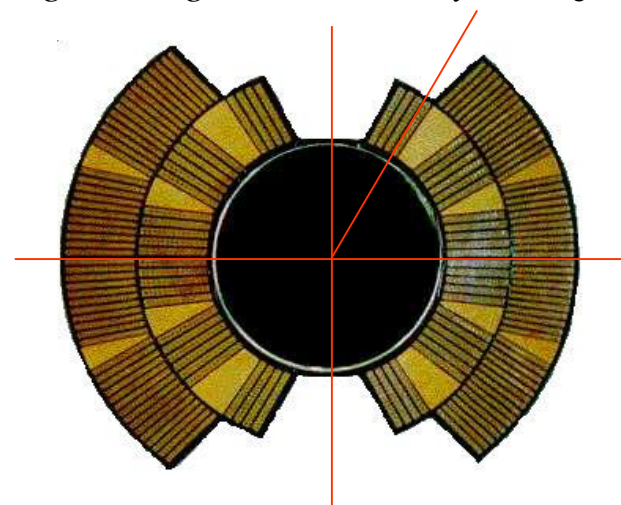
LHC dipole winding
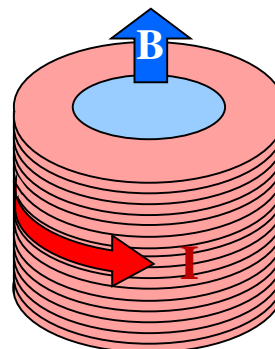
recap solenoid
$$B = \mu_o J_e t \, f(\beta, \tau)$$

Importance of (engineering) current density in dipoles

LHC dipole

$J_e = 375$ Amm^{-2}

120mm

9.5×10^5 Amp turns

$J_e = 37.5$ Amm^{-2}

660mm

9.5×10^6 Amp turns

field produced by a perfect dipole is

$$B = \mu_o \, J_e \, \frac{t}{2}$$

Dipole Magnets
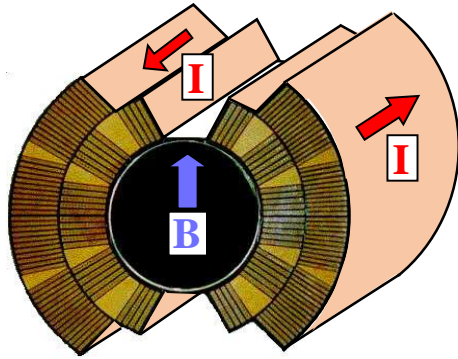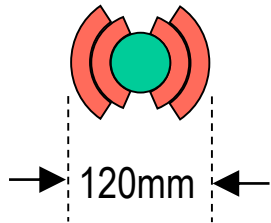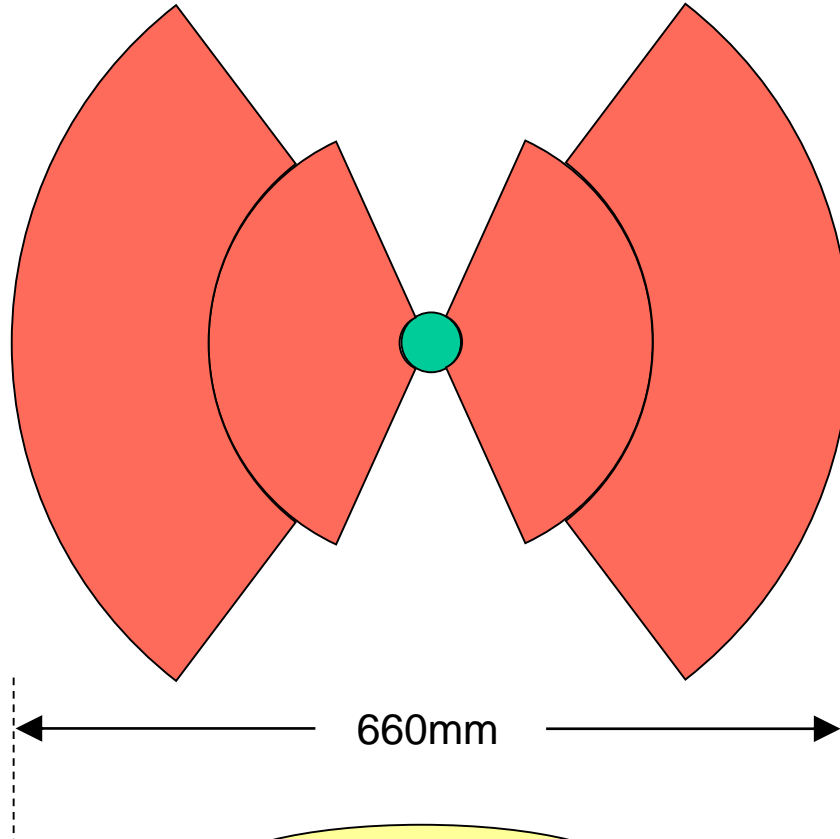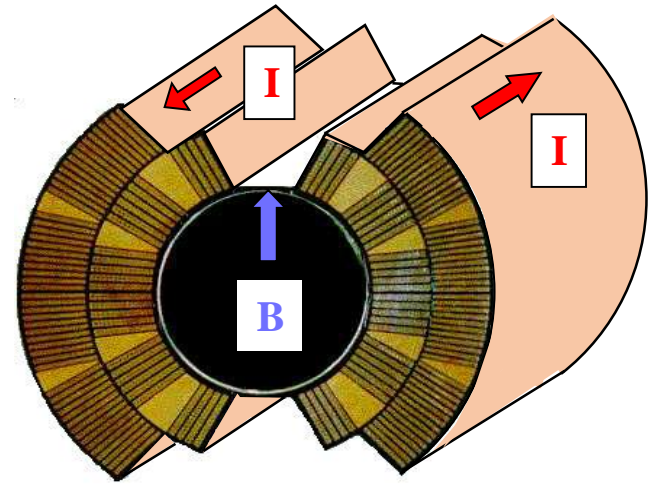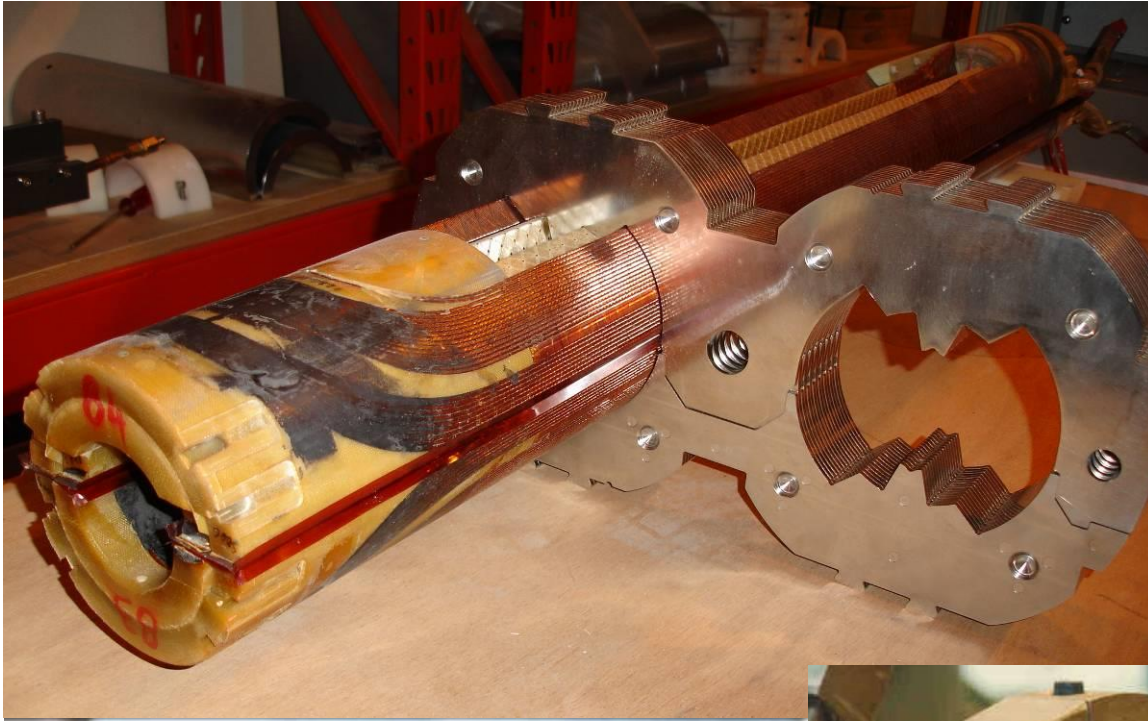
Electromagnetic forces in dipoles

- forces in a dipole are horizontally outwards and vertically towards the median plane

- unlike a solenoid, the bursting forces cannot be supported by tension in the winding

- the outward force must be supported by an external structure

- both forces cause compressive stress and shear in the conductor and insulation

- apart from the ends, there is no tension in the conductor

- simple analysis for thin windings

B_o

F_y F_x

a

F_y F_x
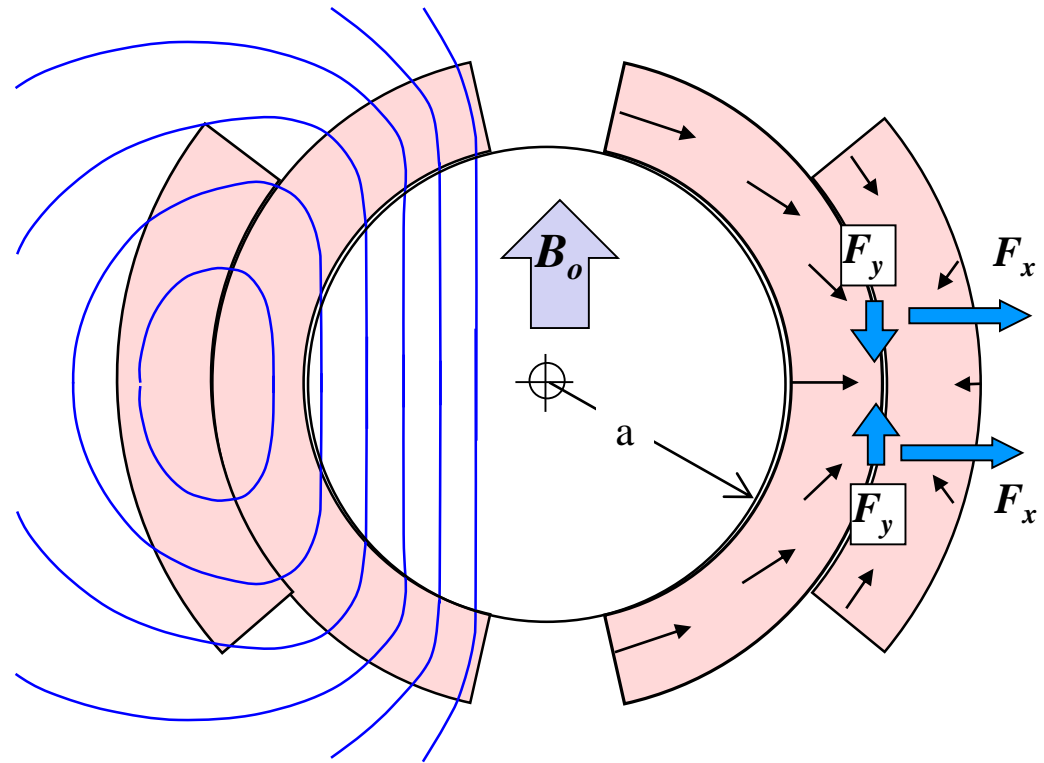
$$F_x = \frac{B_o^2}{2\mu_o} \frac{4a}{3}$$

$$F_y = -\frac{B_o^2}{2\mu_o} \frac{4a}{3}$$

Estimating the iron shield thickness

no iron
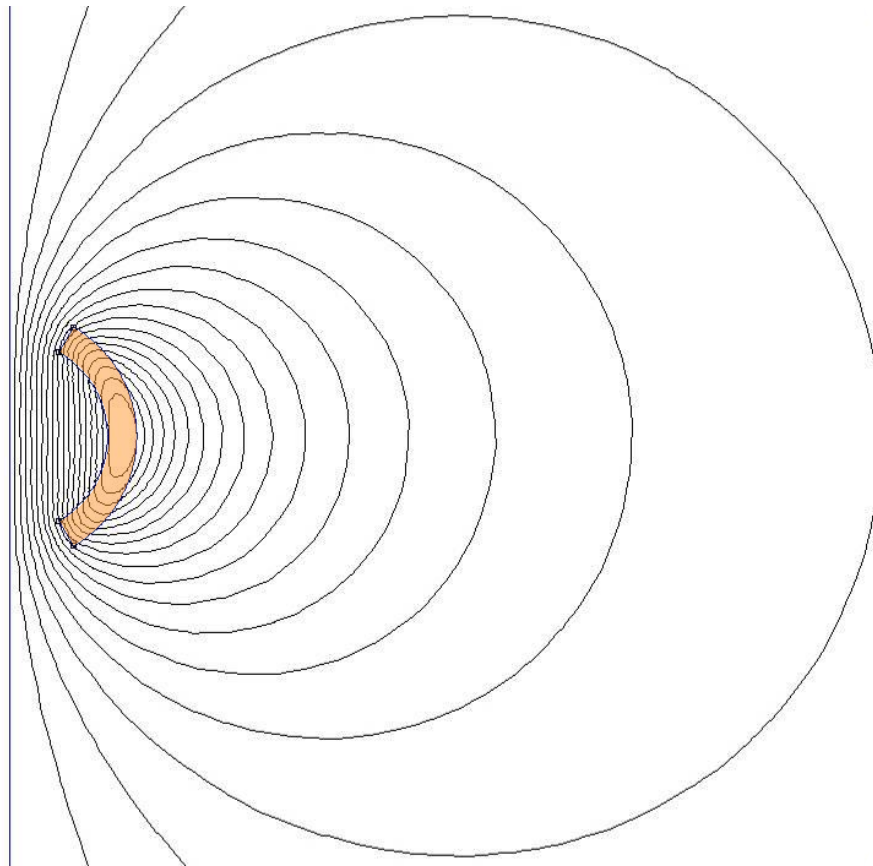
with iron
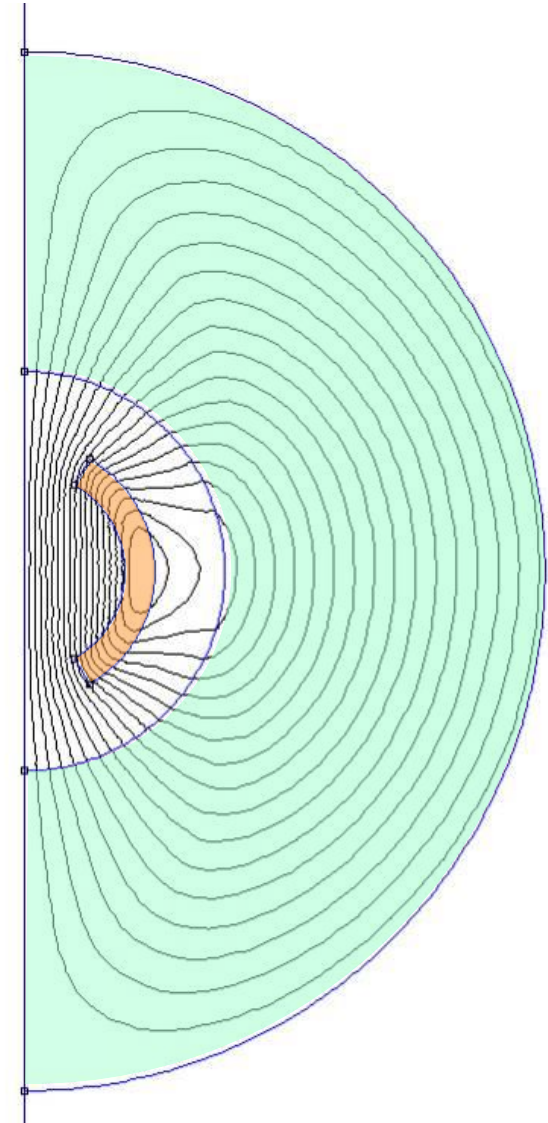
flux through ½ coil aperture $\qquad \phi_c = B_o a$

return flux through iron (one side) $\qquad \phi_i = B_{sat} t$

so $\qquad \boxed{t \sim a\, B_o\, /\, B_{sat}}$ $\qquad a = coil\ radius$
$t = iron\ thickness$

Quadrupole windings
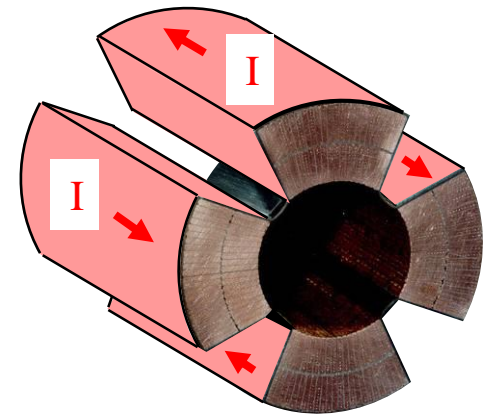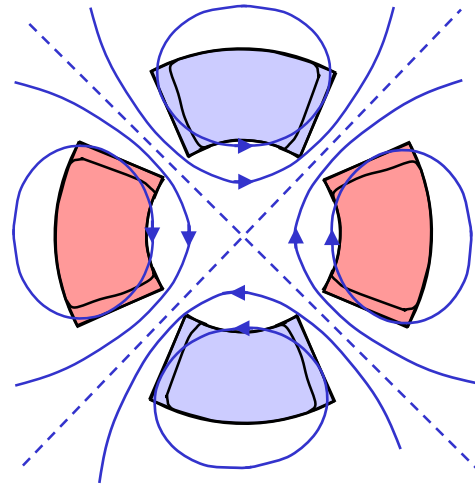
$$B_x(y) = ky \qquad B_y(x) = kx$$
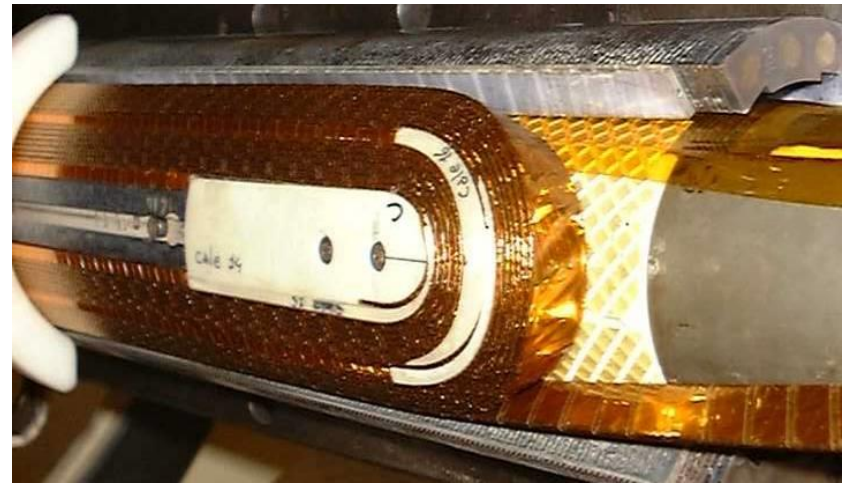
Critical surface and magnet load lines
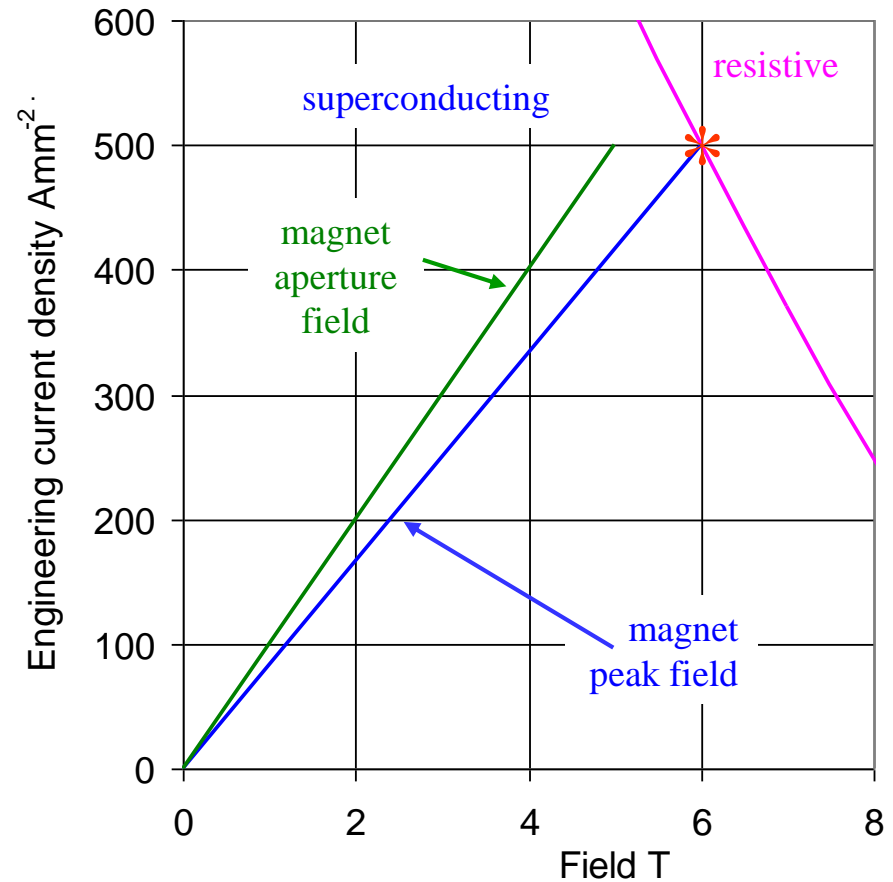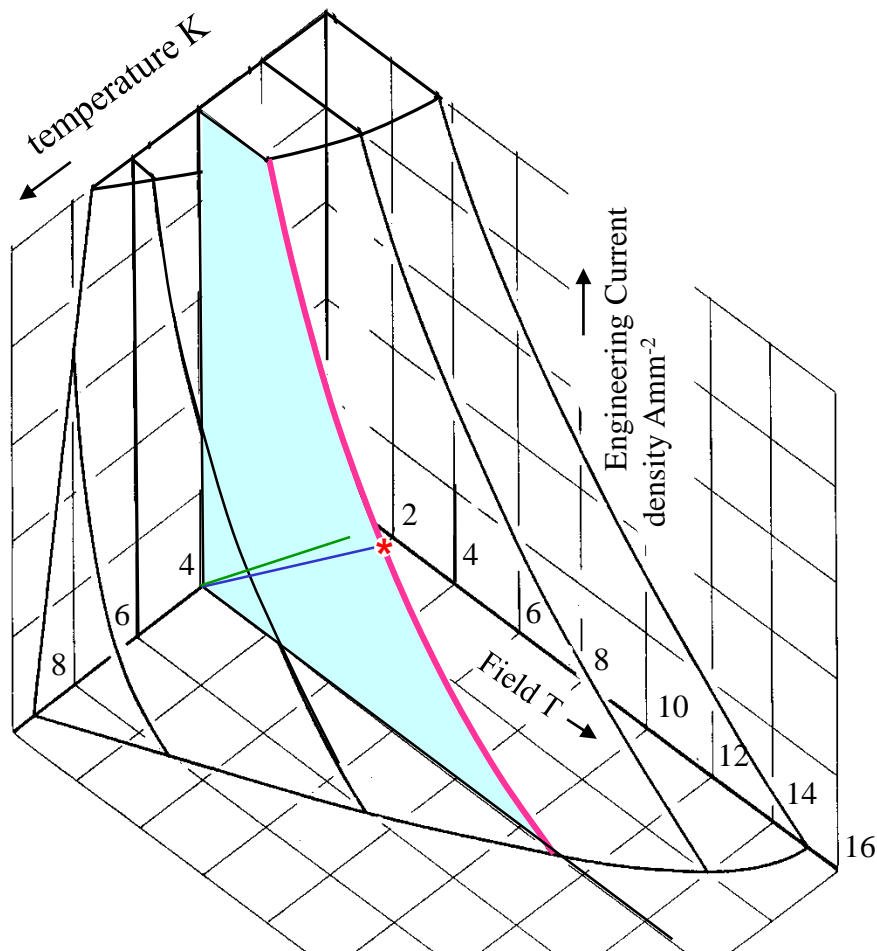
- load line relates magnet field to current

- peak field > aperture (useful) field

- we expect the magnet to go resistive *'quench'* where the peak field load line crosses the critical current line ✳

Degraded performance and 'training' of magnets

- early disappointment for magnet makers when they ramped up the magnet current for the first time

- instead of going up to the critical line, it 'quenched' (went resistive) at less than the expected current

- at the next try it did better

- known as *training*
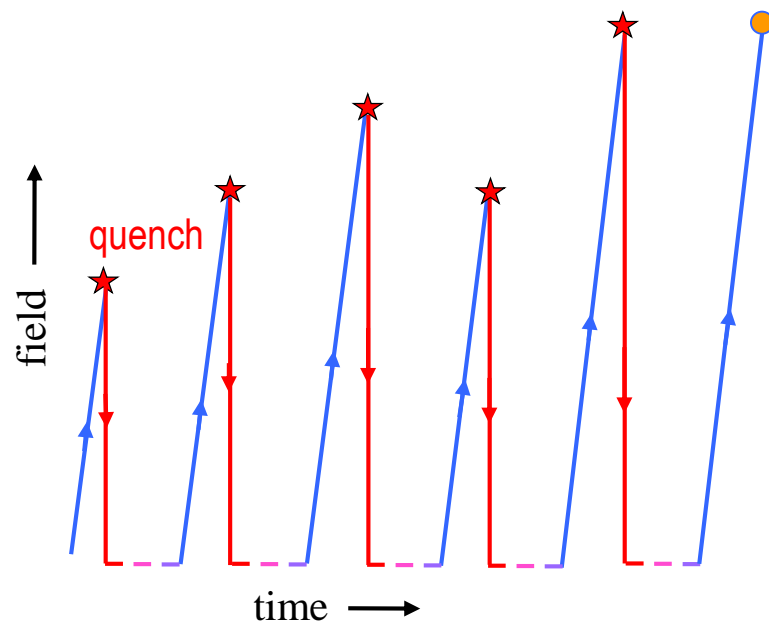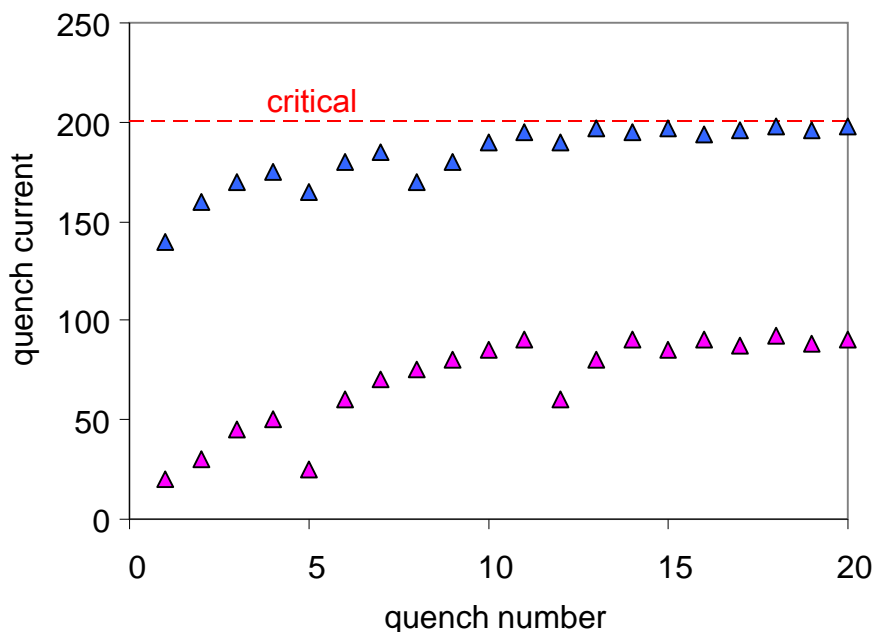
- after a *quench*, the stored energy of the magnet is dissipated in the magnet, raising its temperature way above critical

- you must wait for it to cool down and then try again

- well made magnets ▲ are better than poorly made ▲

'Training' of magnets

- it's better than the old days, but training is still with us

- it seems to be affected by the construction technique of the magnet

- it can be wiped out if the magnet is warmed to room temperature

- 'de-training is the most worrysome feature

operating field

stainless steel collars
stainless steel collars
aluminium collars

field acheived

quench number
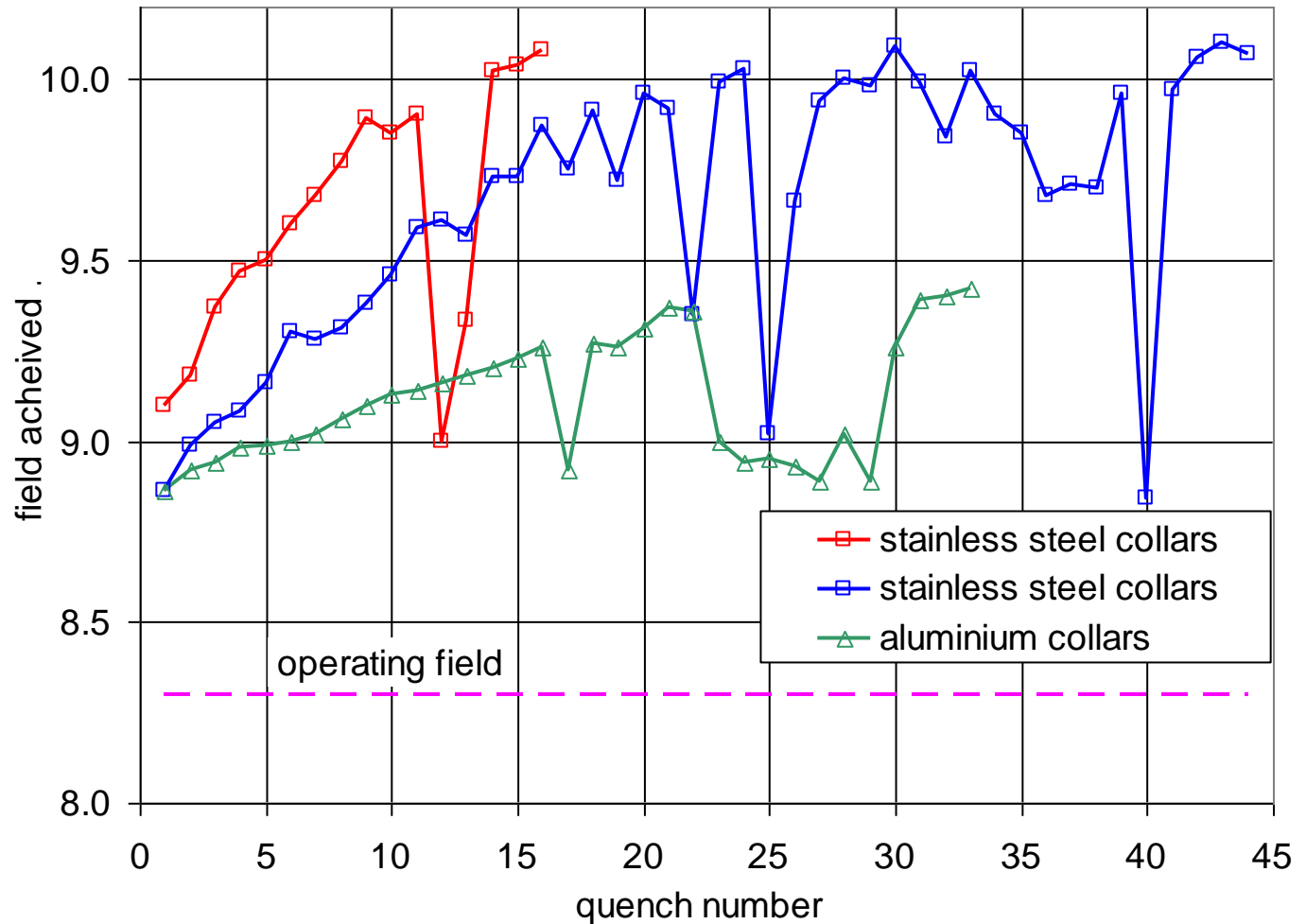
Training of LHC short prototype dipoles *(from A. Siemko)*

Causes of training: (1) low specific heat
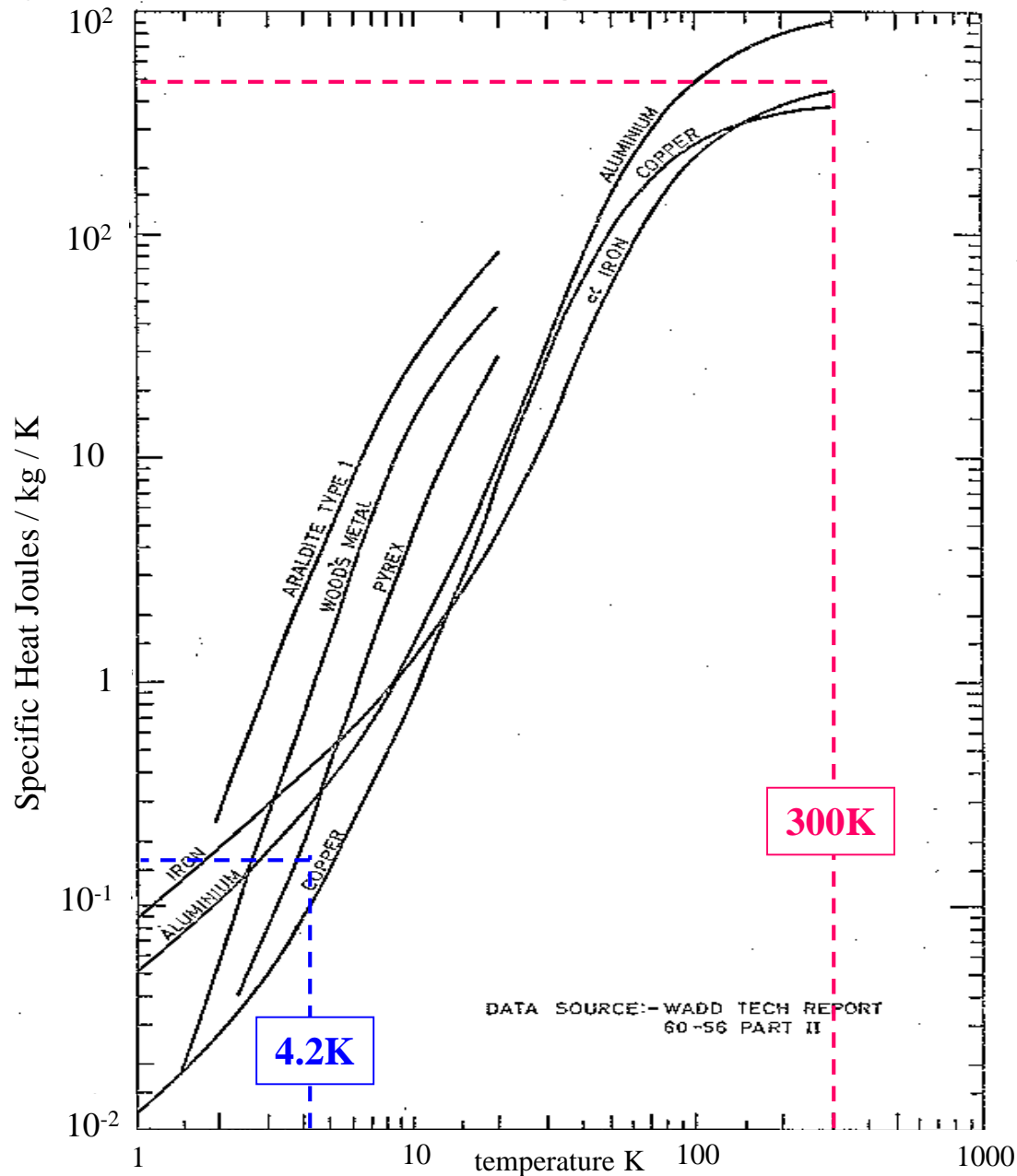
- the specific heat of all substances falls with temperature

- at 4.2K, it is ~2,000 times less than at room temperature

- a given release of energy within the winding thus produce a temperature rise 2,000 times greater than at room temperature

- the smallest energy release can therefore produce catastrophic effects

Causes of training: (2) J_c decreases with temperature
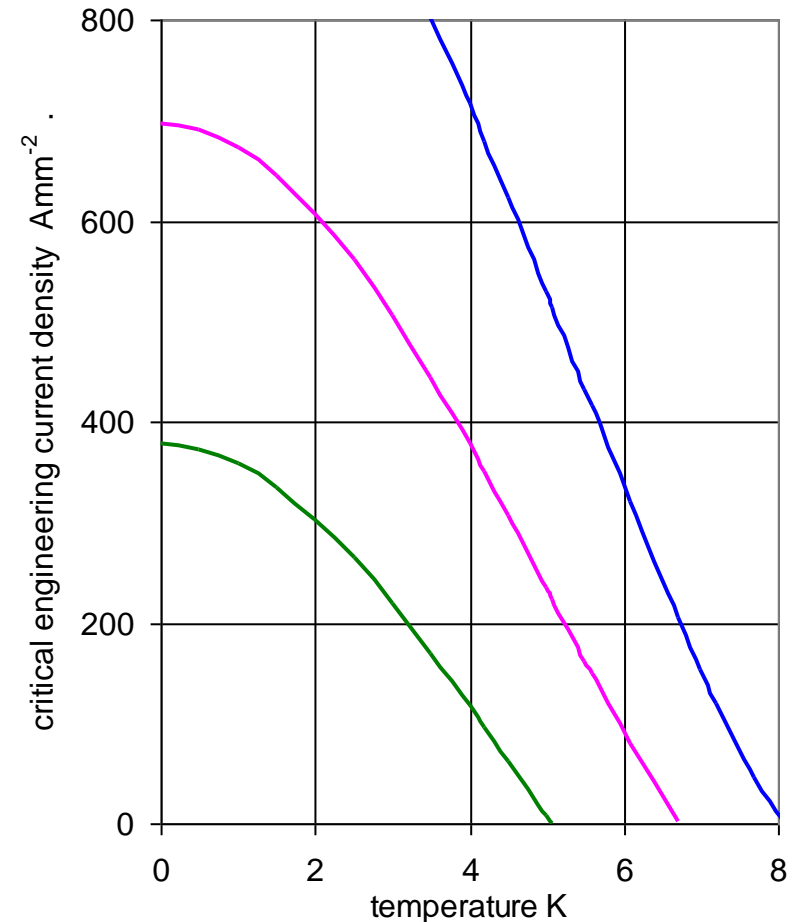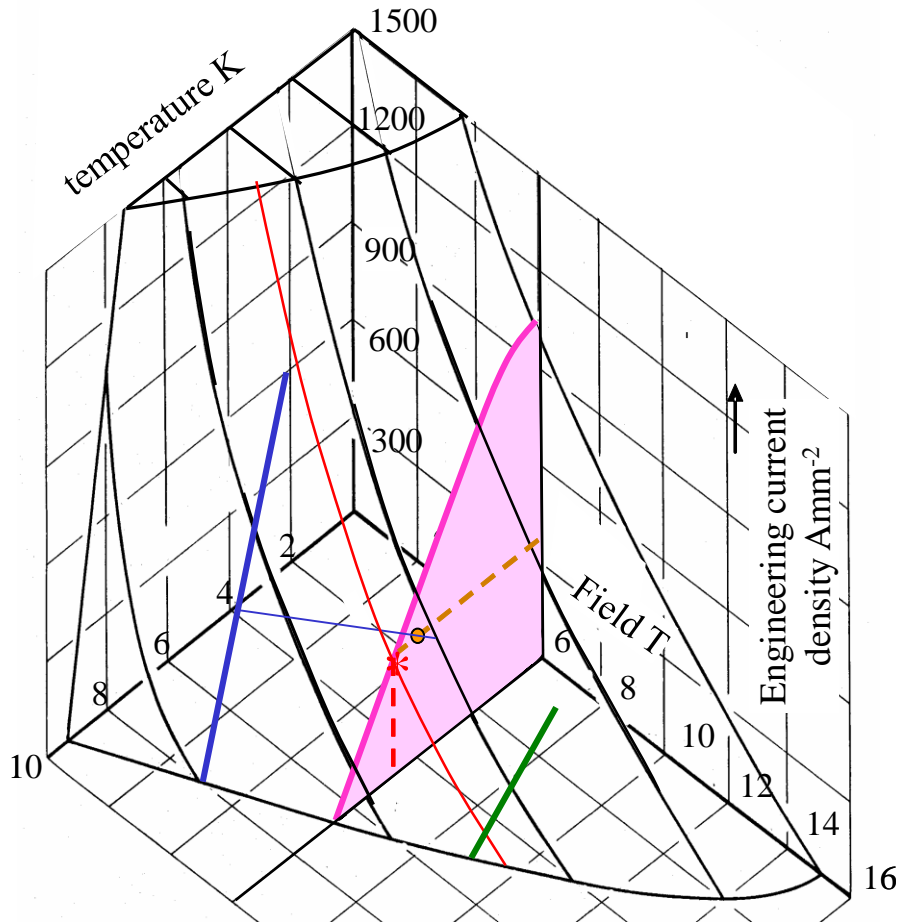
at any given field, the critical current of NbTi falls almost linearly with temperature

- so any temperature rise drives the conductor into the resistive state

Causes of training: (3) conductor motion

- conductors in a magnet are pushed by electromagnetic forces

- sometimes they move suddenly under this force - the magnet 'creaks' as field increases

- large fraction of work done by magnetic field pushing the conductor is released as frictional heating
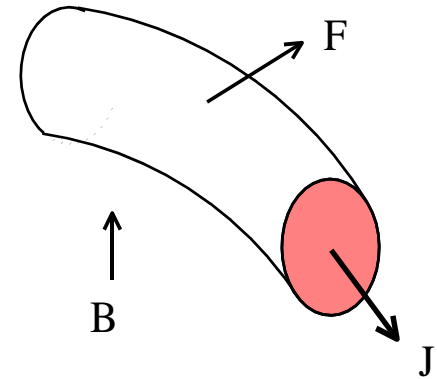
work done per unit length of conductor if it is pushed a distance δz

$$W = F.\delta z = B.I.\delta z$$

frictional heating per unit volume

$$Q = B.J.\delta z$$

typical numbers for NbTi:

$\quad$ B = 5T $\quad$ J_{eng} = 5 x 10^8 A.m^{-2}

$\quad$ so if δ = 10 μm

$\quad$ then Q = 2.5 x 10^4 J.m^{-3}

Starting from 4.2K $\quad$ θ_{final} = 7.5K

can **you** *engineer a winding to better than*

10 μm?

Causes of training: (4) resin cracking

Try to stop wire movement by impregnating the winding with epoxy resin. But resin contracts more than metal, so it goes into tension. Almost all organic materials become brittle at low temperature.

brittleness + tension $\Rightarrow$ cracking $\Rightarrow$ energy release

Calculate strain energy in resin caused by differential thermal contraction

σ = tensile stress   Y = Young's modulus ν = Poisson's ratio

ε = differential strain due to cooling = contraction (resin - metal)

typically: $\varepsilon = (11.5 - 3) \times 10^{-3}$ $Y = 7 \times 10^9$ Pa $\nu = \frac{1}{3}$

uniaxial strain $Q_1 = \dfrac{\sigma^2}{2Y} = \dfrac{Y\varepsilon^2}{2}$ $Q_1 = 2.5 \times 10^5$ J.m^{-3} $\theta_{final} = \mathbf{16K}$

triaxial strain $Q_3 = \dfrac{3\sigma^2(1-2\nu)}{2Y} = \dfrac{3Y\varepsilon^2}{2(1-2\nu)}$ $Q_3 = 2.3 \times 10^6$ J.m^{-3} $\theta_{final} = \mathbf{28K}$
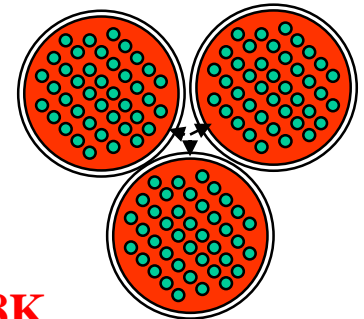
cracking releases most of this stored energy as heat

Interesting fact: magnets impregnated with paraffin wax show almost no training although the wax is full of cracks after cooldown.

Presumably the wax breaks at low σ before it has had chance to store up any strain energy

How to reduce training? – 1) cryostabilization

- conductor with copper joined in parallel with superconductor

- well cooled by liquid helium
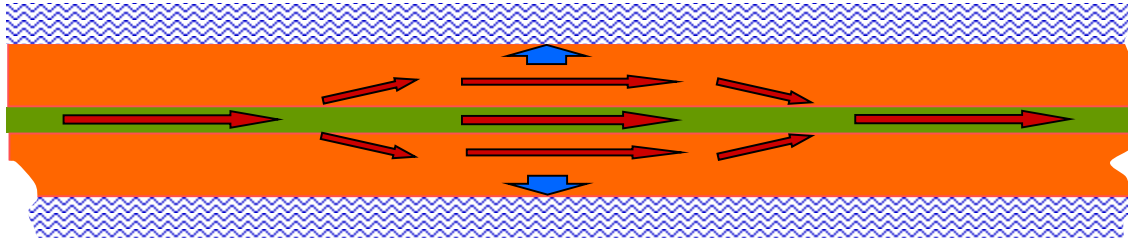
- current normally flows in superconductor

- if superconductor switches off, current diverts to copper

- Ohmic heating in copper

- heat transferred to helium, temperature falls

- current returns to superconductor

made large detector magnets possible

but engineering current density too small for accelerators

How to reduce training?

2) Reduce the disturbances occurring in the magnet winding

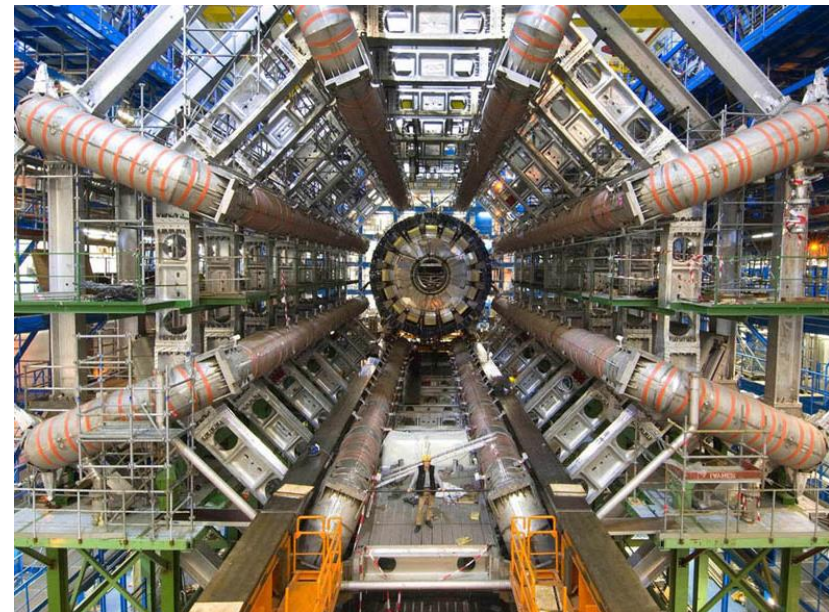- make the winding fit together exactly to reduce movement of conductors under field forces

- pre-compress the winding to reduce movement under field forces

- if using resin, minimize the volume and choose a crack resistant type

- match thermal contractions, eg fill epoxy with mineral or glass fibre

- impregnate with wax - but poor mechanical properties

- most accelerator magnets are insulated using a Kapton film with a very thin adhesive coating on the outer face
  - away from the superconductor

- allows liquid helium to penetrate the cable
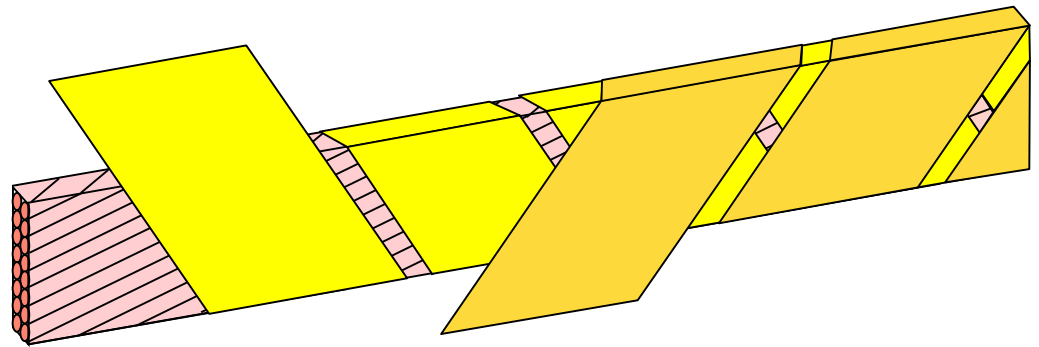
How to reduce training?

3) Make the conductor able to withstand disturbances without quenching

increase the temperature margin
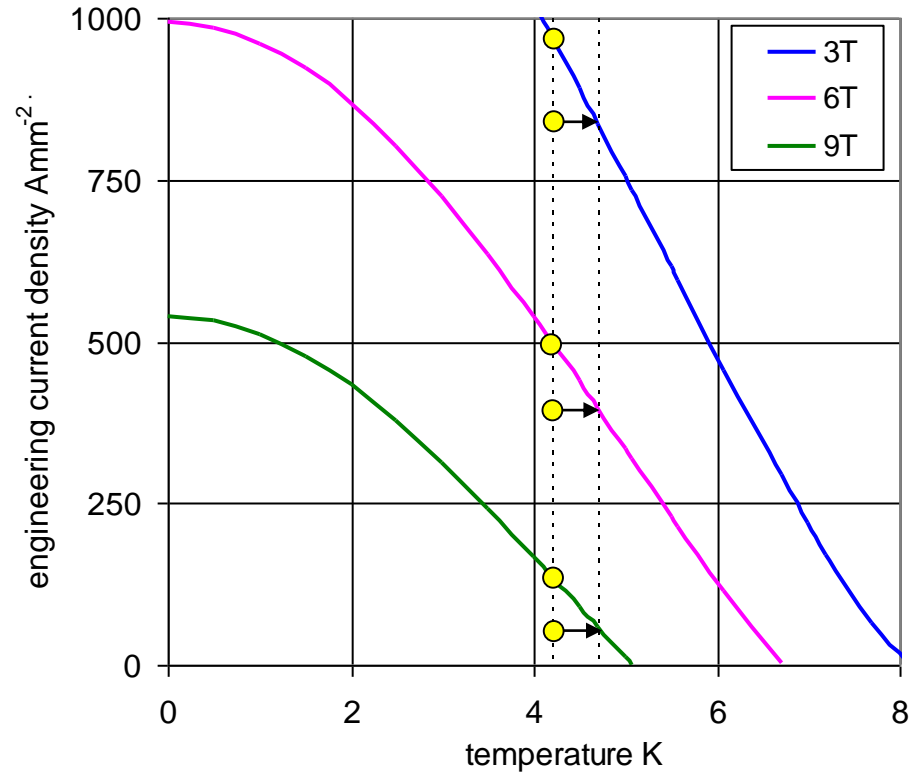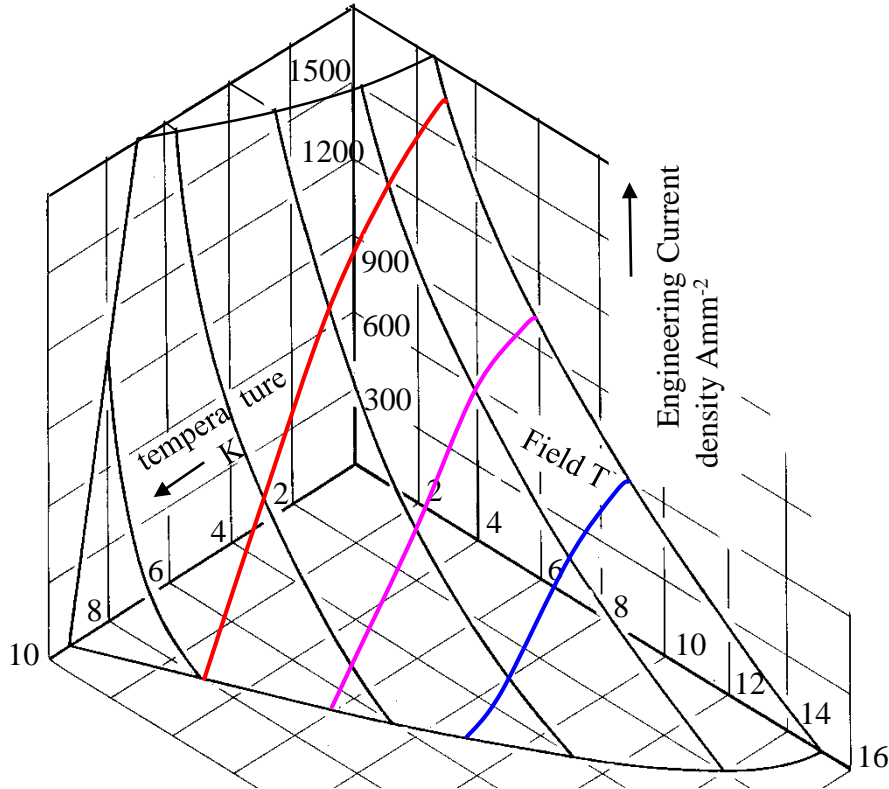
B (T)	$J_{eng}(4.2)$ Amm^{-2}	$J_{eng}(4.7)$ Amm^{-2}	$J_{eng}(4.7) / J_{eng}(4.2)$
3	971	837	**86%**
6	500	397	**79%**
9	136	58	**42%**

gets harder at high fields

How to reduce training?

## 3) Make the conductor able to withstand disturbances without quenching

- increase the temperature margin

- increase the cooling - more cooled surface - better heat transfer - superfluid helium

- increase the specific heat - experiments with Gd_2O_2S $HoCu_2$ etc

- most of this may be characterized by a single number

Minimum Quench Energy MQE

- defined as the energy input at a point in very short time which is just enough to trigger a quench.

- energy input $>$ MQE $\Rightarrow$ quench

- energy input $<$ MQE $\Rightarrow$ recovery

- energy disturbances occur at random as a magnet is ramped up to field

- for good magnet performance we want a high MQE

MQE

energy release

field
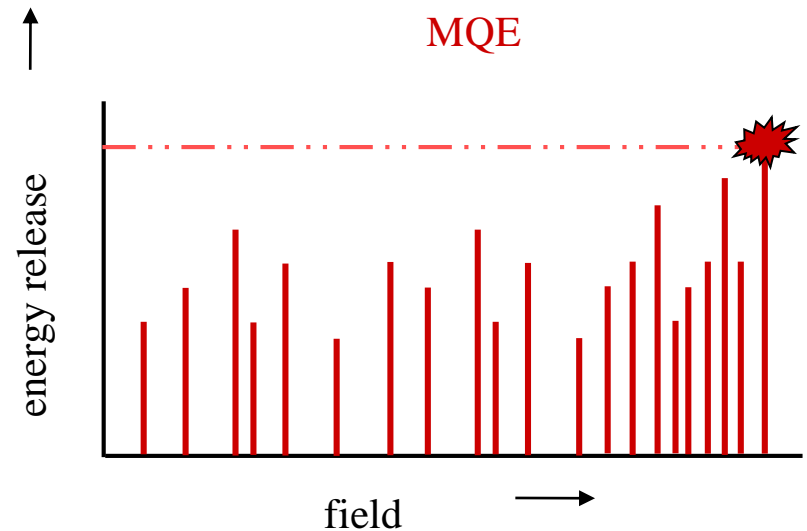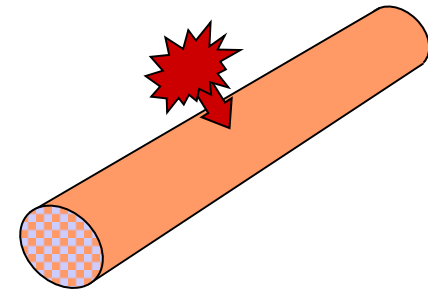
Quench initiation by a disturbance
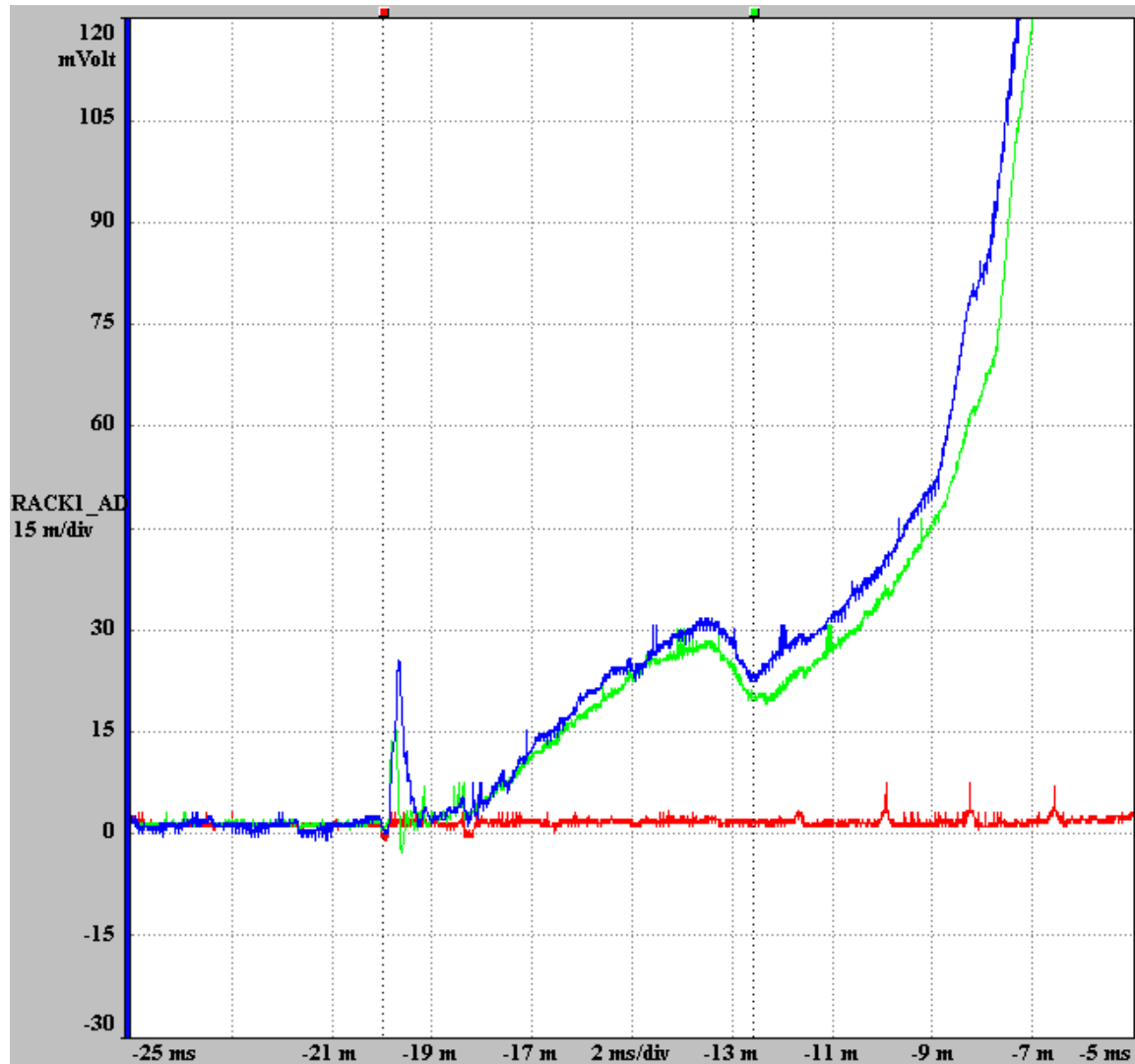
- CERN picture of the internal voltage in an LHC dipole just before a quench

- note the initiating spike - conductor motion?

- after the spike, conductor goes resistive, then it almost recovers

- but then goes on to a full quench

- this disturbance was more than the MQE

Measuring the MQE for a cable

125 μJ

too big!

carbon paste heater

I_{heater}
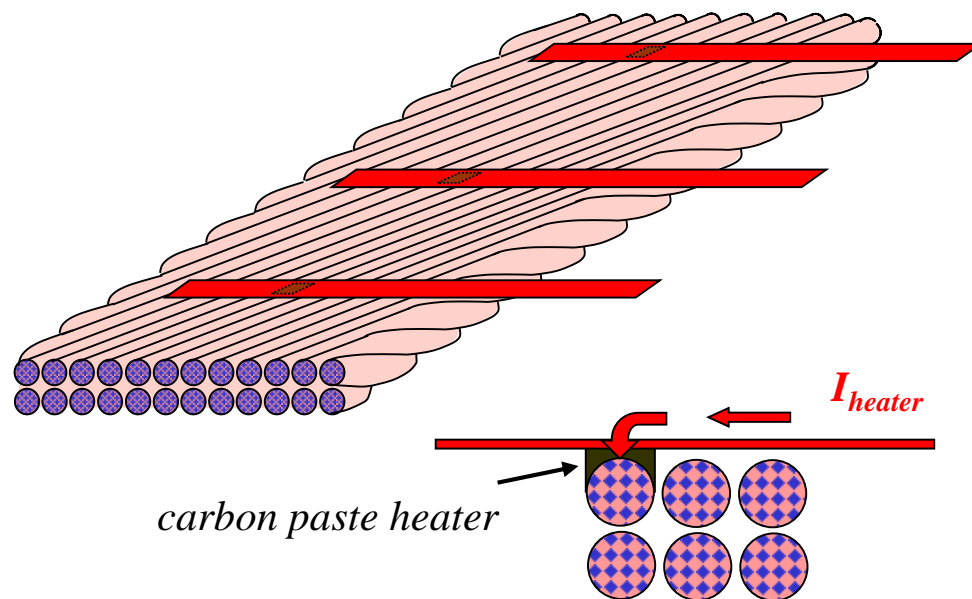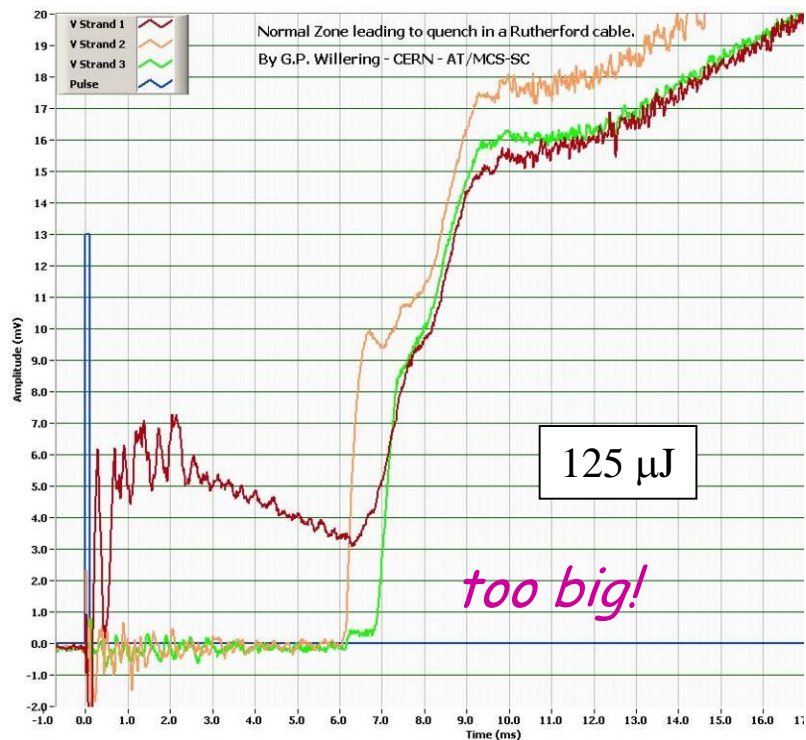
- pass a small pulse of current from the copper foil to the superconducting wire

- generates heat in the carbon paste contact

- how much to quench the cable?

- find the *Minimum Quench Energy MQE*

120 μJ

too small!
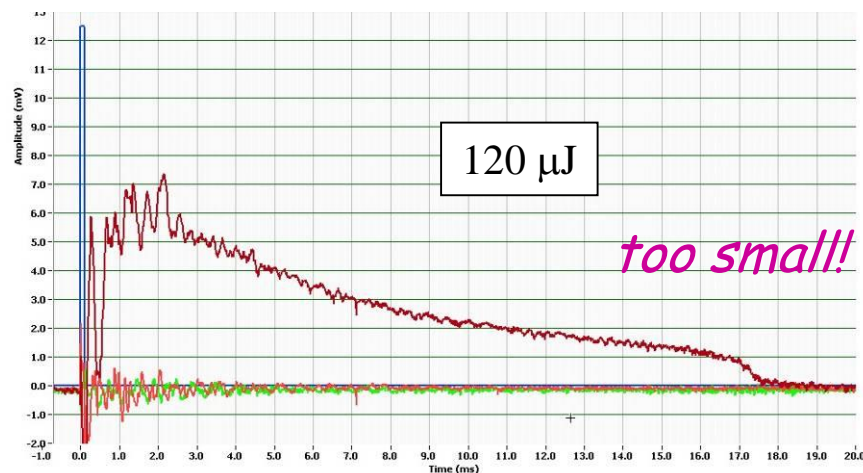
Different cables have different MQEs

- similar cables with different cooling

- better cooling gives higher MQE

- **high MQE is best because it is harder to quench the magnet**
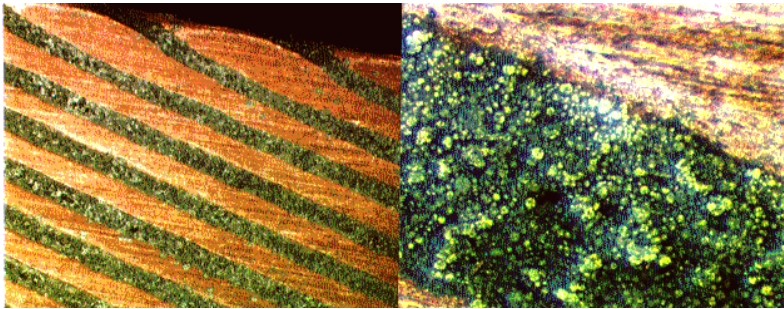
- experimental cable with porous metal heat exchanger

- excellent heat transfer to the liquid helium coolant

40μJ is a pin dropping 40mm

Factors affecting the Minimum Quench Energy

- heat a short zone of conductor $\Rightarrow$ resistive

- heat conducted out > generation $\Rightarrow$ zone shrinks

- heat conducted out < generation $\Rightarrow$ zone grows

- boundary between the two conditions is the
  minimum propagating zone MPZ
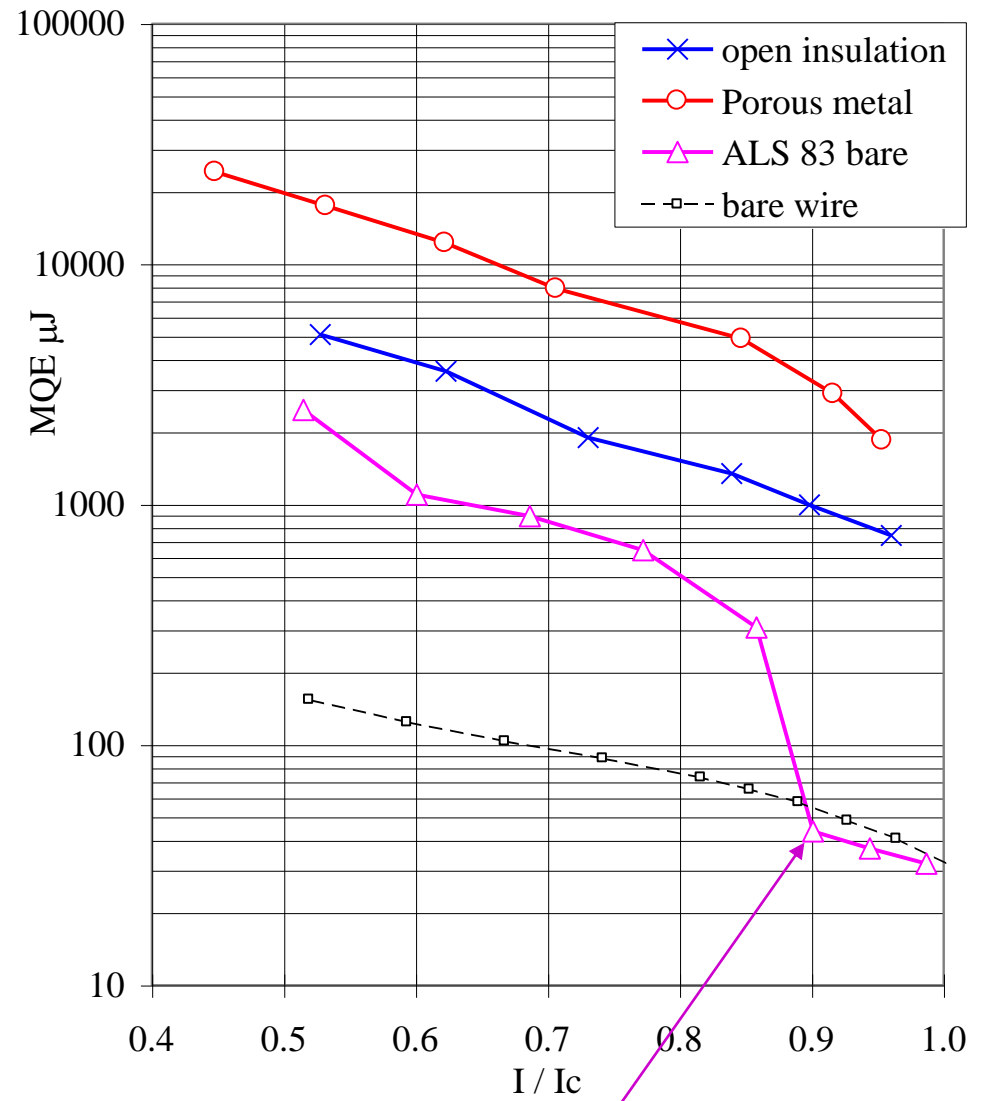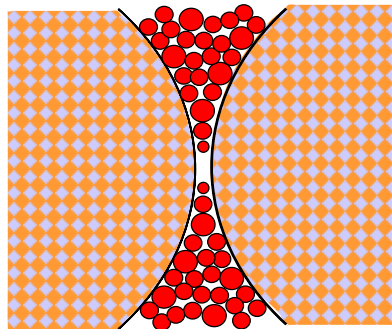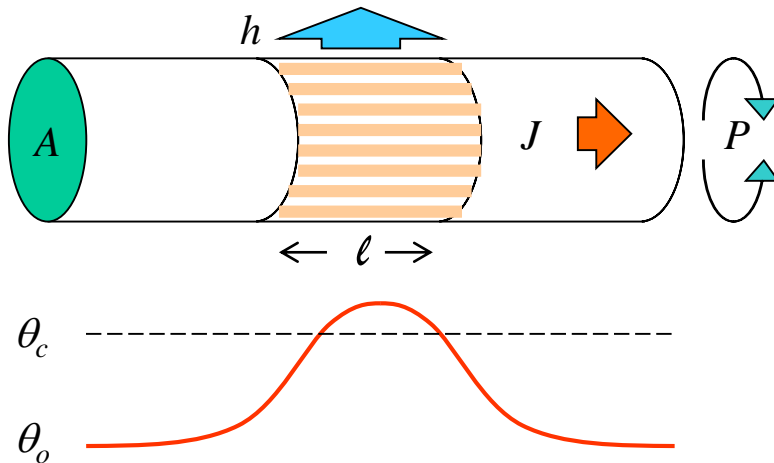
- large MPZ $\Rightarrow$ stability against disturbances

Very approximate heat balance

$$\frac{2kA(\theta_c - \theta_o)}{l} + hPl(\theta_c - \theta_o) = J_c^2 \rho A l$$

so length
of MPZ

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \dfrac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

where: k = thermal conductivity ρ = resistivity A = cross sectional area of conductor

h = heat transfer coefficient to coolant – if there is any in contact

P = cooled perimeter of conductor
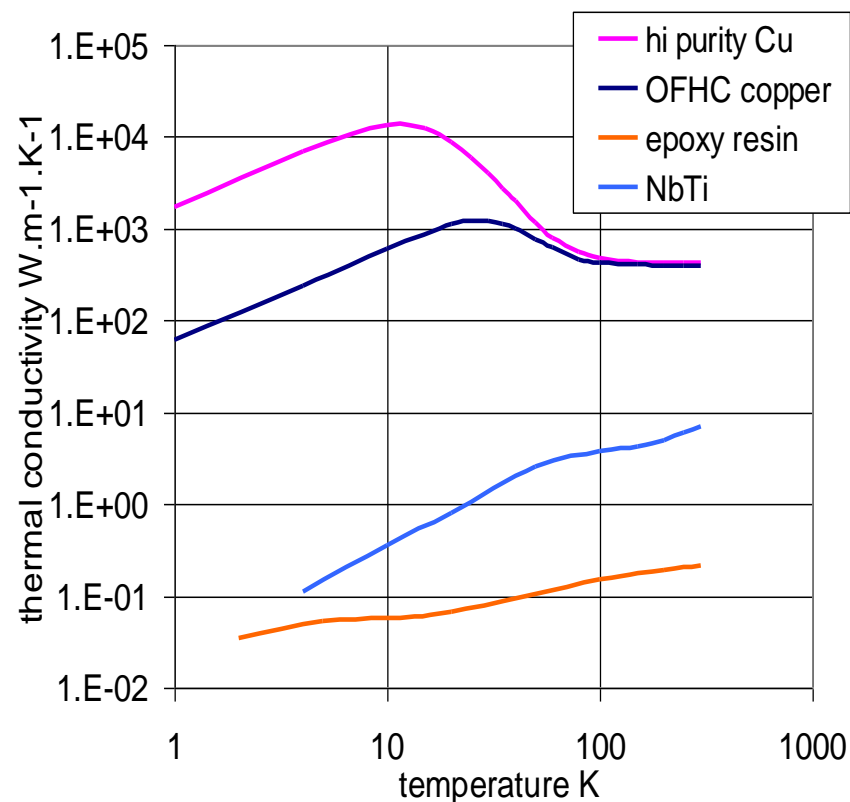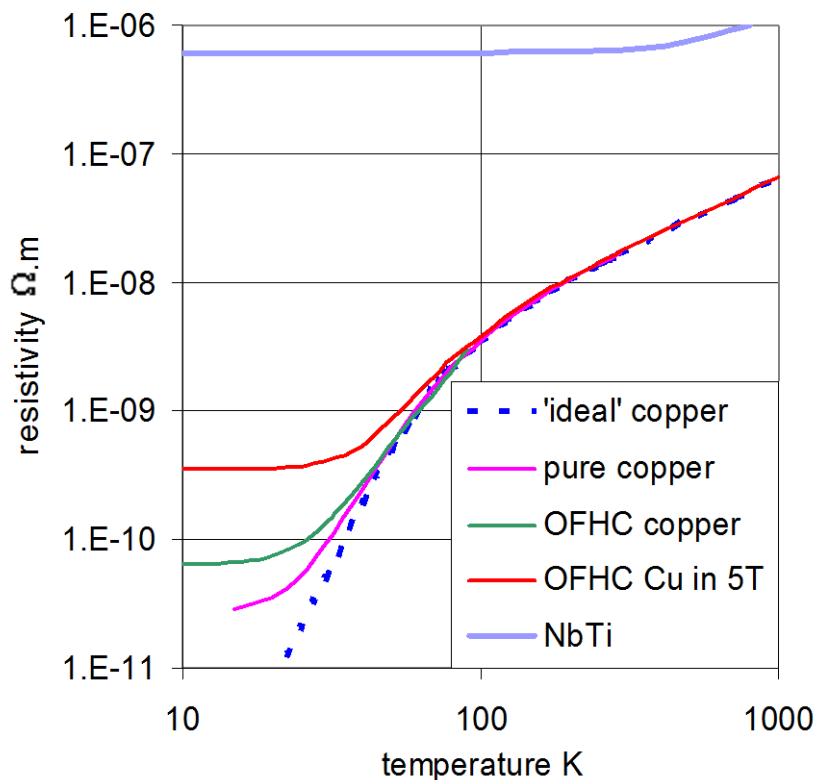
Energy to set up MPZ is the Minimum Quench Energy   *long MPZ $\Rightarrow$ large MQE*

How to make a long MPZ ⇒ large MQE

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \dfrac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

- make thermal conductivity **k** large
- make resistivity **ρ** small
- make heat transfer **hP/A** large (but ⇒ low J_{eng})

Large MPZ ⇒ large MQE ⇒ less training

$$l = \left\{ \frac{2k(\theta_c - \theta_o)}{J_c^2 \rho - \dfrac{hP}{A}(\theta_c - \theta_o)} \right\}^{\frac{1}{2}}$$

- make thermal conductivity k large

- make resistivity ρ small

- make heat transfer term hP/A large

- NbTi has high ρ and low k

- copper has low ρ and high k

- mix copper and NbTi in a filamentary composite wire

- make NbTi in *fine filaments* for intimate mixing

- maximum diameter of filaments ~ 50μm

- make the windings porous to liquid helium
    - superfluid is best

- fine filaments also eliminate flux jumping
    (see next slides)
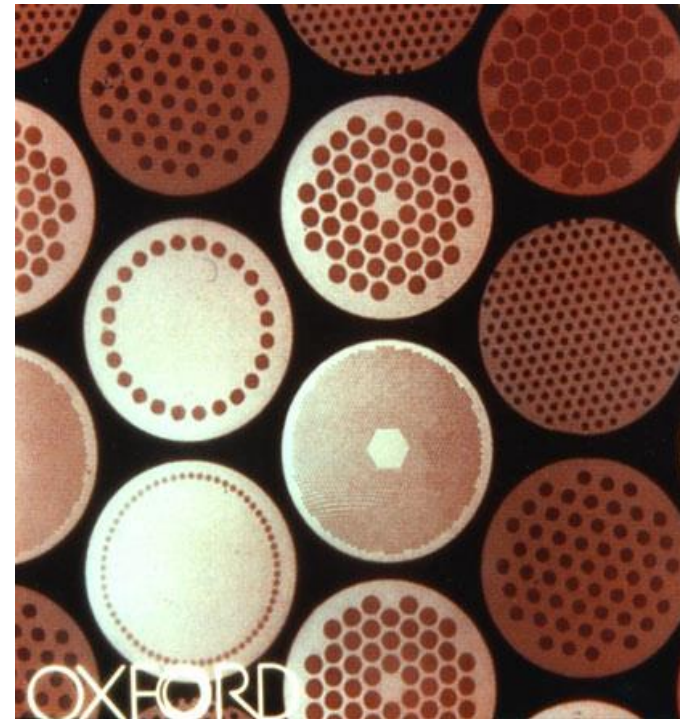
Another cause of training: flux jumping

- recap lecture 2: changing magnetic fields induce screening currents in superconductors

- *screening currents* are in addition to *transport currents*, which come from the power supply

- like eddy currents but don't decay because no resistance,
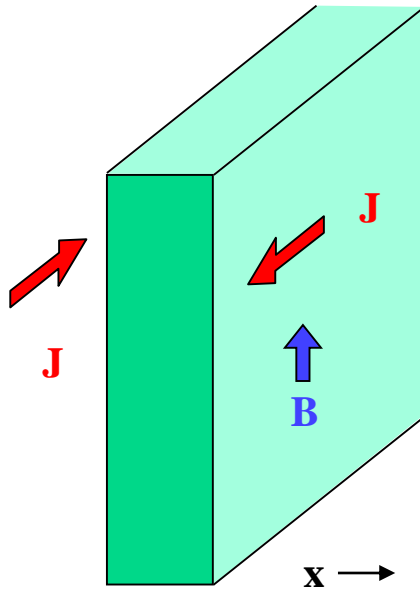
- usual model is a superconducting slab in a changing magnetic field B_y

- assume it's infinitely long in the z and y directions - simplifies to a 1 dim problem

- *dB/dt* induces an electric field E which causes screening currents to flow at critical current density J_c

- known as the *critical state model* or *Bean model*

- in the 1 dim infinite slab geometry, Maxwell's equation says

$$\frac{\partial B_y}{\partial x} = -\mu_o J_z = \mu_o J_c$$

- so uniform J_c means a constant field gradient inside the superconductor

Flux Jumping

a magnetic thermal feedback instability

- **screening currents**

- **temperature rise**

- **reduced critical current density**

- **flux motion**

- **energy dissipation**

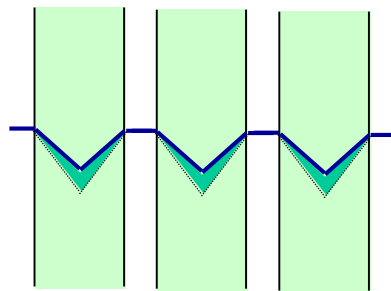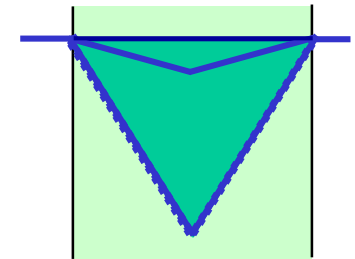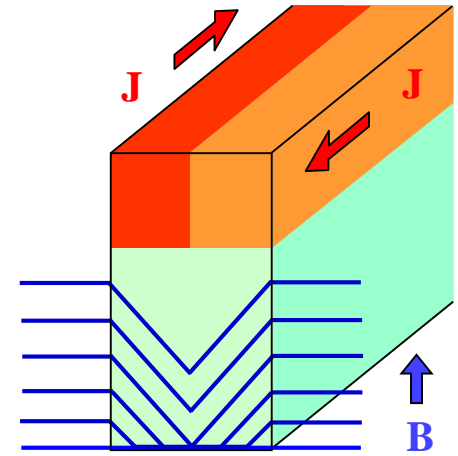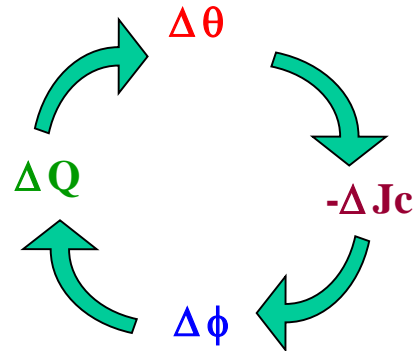- **temperature rise**

- cure flux jumping by weakening a link in the feedback loop

- fine filaments reduce $\Delta\phi$ for a given **-ΔJc**

- for NbTi the stable diameter is ~ 50μm

$\Delta\theta$

ΔQ

-ΔJc

$\Delta\phi$

J   J

B
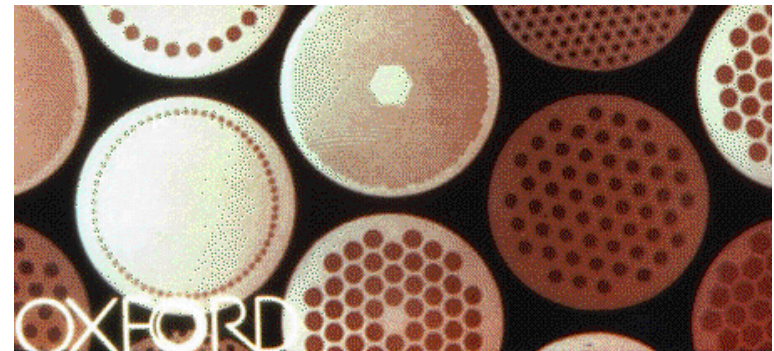
Flux jumping: the numbers

stable against flux jumping when filament diameter d

$$d \leq \frac{2}{J_c} \left\{ \frac{3\gamma\, C(\theta_c - \theta_o)}{\mu_o} \right\}^{\frac{1}{2}}$$

NbTi at 4.2K and 1T

J_c *critical current density* $= 7.5 \times 10^9\ Am^{-2}$

γC *specific heat/volume* $= 3400\ J.m^{-3}K^{-1}$

θ_c *critical temperature* $= 9.0K$

⇨ filament diameter $\boldsymbol{d} < \boldsymbol{52}\mu$m

- least stable at low B because of high J_c

- and at low θ because of high J_c and low C

Nb₃Sn at 4.2K and 2T

J_c *critical current density* $= 1.7 \times 10^{10}\ Am^{-2}$

γC *specific heat/volume* $= 1600\ J.m^{-3}K^{-1}$

θ_c *critical temperature* $= 15.7K$

⇨ filament diameter $\boldsymbol{d} < \boldsymbol{25}\mu$m

Flux jumping is a solved problem ✓

*AK Ghosh et al IEEE Trans App Sup **15** 2 p3484*
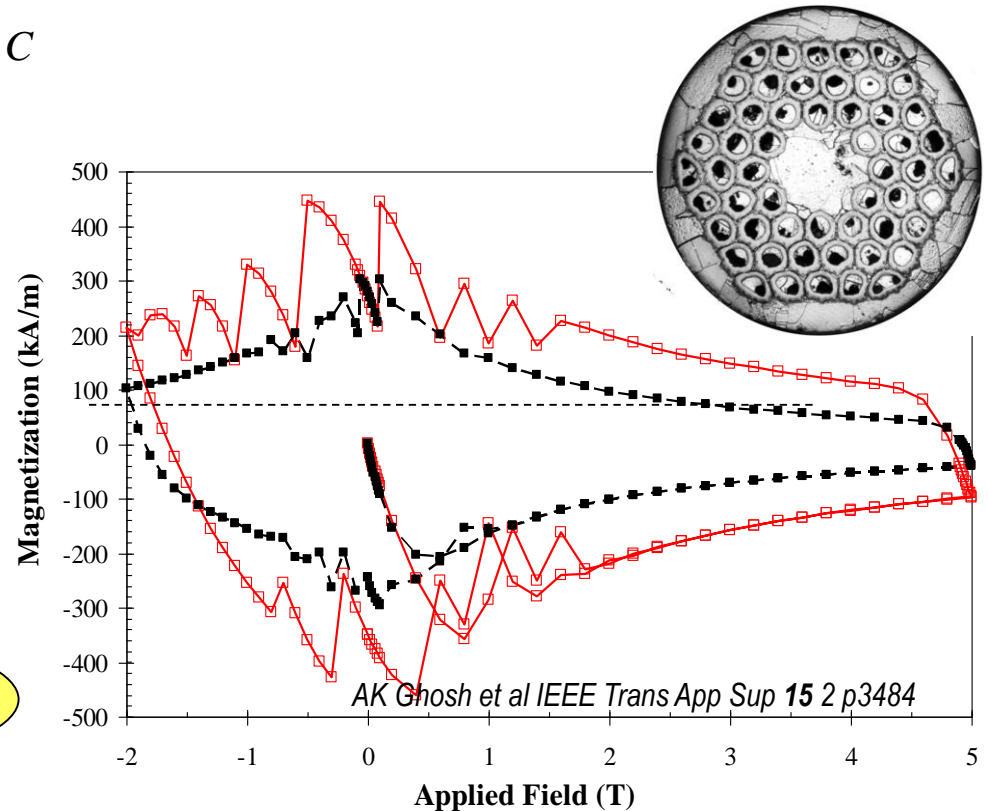
Concluding remarks

- superconducting magnets can make higher fields than conventional because they don't need iron which saturates at 2T - although iron is often used for shielding

- to get different field shapes you have to shape the <u>winding</u> (not the iron)

- practical winding shapes are derived from the ideal overlapping ellipses and/or $J = J_o Cos\theta$

- engineering current density is important for a compact economic magnet design

- expected magnet performance is given by the intersection of the load line and critical surface

- degraded performance and training are still a problem for magnets - and de-training is worse

- improve training by good winding construction
  $\Rightarrow$ no movement, low thermal contraction, no cracking

- improve training by making the conductor have a high MQE
  - temperature margin, high conductivity, good cooling
  - NbTi in good contact with copper $\Rightarrow$ fine filaments

- changing fields induce screening currents in all superconductors $\Rightarrow$ flux jumping

- flux jumping did cause degraded magnet performance but fine filaments have now cured it