BEAM DYNAMICS IN CYCLOTRONS

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Note:

- *Useful expressions are given at the* ***end of the document***
- *The care brought to the redaction of the answers will be taken into account.*

1. Recall the 6 different cyclotron designs

2. Recall the expression of the magnetic rigidity, *Br*, as a function of the particle momentum *P* and its charge Q. Where *B* is the magnetic field at radius *r*.

3. Along the closed orbit, there is equilibrium between the centrifugal force and the Lorentz force. From this equality express the Larmor angular frequency of the particle ω_{rev} in the non relativistic case ($\gamma=1$).

4. The particle in the cyclotron will be accelerated by resonant cavities with a frequency $f_{rf}= \omega_{rf}/2\pi$. Give the expression between the RF frequency f_{rf}, the particle revolution frequency f_{rev} and the harmonic number *h* when the particle is synchronous with the rf accelerating wave.

5. With B0 the magnetic field at the injection and with the previous equations how should the magnetic field B(r) evolve in order to keep the particle synchronous with the RF accelerating wave ω_{rf} in the relativistic case (γ increases) ?

6. In the case of compact cyclotron how can the particles be injected onto the first cyclotron orbit ?

7. In a variable energy isochronous cyclotron with an axial injection system and a fixed geometry of the central region (posts etc…), all beams must follow the same path at least on the first turns. The particles accelerated are ions $^{A}X^{Q+}$ where A is the nucleon number and Q the number of charges.

   a. The kinetic energy of the particle of mass m at the injection is given by a platform potential V_{inj} (ion source for example). r_0 is the first radius of curvature of the orbit at the exit of the inflector. From the expression of the kinetic energy of the ion, find the relationship between the magnetic field B_0 at the injection, the RF frequency f_{rf} , h the harmonic number and V_{inj}: $\frac{1}{r^2{}_0} = \pi \frac{f_{rf} B_0}{h V_{inj}}$

   b. Define the revolution frequency F_{rev} for a particle with an energy per nucleon W=15 MeV/A at a extraction radius of $R_{extraction}$=1.5 m

   c. What is the magnetic field at the extraction as a function of the ratio A/Q ?

   d. What is the magnetic field at injection with a radius r_{inj} = 0.04 m ?

   e. For an ion of Oxygen $^{16}O^{8+}$ find the necessary value of V_{inj}

Useful expressions

CONSTANTS

c = 2,997925x10^{8} m/s
1 u.m.a. = 931,478 MeV= 1,67.10^{-27} kg
e = 1,602x10^{-19} C

W in MeV/nucleon, R en mètres et F_{RF} en MHz

(1) $$\beta = \frac{\sqrt{W(W+1862{,}956)}}{W+931{,}478} = 2{,}0958.10^{-2}\,\frac{F_{RF}R}{h}$$

(2) $$\gamma = \frac{931{,}478+W}{931{,}478}$$ B = 6.5121.10^{-2} F_{rev} (A/Q)

(3) $$W = \sqrt{931.478^2 + \left(\frac{299{,}7925\,BRQ}{A}\right)^2} - 931{,}478 = 96{,}488\left(\frac{QBR}{A}\right)^2 \frac{1}{\gamma+1}$$