Synchrotron radiation

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and

Diamond Light Source

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Contents

Introduction to synchrotron radiation

properties of synchrotron radiation
synchrotron light sources
angular distribution of power radiated by accelerated particles
angular and frequency distribution of energy radiated:
radiation from undulators and wigglers

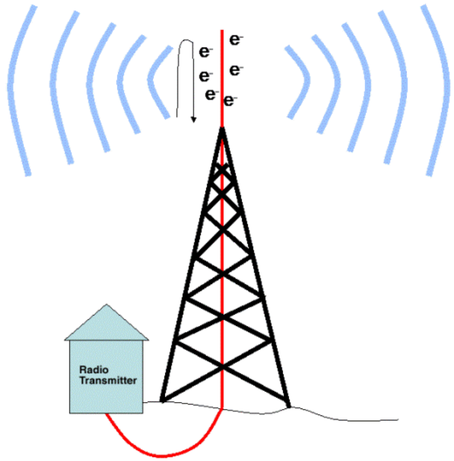
Beam dynamics with synchrotron radiation

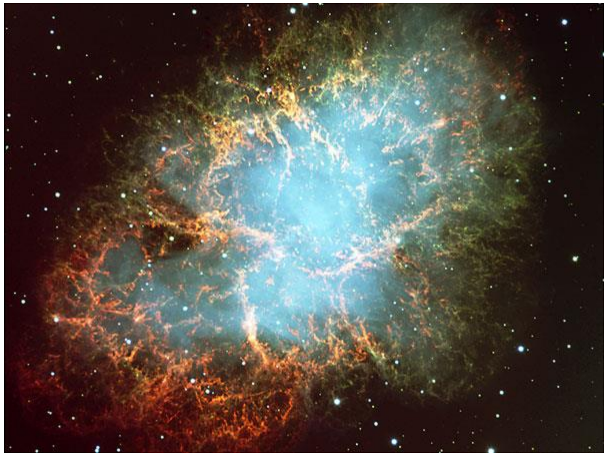
electron beam dynamics in storage rings
radiation damping and radiation excitation
emittance and brilliance
low emittance lattices
diffraction limited storage rings

short introduction to free electron lasers (FELs)

What is synchrotron radiation

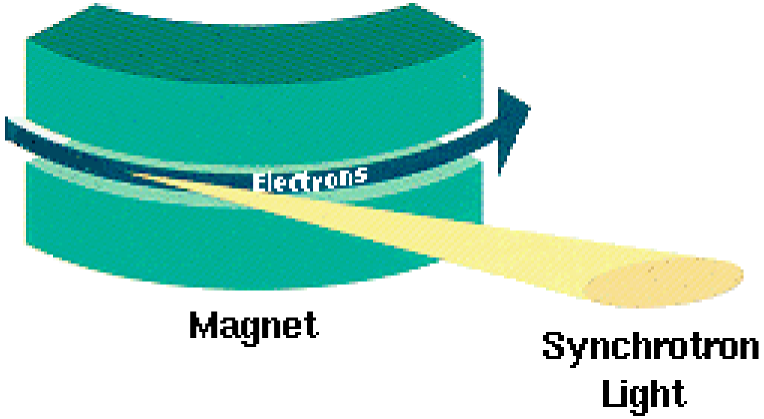
Electromagnetic radiation is emitted by charged particles when accelerated

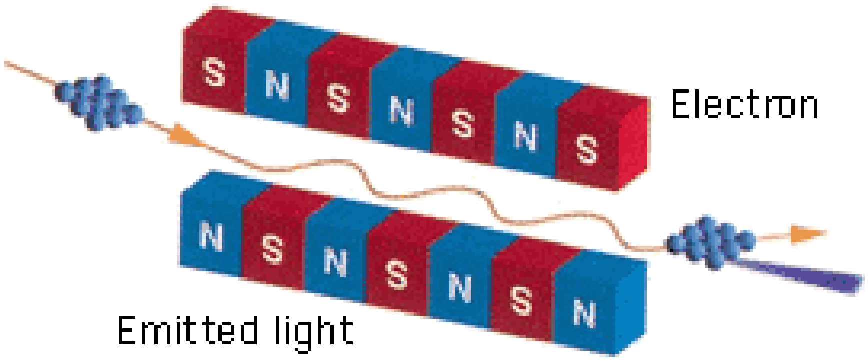




The electromagnetic radiation emitted when the charged particles are accelerated radially ($v \perp a$) is called synchrotron radiation

It is produced in the synchrotron radiation sources using bending magnets undulators and wigglers



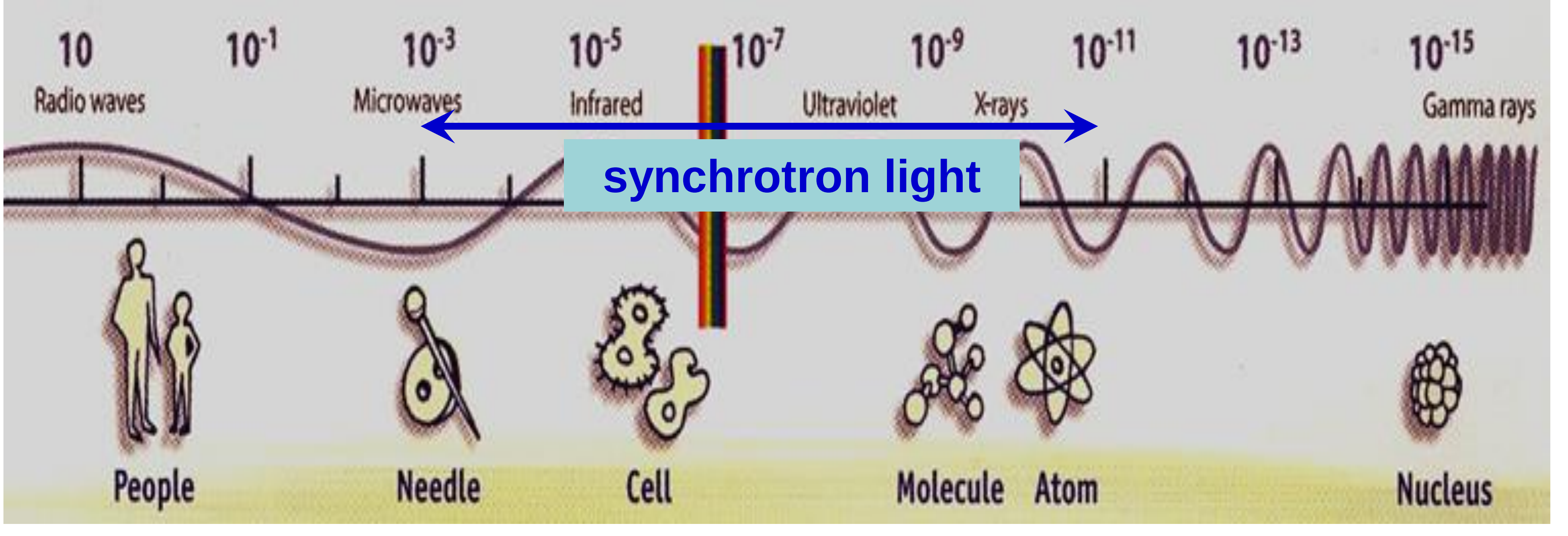


Synchrotron radiation sources properties (I)

Broad Spectrum which covers from microwaves to hard X-rays:

the user can select the wavelength required for experiment;

either with a monochromator
or adjusting the emission wavelength of insertion devices



Synchrotron radiation sources properties (II)

High Flux: high intensity photon beam, allows rapid experiments or use of weakly scattering crystals;

Flux = Photons / (s • BW)

High Brilliance (Spectral Brightness): highly collimated photon beam generated by a small divergence and small size source

Brilliance = Photons / (s • mm^2 • mrad^2 • BW)

Partial coherence in SRs

Full T coherence in FELs

Polarisation: both linear and circular (with IDs)

Pulsed Time Structure: pulsed length down to

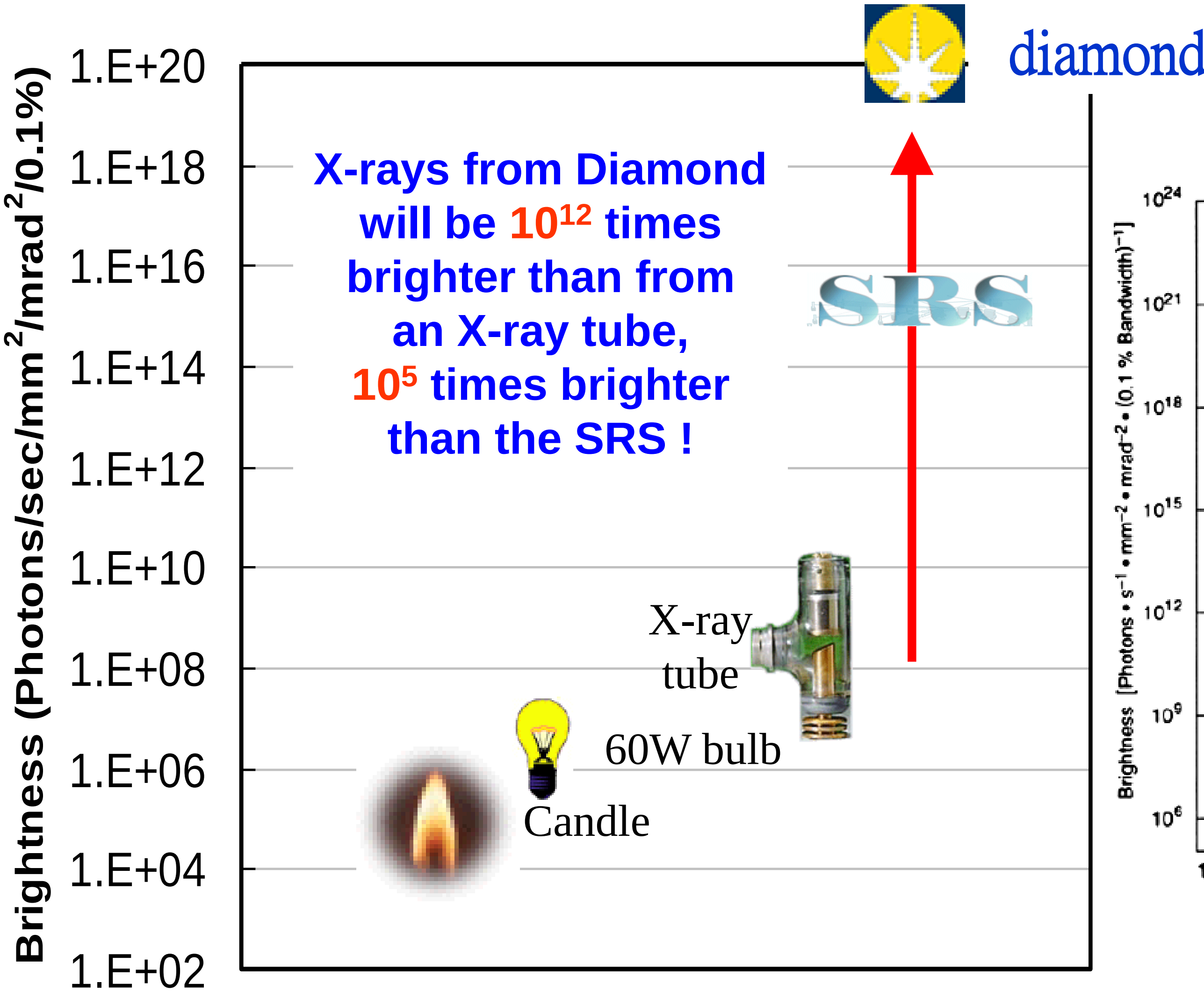
10s ps in SRs

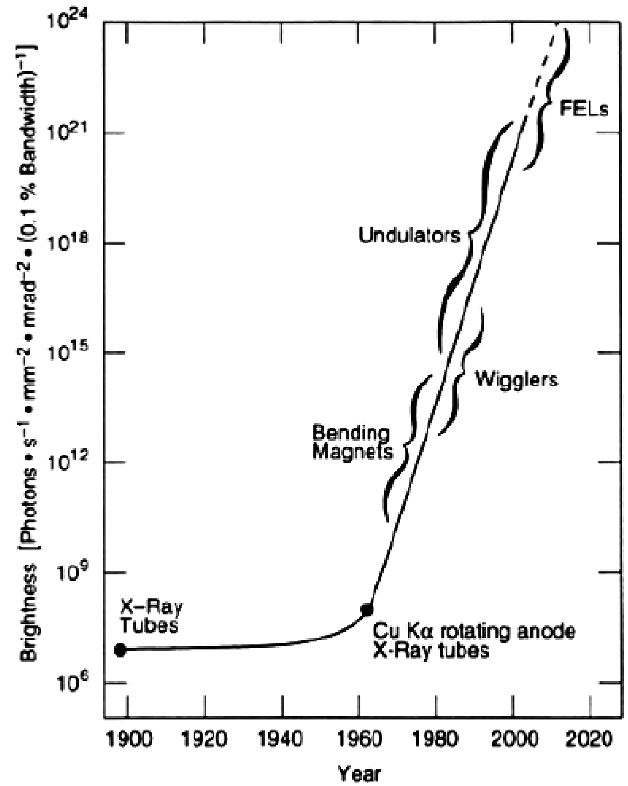
10s fs in FELs

High Stability: submicron source stability in SR

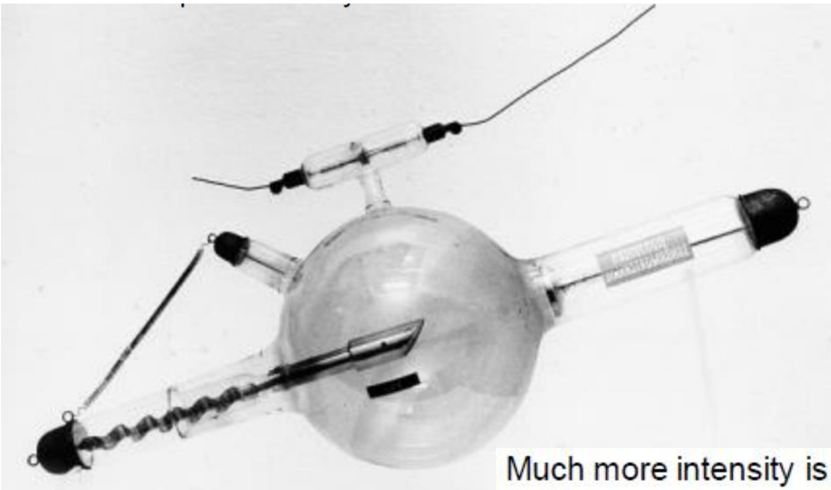
… and **it can be computed!**

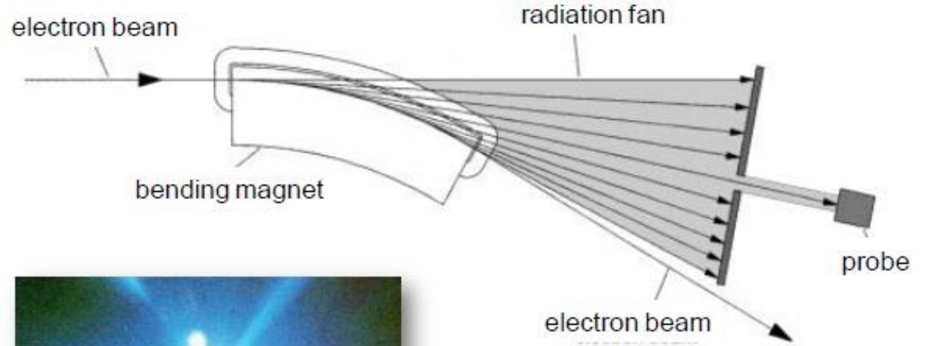
Peak Brilliance

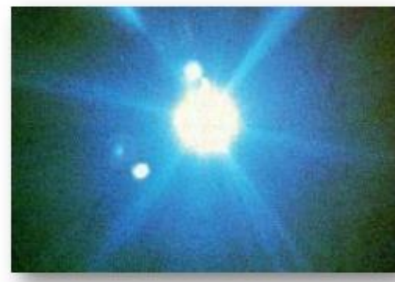




X-ray sources

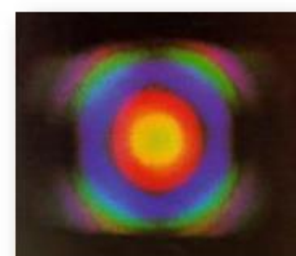


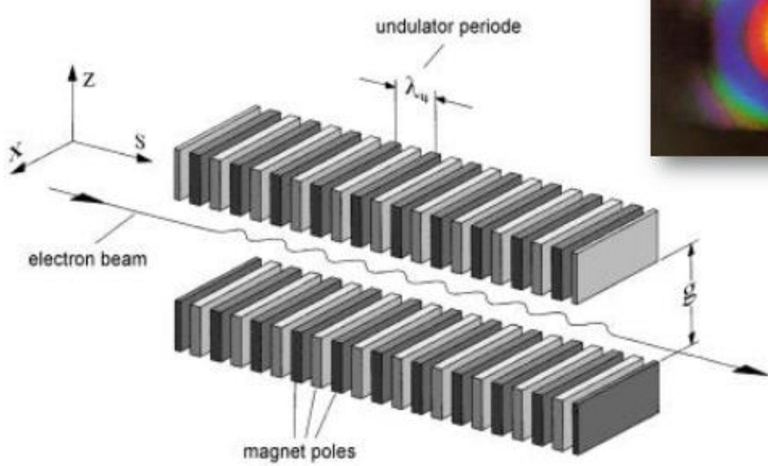




the synchrotron radiation from a bending magnet is horizontally spread out over a wide radiation fan.

Much more intensity is provided by wigglers and undulators



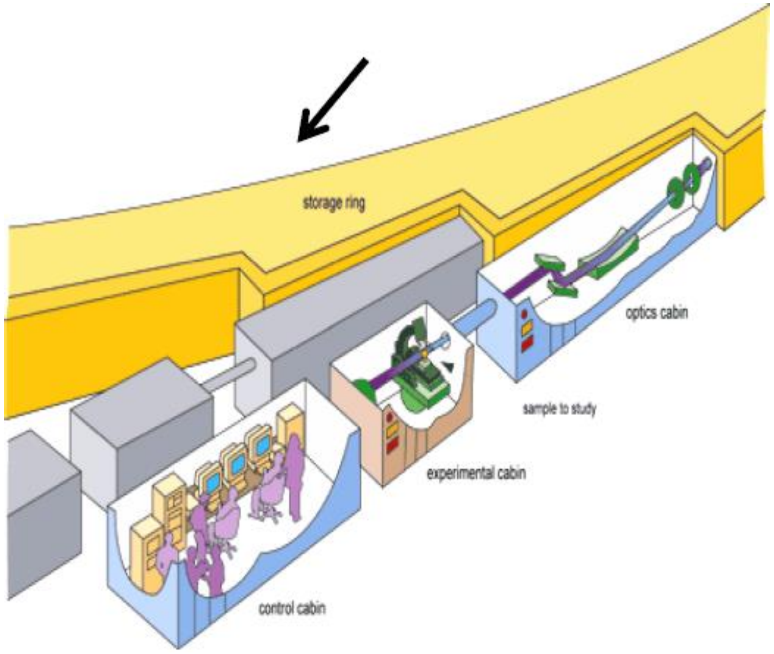


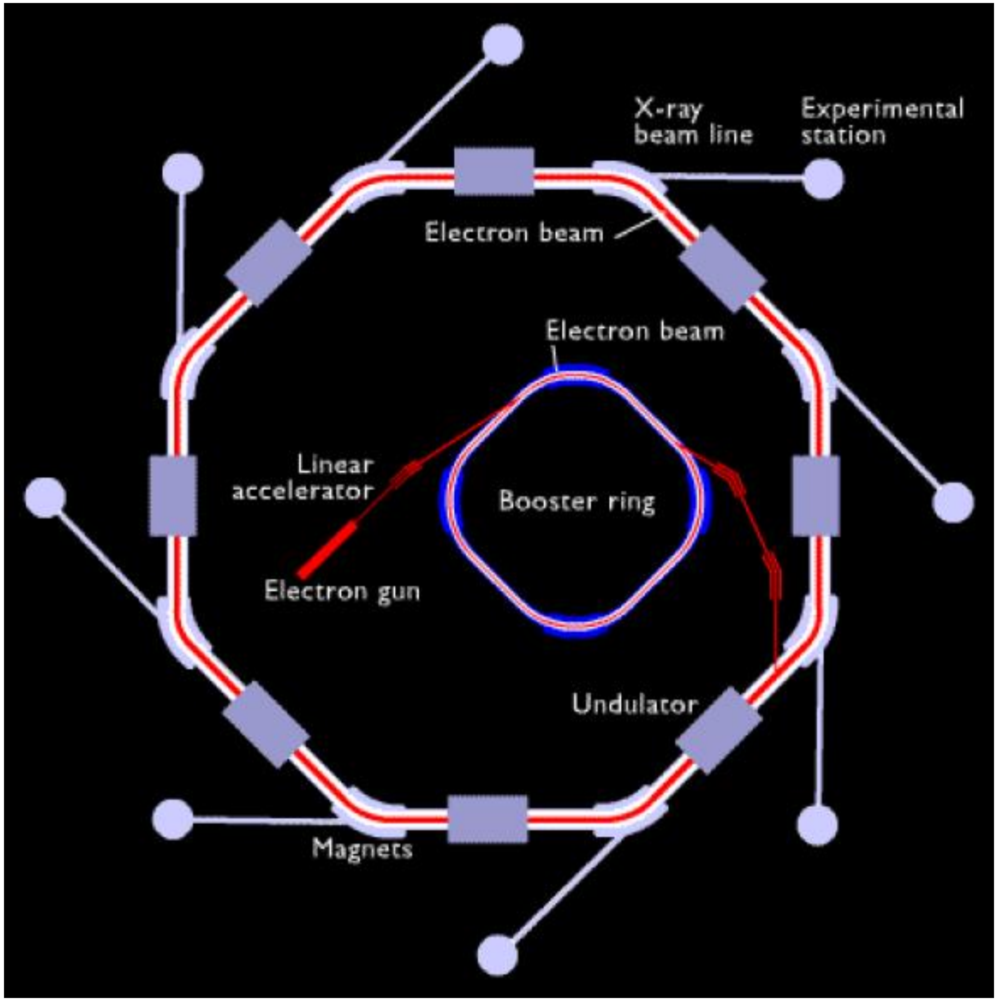
Courtesy K. Wille

Layout of a synchrotron radiation source (I)

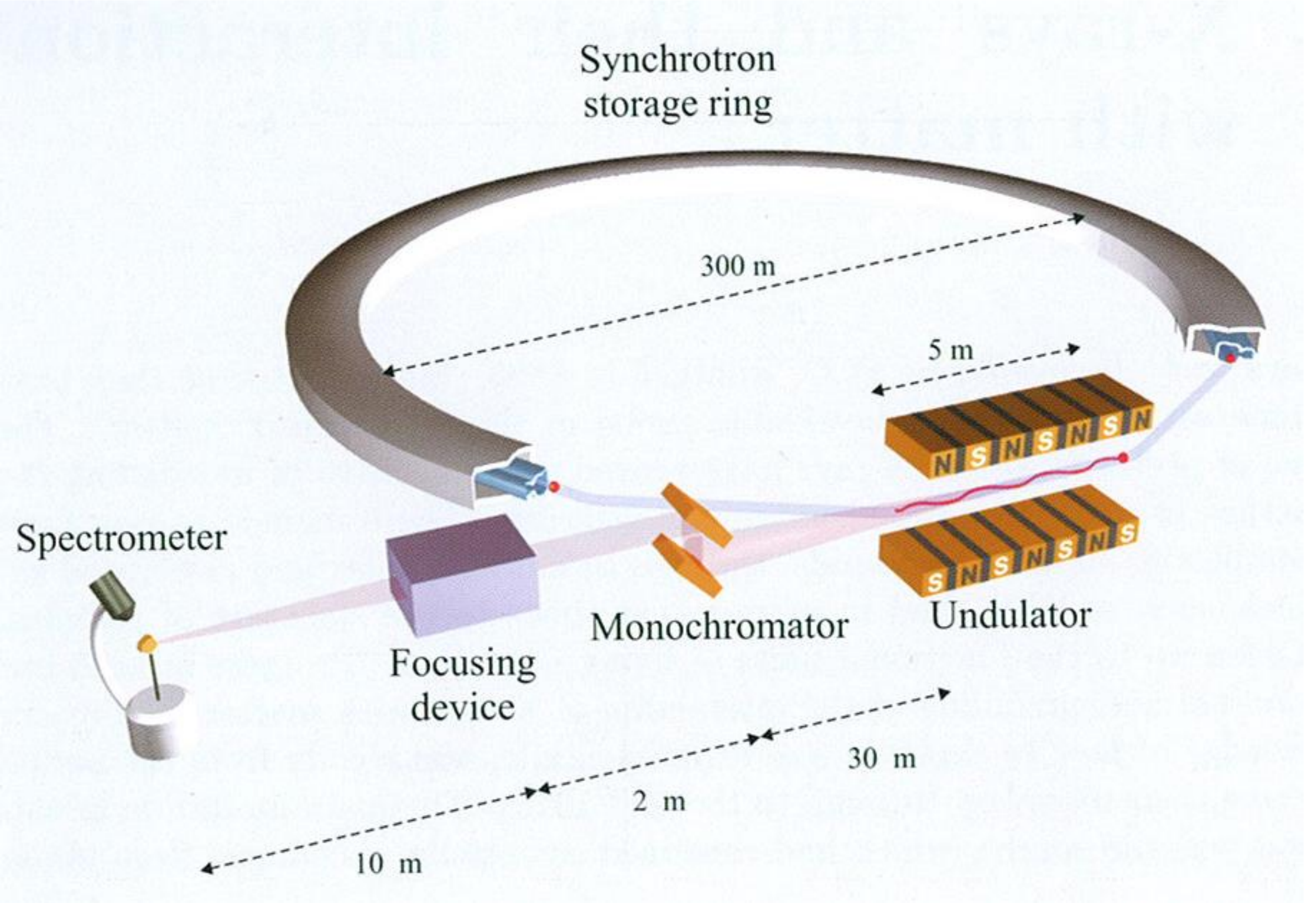
Electrons are generated and accelerated in a linac, further accelerated to the required energy in a booster and injected and stored in the storage ring

The circulating electrons emit an intense beam of synchrotron radiation which is sent down the beamline

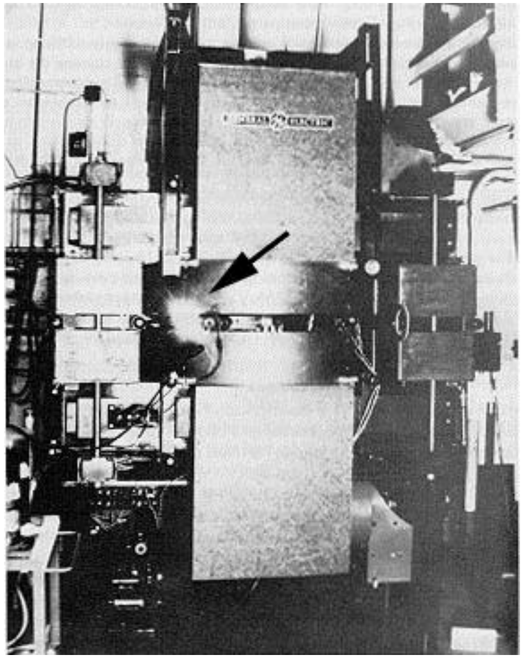




Layout of a synchrotron radiation source (II)



Evolution of synchrotron radiation sources (I)



- **First observation**:

  1947, General Electric, 70 MeV synchrotron

- **First user experiments**:

  1956, Cornell, 320 MeV synchrotron

- **1st generation light sources**: machine built for High Energy Physics or other purposes used parasitically for synchrotron radiation

- **2nd generation light sources**: purpose built synchrotron light sources, SRS at Daresbury was the first dedicated machine (1981 – 2008)

- **3rd generation light sources**: optimised for high brilliance with low emittance and Insertion Devices; ESRF, Diamond, …

Evolution of synchrotron radiation sources (II)

• **4th generation light sources**: photoinjectors LINAC based Free Electron Laser sources;

FLASH (DESY) 2007

LCLS (SLAC) 2009

SACLA (Japan) 2011

Elettra (Italy) 2012

and in the near(?) future

• **4th generation light sources storage ring based**: diffraction limited storage rings

• …and even a **5th generation** with more compact and advanced accelerator technologies e.g. based on laser plasma wakefield accelerators

3rd generation storage ring light sources

1992	**ESRF**, France (EU)	6 GeV
	ALS, US	1.5-1.9 GeV
1993	**TLS,** Taiwan	1.5 GeV
1994	**ELETTRA**, Italy	2.4 GeV
	PLS, Korea	2 GeV
	MAX II, Sweden	1.5 GeV
1996	**APS**, US	7 GeV
	LNLS, Brazil	1.35 GeV
1997	**Spring-8**, Japan	8 GeV
1998	**BESSY II**, Germany	1.9 GeV
2000	**ANKA**, Germany	2.5 GeV
	SLS, Switzerland	2.4 GeV
2004	**SPEAR3**, US	3 GeV
	CLS, Canada	2.9 GeV
2006:	**SOLEIL**, France	2.8 GeV
	DIAMOND, UK	3 GeV
	ASP, Australia3 GeV	
	MAX III, Sweden	700 MeV
	Indus-II, India	2.5 GeV
2008	**SSRF**, China	3.4 GeV
2009	**PETRA-III**, Germany	6 GeV
2011	**ALBA**, Spain	3 GeV

3^{rd} generation storage ring light sources

in commissioning

2014	**NSLS-II**, US	3 GeV
2014	**TPS**, Taiwan	3 GeV

under commissioning

> 2016	**MAX-IV**, Sweden	1.5-3 GeV
	SOLARIS, Poland	1.5 GeV
And then		
	SESAME, Jordan	2.5 GeV
	CANDLE, Armenia	3 GeV

major upgrades

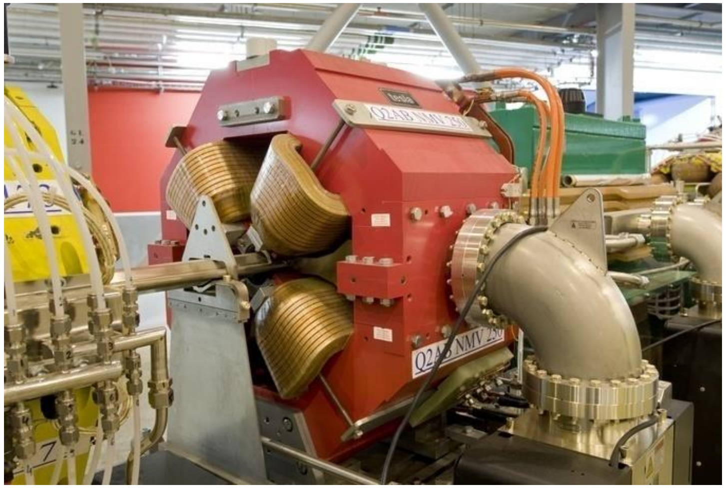
2019	**ESRF-II**, France	6 GeV
> 2020	**Spring8-II**, Japan	6 GeV
	APSU, US	6 GeV

Diamond aerial views

Main components of a storage ring

Dipole magnets to bend the electrons

Quadrupole magnets to focus the electrons

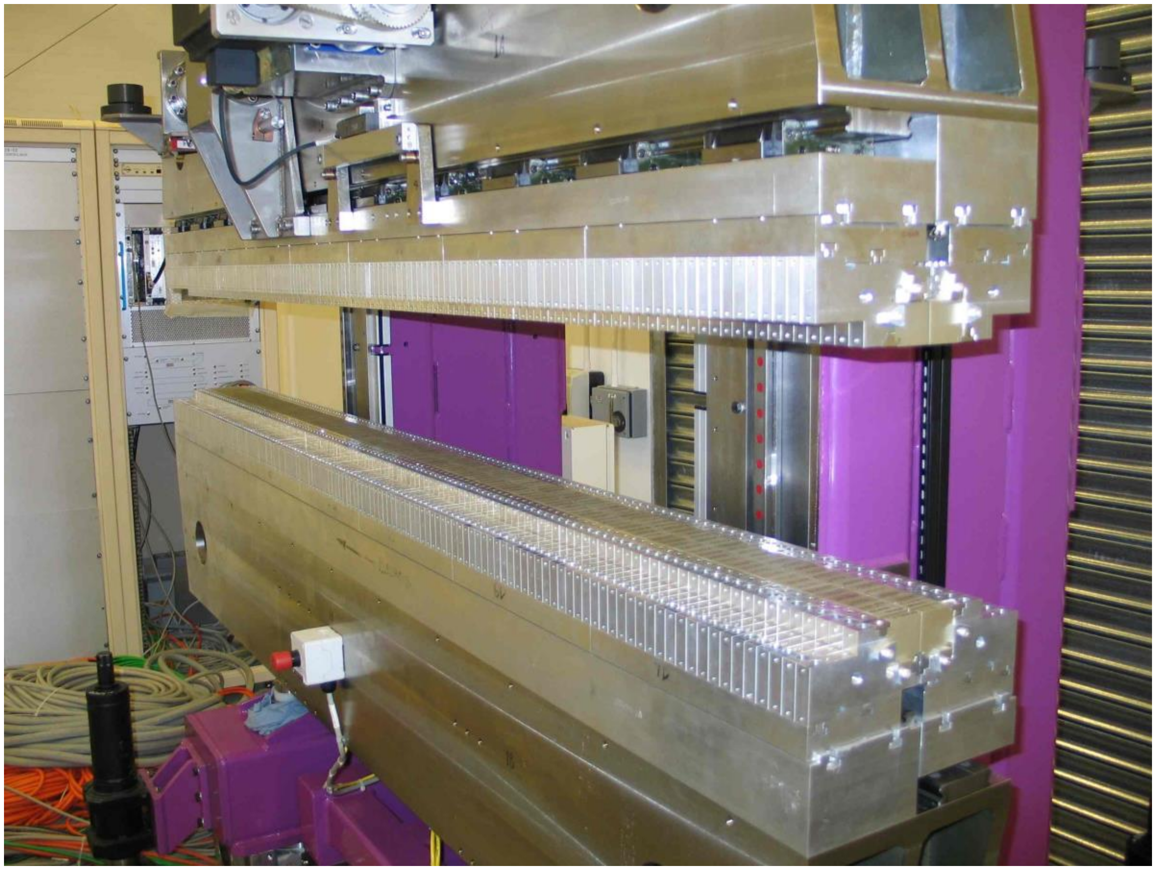


Sextupole magnets to focus off-energy electrons (mainly)

RF cavities to replace energy losses due to the emission of synchrotron radiation

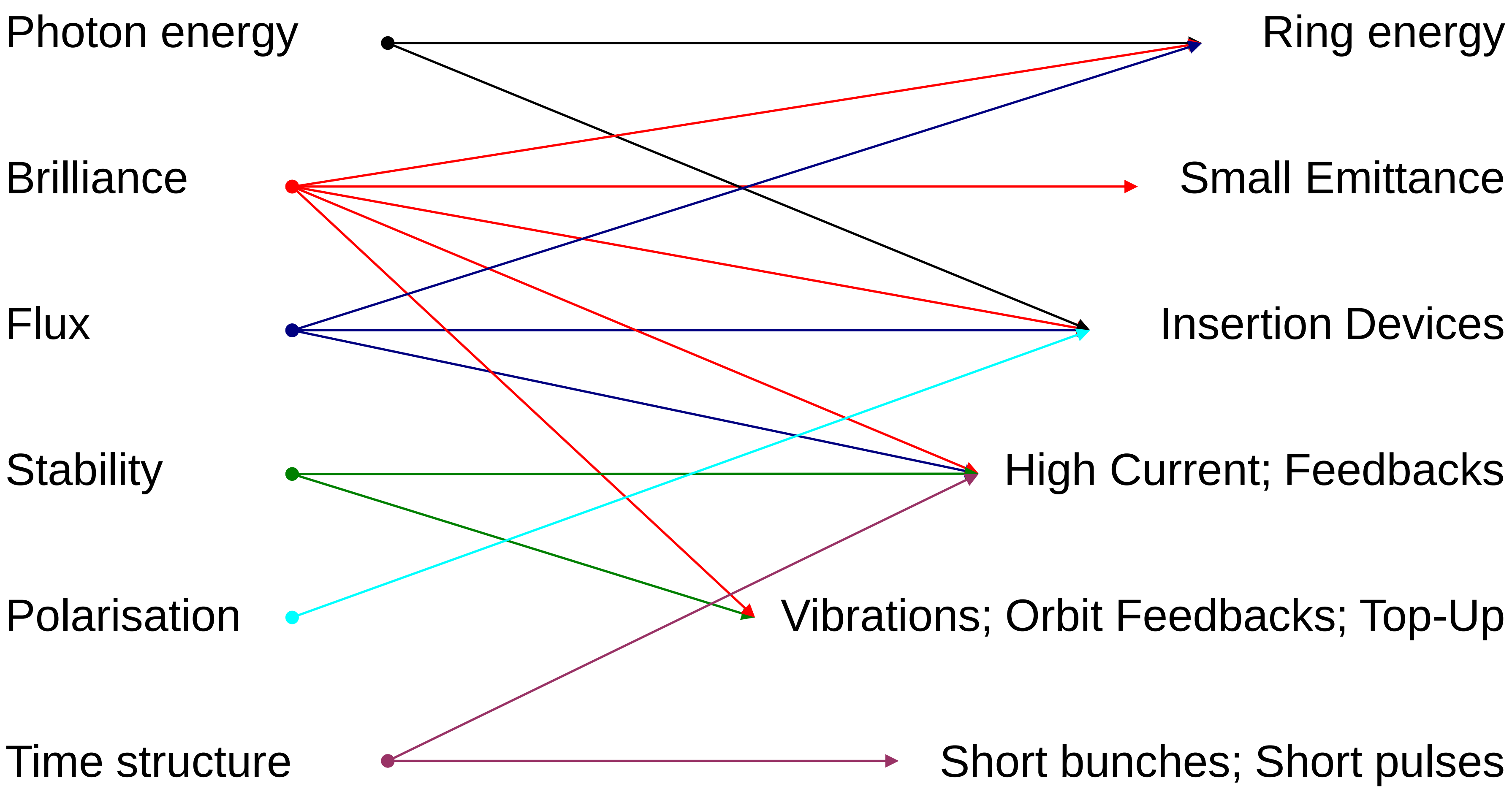
Main components of a storage ring

Insertion devices (undulators) to generate high brilliance radiation



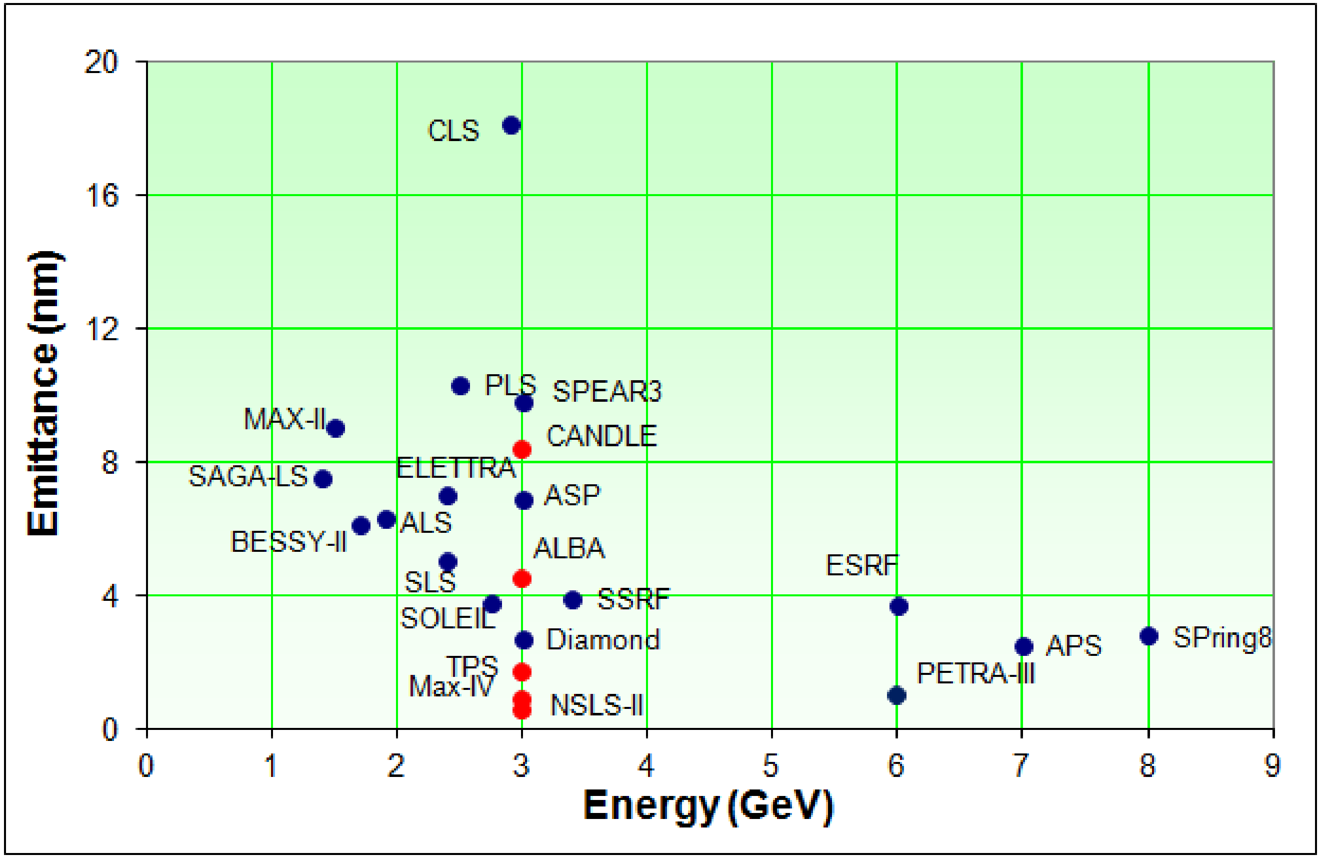
Insertion devices (wiggler) to reach high photon energies

Accelerator physics and technology challenges



Brilliance and low emittance

The brilliance of the photon beam is determined (mostly) by the electron beam



$$\text{brilliance} = \frac{\text{flux}}{4\pi^2 \Sigma_x \Sigma_{x'} \Sigma_y \Sigma_{y'}}$$

$$\Sigma_x = \sqrt{\sigma_{x,e}^2 + \sigma_{ph,e}^2}$$

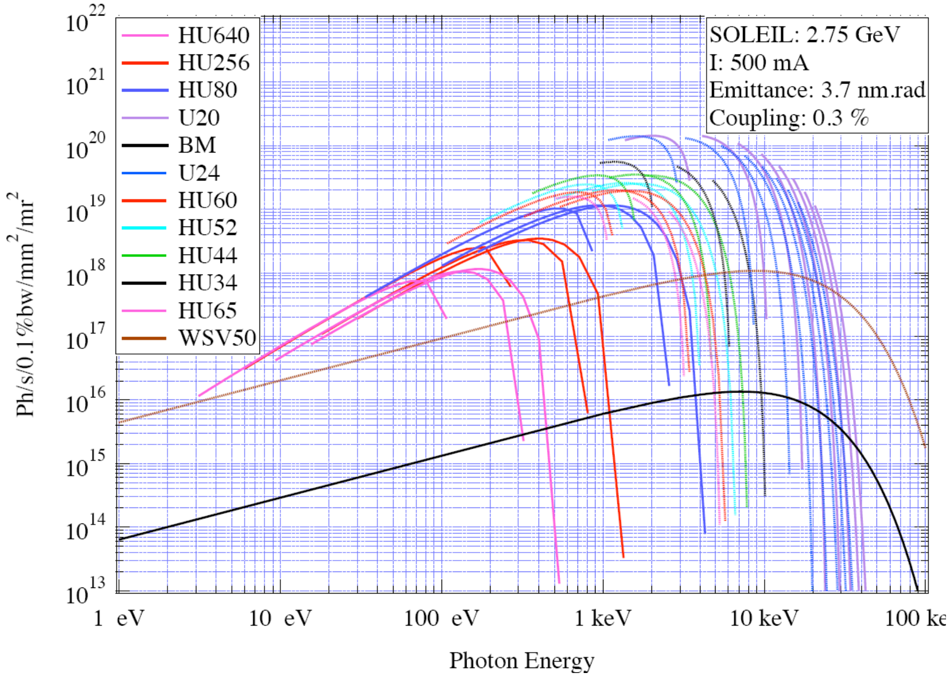
$$\Sigma_{x'} = \sqrt{\sigma_{x',e}^2 + \sigma'^2_{ph,e}}$$

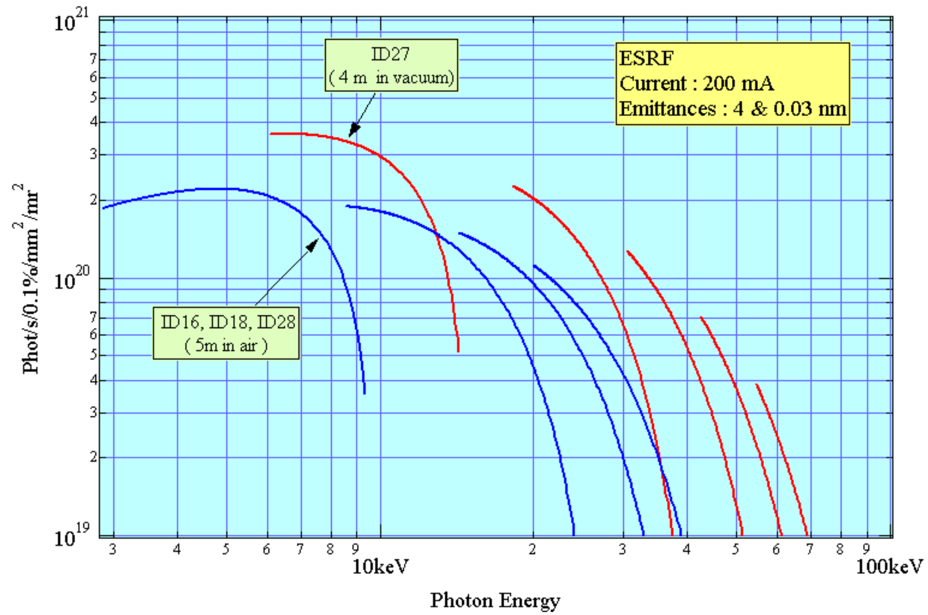
$$\sigma_x = \sqrt{\varepsilon_x \beta_x + (D_x \sigma_\varepsilon)^2}$$

$$\sigma_{x'} = \sqrt{\varepsilon_x / \beta_x + (D'_x \sigma_\varepsilon)^2}$$

Brilliance with IDs

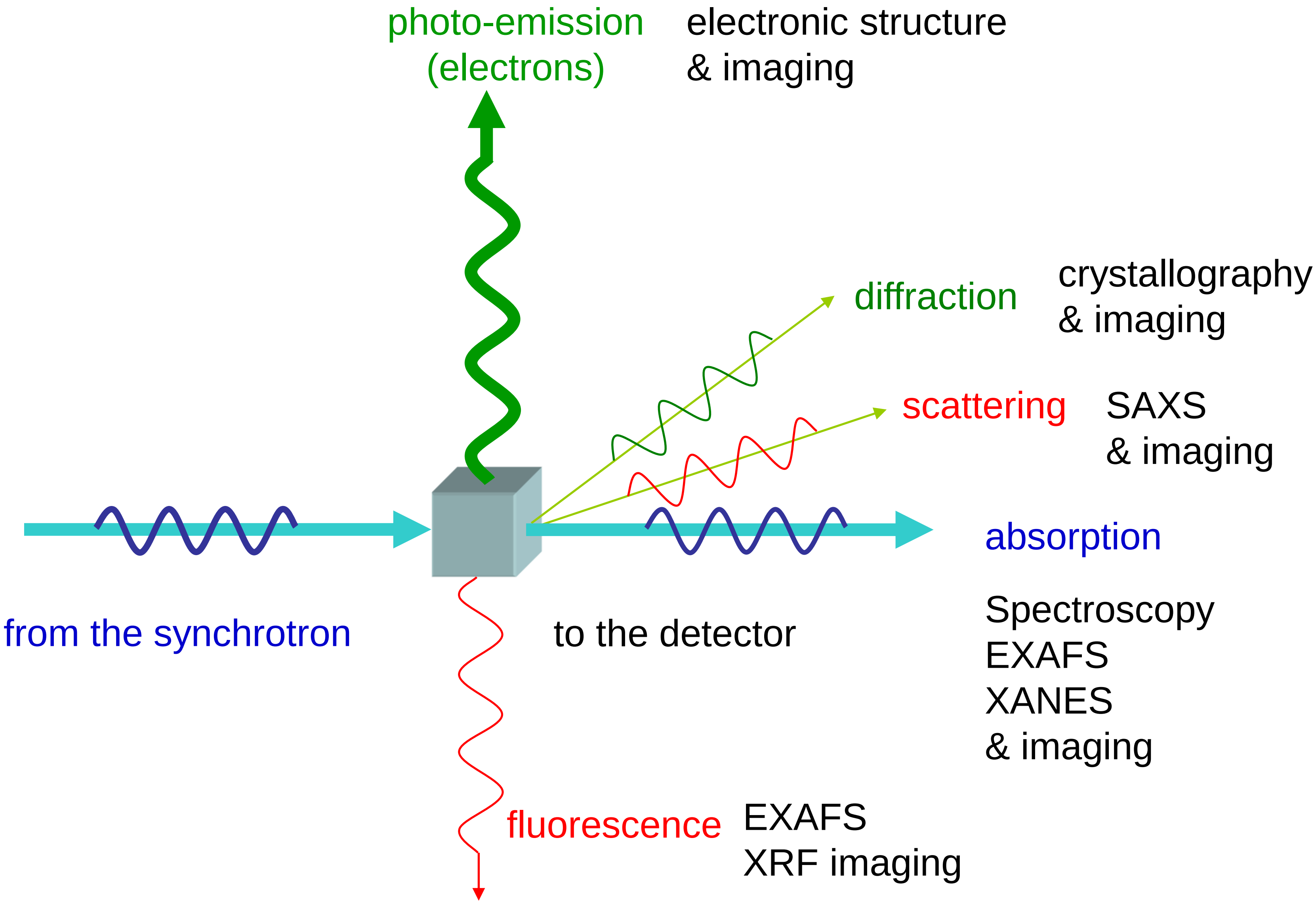
Thanks to the progress with IDs technology storage ring light sources can cover a photon range from few tens of eV to tens 10 keV or more with high brilliance





Medium energy storage rings with In-vacuum undulators operated at low gaps (e.g. 5-7 mm) can reach 10 keV with a brilliance of 10^{20} ph/s/0.1%BW/mm^2/mrad^2

Many ways to use x-rays

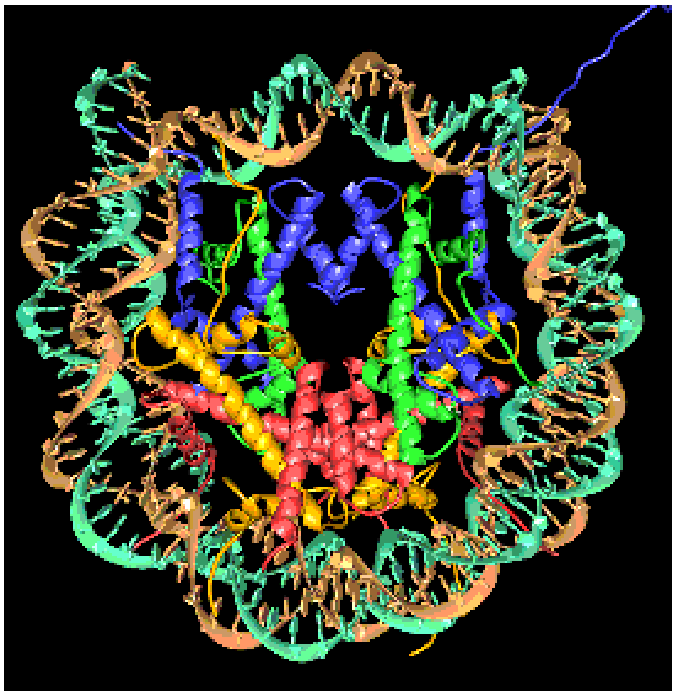


Applications

Medicine, Biology, Chemistry, Material Science, Environmental Science and more

Biology

Reconstruction of the 3D structure of a nucleosome with a resolution of 0.2 nm



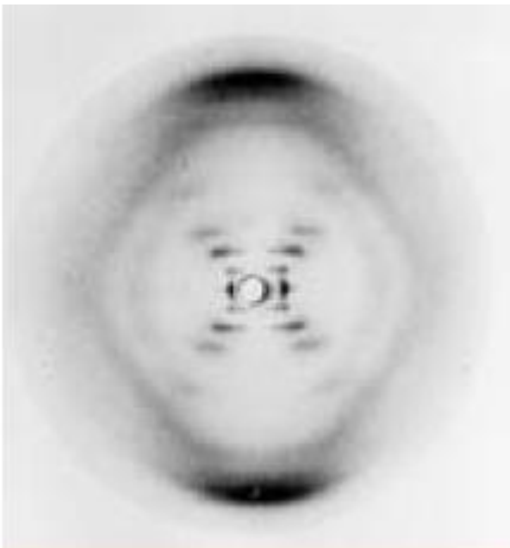
The collection of precise information on the molecular structure of chromosomes and their components can improve the knowledge of how the genetic code of DNA is maintained and reproduced

Archeology

A synchrotron X-ray beam at the SSRL facility illuminated an obscured work erased, written over and even painted over of the ancient mathematical genius Archimedes, born 287 B.C. in Sicily.

X-ray fluorescence imaging revealed the hidden text by revealing the iron contained in the ink used by a 10th century scribe. This x-ray image shows the lower left corner of the page.

Life science examples: DNA and myoglobin



Photograph 51
Franklin-Gosling

DNA (form B)

1952

Franklin and Gosling used a X-ray tube:

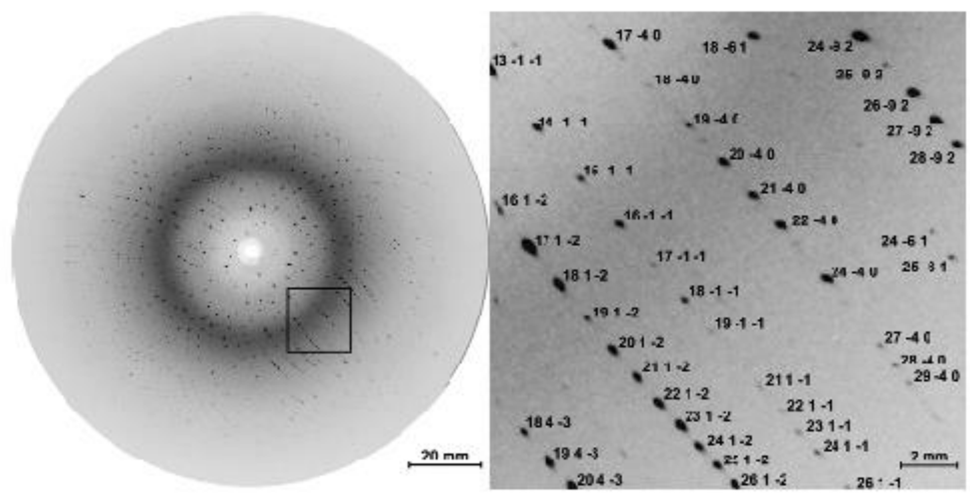
Brilliance was 10^8 (ph/sec/mm^2/mrad^2/0.1BW)

Exposure times of 1 day were typical (10^5 sec)

e.g. Diamond provides a brilliance of 10^{20}

100 ns exposure would be sufficient

Nowadays pump probe experiment in life science are performed using 100 ps pulses from storage ring light sources: e.g. ESRF myoglobin in action



Lienard-Wiechert potentials (I)

We want to compute the em field generated by a charged particle in motion on a given trajectory $\bar{x} = \bar{r}(t)$

The charge density and current distribution of a single particle read

$$\rho(\bar{x}, t) = q\delta^{(3)}(\bar{x} - \bar{r}(t)) \qquad \bar{J}(\bar{x}, t) = q\bar{v}(t)\delta^{(3)}(\bar{x} - \bar{r}(t))$$

We have to solve Maxwell equations driven by such time varying charge density and current distribution.

The general expression for the wave equation for the em potentials (in the Lorentz gauge) reads

$$\bar{\nabla}^2\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \qquad \bar{\nabla}^2\bar{A} - \frac{1}{c^2}\frac{\partial^2\bar{A}}{\partial t^2} = -\mu_0\bar{J}$$

Lienard-Wiechert potentials (II)

The general solutions for the wave equation driven by a time varying charge and current density read (in the Lorentz gauge) [Jackson Chap. 6]

$$\Phi(\bar{x},t)=\frac{1}{4\pi\varepsilon_0}\int d^3\bar{x}'\int dt'\frac{\rho(\bar{x}',t')}{|\bar{x}-\bar{x}'|}\delta\left(t'+\frac{|\bar{x}-\bar{x}'|}{c}-t\right) \qquad A(\bar{x},t)=\frac{1}{4\pi\varepsilon_0 c^2}\int d^3\bar{x}'\int dt'\frac{\bar{J}(\bar{x}',t')}{|\bar{x}-\bar{x}'|}\delta\left(t'+\frac{|\bar{x}-\bar{x}'|}{c}-t\right)$$

Integrating the Dirac delta in time we are left with

$$\Phi(\bar{x},t)=\frac{1}{4\pi\varepsilon_0}\iiint_V \frac{\rho(\bar{x}',t_{ret})}{|\bar{x}-\bar{x}'|}d^3\bar{x}' \qquad \bar{A}(\bar{x},t)=\frac{\mu_0}{4\pi}\iiint_V \frac{\bar{J}(\bar{x}',t_{ret})}{|\bar{x}-\bar{x}'|}d^3\bar{x}'$$

where ret means retarted $t_{ret}=t-\frac{|\bar{x}(t)-\bar{x}(t_{ret})|}{c}$ **(see next slide)**

Now we use the charge density and current distribution of a single particle

$$\rho(\bar{x},t)=q\delta^{(3)}(\bar{x}-\bar{r}(t)) \qquad \bar{J}(\bar{x},t)=q\bar{v}(t)\delta^{(3)}(\bar{x}-\bar{r}(t))$$

Lienard-Wiechert potentials (III)

Substituting we get

$$\Phi(\bar{x},t)=\frac{q}{4\pi\varepsilon_0}\iiint_V \frac{\delta^{(3)}[\bar{x}'-\bar{r}(t_{ret})]}{|\bar{x}-\bar{x}'|}d^3\bar{x}' \qquad \bar{A}(\bar{x},t)=\frac{q\mu_0}{4\pi}\iiint_V \frac{\bar{v}(t_{ret})[\bar{x}'-\bar{r}(t_{ret})]}{|\bar{x}-\bar{x}'|}d^3\bar{x}'$$

Using again the properties of the Dirac deltas we can integrate and obtain the Lienard-Wiechert potentials

$$\Phi(\bar{x},t)=\frac{1}{4\pi\varepsilon_0}\left[\frac{e}{(1-\bar{\beta}\cdot\bar{n})R}\right]_{ret} \qquad \bar{A}(\bar{x},t)=\frac{1}{4\pi\varepsilon_0 c}\left[\frac{e\bar{\beta}}{(1-\bar{\beta}\cdot\bar{n})R}\right]_{ret}$$

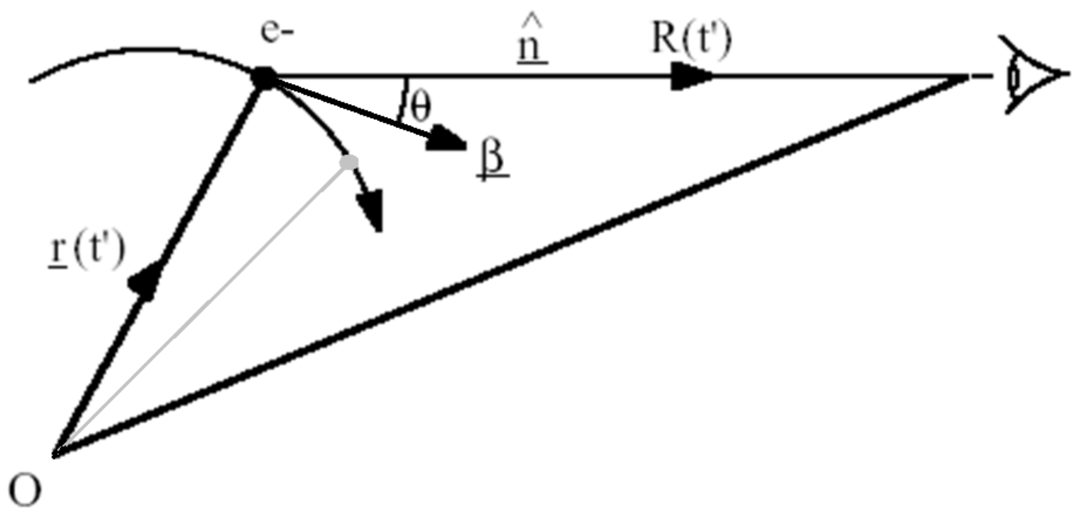
These are the potentials of the em fields generated by the charged particle in motion.

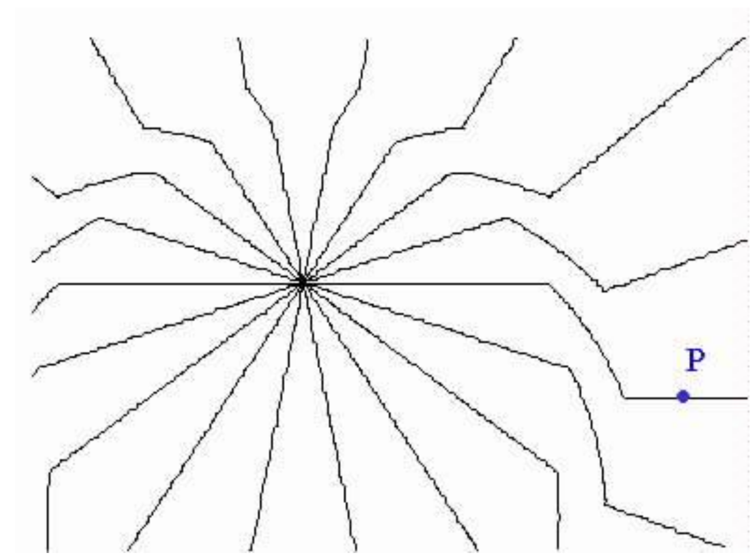
The trajectory itself is determined by external electric and magnetic fields

Lienard-Wiechert Potentials (IV)

[ ]$_{ret}$ means computed at time t'

$$t = t' + \frac{R(t')}{c}$$





Potentials and fields at position x at time t are determined by the characteristic of the electron motion at a time t'

t – t' is the time it takes for the em radiation to travel the distance R(t')

i.e. grey is the position of the electron at time t

The Lienard-Wiechert fields

The electric and magnetic fields are computed from the potentials using

$$\overline{\mathrm{B}} = \overline{\nabla} \wedge \overline{\mathrm{A}} \qquad \mathrm{E} = -\frac{\partial \overline{\mathrm{A}}}{\partial \mathrm{t}} - \overline{\nabla}\phi$$

and are called Lienard-Wiechert fields

The computation has to be done carefully since the potentials depend on *t* via t'*.* The factor *dt/dt'* represents the Doppler factor. We get

$$\overline{E}(\overline{x},t) = \frac{e}{4\pi\varepsilon_0}\left[\frac{\overline{n}-\overline{\beta}}{\gamma^2(1-\overline{\beta}\cdot\overline{n})^3 R^2}\right]_{ret} + \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\overline{n}\times(\overline{n}-\overline{\beta})\times\dot{\overline{\beta}}}{(1-\overline{\beta}\cdot\overline{n})^3 R}\right]_{ret} \qquad \overline{B}(\overline{x},t) = \frac{1}{c}\left[\overline{n}\times\overline{E}\,\right]_{rit}$$

velocity field $\propto \frac{1}{\mathrm{R}^2}$ **acceleration field** $\propto \frac{1}{\mathrm{R}}$

If we consider the acceleration field we have $\vec{E} \perp \overline{B} \perp \hat{n}$

and the correct dependence as 1/R as for radiation field

Power radiated

Power radiated by a particle on a surface is the flux of the Poynting vector

$$\overline{S} = \frac{1}{\mu_0}\overline{E} \times \overline{B} \qquad \Phi_\Sigma(\overline{S})(t) = \iint_\Sigma \overline{S}(\overline{x}, t) \cdot \overline{n}\, d\Sigma$$

Angular distribution of radiated power

$$\frac{d^2P}{d\Omega} = (\overline{S} \cdot \overline{n})(1 - \overline{n} \cdot \overline{\beta})R^2$$

radiation emitted by the particle

We will analyse two cases:

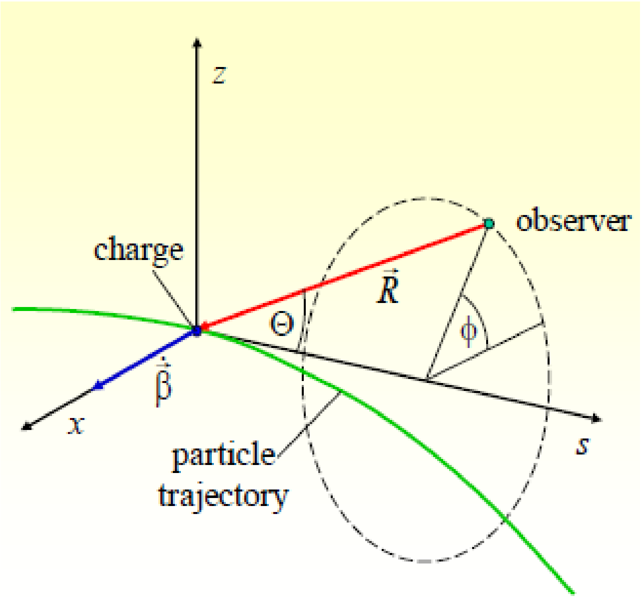
acceleration orthogonal to the velocity → synchrotron radiation

acceleration parallel to the velocity → bremmstrahlung

Synchrotron radiation: non relativistic motion (I)

Assuming $\overline{\beta} \approx \overline{0}$ **and substituting the acceleration field**

$$\overline{E}_{acc}(\overline{x},t) = \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\overline{n}\times(\overline{n}\times\dot{\overline{\beta}})}{R}\right]_{ret}$$



The angular distribution of the power radiated is given by

$$\frac{d^2P}{d\Omega} = \frac{1}{\mu_0 c}\left|R\overline{E}_{acc}\right|^2 = \frac{e^2}{(4\pi)^2\varepsilon_0\, c}\left|\overline{n}\times(\overline{n}\times\dot{\overline{\beta}})\right|^2$$

Working out the double cross product

$$\vec{a}\times\left(\vec{b}\times\vec{c}\right) = \vec{b}(\vec{a}\vec{c}) - \vec{c}\left(\vec{a}\vec{b}\right) \text{ and } \vec{n}\vec{n} = \vec{n}^2 = 1$$

We have

$$\left(\vec{n}\times\left[\vec{n}\times\dot{\vec{\beta}}^*\right]\right)^2 = \left(\vec{n}\left(\vec{n}\,\dot{\vec{\beta}}^*\right) - \dot{\vec{\beta}}^*(\vec{n}\vec{n})\right)^2$$

$$= \vec{n}^2\left(\vec{n}\dot{\vec{\beta}}^*\right)^2 - 2\vec{n}\left(\vec{n}\,\dot{\vec{\beta}}^*\right)\dot{\vec{\beta}}^* + \dot{\vec{\beta}}^{*2} = \dot{\vec{\beta}}^{*2} - \left(\vec{n}\,\dot{\vec{\beta}}^*\right)^2$$

Synchrotron radiation: non relativistic motion (II)

Since

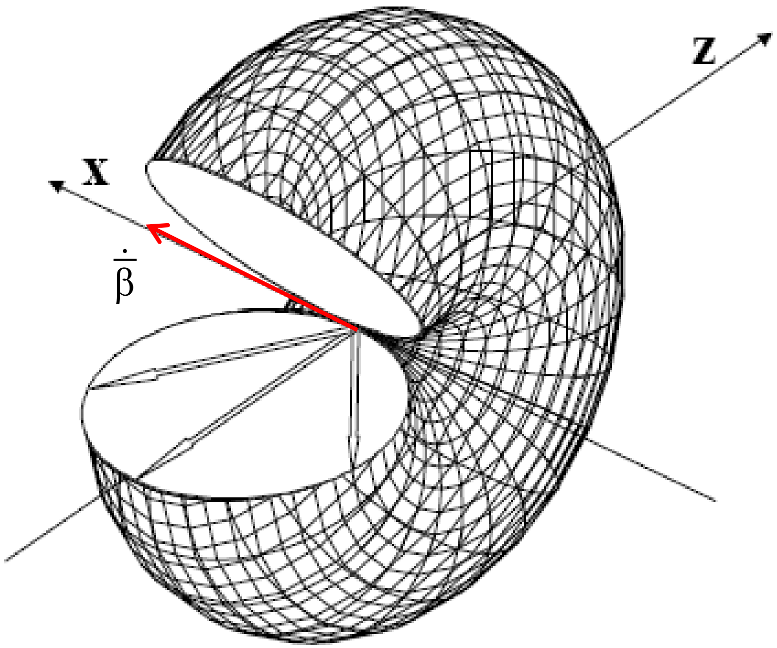
$$\vec{n}\,\dot{\vec{\beta}}^{*} = |\vec{n}|\left|\dot{\vec{\beta}}^{*}\right|\cos\Theta = \left|\dot{\vec{\beta}}^{*}\right|\cos\Theta$$

where θ is the angle between the acceleration and the observation direction, we finally get

$$\left(\vec{n}\times\left[\vec{n}\times\dot{\vec{\beta}}^{*}\right]\right)^{2} = \dot{\vec{\beta}}^{*2} - \dot{\vec{\beta}}^{*2}\cos^{2}\Theta = \dot{\vec{\beta}}^{*2}\left(1-\cos^{2}\Theta\right) = \dot{\vec{\beta}}^{*2}\sin^{2}\Theta$$

The *angular distribution* of power reads

$$\frac{d^{2}P}{d\Omega} = \frac{e^{2}}{(4\pi)^{2}\,\varepsilon_{0}c}\left|\dot{\vec{\beta}}\right|^{2}\sin^{2}\theta$$



Synchrotron radiation: non relativistic motion (III)

Integrating over the angles gives the total radiated power

$$P = \frac{e^2}{(4\pi)^2 c\varepsilon_0} \dot{\vec{\beta}}^{*2} \int_0^{2\pi}\int_0^{\pi} \sin^3\Theta d\Theta d\phi$$

This integral gives the total instantaneous power radiated

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\bar{\beta}}\right|^2$$

Larmor's formula

It shows that radiation is emitted when the particle is accelerated. Using

$$\bar{\beta} = \frac{\bar{p}}{mc}$$

we have (to be used later for the generalisation to the relativistic case)

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{d\bar{p}}{dt}\right|^2$$

Synchrotron radiation: relativistic motion (I)

In the relativistic case the total radiated power is computed in the same way. Using only the acceleration field (large R)

$$\bar{E}(\bar{x},t) \sim \frac{e}{4\pi\varepsilon_0 c}\left[\frac{\bar{n}\times(\bar{n}-\bar{\beta})\times\dot{\bar{\beta}}}{(1-\bar{\beta}\cdot\bar{n})^3 R}\right]_{ret}$$

The angular distribution of the power emitted is (use the retarted time!)

$$\frac{d^2P}{d\Omega} = \frac{1}{\mu_o c}\left|R\bar{E}_{acc}\right|^2(1-\bar{\beta}\cdot\bar{n}) = \frac{e^2}{(4\pi)^2\varepsilon_0\, c}\frac{\left|\bar{n}\times\left[(\bar{n}-\bar{\beta})\times\dot{\bar{\beta}}\right]\right|^2}{(1-\bar{n}\cdot\bar{\beta})^5}$$

The emission is peaked in the direction of the velocity

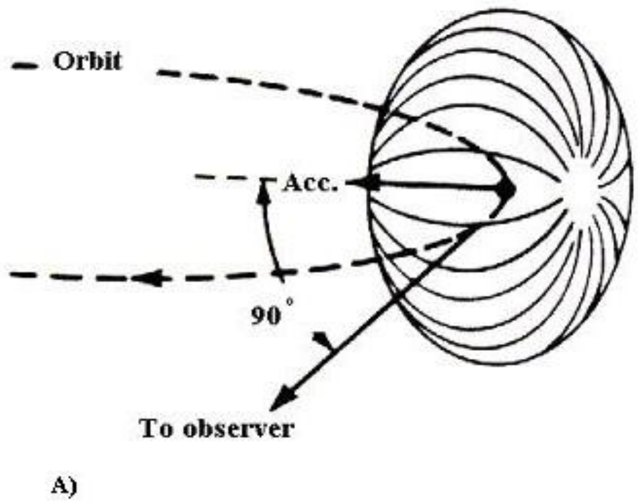
The pattern depends on the details of velocity and acceleration but it is dominated by the denominator

velocity ⊥ acceleration: synchrotron radiation (I)

Assuming $\bar{\beta} \perp \dot{\bar{\beta}}$ **and substituting the acceleration field**

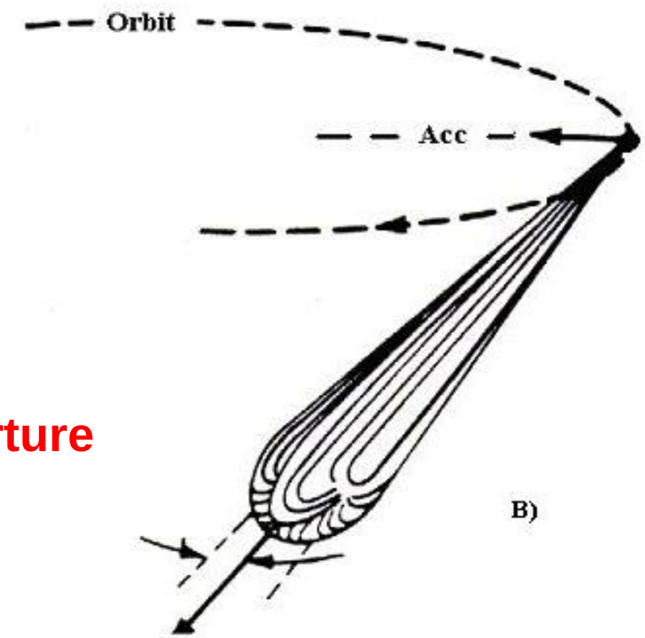
we have the *angular distribution of the radiated power*

$$\frac{d^2P}{d\Omega} = \frac{e^2}{(4\pi)^2 \varepsilon_0 c^2} \frac{\left|\bar{n} \times \left[(\bar{n}-\bar{\beta}) \times \dot{\bar{\beta}}\right]\right|^2}{(1-\bar{n}\cdot\bar{\beta})^5} = \frac{e^2 \left|\dot{\bar{\beta}}\right|^2}{(4\pi)^2 \varepsilon_0 c} \frac{1}{(1-\beta\cos\theta)^3} \left[1 - \frac{\sin^2\theta\cos^2\phi}{\gamma^2(1-\beta\cos\theta)^2}\right]$$



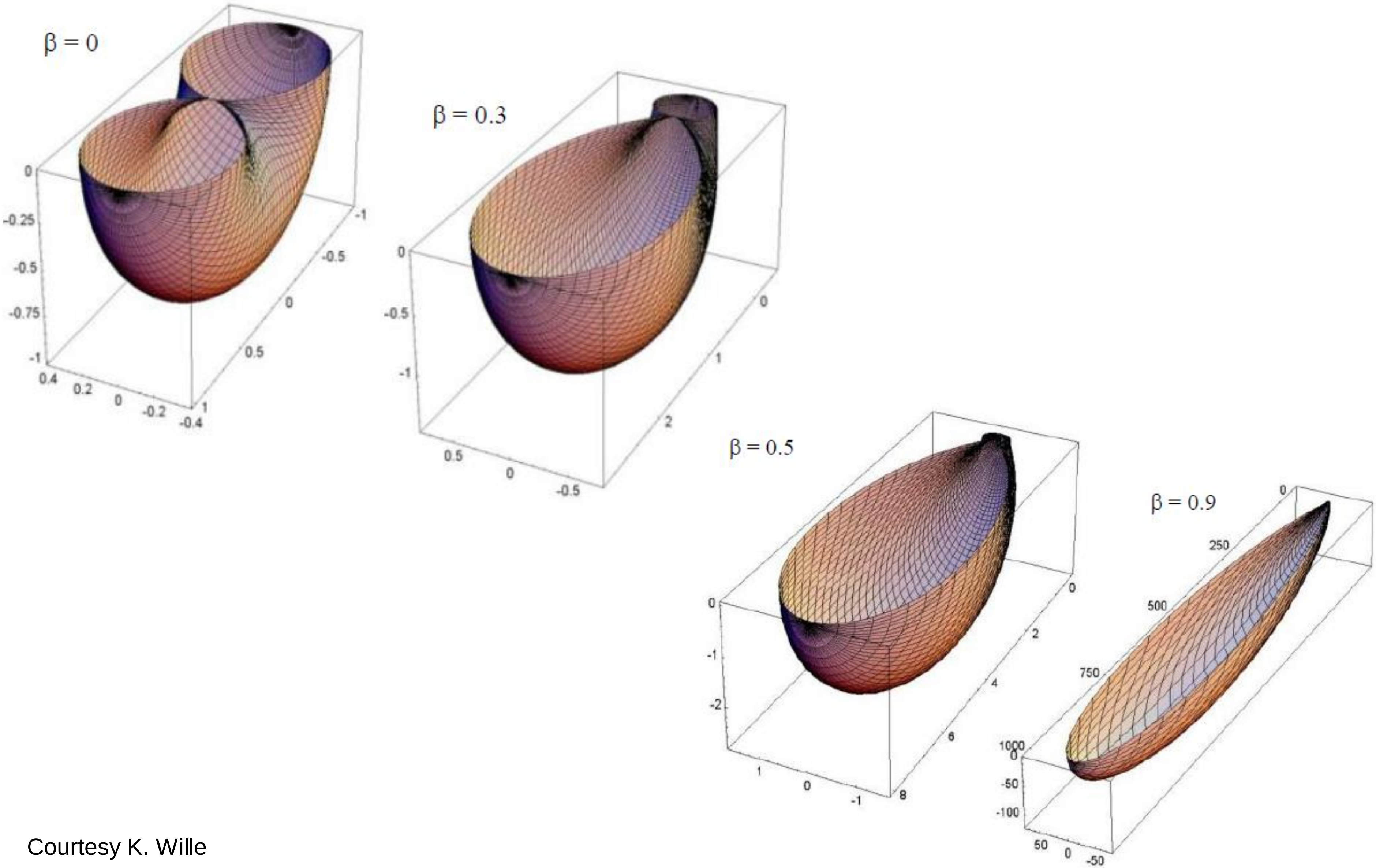
cone aperture

~ 1/γ



When the electron velocity approaches the speed of light, the emission pattern is sharply collimated forward

velocity ⊥ acceleration: synchrotron radiation (II)



Courtesy K. Wille

Total radiated power via synchrotron radiation

Integrating over the whole solid angle we obtain the total instantaneous power radiated by one electron

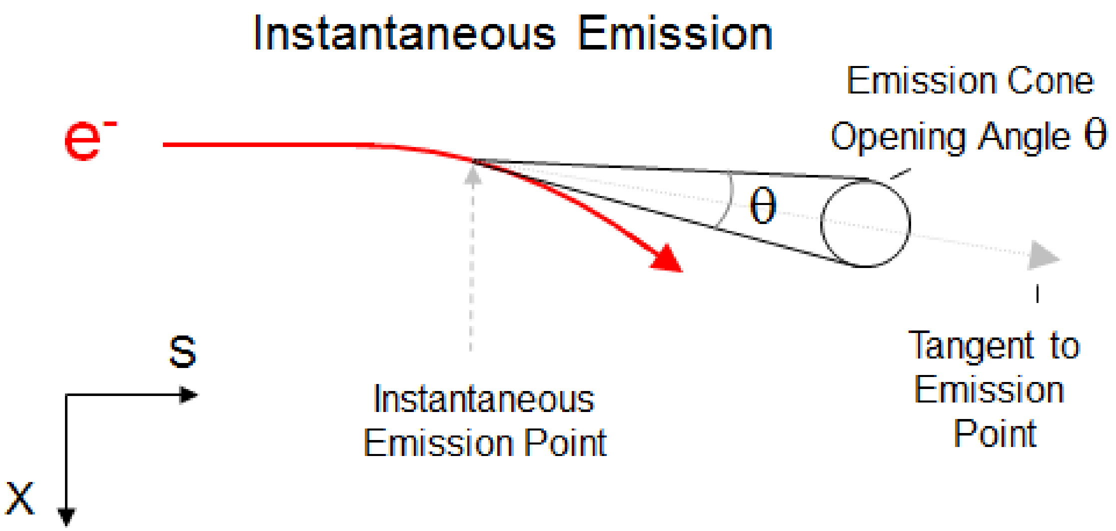
$$P = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\vec{\beta}}\right|^2 \gamma^4 = \frac{e^2}{6\pi\varepsilon_0 c}\left|\dot{\vec{\beta}}\right|^2 \frac{E}{E_o^4}^4 = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{d\vec{p}}{dt}\right|^2 \gamma^2 = \frac{e^2 c}{6\pi\varepsilon_0}\frac{\gamma^4}{\rho^2} = \frac{e^4}{6\pi\varepsilon_0 m^4 c^5} E^2 B^2$$

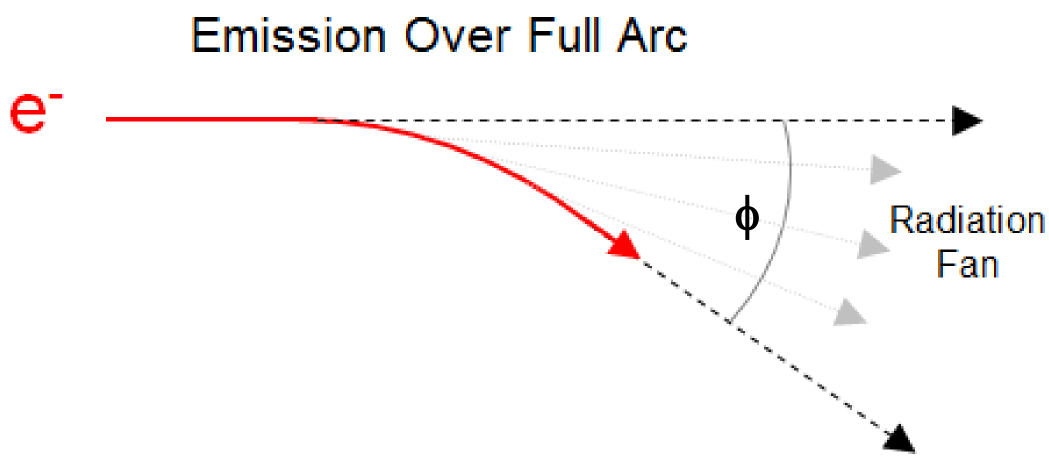
- **Strong dependence $1/m^4$ on the rest mass**
- **proportional to $1/\rho^2$ (ρ is the bending radius)**
- **proportional to B^2 (B is the magnetic field of the bending dipole)**

The radiation power emitted by an electron beam in a storage ring is very high.

The surface of the vacuum chamber hit by synchrotron radiation must be cooled.

Radiation from a bending magnet





Assuming that the total power is radiated in one turn (in a uniform distribution) in the angle ϕ

The angular distribution of the power emitted in ϕ (integrated in the vertical aperture) is

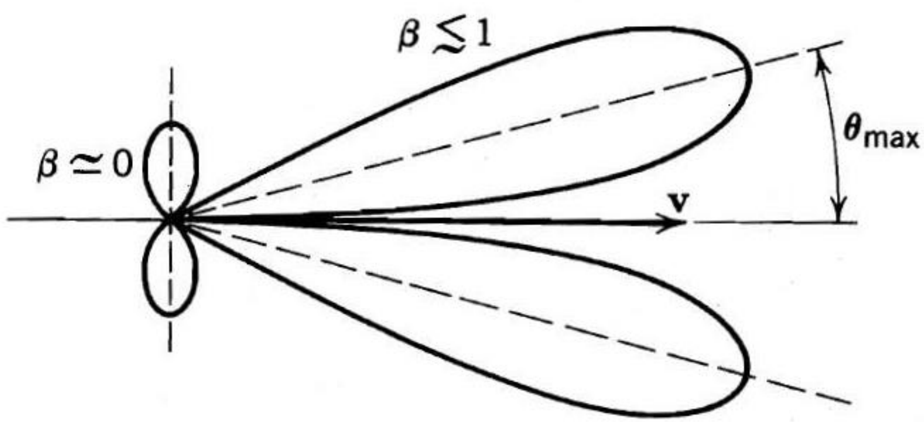
$$\frac{dP}{d\phi} = \frac{P}{2\pi} = \frac{e^2 |\dot{\vec{v}}|^2}{12\pi^2 \varepsilon_0 c^3} \gamma^4$$

Do not mix up ϕ and θ…

velocity || acceleration: bremsstrahlung

Assuming $\bar{\beta} \parallel \dot{\bar{\beta}}$ and substituting the acceleration field

$$\frac{dP}{d\Omega} = \frac{e^2}{(4\pi)^2 \varepsilon_0 c} \frac{\left| \bar{n} \times \left[(\bar{n} - \bar{\beta}) \times \dot{\bar{\beta}} \right] \right|^2}{(1 - \bar{n} \cdot \bar{\beta})^5} = \frac{e^2 \left| \dot{\bar{\beta}} \right|^2}{(4\pi)^2 \varepsilon_0 c} \frac{\sin^2 \theta}{(1 - \beta \cos \theta)^5}$$



$$\theta_{max} = \arccos\left[\frac{1}{3\beta} \left(\sqrt{1 + 15\beta^2} - 1 \right) \right] \rightarrow \frac{1}{2\gamma}$$

Integrating over the angles as before gives the total radiated power

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3} \left| \frac{d\bar{p}}{dt} \right|^2$$

Comparison of radiation from linear and circular trajectories (I)

Back to the general expression for the acceleration field, integrating over the angles gives the total radiated power

$$P = \frac{e^2}{6\pi\varepsilon_0 c}\gamma^6\left[(\dot{\bar{\beta}})^2 - (\bar{\beta}\times\dot{\bar{\beta}})^2\right]$$

Relativistic generalization of Larmor's formula

The total radiated power can also be computed by relativistic transformation of the 4-acceleration in Larmor's formula

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{d\bar{p}}{dt}\right|^2$$

with $t \rightarrow \tau$ and $\frac{d\bar{p}}{dt} \rightarrow \frac{dp_\mu}{dt}$

We build the relativistic invariant

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left[\left|\frac{d\bar{p}}{d\tau}\right|^2 - \frac{1}{c^2}\left(\frac{dE}{d\tau}\right)^2\right]$$

Comparison of radiation from linear and circular trajectories (II)

The particle energy is

$$E^2 = (mc^2)^2 + p^2c^2$$

Therefore

$$E\frac{dE}{d\tau} = c^2 p \frac{dp}{d\tau} \quad \textbf{and} \quad \frac{dE}{d\tau} = v\frac{dp}{d\tau}$$

Inserting in the formula for the power radiated we get

$$P_s = \frac{e^2c}{6\pi\varepsilon_0(m_0c^2)^2}\left[\left(\frac{dp}{d\tau}\right)^2 - \left(\frac{v}{c}\right)^2\left(\frac{dp}{d\tau}\right)^2\right] = \frac{e^2c}{6\pi\varepsilon_0(m_0c^2)^2}(1-\beta^2)\left(\frac{dp}{d\tau}\right)^2$$

For a linear trajectory (mind the proper time in the relativistic invariant)

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2c^3}\left|\frac{d\overline{p}}{dt}\right|^2$$

Comparison of radiation from linear and circular trajectories (III)

Repeating for a circular trajectory

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left[\left|\frac{d\bar{p}}{d\tau}\right|^2 - \frac{1}{c^2}\left(\frac{dE}{d\tau}\right)^2\right]$$

The particle energy is now constant

$$E^2 = (mc^2)^2 + p^2c^2$$

Therefore

$$\frac{dE}{d\tau} = 0$$

Inserting in the formula we get the total radiated power in a circular trajectory

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\gamma^2\left|\frac{d\bar{p}}{dt}\right|^2$$

This is γ^2 larger than the linear case

$P(v \parallel a) \approx 1/\gamma^2\, P(v \perp a)$

Comparison of radiation from linear and circular trajectories (IV)

In the case of linear acceleration

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{d\overline{p}}{dt}\right|^2$$

We get

$$dp/dt = (c\,dp)/(c\,dt) = dE/dx$$

and

$$P = \frac{e^2}{6\pi\varepsilon_0 m^2 c^3}\left|\frac{dE}{dx}\right|^2 \qquad \frac{dE}{dx} \sim 20\,\frac{\text{MeV}}{\text{m}}$$

P is very small!!

$P(v \parallel a) \approx 1/\gamma^2\, P(v \perp a)$

Energy loss via synchrotron radiation emission in a storage ring

In the time T_b spent in the bendings the particle loses the energy U_0

$$U_0 = \int P dt = P T_b = P \frac{2\pi\rho}{c} = \frac{e^2}{3\varepsilon_0} \frac{\gamma^4}{\rho}$$

i.e. Energy Loss per turn (per electron)

$$U_0(keV) = \frac{e^2\gamma^4}{3\varepsilon_0\rho} = 88.46 \frac{E(GeV)^4}{\rho(m)}$$

Power radiated by a beam of average current I_b: this power loss has to be compensated by the RF system

$$N_{tot} = \frac{I_b \cdot T_{rev}}{e}$$

$$P(kW) = \frac{e\gamma^4}{3\varepsilon_0\rho} I_b = 88.46 \frac{E(GeV)^4 I(A)}{\rho(m)}$$

Power radiated by a beam of average current I_b in a dipole of length L (energy loss per second)

$$P(kW) = \frac{e\gamma^4}{6\pi\varepsilon_0\rho^2} L I_b = 14.08 \frac{L(m) I(A) E(GeV)^4}{\rho(m)^2}$$

Spectrum: the radiation integral (I)

The energy received by an observer (per unit solid angle at the source) is

$$\frac{d^2W}{d\Omega} = \int_{-\infty}^{\infty} \frac{d^2P}{d\Omega} dt = c\varepsilon_0 \int_{-\infty}^{\infty} |R\overline{E}(t)|^2 \, dt$$

Using the Fourier Transform we move to the frequency space

$$\frac{d^2W}{d\Omega} = 2c\varepsilon_0 \int_{0}^{\infty} |R\overline{E}(\omega)|^2 \, d\omega$$

Angular and frequency distribution of the energy received by an observer

$$\frac{d^3W}{d\Omega d\omega} = 2\varepsilon_0 c R^2 \left|\hat{\overline{E}}(\omega)\right|^2$$

Neglecting the velocity fields and assuming the observer in the <u>far field</u>: n constant, R constant

$$\frac{d^3W}{d\Omega d\omega} = \frac{e^2}{4\pi\varepsilon_0 4\pi^2 c} \left| \int_{-\infty}^{\infty} \frac{\overline{n} \times \left[(\overline{n} - \overline{\beta}) \times \dot{\overline{\beta}} \right]}{(1 - \overline{n} \cdot \overline{\beta})^2} e^{i\omega(t - \overline{n} \cdot \overline{r}(t)/c)} dt \right|^2$$

Radiation Integral

The radiation integral (II)

The radiation integral can be simplified to [see Jackson]

$$\frac{d^3W}{d\Omega d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\infty}^{\infty} \bar{n}\times(\bar{n}\times\bar{\beta})e^{i\omega(t-\bar{n}\cdot\bar{r}(t)/c)}dt\right|^2$$

How to solve it?

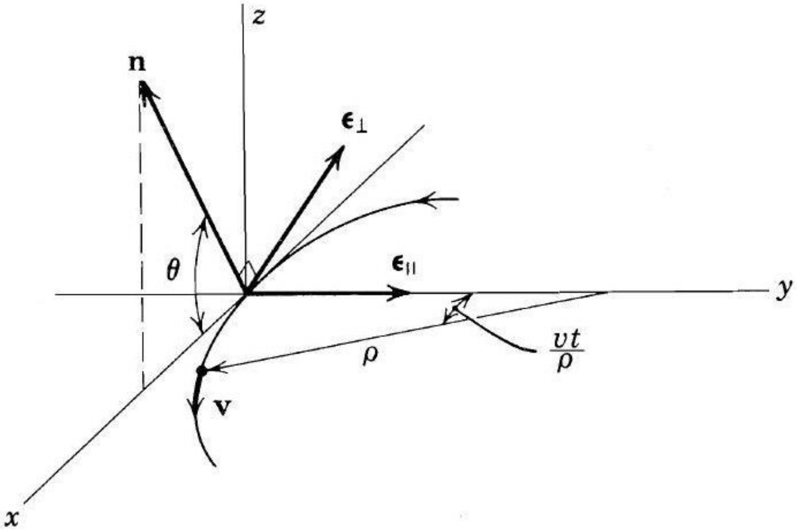
- ✓ **determine the particle motion** $\bar{r}(t);\ \bar{\beta}(t);\dot{\bar{\beta}}(t)$
- ✓ **compute the cross products and the phase factor**
- ✓ **integrate each component and take the vector square modulus**

Calculations are generally quite lengthy: even for simple cases as for the radiation emitted by an electron in a bending magnet they require Airy integrals or the modified Bessel functions (available in MATLAB)

Radiation integral for synchrotron radiation

Trajectory of the arc of circumference [see Jackson]

$$\bar{r}(t) = \left(\rho\left(1-\cos\frac{\beta c}{\rho}t\right), \quad \sin\frac{\beta c}{\rho}t, \quad 0 \right)$$



In the limit of small angles we compute

$$\bar{n}\times(\bar{n}\times\bar{\beta}) = \beta\left[-\bar{\varepsilon}_\parallel \sin\left(\frac{\beta ct}{\rho}\right) + \bar{\varepsilon}_\perp \cos\left(\frac{\beta ct}{\rho}\right)\sin\theta \right]$$

$$\omega\left(t - \frac{\bar{n}\cdot\bar{r}(t)}{c} \right) = \omega\left[t - \frac{\rho}{c}\sin\left(\frac{\beta ct}{\rho}\right)\cos\theta \right]$$

Substituting into the radiation integral and introducing $\xi = \frac{\rho\omega}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2}$

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi) \right]$$

Critical frequency and critical angle

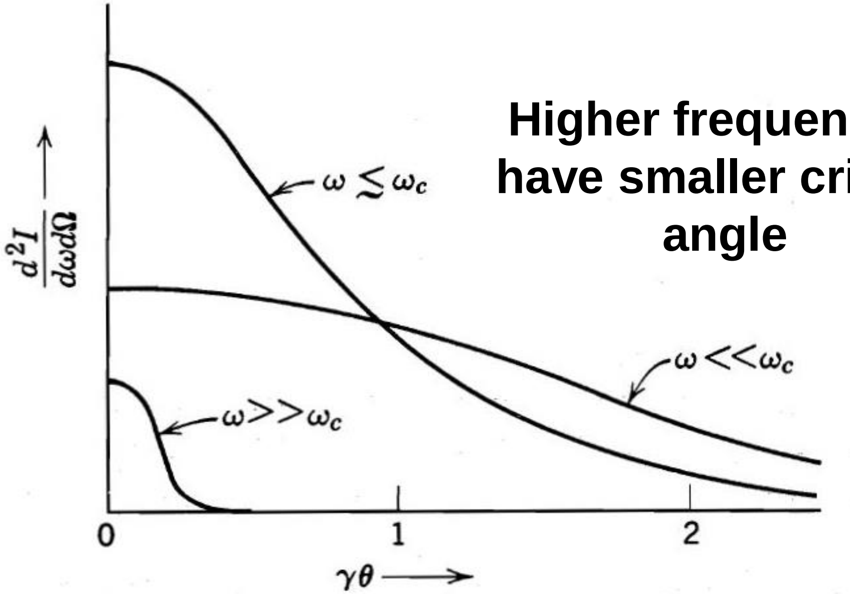
$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi)\right]$$

Using the properties of the modified Bessel function we observe that the radiation intensity is negligible for ξ >> 1

$$\xi = \frac{\omega\rho}{3c\gamma^3}\left(1+\gamma^2\theta^2\right)^{3/2} >> 1$$

Critical frequency $$\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

Critical angle $$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$$



Higher frequencies have smaller critical angle

For frequencies much larger than the critical frequency and angles much larger than the critical angle the synchrotron radiation emission is negligible

Frequency distribution of radiated energy

Integrating on all angles we get the frequency distribution of the energy radiated

$$\frac{dW}{d\omega} = \iint_{4\pi} \frac{d^3 I}{d\omega\, d\Omega} d\Omega = \frac{\sqrt{3}e^2}{4\pi\varepsilon_0 c} \gamma \frac{\omega}{\omega_C} \int_{\omega/\omega_C}^{\infty} K_{5/3}(x)dx$$

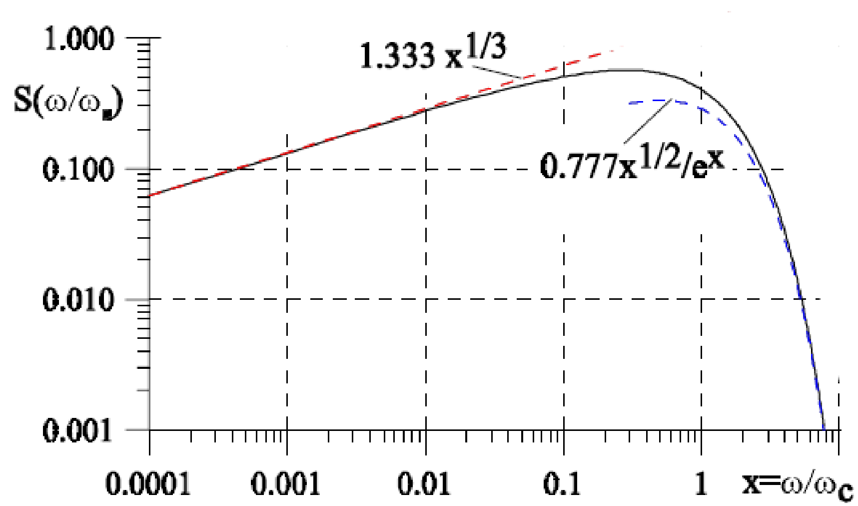
$$\frac{dW}{d\omega} \approx \frac{e^2}{4\pi\varepsilon_0 c}\left(\frac{\omega\rho}{c}\right)^{1/3} \qquad \omega \ll \omega_c$$

$$\frac{dW}{d\omega} \approx \sqrt{\frac{3\pi}{2}} \frac{e^2}{4\pi\varepsilon_0 c} \gamma \left(\frac{\omega}{\omega_c}\right)^{1/2} e^{-\omega/\omega_c} \qquad \omega \gg \omega_c$$

often expressed in terms of the function S(ξ) with $\xi = \omega/\omega_c$

$$S(\xi) = \frac{9\sqrt{3}}{8\pi} \xi \int_{\xi}^{\infty} K_{5/3}(x)dx \qquad \int_0^{\infty} S(\xi)d\xi = 1$$

$$\frac{dW}{d\omega} = \frac{\sqrt{3}e^2\gamma}{4\pi\varepsilon_0 c} \frac{\omega}{\omega_c} \int_{\omega/\omega_c}^{\infty} K_{5/3}(x)dx = \frac{2e^2\gamma}{9\varepsilon_0 c} S(\xi)$$



Frequency distribution of radiated energy

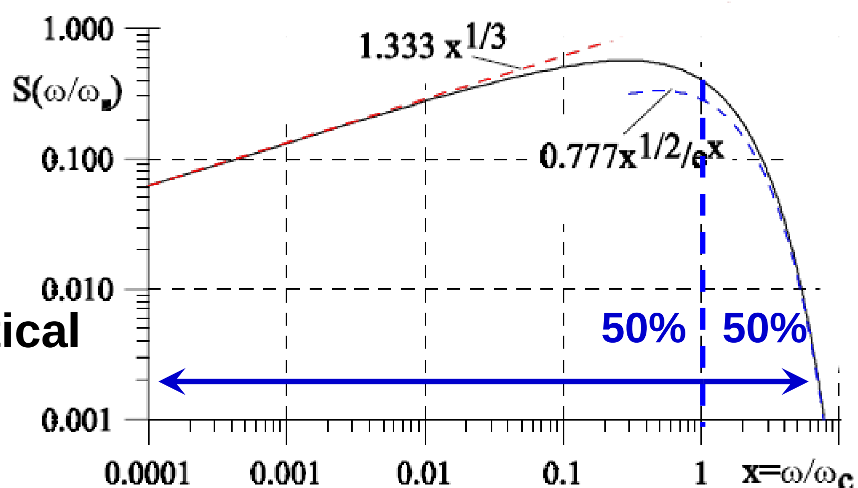
It is possible to verify that the integral over the frequencies agrees with the previous expression for the total power radiated [Hubner]

$$P=\frac{U_0}{T_b}=\frac{1}{T_b}\int_0^{\omega}\frac{dW}{d\omega}d\omega=\frac{1}{T_b}\frac{2e^2\gamma}{9\varepsilon_0 c}\omega_c\int_0^{\omega}\xi\, d\xi\int_{\xi}^{\infty}K_{5/3}(x)dx=\frac{e^2c}{6\varepsilon_0 c}\frac{\gamma^4}{\rho^2}$$

The frequency integral extended up to the critical frequency contains half of the total energy radiated, the peak occurs approximately at $0.3\omega_c$

It is also convenient to define the critical photon energy as

$$\varepsilon_c=\hbar\omega_c=\frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$



For electrons, the critical energy in practical units reads

$$\varepsilon_c[keV]=2.218\frac{E[GeV]^3}{\rho[m]}=0.665\cdot E[GeV]^2\cdot B[T]$$

Heuristic derivation of critical frequency

Synchrotron radiation is emitted in an arc of circumference with radius ρ, Angle of emission of radiation is 1/γ (relativistic argument), therefore

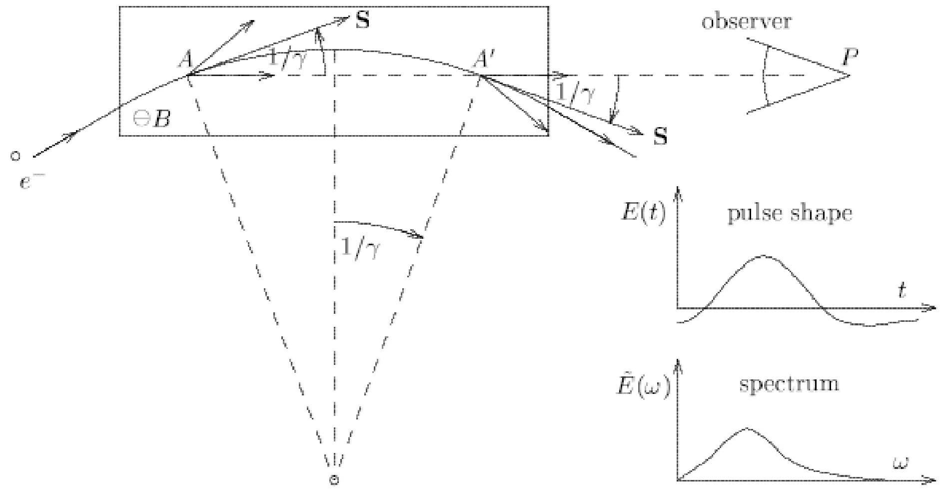
$$\Delta T = \frac{\rho}{\beta c \gamma}$$ transit time in the arc of dipole

During this time the electron travels a distance

$$\Delta s = \beta c \Delta T = \frac{\rho}{\gamma}$$

The time duration of the radiation pulse seen by the observer is the difference between the time of emission of the photons and the time travelled by the electron in the arc

$$\tau = \Delta T - \frac{\Delta s}{\beta c} = \frac{\rho}{c\gamma}\left(\frac{1}{\beta} - 1\right) = \frac{\rho}{c\gamma}\frac{1-\beta^2}{\beta(1+\beta)} \approx \frac{\rho}{2c\gamma^3}$$



The width of the Fourier transform of the pulse is $$\Delta \nu \approx \frac{2c\gamma^3}{\rho}$$

Polarisation of synchrotron radiation

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2}{16\pi^3\varepsilon_0 c}\left(\frac{2\omega\rho}{3c\gamma^2}\right)^2 \left(1+\gamma^2\theta^2\right)^2 \left[K_{2/3}^2(\xi) + \frac{\gamma^2\theta^2}{1+\gamma^2\theta^2} K_{1/3}^2(\xi)\right]$$

Polarisation in the orbit plane — **Polarisation orthogonal to the orbit plane**

In the orbit plane θ = 0, the polarisation is purely horizontal

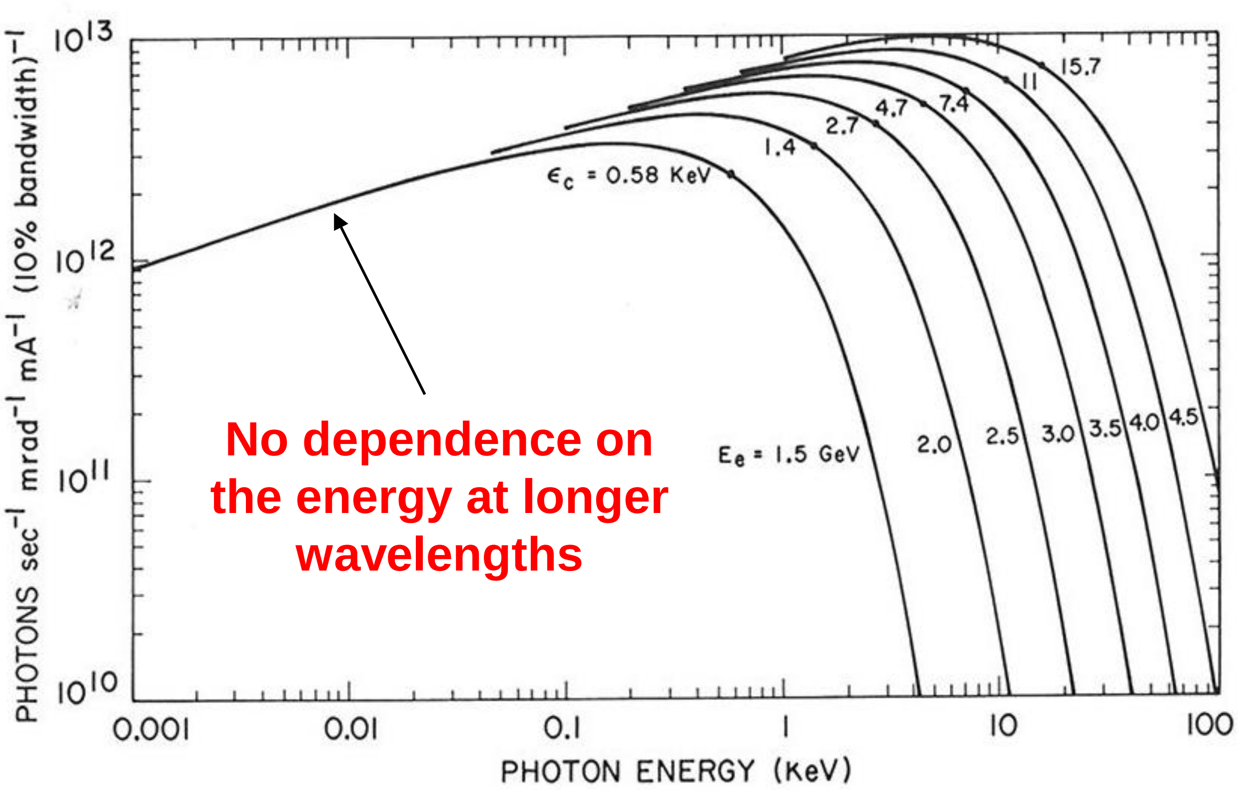
Integrating on all frequencies we get the angular distribution of the energy radiated

$$\frac{d^2W}{d\Omega} = \int_0^\infty \frac{d^3I}{d\omega\, d\Omega} d\omega = \frac{7e^2\gamma^5}{64\pi\varepsilon_0\rho} \frac{1}{(1+\gamma^2\theta^2)^{5/2}} \left[1 + \frac{5}{7}\frac{\gamma^2\theta^2}{1+\gamma^2\theta^2}\right]$$

Integrating on all the angles we get a polarization on the plan of the orbit 7 times larger than on the plan perpendicular to the orbit

Synchrotron radiation emission as a function of beam the energy

Dependence of the frequency distribution of the energy radiated via synchrotron emission on the electron beam energy



Critical frequency

$$\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

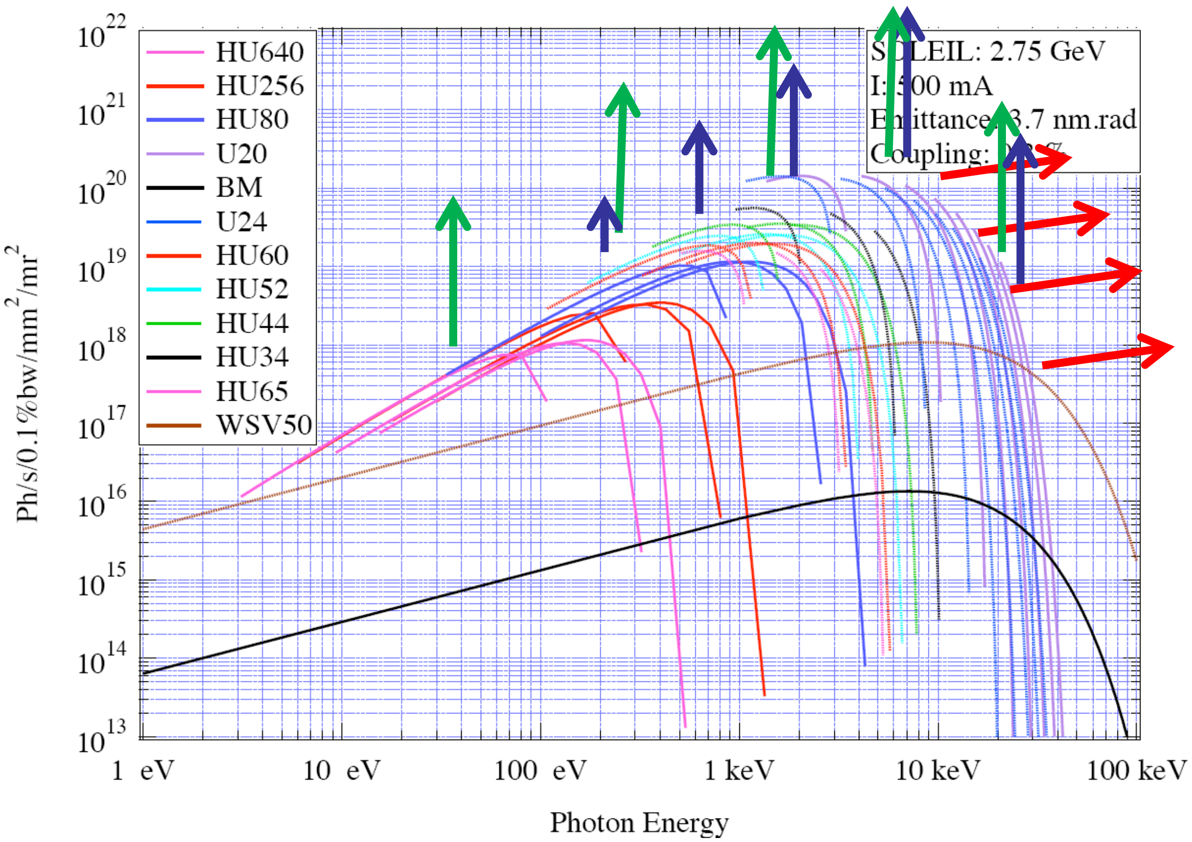
Critical angle

$$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$$

Critical energy

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

Brilliance with IDs (medium energy light sources)



Brilliance dependence

with current

with energy

with emittance

Medium energy storage rings with **in-vacuum undulators** operated at low gaps (e.g. 5-7 mm) can reach 10 keV with a brilliance of 10^{20} ph/s/0.1%BW/mm^2/mrad^2

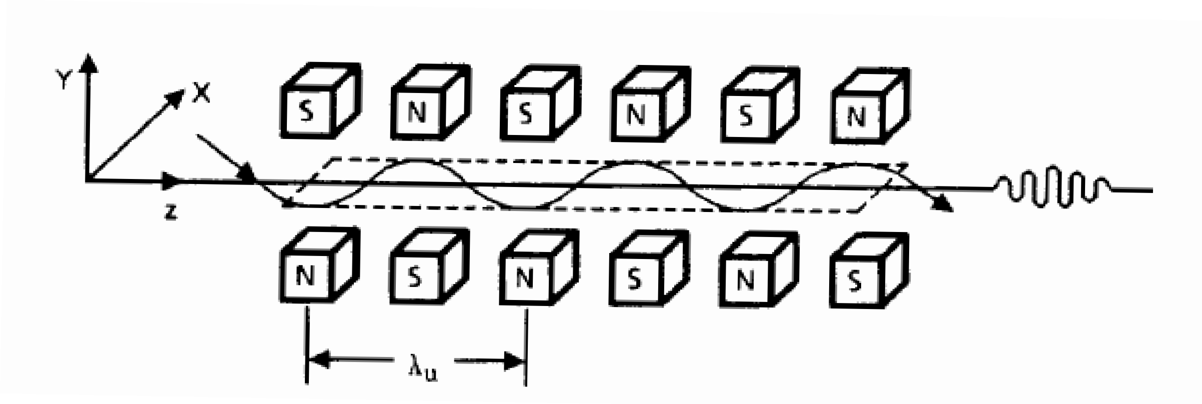
Radiation from undulators and wigglers

Radiation emitted by undulators and wigglers

Types of undulators and wigglers

Undulators and wigglers

Periodic array of magnetic poles providing a sinusoidal magnetic field on axis:



$$B = (0, \quad B_0 \sin(k_u z), \quad 0,)$$

Solution of equation of motions:

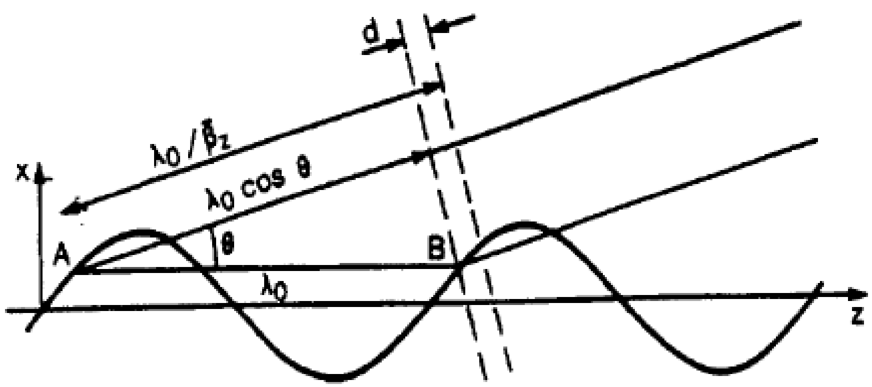
$$K = \frac{eB_0 \lambda_u}{2\pi mc}$$

Undulator parameter

$$\bar{r}(t) = -\frac{\lambda_u K}{2\pi\gamma} \sin \omega_u t \cdot \hat{x} + \left(\bar{\beta}_z ct + \frac{\lambda_u K^2}{16\pi\gamma^2} \cos(2\omega_u t) \right) \cdot \hat{z}$$

$$\bar{\beta}_z = 1 - \frac{1}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Constructive interference of radiation emitted at different poles



$$d = \frac{\lambda_u}{\bar{\beta}} - \lambda_u \cos\theta = n\lambda$$

$$\lambda_n = \frac{\lambda_u}{2\gamma^2 n}\left(1 + \frac{K^2}{2} + \gamma^2\theta^2\right)$$

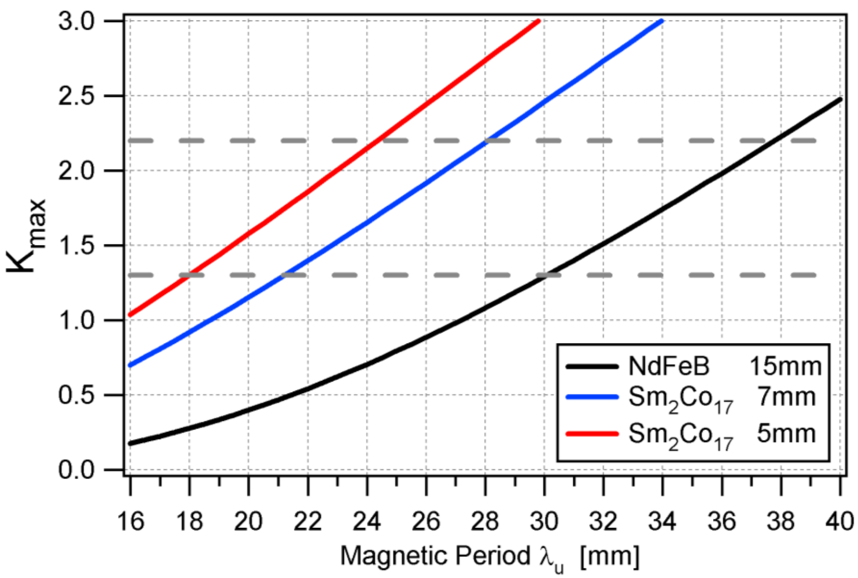
The undulator parameter K

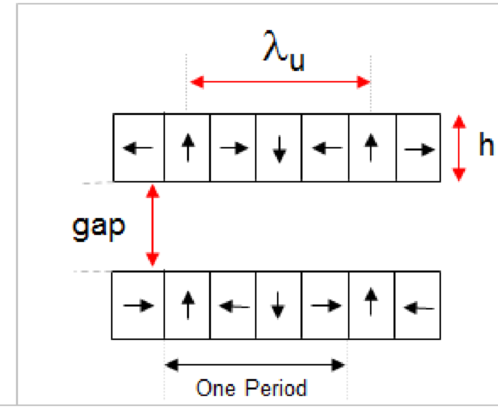
$$K = \frac{eB_0\lambda_u}{2\pi mc}$$ Undulator parameter

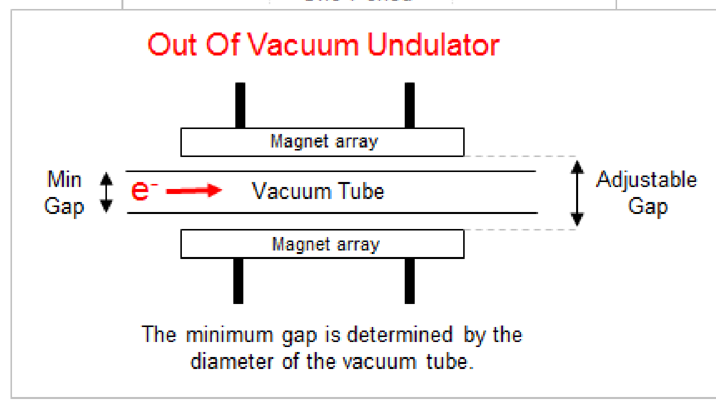
B_0 is the peak magnetic field on axis

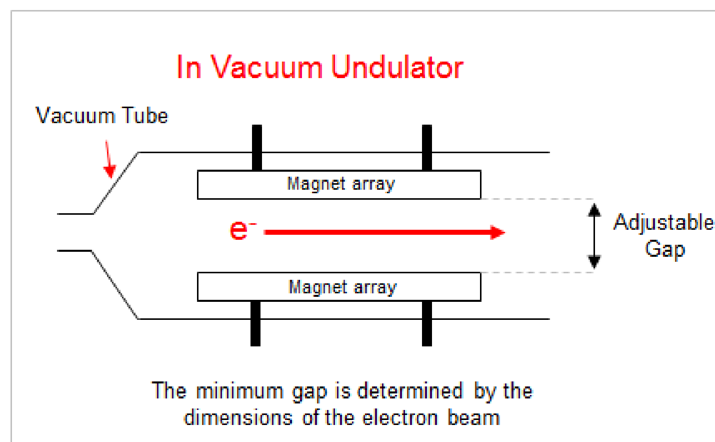
$$K = 0.168\, B_r \lambda_u\, e^{-\frac{\pi gap}{\lambda_u}}$$

lengths in [mm], B_r in [Tesla]
(K expression assumes $h > \lambda_u/2$)

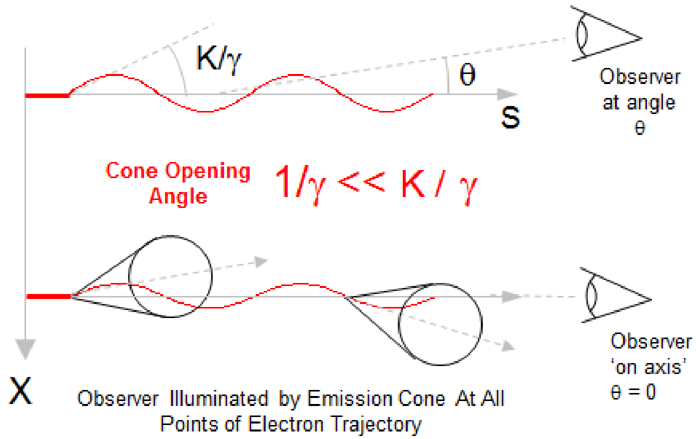






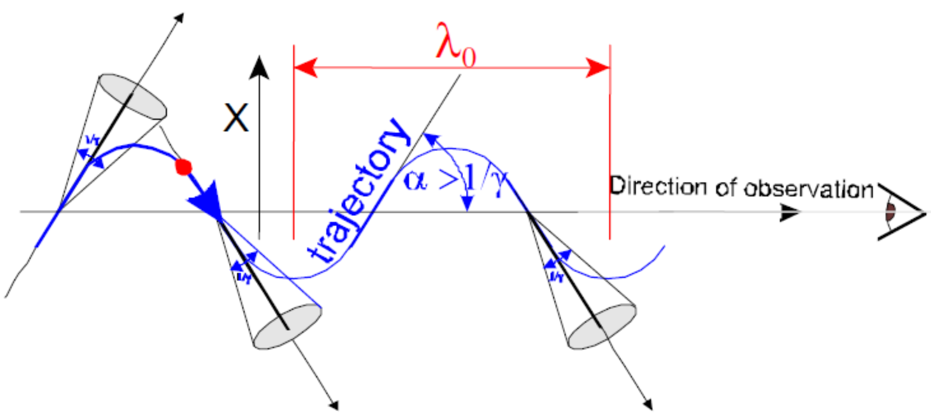


Emission from an undulator (I)



Case 1: K << 1

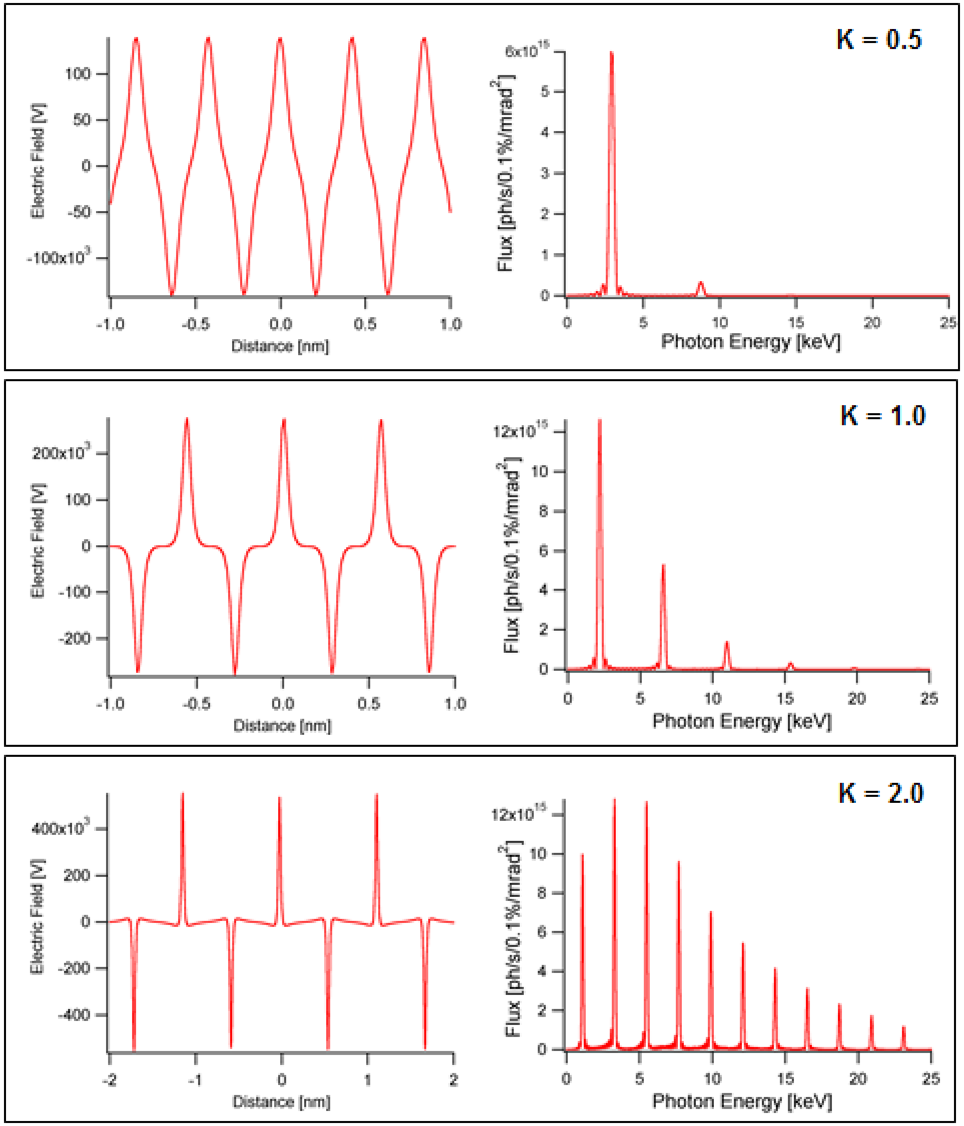
The max angular deflection is much less than the cone opening angle. The observes sees the radiation form the whole undulator length



Case 2: K ~ 1 or K >> 1

The max angular deflection is larger than the cone opening angle. The observer misses part of the radiation as the radiation fan sweeps right/left

Emission from an undulator (II)



Case 1: K << 1

The max angular deflection is much less than the cone opening angle. The observes sees the radiation form the whole undulator length

Case 2: K ~ 1 or K >> 1

The max angular deflection is larger than the cone opening angle. The observer misses part of the radiation as the radiation fan sweeps right/left

Comparison of angular distribution of radiated power

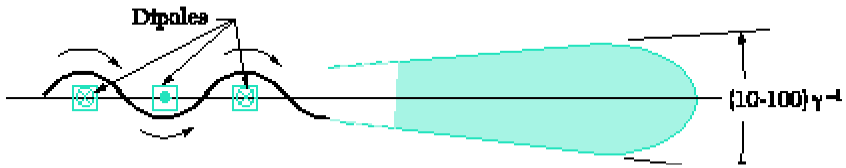
bending magnet - a "sweeping searchlight"

Continuous spectrum characterized by ε_c = critical energy

ε_c(keV) = 0.665 B(T)E^2(GeV)

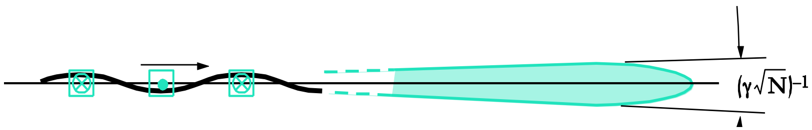
eg: for B = 1.4T E = 3GeV ε_c = 8.4 keV

(bending magnet fields are usually lower ~ 1 – 1.5T)



wiggler - incoherent superposition K > 1
Max. angle of trajectory > 1/γ

Quasi-monochromatic spectrum with peaks at lower energy than a wiggler



undulator - coherent interference K < 1
Max. angle of trajectory < 1/γ

$$\lambda_n = \frac{\lambda_u}{2n\gamma^2}\left(1+\frac{K^2}{2}\right) \approx \frac{\lambda_u}{n\gamma^2}$$

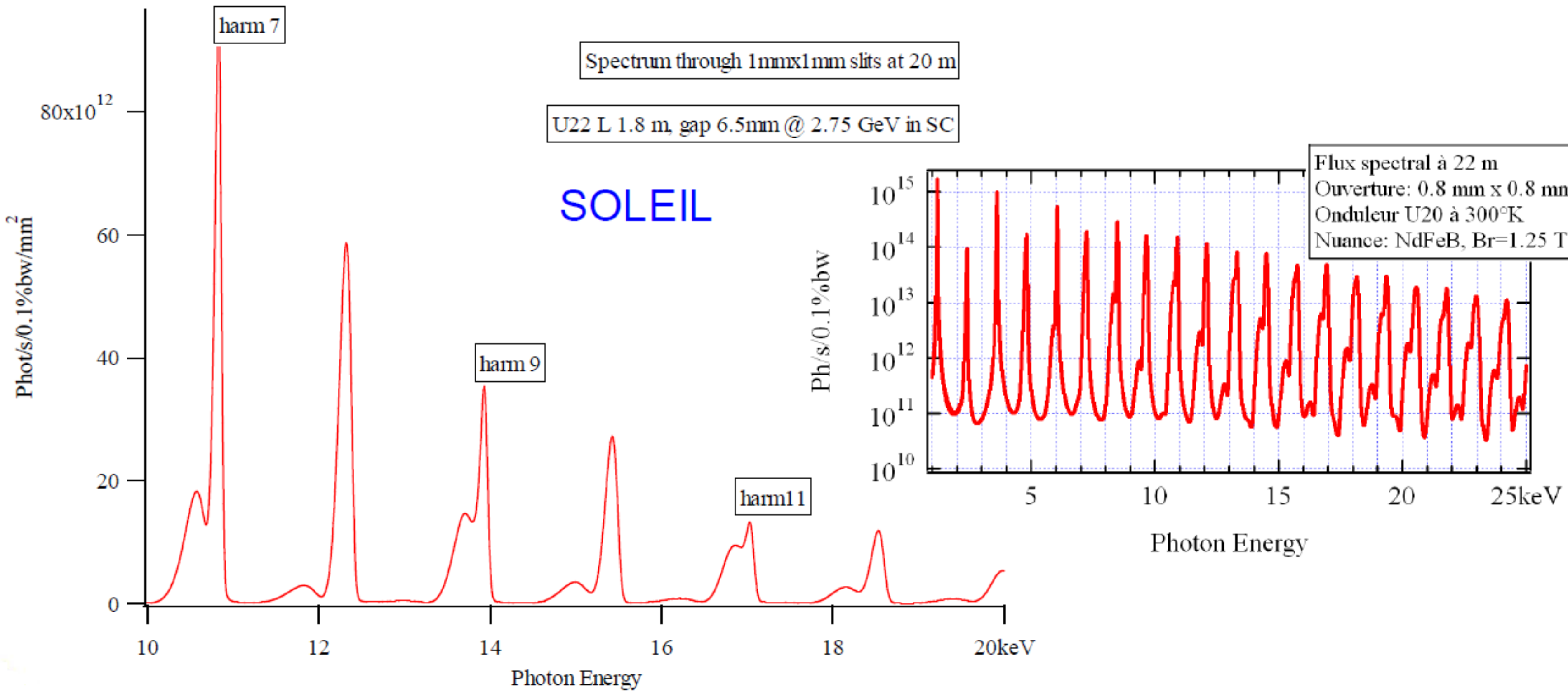
$$\varepsilon_n(eV) = 9.496\frac{nE[GeV]^2}{\lambda_u[m]\left(1+\frac{K^2}{2}\right)}$$

Spectrum of undulator radiation

Interferences along the N periods =>

Discrete lines spectrum with :

- Line width scaling as $(\Delta\lambda/\lambda)_{harm\ n} \sim 1/nN$
- Peak value scaling as N^2



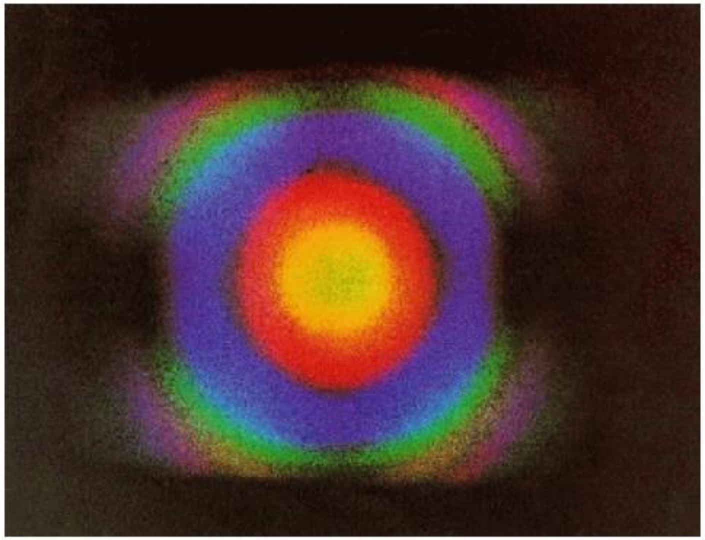
Courtesy J.M Filhol

Angular dependence of undulator radiation

Wave length emitted on harmonic n

$$\lambda_n = \lambda_u (1 + K^2/2 + \gamma^2\theta^2) / (2n\,\gamma^2)$$

λ_u is the undulator magnetic period
θ is the angle of observation

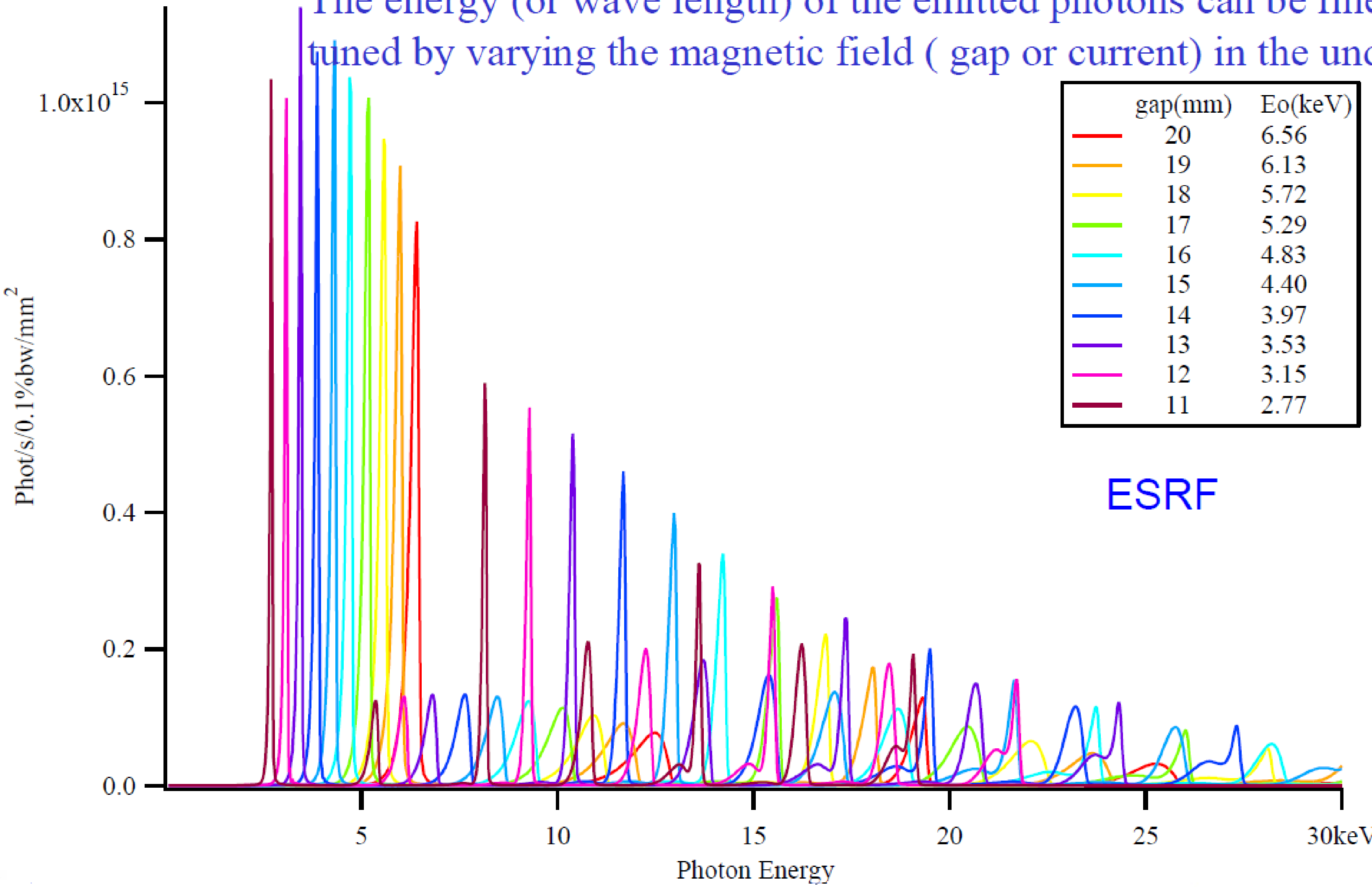


⇒**Photon energy depends on the observation angle**
⇒**Great sensitivity to spread in θ or γ**

Courtesy J.M Filhol

Tunability of undulator radiation

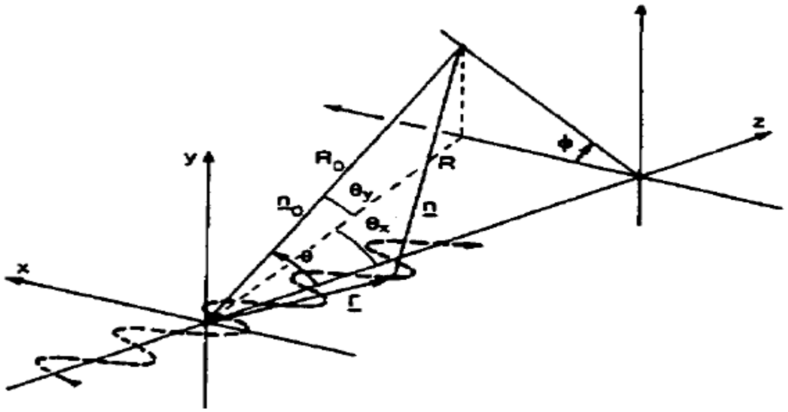
The energy (or wave length) of the emitted photons can be finely tuned by varying the magnetic field (gap or current) in the undulator



Courtesy J.M Filhol

Radiation integral for a linear undulator (I)

The angular and frequency distribution of the energy emitted by a wiggler is computed again with the radiation integral:



$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\infty}^{\infty}\hat{n}\times(\hat{n}\times\overline{\beta})e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}dt\right|^2$$

Using the periodicity of the trajectory we can split the radiation integral into a sum over N_u terms

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\omega^2}{4\pi\varepsilon_0 4\pi^2 c}\left|\int_{-\lambda_u/2\overline{\beta}c}^{\lambda_u/2\overline{\beta}c}\hat{n}\times(\hat{n}\times\overline{\beta})e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}dt\right|^2\left|1+e^{i\delta}+e^{i2\delta}+\ldots+e^{i(N_u-1)\delta}\right|^2$$

where

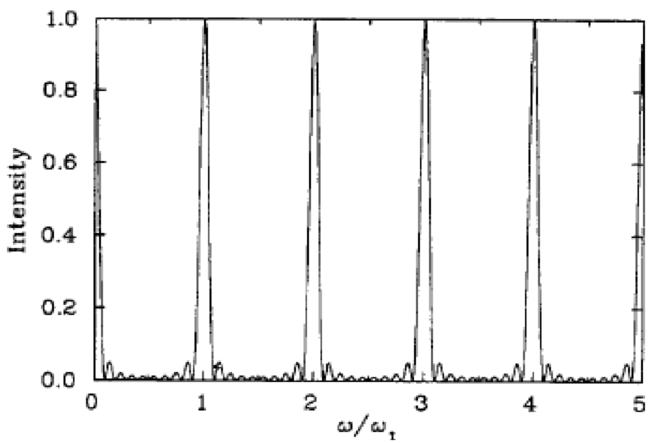
$$\delta = \frac{2\pi\omega}{\omega_{res}(\theta)} \qquad \omega_{res}(\theta) = \frac{2\pi c}{\lambda_{res}(\theta)} \qquad \lambda_{res}(\theta) = \frac{\lambda_u}{2\gamma^2}\left(1+\frac{K^2}{2}+\gamma^2\theta^2\right)$$

Radiation integral for a linear undulator (II)

The radiation integral in an undulator or a wiggler can be written as

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\gamma^2 N^2}{4\pi\varepsilon_0 c} L\left(N \frac{\Delta\omega}{\omega_{res}(\theta)} \right) F_n(K,\theta,\phi)$$

$\Delta\omega = \omega - n\,\omega_{res}(\theta)$

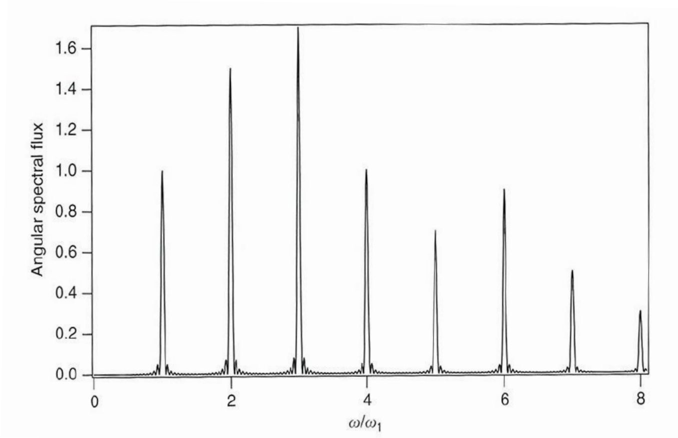


The sum on δ generates a series of sharp peaks in the frequency spectrum harmonics of the fundamental wavelength

$$L\left(N \frac{\Delta\omega}{\omega_{res}(\theta)} \right) = \frac{\sin^2(N\pi\Delta\omega/\omega_{res}(\theta))}{N^2 \sin^2(\pi\Delta\omega/\omega_{res}(\theta))}$$

The integral over one undulator period generates a modulation term F_n which depends on the angles of observations and K

$$F_n(K,\theta,\phi) \propto \left| \int_{-\lambda_0/2\bar{\beta}c}^{\lambda_0/2\bar{\beta}c} \hat{n}\times(\hat{n}\times\bar{\beta})\, e^{i\omega(t-\hat{n}\cdot\bar{r}/c)}\, dt \right|^2$$

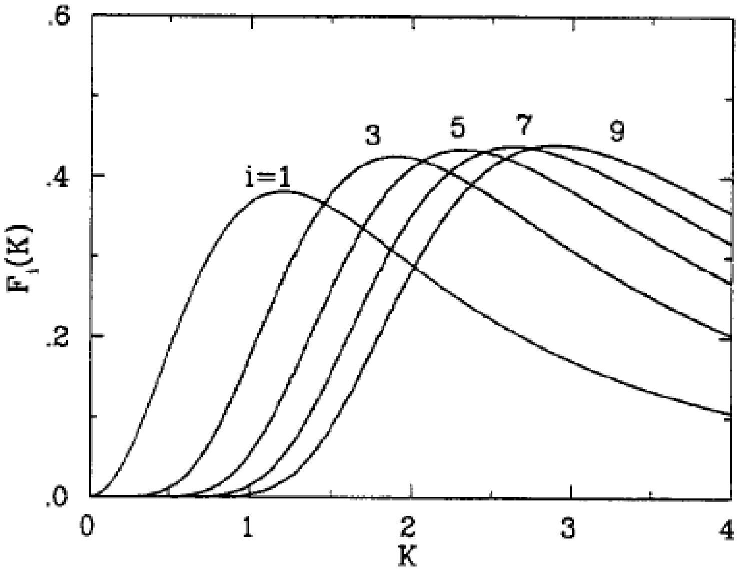


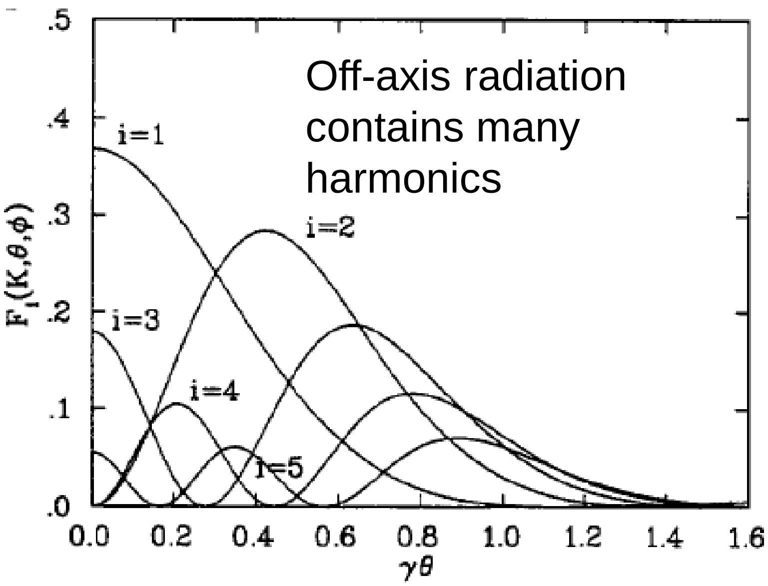
Radiation integral for a linear undulator (II)

e.g. on axis (θ = 0, φ = 0):

$$\frac{d^3W}{d\Omega\, d\omega} = \frac{e^2\gamma^2 N^2}{4\pi\varepsilon_0 c} L\left(N \frac{\Delta\omega}{\omega_{res}(0)} \right) F_n(K,0,0)$$

$$\mathrm{F_n}(\mathrm{K},0,0) = \frac{\mathrm{n}^2\mathrm{K}^2}{(1+\mathrm{K}^2/2)^2}\left[\mathrm{J}_{\frac{n+1}{2}}(\mathrm{Z}) - \mathrm{J}_{\frac{n-1}{2}}(\mathrm{Z}) \right]^2 \qquad Z = \frac{nK^2}{4(1+K^2/2)}$$



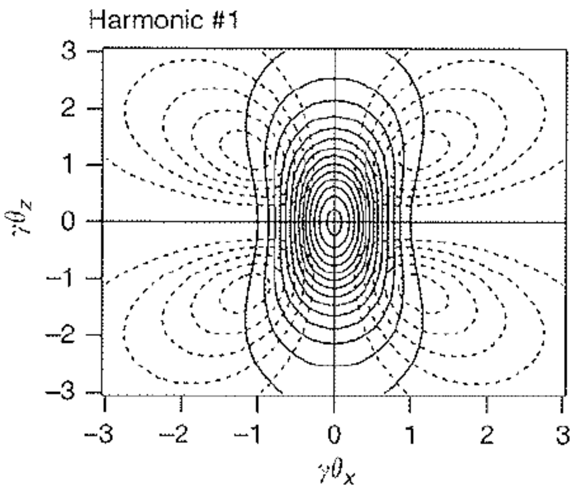


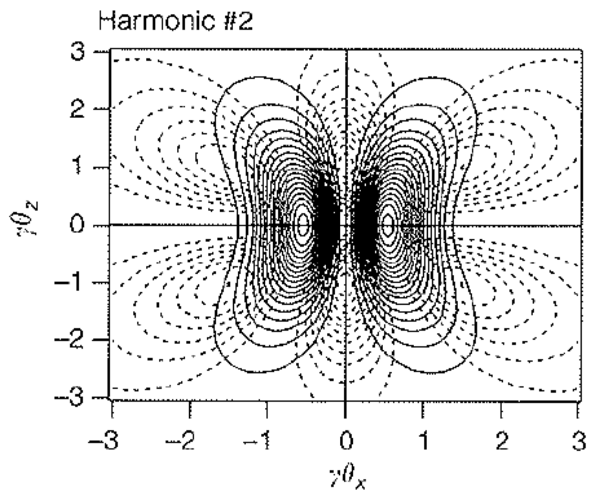
Only odd harmonic are radiated on-axis;

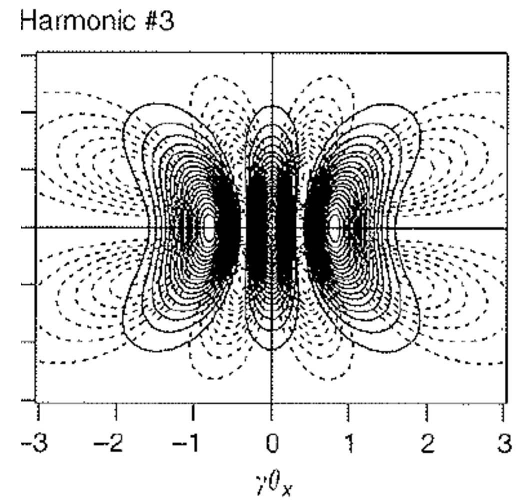
as K increases the higher harmonic becomes stronger

Angular patterns of the radiation emitted on harmonics

Angular spectral flux as a function of frequency for a linear undulator; linear polarisation solid, vertical polarisation dashed (K = 2)







$$\lambda_1 = \frac{\lambda_u}{2\gamma^2}\left(1 + \frac{K^2}{2}\right)$$

Fundamental wavelength emitted by the undulator

$$\lambda_2 = \frac{\lambda_1}{2}$$

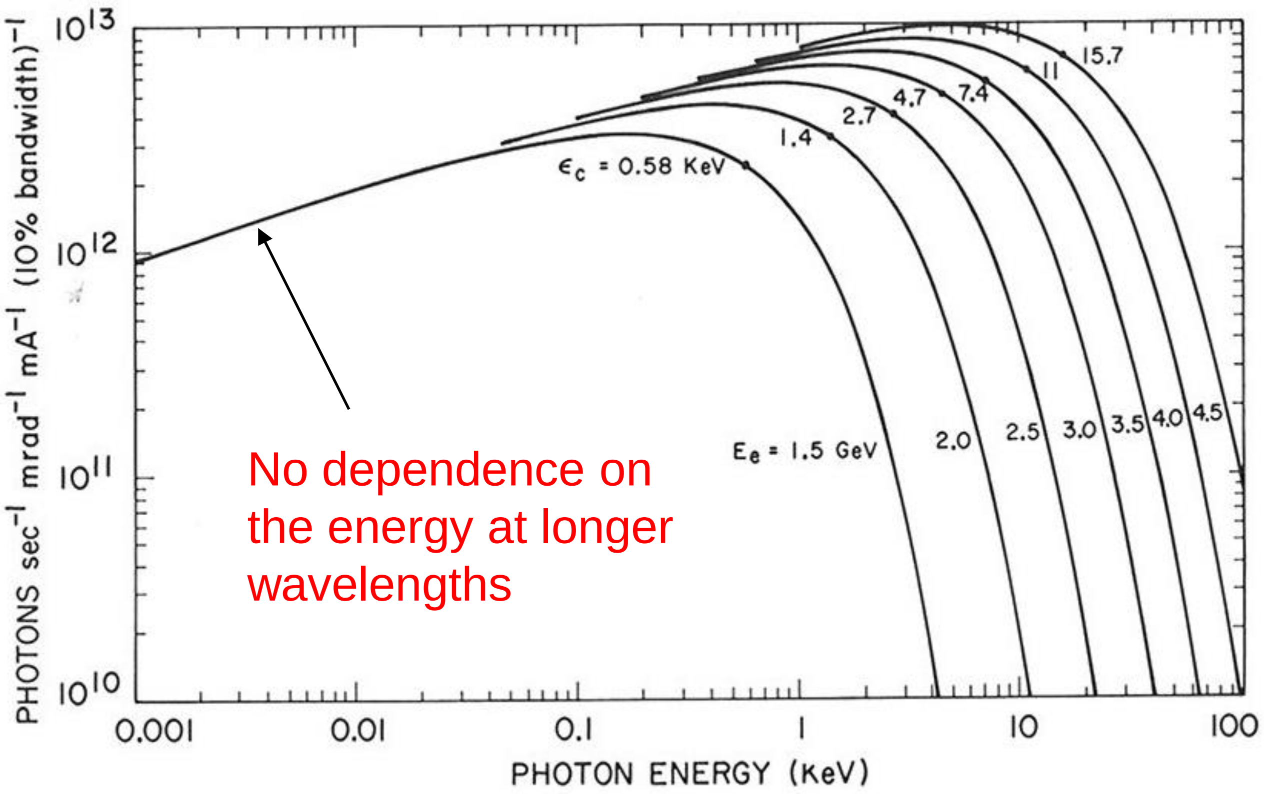
2nd harmonic, not emitted on-axis !

$$\lambda_2 = \frac{\lambda_1}{3}$$

3rd harmonic, emitted on-axis !

Synchrotron radiation emission from a bending magnet

Dependence of the frequency distribution of the energy radiated via synchrotron emission on the electron beam energy



Critical frequency

$$\omega_c = \frac{3}{2}\frac{c}{\rho}\gamma^3$$

Critical angle

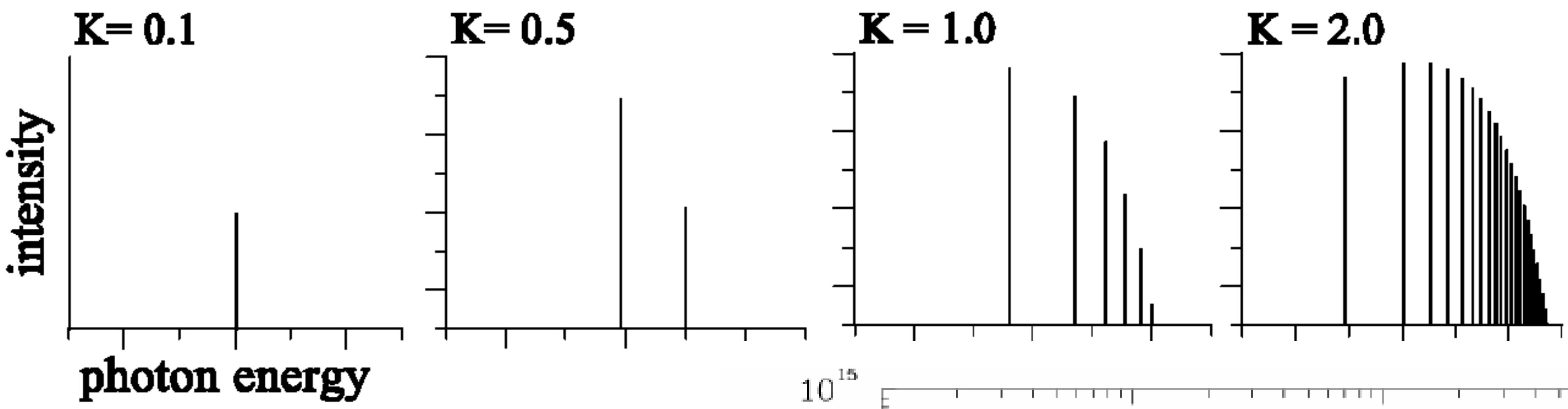
$$\theta_c = \frac{1}{\gamma}\left(\frac{\omega_c}{\omega}\right)^{1/3}$$

Critical energy

$$\varepsilon_c = \hbar\omega_c = \frac{3}{2}\frac{\hbar c}{\rho}\gamma^3$$

Undulators and wigglers (large K)

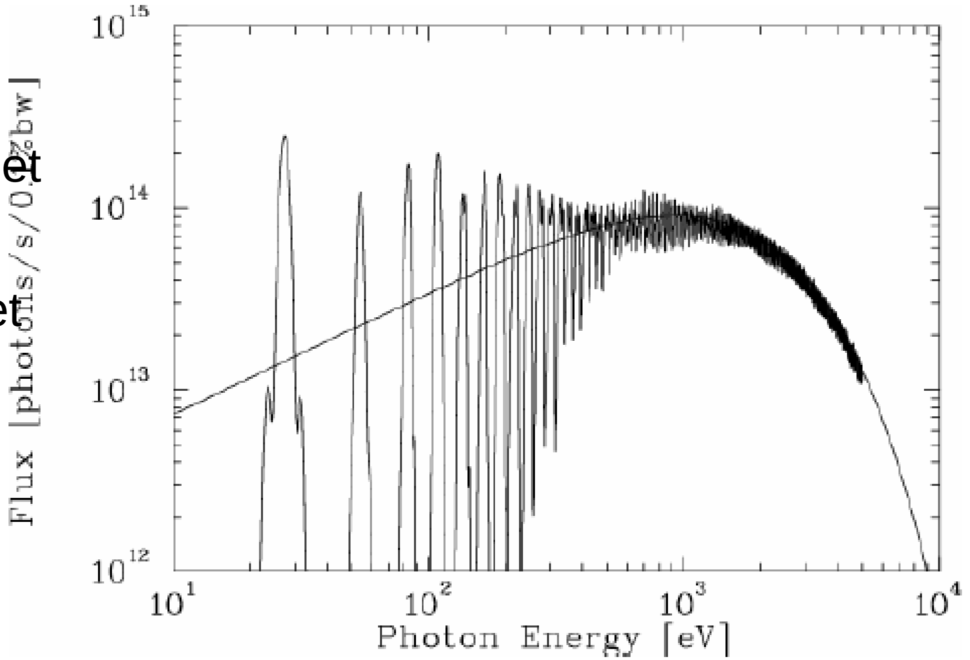
Radiated intensity emitted vs K



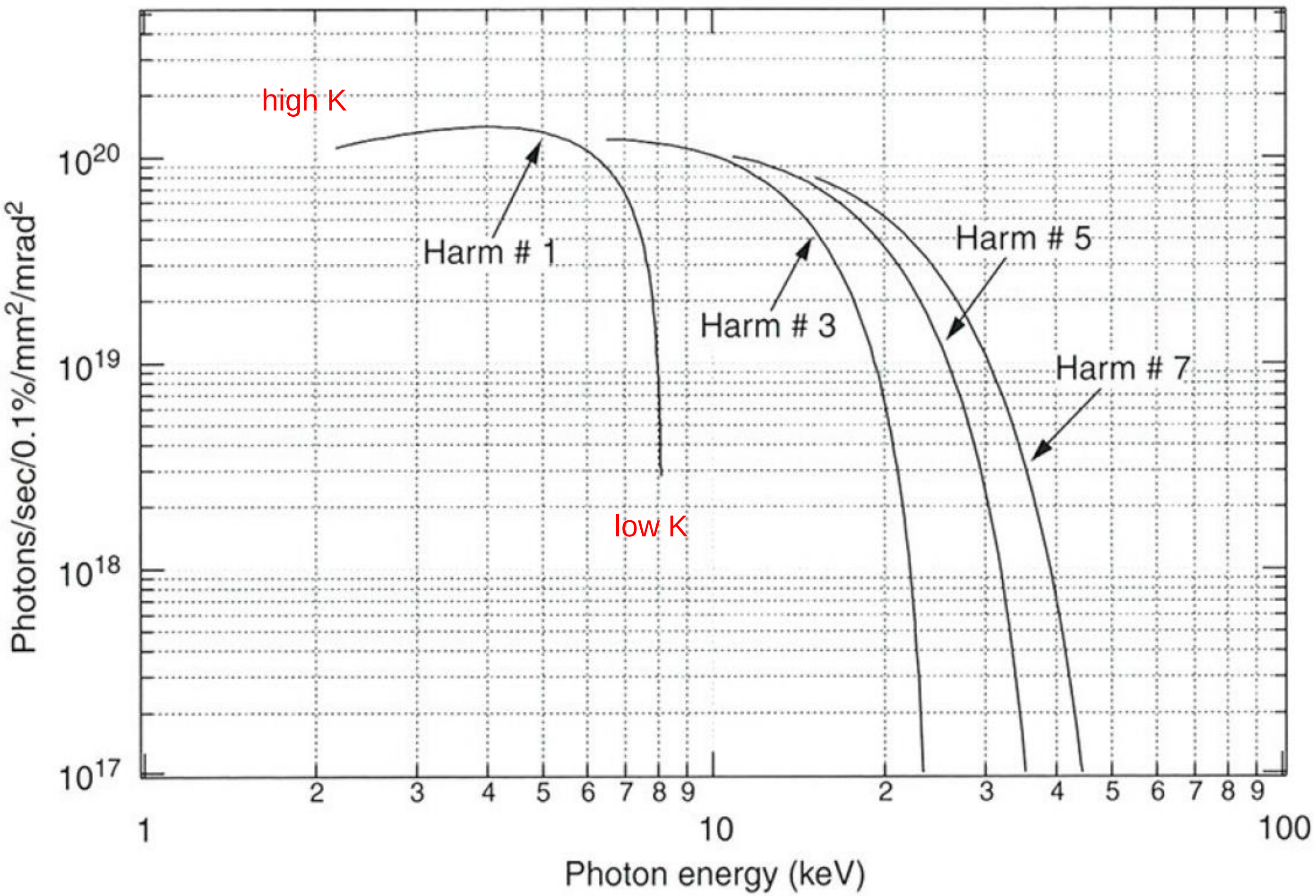
For large K the wiggler spectrum becomes similar to the bending magnet spectrum, $2N_u$ times larger.

Fixed B_0, to reach the bending magnet critical wavelength we need:

K	1	2	10	20
n	1	5	383	3015



Undulator tuning curve (with K)



$$K = \frac{eB_0\lambda_u}{2\pi mc}$$

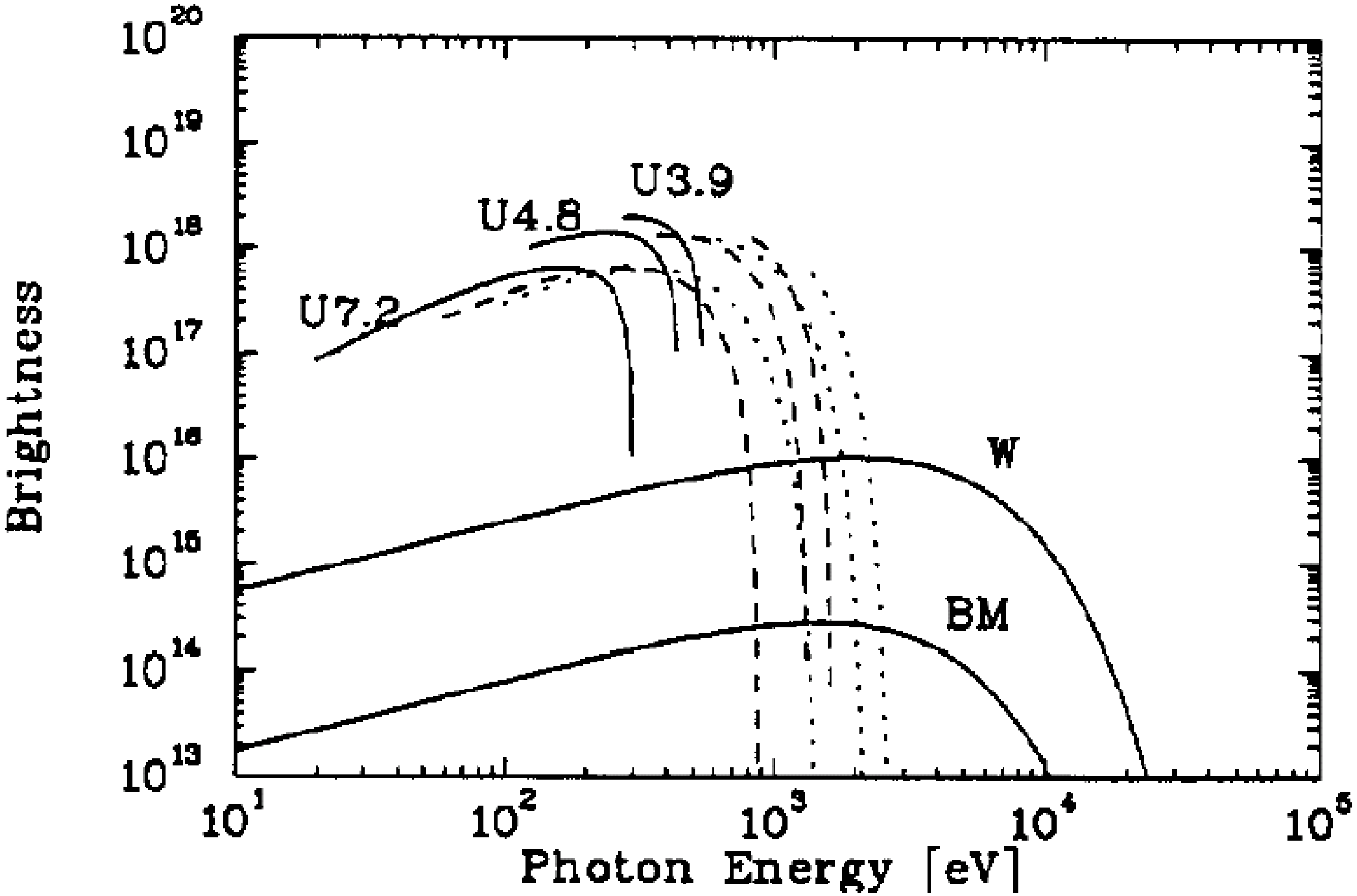
Undulator parameter

K decreases by opening the gap of the undulator (reducing B)

Brightness of a 5 m undulator 42 mm period with maximum K = 2.42 (ESRF)
Varying K one varies the wavelength emitted at various harmonics (not all wavelengths of this graph are emitted at a single time)

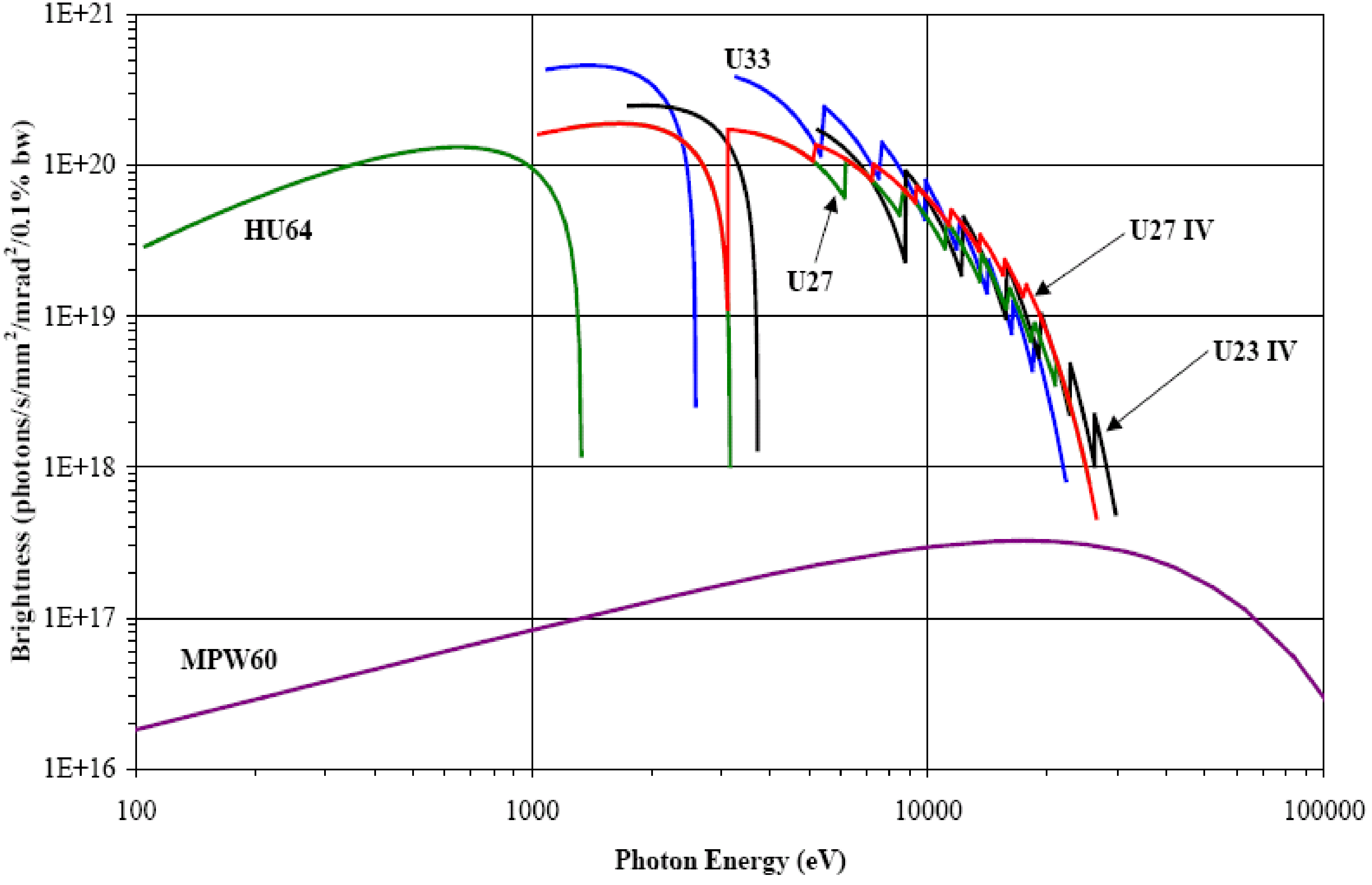
Spectral brightness of undulators of wiggler

Comparison of undulators for a 1.5 GeV ring for three harmonics (solid dashed and dotted) compared with a wiggler and a bending magnet (ALS)



Diamond undulators and wiggler

Spectral brightness for undulators and wigglers in state-of-the-art 3rd generation light sources



Summary of radiation characteristics of undulators or wiggler

Undulators have weaker field or shorter periods (K< 1)

Produce narrow band radiation and harmonics $\Delta\omega/\omega \sim 1/nN_u$

Intensity is proportional to N_u^2

Wigglers have higher magnetic field (K >1)

Produce a broadband radiation

Intensity is proportional to N_u

Type of undulators and wigglers

Electromagnetic undulators: the field is generated by current carrying coils; they may have iron poles;

Permanent magnet undulators: the field is generated by permanent magnets Samarium Cobalt (SmCo; 1T) and Neodymium Iron Boron (NdFeB; 1.4T); they may have iron poles (hybrid undulators);

APPLE-II: permanent magnets arrays which can slide allowing the polarisation of the magnetic field to be changed from linear to circular

In-vacuum: permanent magnets arrays which are located in-vacuum and whose gap can be closed to very small values (< 5 mm gap!)

Superconducting wigglers: the field is generated by superconducting coils and can reach very high peak fields (several T, 3.5 T at Diamond)

Electromagnetic undulators (I)

HU64 at SOLEIL:

variable polarisation electromagnetic undulator

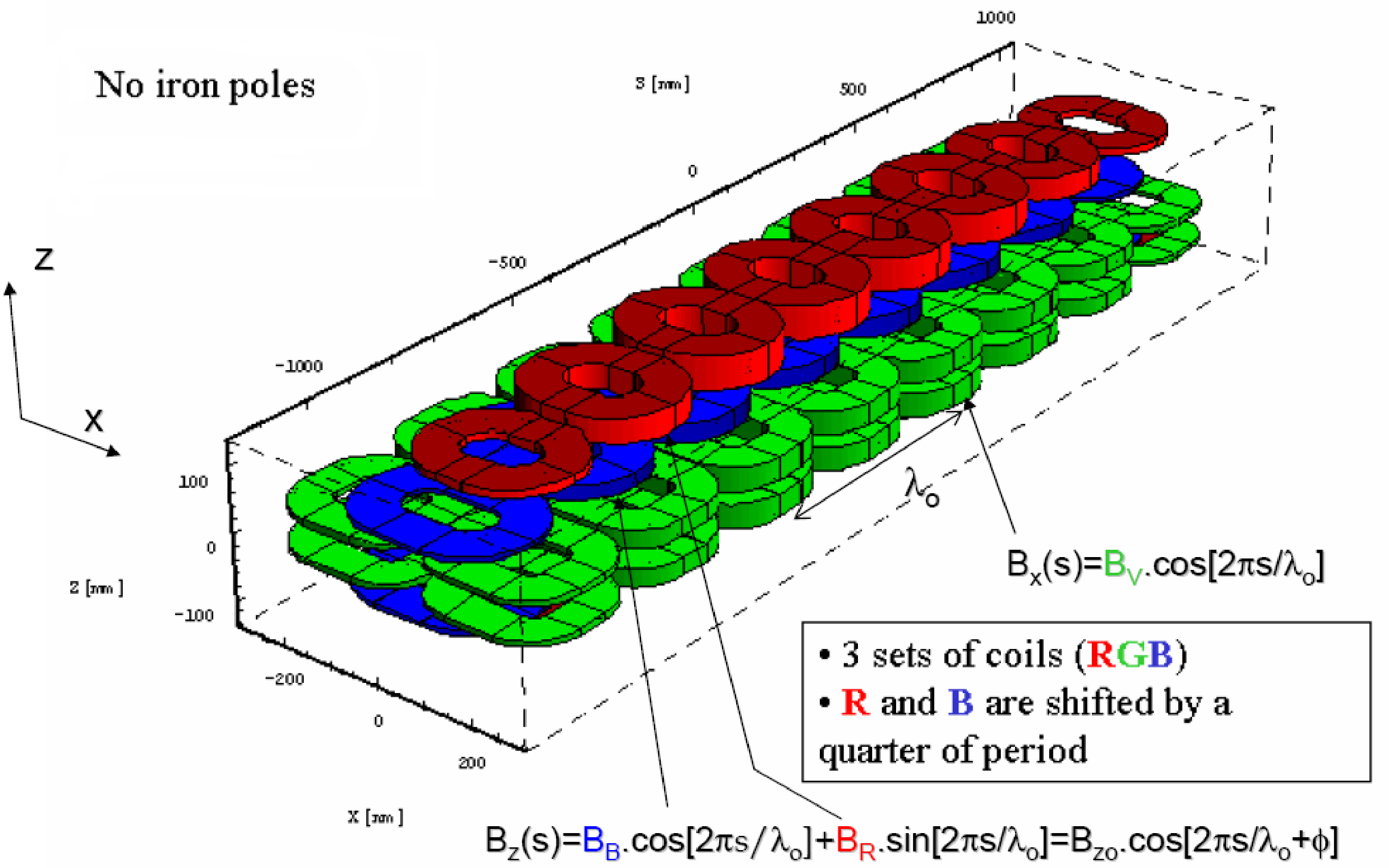
Period 64 mm

14 periods

Min gap 19 mm

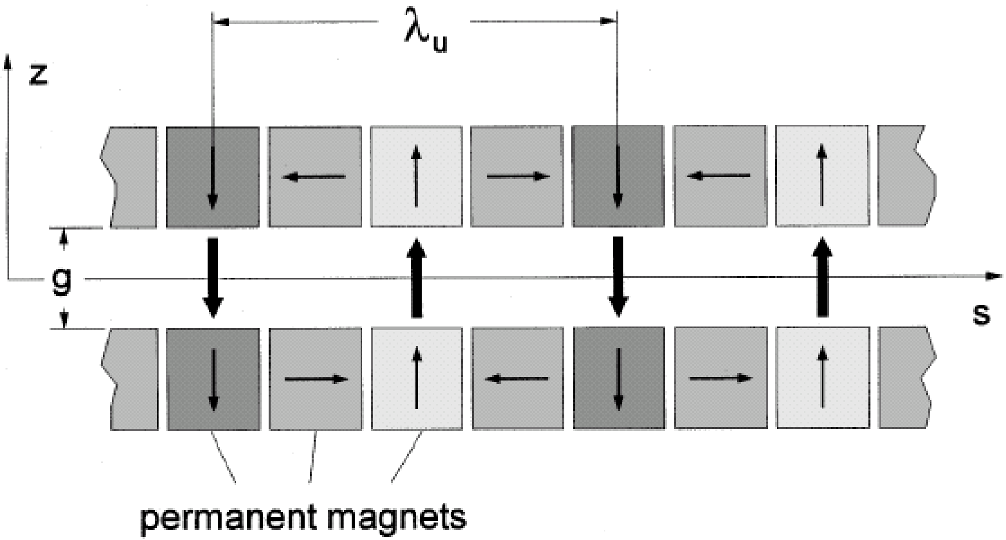
Photon energy < 40 eV (1 keV with EM undulators)

Electromagnetic undulators (II)

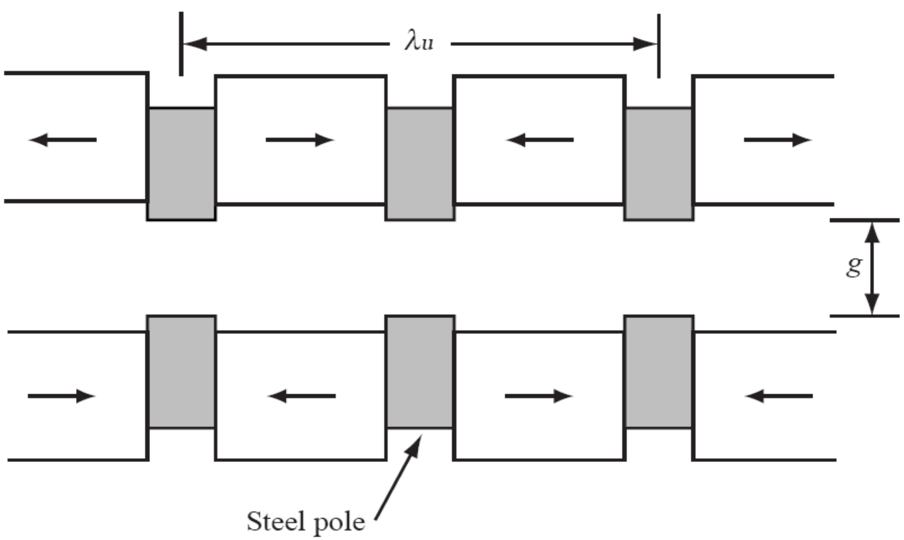


Depending on the way the coil power supplies are powered it can generate linear H, linear V or circular polarisations

Permanent magnet undulators



Halback configuration

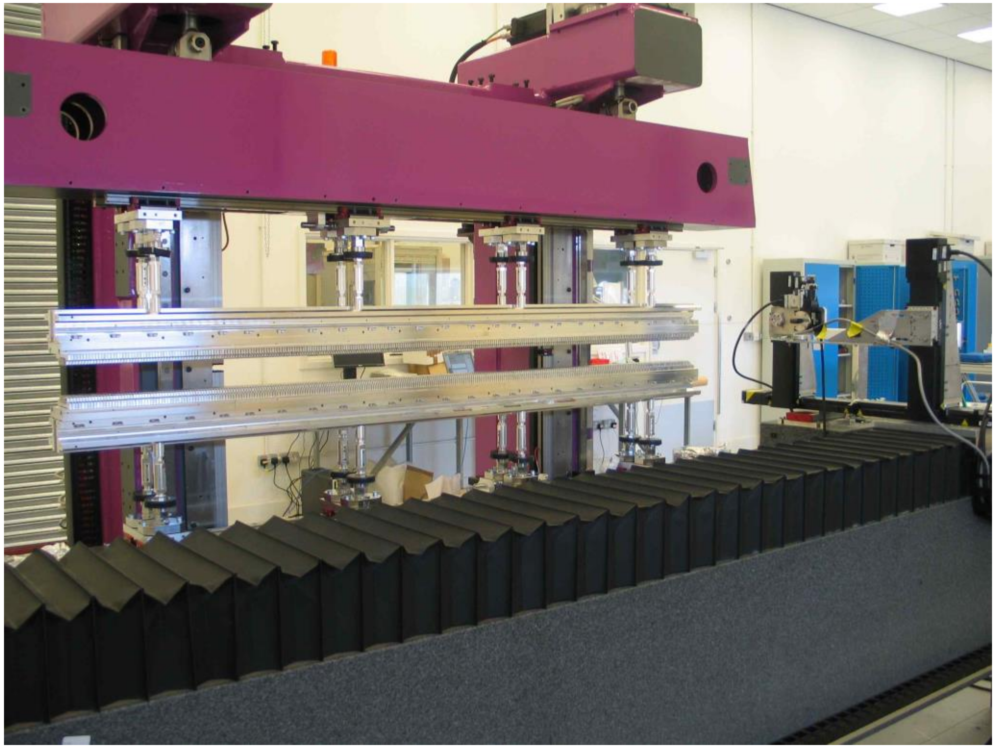


hybrid configuration with steel poles

In-vacuum undulators

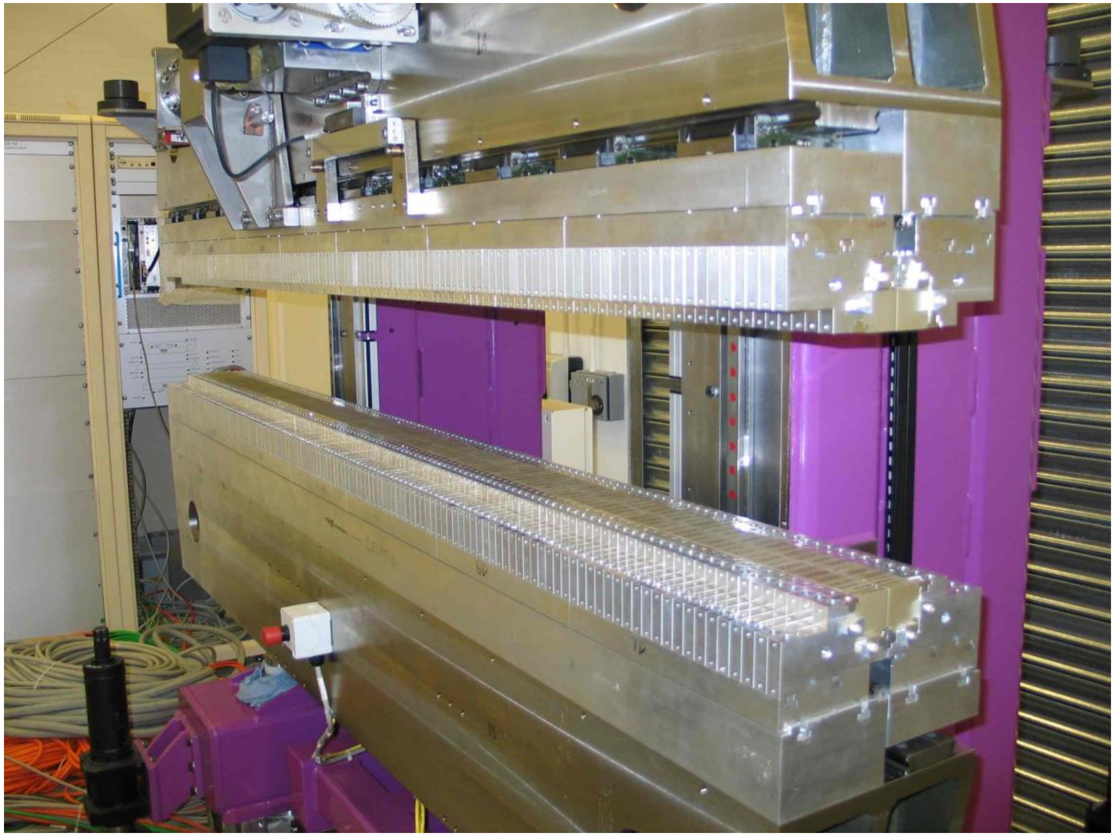
U27 at Diamond

27 mm, 73 periods 7 mm gap, B = 0.79 T; K = 2



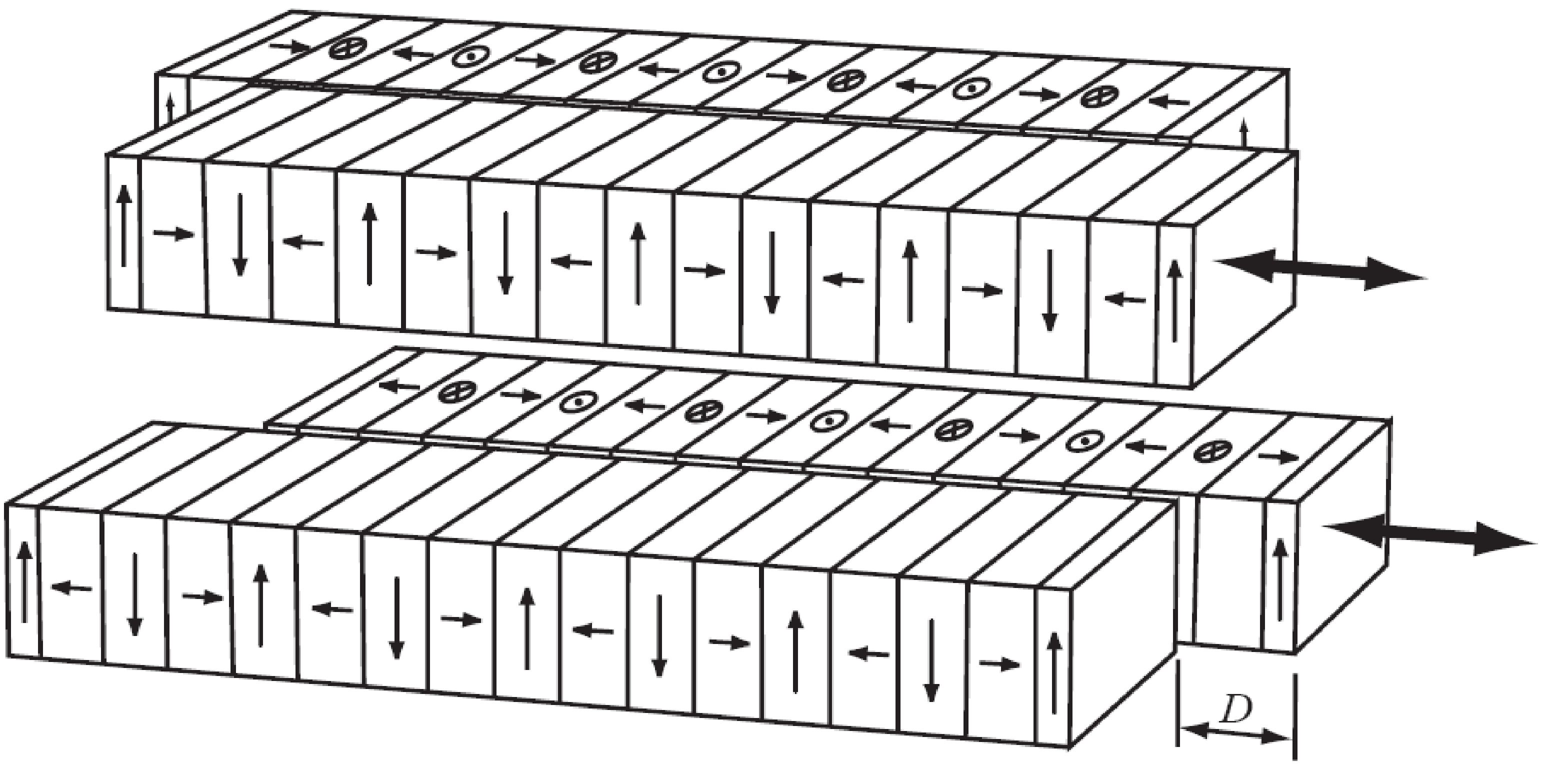
Apple-II type undulators (I)

Advanced Planar Polarized Light Emitter



HU64 at Diamond; 33 period of 64 mm; B = 0.96 T;
gap 15 mm; Kmax = 5.3

Apple-II type undulators (II)



Four independent arrays of permanent magnets

Diagonally opposite arrays move longitudinal, all arrays move vertically

Sliding the arrays of magnetic pole it is possible to control the polarisation of the radiation emitted

Superconducting Wigglers

Superconducting wigglers are used when a high magnetic field is required

3 - 10 T

They need a cryogenic system to keep the coil superconductive

Nb_3Sn and NbTi wires

SCMPW60 at Diamond

3.5 T coils cooled at 4 K

24 period of 64 mm

gap 10 mm

Undulator K = 21

Summary and bibliography

Accelerated charged particles emit electromagnetic radiation

Synchrotron radiation is stronger for light particles and is emitted by bending magnets in a narrow cone within a critical frequency

Undulators and wigglers enhance the synchrotron radiation emission

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