Exercises for Beam Instrumentation and Diagnostics

JUAS 2016

1 Calculation of current

1.1 LINAC

Pulse current: $I_{pulse} = \frac{t_{rep}}{t_{pulse}} = \frac{20 \text{ ms}}{1 \text{ ms}} \cdot I_{dc} = 2$ mA.

Number of particles within one macro-pulse: $N_{pulse} = \frac{I_{pulse} \cdot t_{pulse}}{e} = 1.25 \cdot 10^{13}$.

Number of particles within one bunch: $N_{bunch} = \frac{t_{bunch}}{t_{pulse}} \cdot N_{pulse} = \frac{10 \text{ ns}}{1 \text{ ms}} \cdot 1.25 \cdot 10^{13} = 1.25 \cdot 10^{8}$.

Velocity at 10 MeV: $E_{kin} = 1/2mc^2\beta^2 \Leftrightarrow \beta = \sqrt{\frac{2E_{kin}}{mc^2}} = 0.146$.

Bunch length in time: $t_{bunch} = 0.1/f_{rf} = 1$ ns.

Bunch length in space $l_{bunch} = \beta c t_{bunch} = 4.4$ cm.

Density within the bunch for a rectangular shape of $2x = 1$ cm edge length:

$\rho_{bunch} = \frac{N_{bunch}}{(2x)^2 l_{bunch}} = 2.84 \cdot 10^7$ cm^{-3}.

$\Rightarrow$ average distance $\langle d \rangle = \sqrt[3]{1/\rho_{bunch}} = 30$ μm.

Residual gas density: $n_{gas} = \frac{p}{k_B T} = 2.4 \cdot 10^9$ cm^{-3}.

1.2 Synchrotron

Revolution time at injection: $t_{inj} = \frac{L_{syn}}{\beta c} = 5.0$ μs $\Leftrightarrow f_{rev} = 200$ kHz.

Revolution time at extraction: $E_{kin} = mc^2(\gamma - 1) \Leftrightarrow \gamma = 2.066 \Leftrightarrow \beta = \frac{\sqrt{\gamma^2-1}}{\gamma} = 0.875$.

$\Rightarrow t_{rev} = 0.84$ μs $\Leftrightarrow f_{rev} = 1.19$ MHz.

Multi-turn injection within 100 μs: $I_{inj} = \frac{100\ \mu s}{t_{rev}} \cdot I_{pulse} = 40$ mA and $N_{inj} = 1.25 \cdot 10^{12}$.

After acceleration: $I_{final} = \frac{\beta_{final}}{\beta_{inj}} \cdot I_{inj} = 239$ mA.

Beam size due to multi-turn injection: $x_{inj} = \sqrt{\epsilon} \cdot x_{LINAC} = 5 \cdot x_{LINAC}$ and $y_{inj} = y_{LINAC}$

$\Rightarrow$ density $\rho_{inj} = \frac{N_{inj}}{\pi x_{inj} y_{inj} L_{syn}} = 1.45 \cdot 10^7$ cm^{-3}.

$\Rightarrow \langle d \rangle = \sqrt[3]{1/\rho_{inj}} = 41$ μm.

Emittance shrinkage during acceleration: $\epsilon_{extr} = \frac{\beta_{inj}\gamma_{inj}}{\beta_{extr}\gamma_{extr}} \cdot \epsilon_{inj} = 0.081\epsilon_{inj}$

and beam size shrinkage $x_{extr} = \sqrt{\epsilon_{extr}/\epsilon_{inj}} \cdot x_{inj} = 0.7$ cm.

Density at extraction with emittance shrinkage in both directions:

$\rho_{extr} = \frac{\rho_{inj}}{0.081} = 1.79 \cdot 10^8$ cm^{-3}.

$\Rightarrow$ average distance $\langle d \rangle = \sqrt[3]{1/\rho_{extr}} = 18$ μm.

Residual gas density: $n_{gas} = \frac{p}{k_B T} = 2.7 \cdot 10^6$ cm^{-3}.

1.3 Extraction

a) Slow extraction: Current in transfer line: $I_{trans} = \frac{t_{rev}}{t_{extr}} \cdot I_{final} = 20$ nA.

Total length of current string: $L_{string} = \beta_{extr} c \cdot t_{extr} = 2.6 \cdot 10^9$ m

$\Rightarrow$ density: $\rho = \frac{1}{\pi x_{extr} \cdot y_{extr} \cdot L_{string}} = 16$ cm^{-3}

$\Rightarrow$ average distance $\langle d \rangle = \sqrt[3]{1/\rho_{extr}} = 4.0$ mm.

Residual gas density: $n_{gas} = \frac{p}{k_B T} = 2.7 \cdot 10^7$ cm^{-3}.

b) Fast extraction: Current: $I_{bunch} = \frac{t_{rev}}{t_{bunch}} \cdot I_{extr} = 2.0$ A.

Bunching to $t_{bunch} = 100$ ns leads to a bunch length of $l_{bunch} = \beta_{extr} c t_{bunch} = 26.3$ m

$\Rightarrow$ density: $\rho_{bunch} = \frac{L_{syn}}{l_{bunch}} \cdot \rho_{extr} = 1.5 \cdot 10^9$ cm^{-3}.

$\Rightarrow$ average distance $\langle d \rangle = \sqrt[3]{1/\rho_{bunch}} = 9$ μm.

Table of results

	LINAC 10 MeV	Synchrotron inj. 10 MeV	Synchrotron extr. 1 GeV	Extraction slow	Extraction fast
beam current I	2 mA	40 mA	240 mA	20 nA	2.0 A
beam velocity β [% of c]	15	15	88	88	88
protons per bunch	$1.3 \cdot 10^8$	$1.3 \cdot 10^{12}$	$1.3 \cdot 10^{12}$	$1.3 \cdot 10^{12}$	$1.3 \cdot 10^{12}$
bunch duration t_{bunch}	1 ns	(5 μs)	(840 ns)	(10 s)	100 ns
bunch length l_{bunch}	4.4 cm	(220 m)	(220 m)	($3 \cdot 10^9$ m)	26 m
horizontal size $2x$ [cm]	1	5	1.4	1.4	1.4
vertical size $2y$ [cm]	1	1	0.3	0.3	0.3
average distance $\langle d \rangle$ [μm]	30	41	18	4000	9
beam density ρ_{beam} [cm^{-3}]	$3.6 \cdot 10^7$	$1.5 \cdot 10^7$	$1.8 \cdot 10^8$	$1.6 \cdot 10^1$	$1.5 \cdot 10^9$
vacuum pressure p [mbar]	10^{-7}	10^{-10}	10^{-10}	10^{-9}	10^{-9}
res. gas density ρ_{gas} [cm^{-3}]	$2.7 \cdot 10^9$	$2.7 \cdot 10^6$	$2.7 \cdot 10^6$	$2.7 \cdot 10^7$	$2.7 \cdot 10^7$

1.4 Changes for an ion accelerator

Current up to 1 MeV/u with $q = 4$: $I_{pulse} = 2$ mA $\Rightarrow N_{pulse} = \frac{I_{pulse} \cdot t_{pulse}}{4e} = 3.1 \cdot 10^{12}$
Current for 1 MeV/u to 10 MeV/u with $q = 28$: $I_{pulse} = 2$ mA $\Rightarrow N_{pulse} = \frac{I_{pulse} \cdot t_{pulse}}{28e} = 4.5 \cdot 10^{11}$
Current at 10 MeV/u with $q = 70$: $I_{pulse} = 2$ mA $\Rightarrow N_{pulse} = \frac{I_{pulse} \cdot t_{pulse}}{70e} = 1.8 \cdot 10^{11}$
Injection into the synchrotron $q = 70$: $I_{pulse} = 2$ mA $\Rightarrow N_{inj} = \frac{0.1\ ms}{1\ \text{ms}} \cdot N_{pulse} = 1.8 \cdot 10^{10}$.

2 Transformer for a pulsed LINAC

Droop of a transformer $U(t) = I_{beam} \frac{R}{N} \cdot e^{-t/\tau_{droop}} = U_0 \cdot e^{-t/\tau_{droop}}$
3 % within 1 ms: $\frac{U(t)}{U_0} = e^{-1ms/\tau_{droop}} = 0.97 \Leftrightarrow \tau_{droop} = 32.8$ ms.
The lower cut-off frequency is $f_{low} = \frac{1}{2\pi \cdot \tau_{droop}} = 4.85$ Hz.
The bandwidth of 100 kHz is required to monitor the changes in the pulse current on a 10 μs time scale, e.g. given by the ion source fluctuation.
Passive transformer:
The droop is determined by the inductance L and the resistors R and R_L: $\tau_{droop} = \frac{L}{R+R_L} \Leftrightarrow L = 33.13$ Hy.
The inductance of a given core is: $L = \frac{\mu_0 \mu_r}{2\pi} \cdot l N^2 \cdot \ln \frac{r_o}{r_i} \Leftrightarrow N = \sqrt{\frac{2\pi L}{\mu_0 \mu_r l \cdot \ln r_0/r_i}} = 244.5 \simeq 250$.
Sensitivity: $S = \frac{U}{I_{beam}} = \frac{R}{N} = 4$ V/A (here $\tau_{droop} \gg t$).
Signal-to-noise: $U_{noise} = \sqrt{4 k_B T (R_L + R) \Delta f} = 1.3\ \mu$V
$\Rightarrow$ for S-to-N=1: $I_{beam}^{\text{S-to-N=1}} = \frac{1}{S} \cdot U_{noise} = \frac{N}{R} \cdot U_{noise} = 0.315\ \mu$A.
Active transformer:
$\tau_{droop} = \frac{L}{R_L}$ for $R_f/A \ll R_L \Rightarrow L = 0.315$ Hy
The number of windings is $N = \sqrt{\frac{2\pi L}{\mu_0 \mu_r l \cdot \ln r_0/r_i}} = 25$.
Sensitivity: $S = \frac{R_f}{N} = 4.2 \cdot 10^4$ V/A.
The noise voltage is $U_{noise} = \sqrt{4 k_B T (R_L + R_f/A) \Delta f} = 0.135\ \mu$V and
the noise current $I_{noise} = \frac{A}{R_f} \cdot U_{noise} = 0.14\ \mu$A.
The corresponding beam current for S-to-N=1 $\Rightarrow I_{beam}^{\text{S-to-N=1}} = I_{noise}/N = 6$ nA i.e. a factor 50 lower

than for the passive case. This 6 nA threshold concerns only the amplifier input contribution, which is indeed lower than other limiting factors, like Barkhausen noise and magnetostriction.

Fast passive transformer:

3 % droop within 1 μs: $\frac{U(t)}{U_0} = e^{-1\mu s/\tau_{droop}} = 0.97 \Leftrightarrow \tau_{droop} = 32.8\ \mu$s

$\Rightarrow f_{low} = \frac{1}{2\pi \cdot \tau_{droop}} = 4.9$ kHz.

Inductance L: $\tau_{droop} = \frac{L}{R+R_L} \Leftrightarrow L = 1.97$ mHy.

The number of windings for $\mu_r = 10^3$: $N = 19 \simeq 20$.

Sensitivity: $S = \frac{U}{I_{beam}} = \frac{R}{N} = 2.65$ V/A.

Signal-to-noise: $U_{noise} = 9.09\ \mu$V $\Rightarrow$ for S-to-N=1: $I_{beam}^{\text{S-to-N=1}} = \frac{1}{S} \cdot U_{noise} = 3.8\ \mu$A. Due to the wide bandwidth, the minimal detectable beam current is larger than for the case above.

3 Slow extraction current measurement

Density of the Ar filling: $\rho = \frac{N_A \cdot 40\text{ g}}{22.4\text{ l}} = 1.79$ mg/cm^3

Energy loss of protons: $\Delta E_p = \frac{dE}{\rho dx} \cdot \rho \cdot \Delta x = 1289$ eV per proton, $\Delta x = 0.5$ cm.

Number of secondary e^-: $N_e = \frac{\Delta E}{26.3\text{ eV}} = 49\ e^-$/ion.

For the extraction of $N_i = 1.25 \cdot 10^{12}$ ions/s it is $N_e^{tot} = N_i \cdot N_e = 6.13 \cdot 10^{13}\ e^-$/s,

the corresponding current is: $I_e = e \cdot N_e^{tot} = 9.77 \cdot 10^{-6}$ A $\simeq 10\ \mu$A.

For Uranium: $\Delta E_u = (92)^2 \cdot \Delta E_p = 10.9$ MeV.

Number of secondary e^-: $N_e = 4.14 \cdot 10^5\ e^-$/ion,

the e^--rate for the Uranium case is: $N_e^{tot} = N_i \cdot N_e = 7.45 \cdot 10^{15}\ e^-$/s,

the corresponding current is: $I_e = 1.19$ mA $\rightarrow$ saturation of the IC!

For the SEM and protons: $N_e = 0.048\ e^-$/ion $\Rightarrow I_e = 9.69 \cdot 10^{-9}$ A $\simeq 10$ nA.

For Uranium: $N_e = 406\ e^-$/ion $\Rightarrow I_e = 1.18 \cdot 10^{-6}$ A $\simeq 1\ \mu$A.

4 Beam power at a LINAC

The macro-pulse current is: $I_{macro} = 2$ mA,

the average current is $I_{aver} = \frac{1\text{ ms}}{20\text{ ms}} \cdot I_{macro} = 100\ \mu$A.

With $I = \frac{qeN}{t}$ the corresponding average rate is:

protons: $N_p = 6.25 \cdot 10^{14}$ 1/s and

Uranium: $N_U = 2.23 \cdot 10^{13}$ 1/s.

The absorbed average power is $P^{aver} = eN \cdot E_{kin} \cdot A$, A mass of ions

$\Rightarrow P_p^{aver} = 1.0$ kW and $P_U^{aver} = 8.5$ kW.

The *peak* power is higher than the average value by the factor $\frac{20\text{ ms}}{1\text{ ms}}$

$\Rightarrow P_p^{peak} = 20$ kW and $P_U^{peak} = 170$ kW.

The average power has to be transported by the flow of cooling water:

$P_{water} = \frac{dV}{dt} \cdot \rho c \cdot \Delta T$

($\frac{dV}{dt}$ is the flow, ρ the density, c the heat capacitance and $\Delta T = 60$ K the temperature increase.)

Protons: $P_{water} = P_p^{aver} \Rightarrow \frac{dV}{dt} = 3.97$ cm^3/s $= 0.23$ l/min

Uranium: $P_{water} = P_U^{aver} \Rightarrow \frac{dV}{dt} = 33.7$ cm^3/s $= 2.0$ l/min.

5 Material destruction for intense beams

Power within a macro-pulse fo an ion of mass $A = 238$: $P_{macro} = \frac{I_{macro}}{q} \cdot E_{kin} \cdot A = 595$ kW
Energy within the macro-pulse: $W_{macro} = P_{macro} \cdot t_{macro} = 119$ J.
The specific energy for heating the metal to the melting temperature is $Q_{heat} = \rho c(T_{melt} - T_0)$ and the specific energy to melt the metal is $Q_{melt} = \rho \cdot a_{melt}$. The total specific energy is the sum for heating and melting $Q_{tot} = Q_{heat} + Q_{melt}$

	Cu	Fe	W
Q_{heat} [kJ/cm^3]	3.1	5.3	8.4
Q_{melt} [kJ/cm^3]	1.9	2.2	3.7
Q_{tot} [kJ/cm^3]	5.0	7.5	12.1

The beam is stopped in a volume $V = \pi r_{min}^2 \cdot R_{mat}$ (assuming a round beam with constant density as a first approximation). The minimum beam radius to prevent melting is $r_{min} = \sqrt{\frac{W_{macro}}{Q_{tot} \cdot \pi \cdot R_{mat}}}$, with the material dependent ranges R_{mat} of Uranium ions (i.e. $R_{Cu} = 9.5$ μm, $R_{Fe} = 9.7$ μm, $R_W = 7.4$ μm):

	Cu	Fe	W
r_{min} [mm]	28.3	22.8	20.6

Having now a beam with $r_{beam} = 5$ mm the maximal allowed beam energy
$W_{macro} = \pi r_{beam}^2 \cdot Q_{tot} \cdot R_{mat}$ is:

	Cu	Fe	W
W_{macro} [J]	4.6	5.7	7.0

and the time is $t_{pulse} = \frac{W_{macro}}{P_{macro}}$

	Cu	Fe	W
t_{pulse} [μs]	7.7	9.6	11.8

The estimation above assumes a constant beam density; for a Gaussian distribution the central part of the beam spot is heat more server. For a save operation of an accelerator, the above given value for t_{pulse} have to be about a factor 2 or 3 lower.
The penetration depth R (or range) of ions in material scales approximately with $R_{mat} \propto E_{kin}^{1.75}$
$\Rightarrow r_{min} \propto \sqrt{E_{kin}/R} \propto E_{kin}^{-0.375}$, i.e. for high beam energies the temperature increase is lower.
The corresponding values for a 1 MeV proton beam of 10 mA current and 200 μs duration are $P_{macro} = 10$ kW and $W_{macro} = 2$ J.
The minimum radius is:

	Cu	Fe	W
r_{min} [mm]	4.0	3.6	3.1

To prevent melting assuming a beam radius of 5 mm it is:

	Cu	Fe	W
W_{macro} [J]	3.2	3.8	5.1
t_{pulse} [μs]	320	378	510

In contradiction to the ion case, no destruction is expected for the full beam duration.

6 Transverse profile by flying wire scanner

The energy loss per passage of the wire with thickness $\Delta x = 50$ μm as given by $\Delta E_{pass} = \frac{dE}{dx} \cdot \Delta x$ is for the two ions:

	p	U
ΔE_{pass} [MeV]	0.021	178
$\Delta E_{pass}/E_{kin}$	$2.1 \cdot 10^{-5}$	$7.0 \cdot 10^{-4}$

$\Rightarrow$ even after 2 passages the Uranium is out of the typical longitudinal acceptance of typically about $(\Delta E/E)_{accept} \simeq 10^{-3}$ and they will be lost. For the proton case the particles energy loss is below the acceptance value and they will be stored.

Kinematics of the 'flying wire':

total time to pass the beam: $t_{tot} = \frac{d_{beam}}{v} = 1$ ms

time to move by *one* wire thickness $\Delta x = 50$ μm: $t_{\Delta x} = \frac{\Delta x}{v} = 5.0$ μs

number of passages of particles through the wire: $N_{pass} = \frac{t_{\Delta x}}{t_{rev}} = 5.0$ with $t_{rev} = 1.0$ μs, i.e. 5 passes through the wire.

Using the number of stored particle N_{stored} (protons $N_{stored} = 10^{12}$ and Uranium $N_{stored} = 10^9$)

the total energy is: $W = e \cdot \Delta E_{pass} \cdot N_{pass} \cdot N_{stored}$,

the total power: $P = \frac{W}{t_{tot}}$,

and the power per mm: $P_{mm} = \frac{P}{10 \text{ mm}}$. It follows for both ions:

	p	U
W [J]	0.017	0.142
P [W]	17	142
P_{mm} [W/mm]	1.7	14.2

$\Rightarrow$ For protons the heating of the wire is just above the safty-border of 1 W/mm ($\Rightarrow$ a more realistic calculation is required).

$\Rightarrow$ For Uranium the wire melts; note, that 10^{-3} less Uranium ions are stored.

If not fully striped ions are stored, than they change their charge state according to an energy-dependence distribution, i.e. a noticeable fraction of ions is lost after the next dipole due to the non-matched charge state.

7 Signal estimation for a residual gas monitor

Target thickness of H_2: $x_H = \rho_H \cdot \frac{p}{p_{norm}} \cdot l = 9.0 \cdot 10^{-11}$ mg/cm^2

(p vacuum pressure, $l = 100$ mm monitor length)

Target thickness of N_2: $x_N = \rho_N \cdot \frac{p}{p_{norm}} \cdot l = 1.25 \cdot 10^{-9}$ mg/cm^2

Energy loss per ion in H_2: $E_H = \frac{dE}{\rho_H dx} \cdot x_H = 5.94$ meV

Energy loss per ion in N_2: $E_N = \frac{dE}{\rho_N dx} \cdot x_N = 27.5$ meV

Number of ions per pulse: $N_{macro} = \frac{I}{qe} \cdot t_{macro} = 6.25 \cdot 10^{11}$.

Total energy loss within the monitor per pulse:

for H_2: $E_H^{tot} = N_{macro} \cdot E_H = 3.71 \cdot 10^9$ eV

for N_2: $E_N^{tot} = N_{macro} \cdot E_N = 17.2 \cdot 10^9$ eV.

Numbers of e^--ion pairs $N^{pair} = \frac{E^{tot}}{W}$, with $W = 36$ eV:

for H_2: $N_H^{tot} = 1.0 \cdot 10^8$
for N_2: $N_N^{tot} = 4.8 \cdot 10^8$.
For the case of the LINAC: 90 % N_2 and 10 % H_2 it is:
$I_{sec} = e\left(0.1N_H^{pair} + 0.9N_N^{pair}\right)/t_{macro} = 70.4$ nA
10 % of the beam is covered by one strip $\Rightarrow I_{sec}^{strip} = 7.04$ nA
$\Rightarrow U_{sec}^{strip} = 0.704$ V after I/U-conversion.
(Current down to 1 nA can be measured with sensitive pre-amplifiers even in the electronically noisy environment close to an accelerator.)
For the case of the synchrotron: Target thickness $x_H = 9.0 \cdot 10^{-14}$ mg/cm^2 and $p = 10^{-10}$ mbar
$\Rightarrow N_H^{pair} = 1.0 \cdot 10^6$
$\Rightarrow I_{sec} = 0.16$ nA and per strip $I_{sec}^{strip} = 16$ pA.
This cannot be converted by an I/U-converter and a MCP as a 'pre-amplifier' with up to 10^6 fold amplification is required.

8 Signal estimation for broad-band BPM

Transfer impedance for 1 MΩ: $Z_t = \frac{1}{\beta c C} \cdot \frac{A}{\pi a} = \frac{l}{\beta c C} = 6.7\ \Omega$, with area $A = \pi a l$.
Sum voltage for $I_{beam} = 1$ A: $U_\Sigma = 2Z_t \cdot I_{beam} = 13.3$ V.
Difference voltage for $I_{beam} = 1$ A: $U_\Delta = Z_\perp \cdot x I_{beam} = \frac{x}{a} \cdot Z_t \cdot I_{beam}$ with $Z_\perp = \frac{Z_t}{a}$
$\Rightarrow U_\Delta(x = 1\text{mm}) = 67$ mV.
For 50 Ω termination and $Z_t(50\Omega) = Z_t(1\text{M}\Omega)/20$:
Sum voltage for $I_{beam} = 1$ A : $U_\Sigma = 2Z_t \cdot I_{beam} = 0.67$ V and
difference voltage $U_\Delta(x = 1\text{mm}) = 3.3$ mV.
The thermal noise voltage is given by:
$R = 1\text{M}\Omega$: $U_{eff}(R = 1\text{M}\Omega) = \sqrt{4k_BTR\Delta f} = 1.3$ mV
$R = 50\Omega$: $U_{eff}(R = 50\Omega) = \sqrt{4k_BTR\Delta f} = 9.2\ \mu$V.
The minimum beam current for $S/N = 2$ is:
$R = 1$ MΩ: $U_\Delta = 2 \cdot U_{eff}(R = 1\text{M}\Omega) = 2.7$ mV $\Rightarrow I_{beam} = \frac{a}{x}\frac{1}{Z_t(1M\Omega)} \cdot U_\Delta = 40$ mA.
$R = 50\Omega$: $U_\Delta = 2 \cdot U_{eff}(R = 50\Omega) = 18.4\ \mu$V $\Rightarrow I_{beam} = \frac{a}{x}\frac{1}{Z_t(50\Omega)} \cdot U_\Delta = 5.5$ mA.
The scaling of Z_t for 50 Ω is given by $U_\Delta \propto \omega \propto 1/\sigma_t$
$\Rightarrow$ 50 Ω has a better S/N for bunches shorter than 720 ns.
By a narrow-band analysis the band-width Δf is much lower (typically 10 kHz for the narrow-band case instead of 100 MHz in the broad-band case) and therefore the thermal noise voltage via $U_{eff} \propto \sqrt{\Delta f}$ (i.e. the noise contribution is about a factor 100 lower).
Remark concerning the value of minimal current I_{beam} for termination with $R = 1$ MΩ:
The minimal values of $I_{beam} = 40$ mA calculated above is unrealistically high due to the fact that the BPM's capacitance of $C = 100$ pF together with the resistor $R = 1$ MΩ form a low-pass filter with the cut-off frequency $f_{cut} = (2\pi RC)^{-1} = 1.6$ kHz. High frequencies $f \gg 10$ kHz will therefore be damped. A more realistic value for the minimal beam current related to that effect is $I_{beam} \simeq 1$ mA. For the estimation of this value the noise of an amplifier is taken into account as well. Generally, the amplifiers contribution is characterized by the so called noise figure $F = U_{eff}(\text{amplifier})/U_{eff}(\text{thermal})$, which describes the ratio between the real amplifier noise U_{eff}(amplifier) with respect to the thermal noise U_{eff}(thermal) at a corresponding resistor and temperature. Typical values for a low-noise amplifier

are $F \geq 2$.

9 Profile Measurement with a scraper

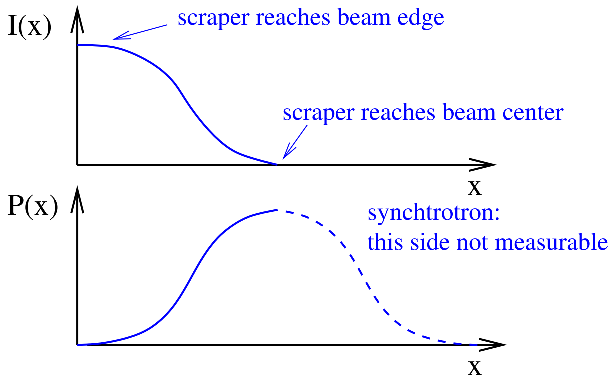

Figure 1: The circulating current $I(x)$ and the calculated beam profile $P(x)$ as a function of the scraper position x in the case of a synchrotron.

a) The primary signal is the loss of stored beam particles.

b) One could measure the fraction of lost particles either by determine the current hitting the scraper (i.e. the scraper acts like a Faraday cup) or with a beam loss monitor outside of the beam pipe. These methods have the disadvantage of low signal strength, because no refilling occur. Therefore it is more suitable to measure the remaining circulating current inside the synchrotron with a dc-transformer (be aware that the scraper movement last typically 1 s, while the revolution time is typically 1 μs). This later method is proposed due to the larger signal strength.

c) Because the remaining circulating current $I(x)$ as a function of scraper position x is determined, the profile $P(x)$ is given by the negative derivative $P(x) = -\frac{dI(x)}{dx}$.

d) The movement of the scraper is slow compared to the revolution time. Due to the betatron oscillation the beam particles reach after a few turns the side of the scraper in any case. If their beatatron amplitudes are larger than the scraper position, they are lost. When the scraper reaches the middle of the beam path, no particles are circulating any more.

e) In a transport-line of a LINAC new particles are passing the scraper location. Therefore the argument of item **d)** is not valid. For recording the full beam profile, the scraper has to transverse the full beam profile.

10 Emittance measurement

For 1 GeV protons the range in metal is about 0.5 m. Therefore the cutting of 'beamlets' with destructive methods like slit-grid or pepper-pot devices is not possible. The quadrupole variation is a suitable method in a transfer-line. The focusing strength of a quadrupole is varied and the profile for each setting is measured after a certain drift. The square of the profile width is plotted as a function of the focusing strength. The transfer matrices **R** can be calculated from the quadrupole settings and are used for a fit of the beam matrix elements at the location of the quadrupole by using the measured profile widths. At least three focusing settings have to be used, to get a unique solution for the three Twiss parameters of each plane. For a good fit accuracy, more than three measurements have be performed and it should include values with the beam-waist in front and behind the profile measurement location.

11 Beam energy measurement by time-of-flight

Non-relativistic formulas can be used for the given energies.
For the lower boundary of $E = 5$ MeV it is:
$\beta = \sqrt{\frac{2E}{mc^2}} = 0.103$, time required for $L = 4$ m distance $T = \frac{L}{\beta c} = 129$ ns, number of bunches N between the pick-ups $N = T \cdot f_{rf} \simeq 13$ for 100 MHz rf.
For the upper boundary of $E = 10$ MeV it is:
$\beta = \sqrt{\frac{2E}{mc^2}} = 0.146$, time required for $L = 4$ m distance $T = \frac{L}{\beta c} = 91$ ns, number of bunches N between the pick-ups $N = T \cdot f_{rf} \simeq 9$.
a) The error propagation of $\Delta\beta/\beta$ as a function of number of bunches N between the pick-ups is given by $\frac{\partial \beta c}{\partial N} = \frac{\partial}{\partial N}\left(\frac{L}{NT + t_{scope}}\right) \simeq -\frac{L}{N^2 T}$ due to $T \gg t_{scope}$, with T equals the total time to travel the distance L between the pick-ups.
The error of the energy reading is $\frac{\Delta E}{E} = 2\frac{\Delta\beta}{\beta} = 2\frac{\Delta N}{N}$.
For a unique solution $\Delta N = 1$ and therefore:
Lower boundary $E = 5$ MeV $\Rightarrow \frac{\Delta E}{E} = \frac{2}{N} = 15$ % with $N = 13$.
Upper boundary $E = 10$ MeV $\Rightarrow \frac{\Delta E}{E} = \frac{2}{N} = 22$ % with $N = 9$.
b) The third pick-up should be installed within the distance between two successive bunches:
Lower boundary $E = 5$ MeV: $d_{bunch} = \frac{\beta c}{f_{rf}} = 31$ cm.
Upper boundary $E = 10$ MeV: $d_{bunch} = \frac{\beta c}{f_{rf}} = 44$ cm.
The lower boundary $E = 5$ MeV gives the shorter distance and therefore the third pick-up should be installed less than 31 cm apart from the first one.