# Superconducting RF cavities for accelerators

### Fritz Caspers

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**CERN** 

JUAS lecture 2016

### OUTLINE OF LECTURE

- Basics of superconductivity
- Basics of RF cavities
- Interaction of the cavity with the beam
- Technological issues
- Diagnostics
- State of the art SRF research

## Basics of superconductivity

- Recommended Literature
- Historical remarks
- Meissner effect
- Two kinds of superconductors
- Materials
- Two fluid model
- Basics of RF superconductivity
  - Penetration depth
  - The surface resistance
  - Critical fields
- Summary

## Recommended literature

### Literature

- W. Buckel and R. Kleiner, « Superconductivity: Fundamentals and applications, Wiley VCH 2004
- V. V. Schmidt «The physics of superconductors », Springer 1997
- M. Tinkham, « Introduction to superconductivity », McGraw-Hill 1996, and many others
- Nobel lectures (<a href="http://nobelprize.org/nobel-prizes/physics/laureates/">http://nobelprize.org/nobel-prizes/physics/laureates/</a>)

## Historical remarks 1/4

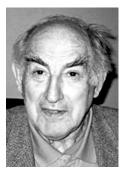
- 1908 Liquefaction of helium (4.2 K)
- 1911 Zero resistance
- 1933 Meissner effect
- 1935 Phenomenological theory of H & F. London
- 1950 Ginzburg Landau theory
- 1951 − 2 TYPE II superconductors (Abrikosov)
- 1957 Bardeen Cooper Schrieffer theory
- 1960 Magnetic flux quantisation
- 1962 Josephson effect
- 1986 High temperature superconductors (Bednorz – Müller)







Bardeen - Cooper - Schrieffer (BCS)



Ginzburg



**Bednorz** 



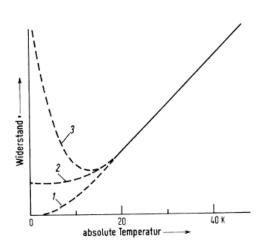
Abrikosov

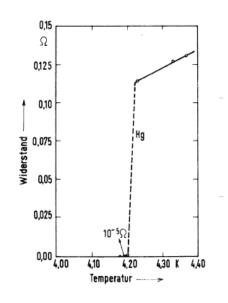


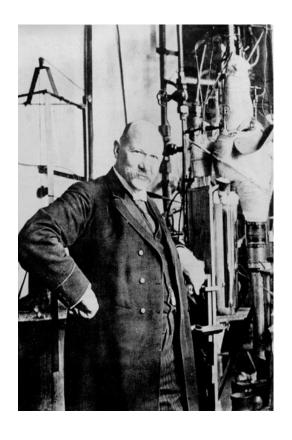
Müller

## Historical remarks 2/4

Nobel prize for « his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium »



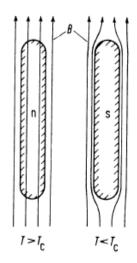


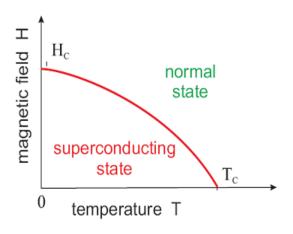


H. Kamerling – Onnes in his laboratory at Leiden (NL)

## Historical remarks 3/4

- Zero resistivity
- Meissner effect





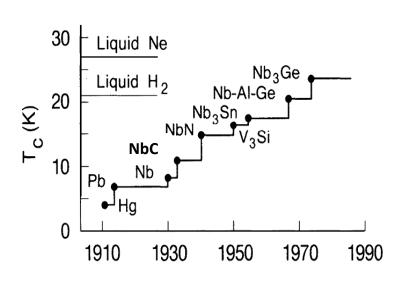
H - T diagram for the superconducting state

### Superconductivity is destroyed:

- by increasing temperature at  $T > T_c$
- by large magnetic field  $H > H_c$

$$\frac{H_c(T)}{H_c(0)} = \left[1 - \left(\frac{T}{T_c}\right)^2\right]$$

## Historical remarks 4/4



Development of the superconducting transition temperatures after the discovery of the phenomenon in 1911. The materials listed are metals or inter-metallic compounds and reflect the respective highest T<sub>c</sub>'s - Adapted from G. Bednorz – Nobel lecture

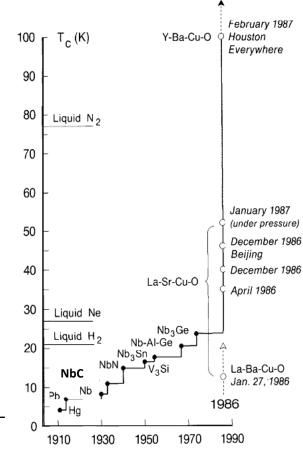
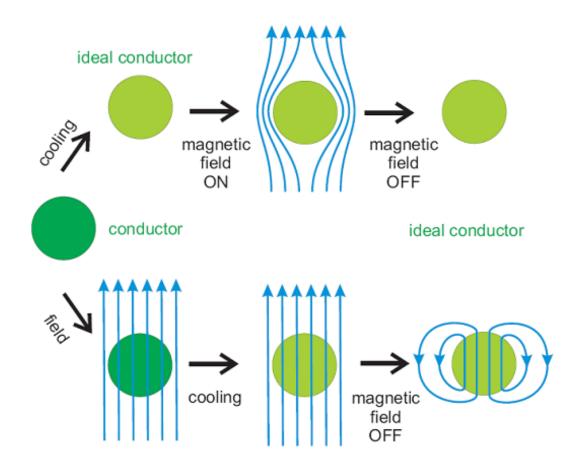


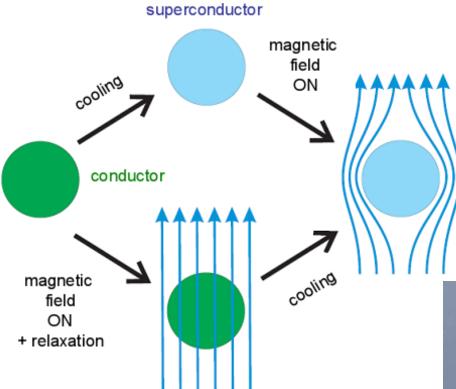
Figure 1.13. Evolution of the superconductive transition temperature subsequent to the discovery of the phenomenon. From [1.29], © 1987 by the American Association for the Advancement of Science.

## Meissner effect 1/3



An ideal conductor in magnetic field

## Meissner effect 2/3



A superconductor in magnetic field

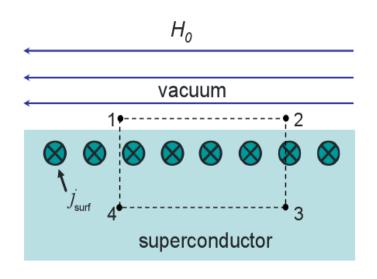


## Meissner effect 3/3

1. Magnetic lines of force outside a superconductor are always tangential to its surface

$$\operatorname{div} \overrightarrow{B} = 0;$$
Since  $\overrightarrow{B}_n = 0 \Rightarrow \overrightarrow{B}_n = 0$ 

2. A superconductor in an external magnetic field always carries an electric current near its surface



$$\nabla \times \vec{B} = \mu_0 \vec{j} \Rightarrow \vec{j} = 0 \text{ inside the superconductor}$$

$$\oint \vec{B} \cdot \vec{d} \vec{l} = \mu_0 j_{\text{surf}} \cdot l_{1-2}$$

$$\oint \stackrel{\rightarrow}{B} \cdot \mathrm{d} \stackrel{\rightarrow}{l} = \mu_0 H_0 \cdot l_{1-2} \ \, \Longrightarrow \stackrel{\rightarrow}{j}_{\mathrm{surf}} = \stackrel{\rightarrow}{n} \times \stackrel{\rightarrow}{H}_0$$

Thus, the surface current is completely defined by the magnetic field at the surface of a superconductor.

# Two kinds of superconductors 1/3

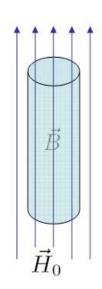
# Magnetic properties of a **type I** superconductor

Magnetic properties of a superconductor  $^B$  can be derived from  $\rho=0$  and B=0

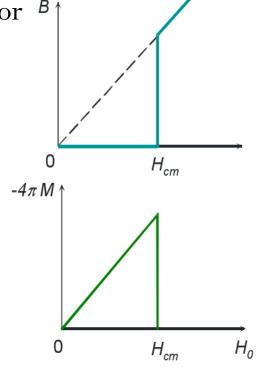
Type I superconductors are all elemental superconductors (except niobium)

$$\overrightarrow{B} = \mu_0 \cdot \left( \overrightarrow{H}_0 + \overrightarrow{M} \right)$$

The magnetization  $\stackrel{\frown}{M}$  compensates the external field  $\stackrel{\frown}{H_0}$ , thus the SC bulk is field free.



### Magnetization curve



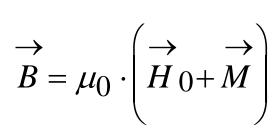
H<sub>cm</sub> ... critical field

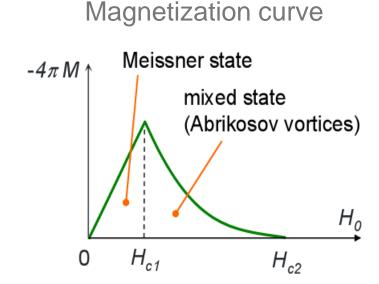
# Two kinds of superconductors 2/3

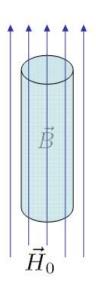
Type II superconductor

Magnetic properties of a type II superconductor

Above the 1st critical field  $H_{c1}$  magnetic flux penetrates into the bulk Above the 2nd critical field  $H_{c2}$  the material is normal conducting (except for a thin surface layer that remains superconducting until the 3rd critical field  $H_{c3}$ )

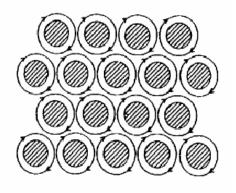






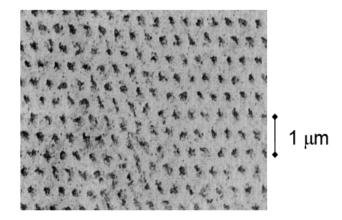
# Two kinds of superconductors 3/3

### Mixed state of a type II superconductor



Mixed state (Shubnikov phase) of a type II superconductor consists of a regular lattice of Abrikosov vortices.

Magnetic decoration image of a vortex lattice



## Surface tension at nc-sc interface 1/2

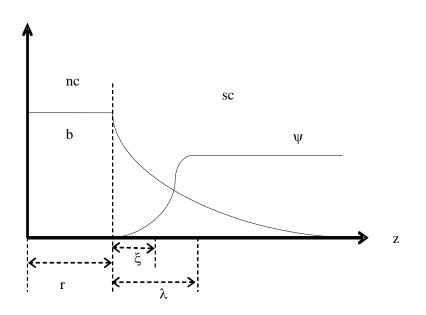


Fig. 1: Interface between normal to superconducting metal for a type II superconductor with  $\lambda > \xi$ . b denotes the (microscopic) magnetic field and  $\psi$  describes the wave function of the superconducting condensate.

Inspecting Fig. 1, the energy balance DE between the condensation energy  $E_c$  and the diamagnetic energy  $E_B$  for a planar interface area A and an applied magnetic field B, is

$$\Delta E = \Delta E_c - \Delta E_B =$$

$$= \frac{1}{2\mu_0} B_{th}^2 A(r + \xi) - \frac{1}{2\mu_0} B^2 A(r + \lambda)$$

## Surface tension at nc-sc interface 2/2

For a type II superconductor, as the penetration of magnetic fields starts from small filaments of cylindrical shape located parallel to the interface, a more realistic way to describe the energy balance is based on a small half-cylinder of radius r instead of a plane, which will become normal:

$$\Delta E = \Delta E_c - \Delta E_B = \frac{1}{2\mu_0} \cdot B_{th}^2 \cdot \frac{\pi}{2} \cdot (r + \xi)^2 - \frac{1}{2\mu_0} \cdot B^2 \cdot \frac{\pi}{2} \cdot (r + \lambda)^2 < 0,$$

from which the threshold B\*c1 of the magnetic field for penetration is defined as

$$B_{c1}^* \geq \frac{r + \xi}{r + \lambda} \cdot B_{th} \xrightarrow{r \to 0} \frac{\xi}{\lambda} \cdot B_{th} = \frac{1}{\kappa} \cdot B_{th}.$$

In a type II superconductor, the lowest value of the applied magnetic field B which induces penetration as filaments of magnetic field into the bulk is called the lower critical field  $B_{c1}$ , for which the microscopic theory gives as exact result:

$$B_{c1} = \frac{\ln \kappa}{\sqrt{2} \cdot \kappa} \cdot B_{th}$$

very close to the previous one. In a type I superconductor, the lowest value of the applied magnetic field B which induces bulk penetration of magnetic field is called the thermodynamic critical field B<sub>th</sub>

## Materials 1/2

#### pure metals

material	$T_c$ ,K	$H_c,$ Oe	year
Al	1.2	105	1933
In	3.4	280	
Sn	3.7	305	
Pb	7.2	803	1913
Nb	9.2	2060	1930

Cold liquids required for reaching low temperatures:

helium  $^4$ He (4.2 K) hydrogen  $H_2$  (20 K) neon Ne (27 K) nitrogen  $N_2$  (77 K)

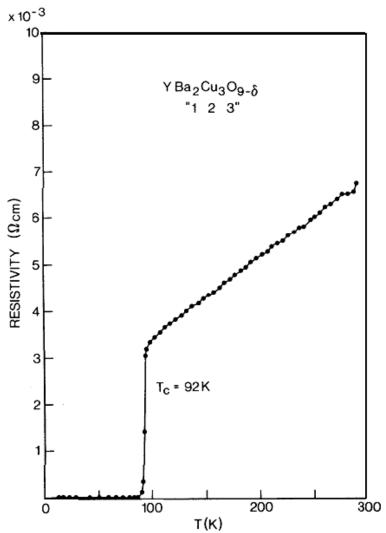
alloys

NbN	15	1.4 10 <sup>5</sup>	1940
$Nb_3Ge$	23	3.7 10 <sup>5</sup>	1971

ceramics

material	$T_c,$ K	year
$La_{1.85}Ba_{0.15}CuO_4$	35	1986
$YBa_2Cu_3O_7$	93	1987
$Bi_2Sr_2CaCu_2O_{8+x}$	94	1988
$Ta_2Ba_2Ca_2Cu_3O_{10+x}$	125	1988

## Materials 2/2



Resistivity of a single-phase YBa<sub>2</sub>Cu<sub>3</sub>0<sub>7</sub> sample as a function of temperature.

## Two fluid model

# Basic ingredients for RF superconductivity

Two fluid model (Gorter-Casimir)
Maxwell electrodynamics

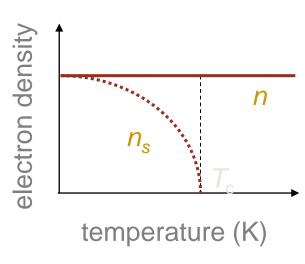


- all free electrons of the superconductor are divided into two groups
  - $\square$  superconducting electrons of density  $n_s$
  - $\square$  normal electrons of density  $n_n$

The total density of the free electrons is

$$n = n_s + n_n.$$

As the temperature increases from 0 to  $T_c$ , the density  $n_s$  decreases from n to 0.



$$n_s/n = 1 - (T/T_c)^4$$

## RF Superconductivity

1st London equation (Newton's force law without friction)

$$n_{\rm s} m \frac{\mathrm{d} \vec{v}_{\rm s}}{\mathrm{d} t} = n_{\rm s} e \vec{E}$$
  $\vec{j}_{\rm s} = n_{\rm s} e \vec{v}_{\rm s}$   $\vec{E} = \frac{\mathrm{d}}{\mathrm{d} t} (\Lambda \vec{j}_{\rm s})$   $\Lambda = m/n_{\rm s} e^2$ 

In the stationary state  $dj_s/dt = 0$  and hence E = 0 everywhere in the superconductor.

2nd London equation (Meissner effect)

$$\vec{\nabla} \times \vec{E} - \Lambda \cdot \vec{\nabla} \times \frac{d}{dt} \vec{j}_{s} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{d}{dt} \vec{B} \implies \frac{d}{dt} \vec{B} + \Lambda \cdot \vec{\nabla} \times \frac{d}{dt} \vec{j}_{s} = 0$$

After integration and taking the integration constant = 0

$$\Rightarrow \vec{B} = -\Lambda \cdot \vec{\nabla} \times \vec{j}_s$$

## Penetration depth

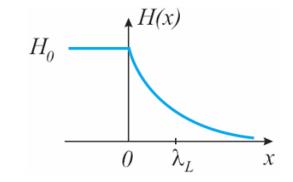
On introducing the vector potential  $\vec{A}$  via  $\vec{B} = \vec{\nabla} \times \vec{A}$  one obtains a relationship between the supercurrent and the vector potential, very similar to Ohm's law  $\vec{j} = \sigma \vec{E}$ 

$$\Rightarrow \vec{j}_s = -\frac{1}{\Lambda} \vec{A} = -\frac{n_s e^2}{m} \vec{A}$$

London penetration depth:

Starting from the 2nd London equation

$$\vec{B} + \Lambda \vec{\nabla} \times \vec{j}_s = 0 \implies \vec{B} + (\Lambda/\mu_0) \underbrace{\vec{\nabla} \times \vec{\nabla} \times \vec{B}}_{\vec{\nabla} \vec{\nabla} \cdot \vec{B} - \Delta \vec{B}} \vec{B} = 0$$



$$\vec{B} - (\Lambda/\mu_0)\Delta\vec{B} = 0 \implies \vec{B} = \vec{B}_0 \exp(x/\lambda_L)$$

with the London penetration depth  $\lambda_{\rm L}^2 = \Lambda/\mu_0 = m/(n_s e^2 \mu_0)$ 

Element	Al	Nb (crystal)	Nb (film)	Pb	Sn	YBCO
$λ_L$ [nm]	50	47	90	39	51	170

Compare to NC Cu skin depth: 2 µm @ 2 GHz

## The surface resistance 1/2

The RF magnetic field penetrates this sheet to within the penetration depth  $\lambda_{\rm L}$ .

According to the Maxwell equation curl E = -dB/dt, the RF magnetic field is accompanied by an electric field  $E_v = i\omega \lambda_1 B_z = i\omega \lambda_1 \mu_0 H_z = i\omega \lambda_1 \mu_0 H_{z0} \exp(-x/\lambda_1).$ 

The electric field interacts with the nc electrons (still present at non-zero temperatures) and gives rise to a power dissipation per square meter

$$P_{c} = (\sigma_{n}/2) \int_{0}^{\infty} E_{y}^{2}(x) dx = (1/4) \lambda_{L} \sigma_{n} E_{y0}^{2} = (1/4) \omega^{2} \mu_{0}^{2} \lambda_{L}^{3} \sigma_{n} H_{z0}^{2}$$

with  $\sigma_{\rm n}=\sigma_{\rm 0}(T/T_{\rm c})^4$ ,  $\sigma_{\rm 0}$  being the conductivity of the nc electrons just above  $T_{\rm c}$ , By definition,  $P_{\rm c}=(1/2)~R_{\rm s}~H_{\rm z}^{2}$ , and we obtain for the surface resistance  $R_s$  in the two-fluid model approximation,

$$R_s = \frac{1}{2}\omega^2 \mu_0^2 \lambda_L^3 \sigma_n = \frac{1}{2}\omega^2 \mu_0^2 \lambda_L^3 \sigma_0 \left(\frac{T}{T_c}\right)^4$$

which can be approximated for  $T < T_c/2$ 

$$R_{\rm s} = (A/T)\omega^2 \exp(-\Delta/kT), 2\Delta \approx 3.5kT_{\rm c}$$

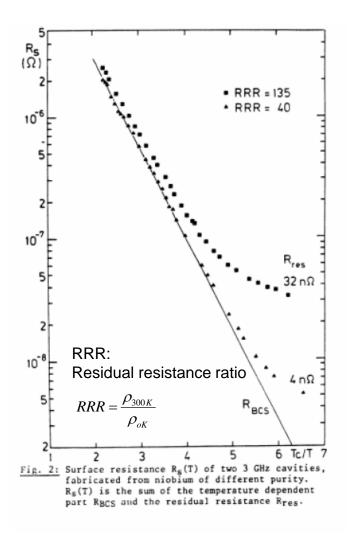
An approximate expression for Niobium: 
$$R_s^{BCS} \approx 3 \times 10^{-4} \, \Omega \cdot \left(\frac{f[MHz]}{1500MHz}\right)^2 \cdot \frac{\exp\left(-\frac{17.67\,K}{T[K]}\right)}{T[K]} \quad \text{Developed by \textit{Mattis} and \textit{Bardeen, based on the SC theory of \textit{Bardeen, Cooperations}}$$

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theory of Bardeen, Cooper and Schrieffer (BCS theory).

### The surface resistance 2/2

• Material parameter dependence of  $R_s$  (T, f)



23

$$R_s^{BCS}(\omega, T) = \frac{A}{T}\omega^2 \exp\left(-\frac{\Delta}{kT}\right)$$

$$2\Delta \approx 3.5 \cdot kT_c$$

A depends on the material

$$R_{s} = R_{s}^{BCS} + R_{res}$$

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## Critical field (s)

### Critical field of superconductors studied for RF

Material	T <sub>c</sub> [K]	B <sub>cth0</sub> [mT]	B <sub>c10</sub> [mT]	B <sub>c20</sub> [mT]	B <sub>sh0</sub> [mT]	B <sub>exp0</sub> [mT]
Sn	3.7	30.9		_	68	30.6
In	3.4	29.3			104	28.4
Pb	7.2	80.4	_	_	105	112
Nb	9.2	200	185	420	240	160
Nb <sub>3</sub> Sn	18.2	535	≈ 20	2400	400	106

#### Critical fields in DC and RF superconductivity

B <sub>c</sub>	Critical magnetic field of type-I superconductor			
B <sub>c1</sub>	Lower critical magnetic field of type-II superconductor			
$B_{c2}$	Upper critical magnetic field of type-II superconductor			
B <sub>cth</sub> Thermodynamic critical field				
B <sub>sh</sub> Superheating critical field				
Bexp				
An index "0" following any of the above symbols refers to the temperature $T = 0$ K, tacitly assuming $B(T) = B_0 [1 - (T/T_c)^2]$ .				

### Other deterministic parameters for cavity performance

Until till now we discussed the role of the RF frequency, lHe bath temperature, and sc material with its characteristic critical field and temperature. There are still other (less important) parameters that determine the performance of the cavity as well:

Influencing quantity	Impact quantity	Physical explanation	Cure
External static magnetic field $B_{\text{ext}}$	Residual surface resistance	Creation of vortices	Shielding of ambient magnetic field by Mu- metal / Cryoperm
Residual resistivity ratio <i>RRR</i>	BCS surface resistance	Mean free path dependence of $R_{\rm res}$	Annealing steps during ingot production/after cavity manufacture
Ratio peak magnetic field to accelerating gradient $B_p/E_a$	Max. accelerating gradient	Critical magnetic field as ultimate gradient limitation	Optimization of cavity shape
Nb-H precipitate	Q-value / acc. gradient (Q-disease)	Lowering of $T_c/B_c$ at precipitates of Nb-H	T-control during chemical polishing Degassing @ 700 °C Fast cool-down

# Summary

### Superconducting materials:

- are characterized by zero resistivity (in DC) and the Meissner effect;
- Show the (thermodynamic) phase transition into the superconducting state below a critical temperature and below a critical field;
- have a non-zero surface resistance for RF which can be understood by the two-fluid model and the London theory
- lacksquare are subdivided into type I and type II, depending on the value of the Ginzburg-Landau parameter  $\kappa$ ;
- may be alloys or elements, for which they are of type I, except Nb, the technically most important one, which is type II and has the largest critical temperature and critical field;

### Basics of RF cavities

- Variety of RF cavities (examples)
- Cavity characteristics
  - Cavity characteristics (peak fields, stored energy, ...)
  - Pillbox resonator —basics, field distribution
  - Computer codes to determine the cavity parameters
  - Different mode families
- Transmission line
- Response of a sc cavity to RF (determination of  $Q_0, E_{acc}, \ldots$ )
- Measuring setup  $(Q(E_{acc}) \text{ curve}, \ldots)$
- Pass-band modes
- Typical example of storage ring cavity (LEP)
- Summary

# **Examples of RF cavities**



Fig. 1 A spectrum of superconducting cavities.

(from H. Padamsee, CERN-2004-008)

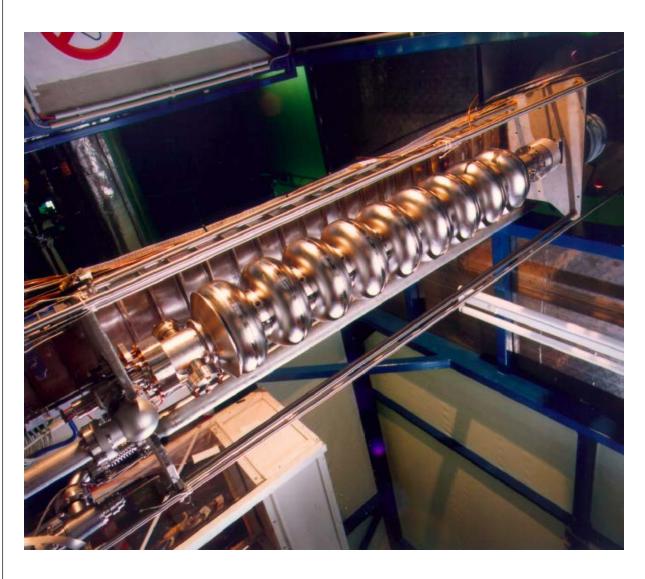
### LHC - CERN



JUAS lecture 2016: SC RF cavities Caspers/Weingarten/Junginger/Aull/Peters/Bartel/Wendt

- $f_{res} = 400 \text{ MHz}$ 
  - $R/Q = 89 \Omega$
  - $Q_0 = 2 \times 10^9$
  - $E_{acc} = 5.33 \text{ MV/m}$
  - $P_{in} = 116 \text{ kW (CW)}$
- Niobium-film on Cu
  - 1-2 μm thickness
- 4 single-cell cavities per cryomodule
  - Each resonator delivers2 MV
  - Blade tuner
  - «Doorknob» power coupler, 75  $\Omega$  coaxial
- Total of 8 cavities per beam: 16 MV

### XFEL - DESY 1/3



- $f_{res} = 1300 \text{ MHz}$ 
  - $R/Q = 89 \Omega$
  - $Q_o = 2 \times 10^9$
  - $E_{acc} = 5.33 \text{ MV/m}$
  - $P_{in} = 116 \text{ kW (CW)}$
- Niobium-film on Cu
  - 1-2 μm thickness
- 4 single-cell cavities per cryomodule
  - Each resonator delivers2 MV
  - Blade tuner
  - «Doorknob» power coupler, 75  $\Omega$  coaxial
- Total of 8 cavities per beam: 16 MV

## XFEL - DESY 2/3



XFEL cryomodule assembly of eight 9-cell cavities, quadrupole and BPM in a cleanroom

## XFEL - DESY 3/3



Cryomodule installation in the tunnel (the XFEL consists out of 101 cryomodules)

## CEBAF - JLAB

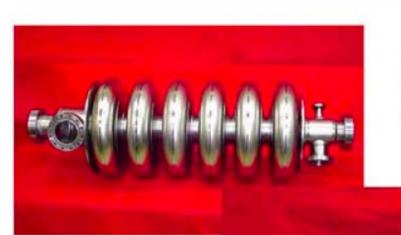


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## SPL - CERN/ SNS - ORNL

### Prototype Beta 0.61 and .81 Cavities





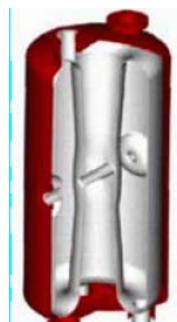
Development led by P. Kneisel with aid from colleagues all over the world

704 MHz SRF cavity R&D study to upgrade the CERN injectors with a high intensity superconducting RF proton linac (SPL)

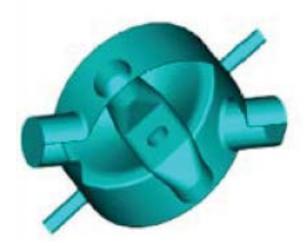
## Heavy Ion accelerators ATLAS - ANL

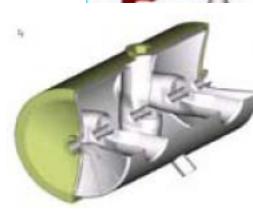


# Shapes of heavy ion accelerator cavities







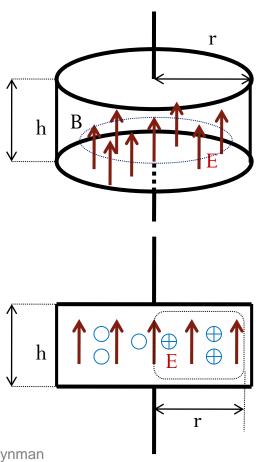




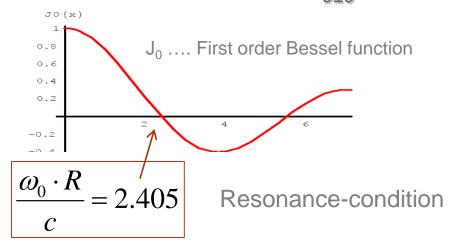


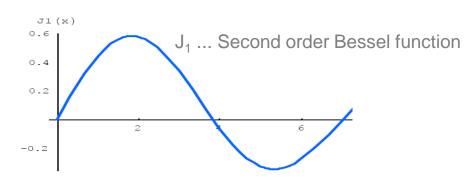
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#### Pill box resonator



#### Field distribution TM<sub>010</sub> mode:





Source: The Feynman Lectures on Physics, Vol. II

## Pill box resonator

$$h = \beta \cdot \frac{\lambda}{2}$$

$$B$$

$$C$$

$$E$$

$$\oplus$$

$$\oplus$$

$$E(r,t) = J_0 \left( \frac{2.405 \cdot r}{R} \right) \cdot E_0 \sin(\omega_0 \cdot t) \qquad B_{\varphi}(r) = \frac{i}{c} J_1 \left( \frac{2.405 \cdot r}{R} \right) \cdot E_0 e^{i\omega_0 t}$$

$$B_{\varphi}(r) = \frac{\mathrm{i}}{c} J_{1} \left( \frac{2.405 \cdot r}{R} \right) \cdot E_{0} \epsilon$$

$$t(z) = z/(\beta \cdot c)$$

Stored energy U: 
$$U = \frac{\varepsilon_0}{2} \int_0^{2\pi} d\phi \int_0^h dz \int_0^R r dr \cdot |E(r)|^2 = \frac{\varepsilon_0}{2} \cdot \underline{\pi R^2 h} |E_0|^2 \cdot |J_1(2.405)|^2$$

$$\boxed{\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}}$$

**Dissipated power P:** 

$$P = \frac{R_s}{2\mu_0^2} \int_0^{2\pi} d\varphi \cdot \left( \int_0^h dz |B_{\varphi}(R)|^2 + 2 \cdot \int_0^R r dr |B_{\varphi}(r)|^2 \right) = R_s \frac{1}{\eta^2} |E_0|^2 \cdot \pi R \cdot (h+R) \cdot \left| \underbrace{J_1(2.405)}_{0.581865} \right|^2$$

$$Q = \frac{\text{Stored energy } U}{\text{Energy lost during 1 RF period}} = \omega \frac{U}{P} \qquad Q = \omega \cdot \frac{\varepsilon_0 \cdot \eta^2}{2R_s \cdot \left(\frac{1}{R} + \frac{1}{h}\right)}$$

$$Q = \omega \cdot \frac{\varepsilon_0 \cdot \eta^2}{2R_s \cdot \left(\frac{1}{R} + \frac{1}{R}\right)}$$

# Cavity characteristics

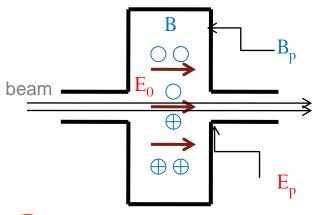
shunt impedance R:

$$R = \frac{V_a^2}{2 \cdot P}$$

**R/Q** measures the interaction of the cavity with the beam:

$$\frac{R}{Q} = \frac{V_a^2}{\underbrace{2 \cdot P}_{R}} \cdot \underbrace{\frac{P}{\omega \cdot U}}_{1/Q} = 7.10 \frac{4}{\pi^3} \cdot \frac{h}{R} \cdot \eta$$

The peak surface electric and magnetic fields constitute the ultimate limit for the accelerating gradient => minimize the ratio  $E_p/E_a$  and  $B_p/E_a$ .



$$E_a = \frac{2E_0}{\pi}$$

$$\frac{B_p}{E_a} = 3.07 \left[ \frac{\text{mT}}{\text{MV/m}} \right] \qquad \frac{E_p}{E_a} = \frac{\pi}{2} \approx 1.57$$

$$\frac{E_p}{E_a} = \frac{\pi}{2} \approx 1.57$$

## Cavity characteristics - Summary

Symbol	Name	Definition	Pillbox cavity [0.35 GHz, 4.2 K, Nb]	Accelerating cavity [0.35 GHz, 4.2 K, Nb]
$E_p/E_a$	Peak normalized surface electric field	n/a	1.6	2
$\frac{B_p/E_a}{[mT/(MV/m)]}$	Peak normalized surface magnetic field	n/a	3.1	4
$R_s [n\Omega]$	Surface resistance	$E_x/H_y$	40	40
h [m]	Cavity length	$h=\lambda/2$	0.43	0.43
$E_a [MV/m]$	Accelerating gradient	(1/e) ·Energy gain/length	10	10
V [MV]	Accelerating voltage	$V=E_a \cdot h$	4.3	4.3
$G\left[\Omega ight]$	Geometry factor	$G=R_s\cdot Q$	260	275
$Q[10^9]$	Quality factor	Q=ωU/P	6.5	6.9
$R/Q[\Omega]$	(R/Q) factor	$(R/Q)=V^2/(2\omega U)$	450	280
R [M $\Omega$ ]	Shunt impedance	$R=V^2/(2P)$	$3.10^{6}$	$2 \cdot 10^6$
ս [J]	Stored energy	$U=V^2/[2\omega(R/Q)]$	9	15
P [W]	Dissipated power	P=ωU/Q	3	5
h/R	Ratio cavity length to radius	n/a	1.3	0.5

# Computer codes for RF cavities

#### Computer codes to determine the cavity parameters

For real structures with contoured shapes, beam apertures and beam pipes, it is necessary to use field computation codes, such MAFIA and Microwave Studio. Figure 10 shows the electric and magnetic fields computed by Microwave Studio for the accelerating mode of a pillbox cavity with a beam hole, and for a round wall cavity. Such codes are also necessary for computing the fields in the higher order modes of a cavity that can have an adverse effect on beam quality or cause instabilities. Figure 11 shows the electric and magnetic fields of the first monopole HOM. Beam induced voltages are also proportional to the R/Q of HOMs.

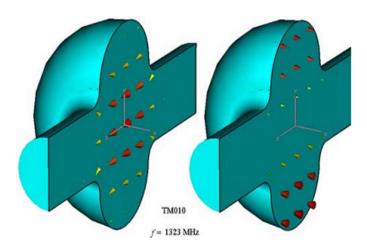


Fig.10 (Left) Electric and (Right) Magnetic fields for a round cavity with beam holes.

### Cavity characteristics - Summary table

Table: Equivalence of cavity and lumped-element circuit parameters

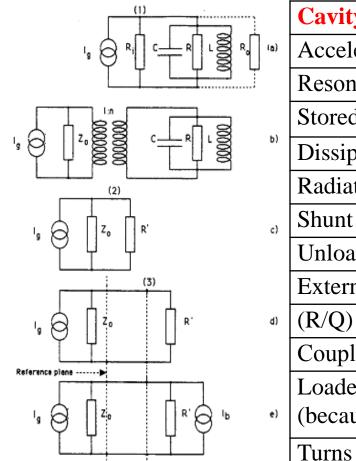


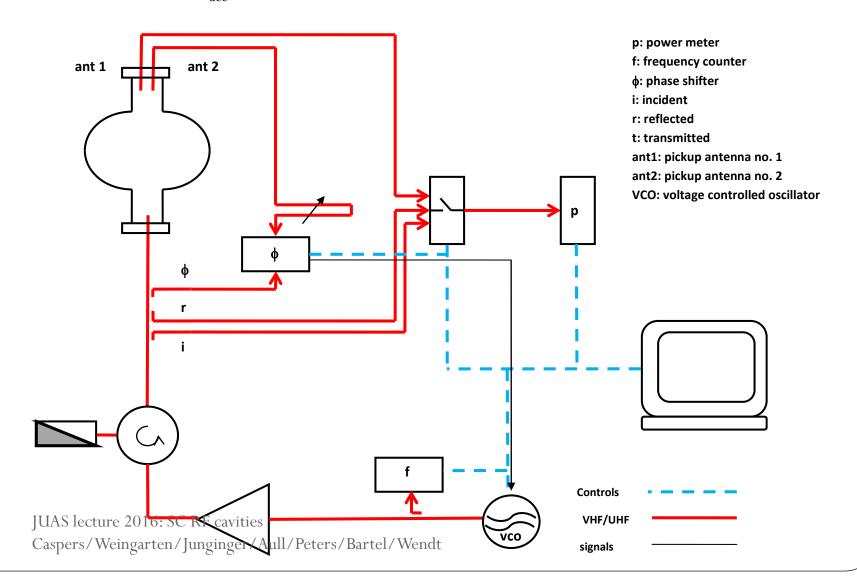
Fig. 8 Lumped element equivalent circuit model of an accelerating cavity resonator

Cavity	<b>Lumped-element circuit</b>	
Accelerating voltage V	Peak voltage V	
Resonant frequency $\omega_0$	$\omega_0 = 1/V(LC)$	
Stored energy U	$U = (1/2)CV^2$	
Dissipated power P <sub>c</sub>	$P_c = (1/2) V^2/R$	
Radiated power P <sub>rad</sub>	$P_{rad} = (1/2) V^2/R_i$	
Shunt impedance $R = V^2/(2 \cdot P_c)$	R	
Unloaded Q - value $Q_0 = \omega_0 \cdot U/P_c$	$Q_0 = \omega_0 \cdot RC$	
External Q - value $Q_{ext} = \omega_0 \cdot U/P_{rad}$	$Q_{\rm ext} = \omega_0 \cdot R_i C = R_i / (R/Q)$	
(R/Q) value R/Q = $V^2/(2 \omega_0 \cdot U)$	$R/Q = V(L/C) = 1/(\omega_0 \cdot C)$	
Coupling factor $\beta = Q_0/Q_{ext}$	$\beta = R/R_i$	
Loaded Q - value $Q_L = Q_0/(1+\beta)$	$Q_{L} = \omega_{0} \cdot RC/(1+\beta)$	
(because $Q_L^{-1} = Q_0^{-1} + Q_{ext}^{-1}$ )		
Turns ratio $n = V[(R/Q) \cdot Q_{ext}/Z_0]$	$n = V(R_i/Z_0)$	
Wave impedance $Z_0 = 50 \Omega$		

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# Measuring setup

- Q determined by measuring the decay time of the cavity response
- Measurement of Q vs E<sub>acc</sub>



## Q(E<sub>acc</sub>) curve

#### Electropolishing and in-situ Baking of 1.3 GHz Niobium Cavities

L. Lilje<sup>+</sup>, D. Reschke, K. Twarowski, DESY, Notkestraße 85, 22607 Hamburg
P. Schmüser, Universität Hamburg

D. Bloess, E. Haebel, E. Chiaveri, J.-M. Tessier, H. Preis, H. Wenninger, CERN, Geneva H. Safa, J.-P. Charrier, CEA, Saclay

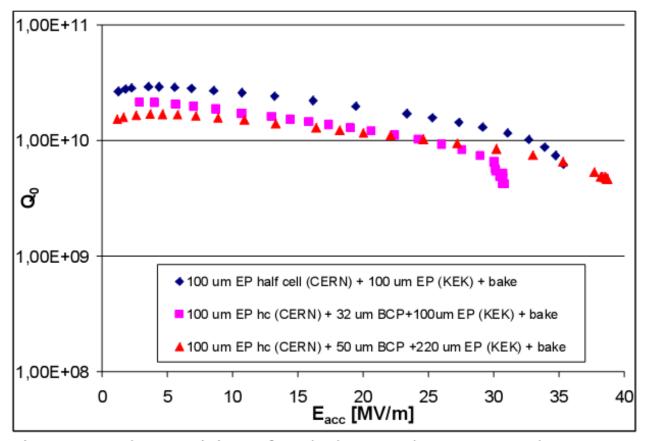
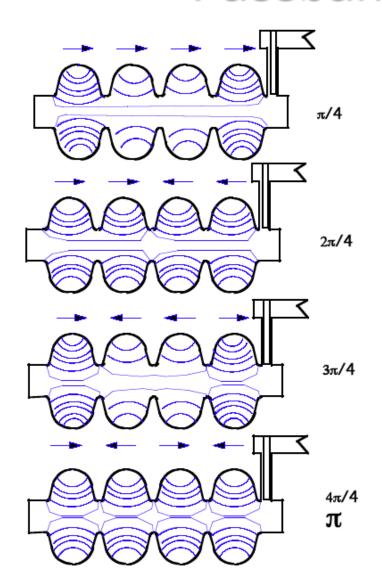


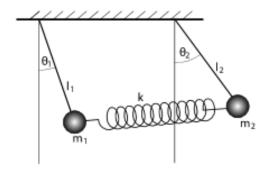
Figure 6: The cavities after bakeout show no Q-drop. One cavitiy is limited at 30 MV/m due to strong field emission and available RF power.

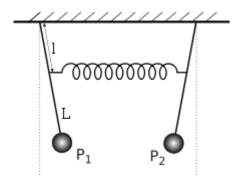
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### Passband modes







http://de.wikipedia.org/wiki/Gekoppelte\_Pendel



• <a href="http://www.youtube.com/watch?v=IAPWpViY19A">http://www.youtube.com/watch?v=IAPWpViY19A</a>

### Typical storage ring cavity (LEP)



# Summary

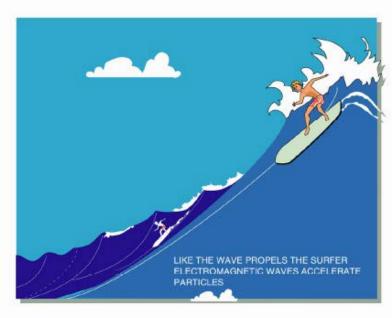
- The pillbox resonator ( $TM_{010}$  mode) allows as a paradigm the analytical description of typical accelerator parameters, such as peak surface fields (E and H), power loss and Q-value, shunt impedance, geometrical shunt impedance, geometry factor, etc.
- « Real » accelerator cavities are designed by making use of computer codes such as Microwave Studio, MAFIA, SUPERFISH, etc.
- The response of a cavity to an RF pulse is well described by lumped circuit networks, in particular by the transmission and reflection of an electromagnetic wave at a discontinuity in the line.
- An algorithm is presented to determine the coupling factor  $\beta$  (or the reflection factor  $\rho$ ), and finally the unloaded Q-value  $Q_0$ , the accelerating voltage V (accelerating gradient  $E_a$ ) and the surface resistance  $R_s$ .

# Interaction of cavity with beam

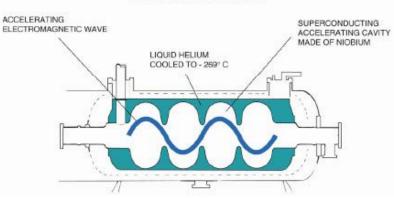
- Descriptive introduction
- Analytical introduction
- Transfer of RF power from the cavity to the beam
  - The fundamental mode power coupler
- Transfer of RF power from the beam to the cavity
  - Higher order modes and their damping
- The frequency tuner
- Summary

### Descriptive Introduction

#### THE SUPRACONDUCTIVITY



THE USE OF SUPRACONDUCTIVITY TO INCREASE PERFORMANCES AND CONSIDERABLY REDUCE ELECTRICITY CONSUMPTION

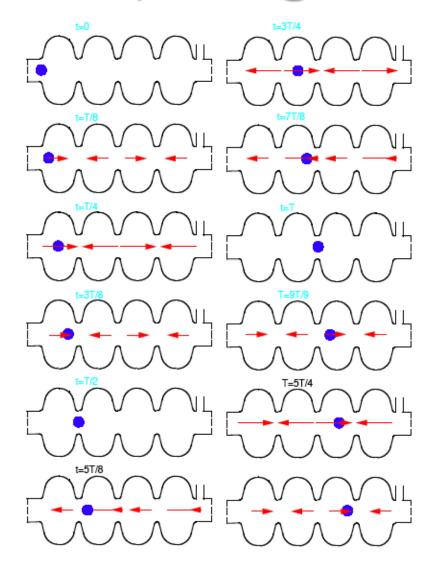


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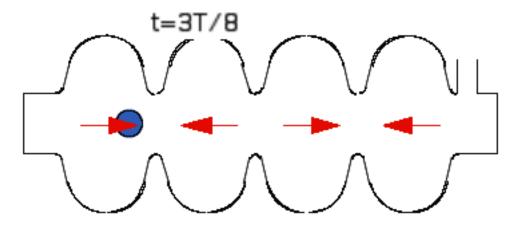
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# Particle passing through cavity



## **Analytical Introduction**



$$V = E(z_1,t_1)\Delta z_1 + E(z_2,t_2)\Delta z_2 + \dots$$

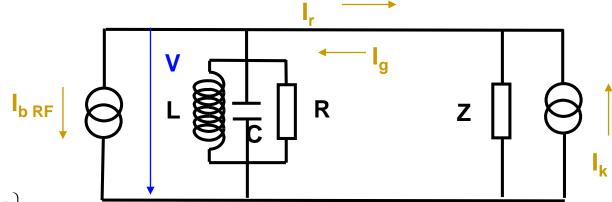
$$\Rightarrow V = \int_{-\infty}^{\infty} E(z, t(z)) dz$$

$$E(z,t) = E(z) * \cos(\omega t + \varphi)$$

$$t = z/v$$

$$V(\varphi) = \int_{-\infty}^{+\infty} E(z) * \cos(z\omega/v + \varphi) dz$$

#### Transfer of RF power from the cavity to the beam 1/3



$$I_r - I_g + I_k = \frac{V}{Z}$$

$$I_k = 2I_g$$

$$\Rightarrow I_r + I_g = \frac{V}{Z}$$

$$\begin{split} I_{LCR} &= I_g - I_r - I_{b,RF} = 2I_g - I_{b,RF} - \frac{V}{Z} \\ I_{LCR} &= V \bigg( \frac{1}{i\omega L} + i\omega C + \frac{1}{R} \bigg) \end{split}$$

With 
$$\omega_0^2 - \omega^2 \approx 2\omega_0 \Delta \omega$$
 and  $\Delta \omega \prec \omega$ 

$$\Rightarrow V \left( -\frac{2\Delta \omega}{\omega} + \frac{1}{i\omega C} \left( \frac{1}{R} + \frac{1}{Z} \right) \right) = \frac{2I_g - I_{b,RF}}{i\omega C}$$

A circulator guarantees that under no circumstances there is no reflected wave impinging to the RF generator

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

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#### Transfer of RF power from the cavity to the beam 2/3

Re-write preceding equation

$$V\left(-\frac{2\Delta\omega}{\omega} + \frac{1}{i\omega C}\left(\frac{1}{R} + \frac{1}{Z}\right)\right) = \frac{2I_g - I_{b,RF}}{i\omega C}$$

in cavity parameters

$$V\left(\frac{1}{2(R/Q)}\left(\frac{1}{Q_{ext}} + \frac{1}{Q_0}\right) - i\frac{\Delta\omega}{\omega(R/Q)}\right) = I_g - \frac{1}{2}I_{b.RF}$$

#### Transfer of RF power from the cavity to the beam 3/3

$$V\left(\frac{1}{2(R/Q)}\left(\frac{1}{Q_{ext}} + \frac{1}{Q_0}\right) - i\frac{\Delta\omega}{\omega(R/Q)}\right) = I_g - \frac{1}{2}I_{bRF}$$
• Minimize reflected power

$$\Rightarrow I_g = \frac{V}{2(R/Q)} \left( \frac{1}{Q_{ext}} + \frac{1}{Q_0} \right) + I_{DC} \cos \Phi - i \left( I_{DC} \sin \Phi + \frac{V\Delta \omega}{\omega (R/Q)} \right) + \frac{1}{Q_0} \prec \frac{1}{Q_{ext}} \text{ for sc cavities}$$

$$I_r = \frac{V}{Q_{ext} \cdot (R/Q)} - I_g = \frac{V}{2 \cdot (R/Q)} \left( \frac{1}{Q_{ext}} - \frac{1}{Q_0} \right) - I_{DC} \cos \Phi - i \left( I_{DC} \sin \Phi + \frac{V\Delta \omega}{\omega (R/Q)} \right)$$

**Actions:** 1) compensate « reactive beam loading » to zero by detuning  $\Delta \omega$ 

$$\Delta \omega = -\omega \frac{(R/Q) \cdot I_{DC}}{V} \sin \Phi$$

2) define optimum  $Q_{ext}$  for nominal beam current for  $I_r = 0$ 

$$Q_{ext,opt} = \frac{V}{2 \cdot (R/Q) I_{DC} \cos \Phi}$$

RF power 
$$P_{g,r} = \frac{1}{2}Z|I_{g,r}|^2 = \frac{1}{2}(R/Q)\cdot Q_{ext}\cdot |I_{g,r}|^2$$

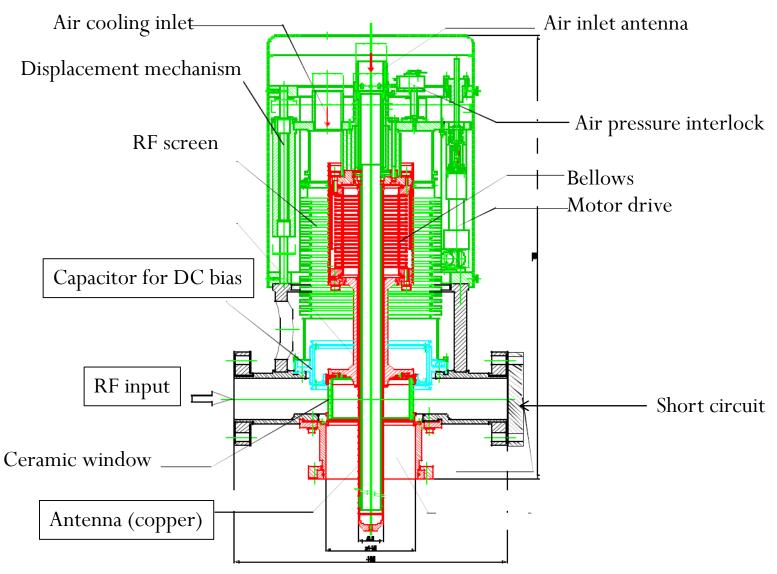
Check:

$$P_{beam,r} = P_{g,r} - P_{r,r} = V \cdot I_{DC} \cdot \cos \Phi$$

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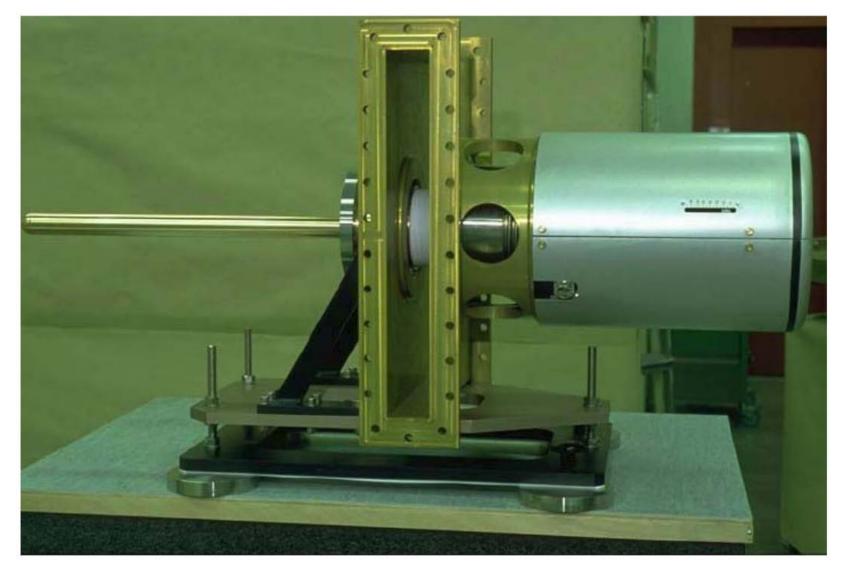
## The fundamental mode power coupler



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# LHC solution of the power coupler



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#### Transfer of RF power from the beam to the cavity

• Need for Higher Order Mode (HOM) coupler

Imagine worst case

- the cavity resonant frequency is « tuned » to a spectral line of the beam
- 2. Generator switched off,  $I_a=0$ .

$$V\left(\frac{1}{2(R/Q)}\left(\frac{1}{Q_{ext}} + \frac{1}{Q_0}\right) - i\frac{\Delta\omega}{\omega(R/Q)}\right) = I_g - \frac{1}{2}I_{b.RF}$$

$$\Rightarrow \Delta \omega = 0; \Phi = 0$$

$$\Rightarrow V = -I_{b.RF} \cdot (R/Q) \cdot Q_{ext} = -2 \cdot I_{DC} \cdot (R/Q) \cdot Q_{ext}$$

This means that the beam is decelerated.

Remedy: keep Q<sub>ext</sub> as low as possible.

Output power (reflected):

$$P_r = \frac{V^2}{2 \cdot Z} = \frac{V^2}{2 \cdot (R/Q) \cdot Q_{ext}} = 2 \cdot (R/Q) \cdot Q_{ext} \cdot I_{DC}^2$$

1<sup>st</sup> example (LEP); RF Generator trip.

We obtain for the **accelerating mode**  $R/Q = 232 \Omega$ ;  $Q_{ext} = 2 \cdot 10^6$ ;  $I_{DC} = 6 \text{ mA}$ ;  $P_r = 33 \text{ kW}$ 

2<sup>nd</sup> example;

We obtain for the **higher order mode** with  $(R/Q) = 10 \Omega$ , Qext = 20000

$$V = -2.4 \text{ kV} \Rightarrow P_r = 14.4 \text{ W}$$

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### Higher order modes

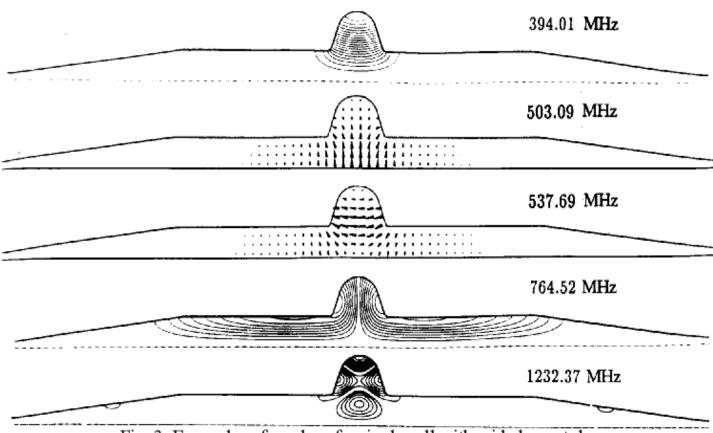
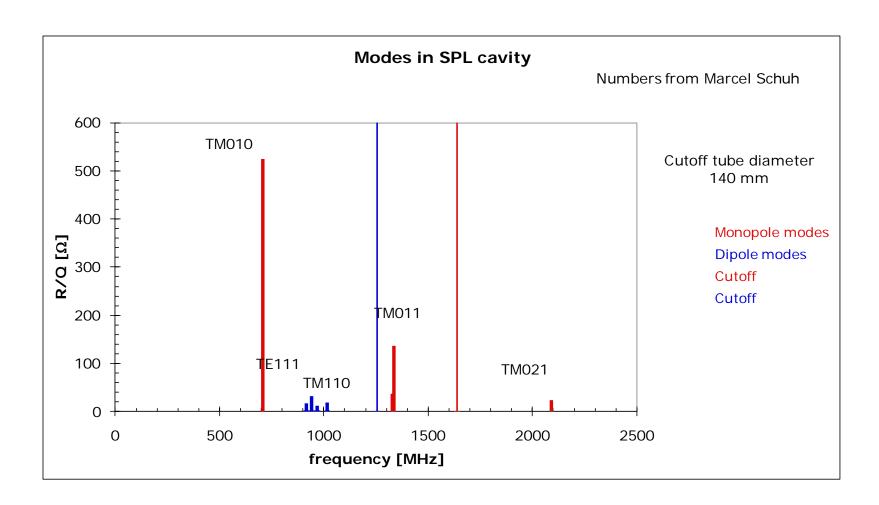


Fig. 3 Examples of modes of a single cell with wide beam tubes

## A typical HOM spectrum



#### How to deconfine HOMs<sup>1</sup>

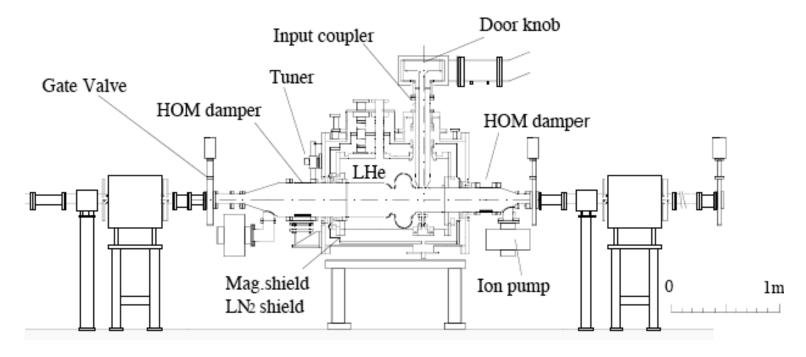


Figure: 1 A sketch of the prototype module in TRISTAN Accumulation Ring.

Open beam tube

OK for single cell cavity, but high cryo-load by thermal radiation

<sup>1</sup>http://www.lns.cornell.edu/Events/HOM10/Agenda.html

#### Damping HOMs 1/2: Beam tube loads

#### **Ferrites**

low power handling capacity if cold higher power handling capacity if warm mechanical and vacuum design not easy

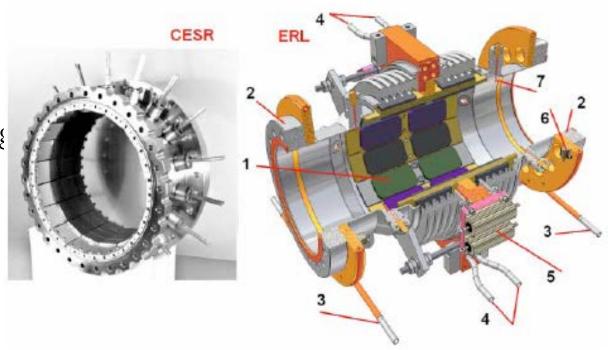


Figure 1: CESR and ERL HOM loads. 1 – absorber plates, 2 – flange to cavity, 3 – 5 K He cooling loop, 4 – 80 K cooling loop, 5 – 80 K heater, 6 – 5 K heaters, 7 – HOM pickup.

# Damping HOMs 2/2: Resonant coaxial transmission line dampers

• Compensate internal impedances: The HOM coupler becomes a resonator coupled to the cavity resonator. It may have two eigenfrequencies. Obtainable  $Q_{\rm ext}$ : 50

• Pros:

- Couplers with several resonances possible (HERA, LEP, LHC, ILC are of this type)
- Demountability
- Fundamental mode rejection:
  - LEP: Fundamental mode E-field rejected by stop-filter in front of HOM coupler
  - Fundamental mode H-field rejected by loop plane perpendicular to cavity axis
  - Risk of detuning of notch filter
- BUT: High currents request for superconducting material prepared under ultra-clean conditions (like the cavity) and lHe cooling
- Prone to electron emission from inside cavity

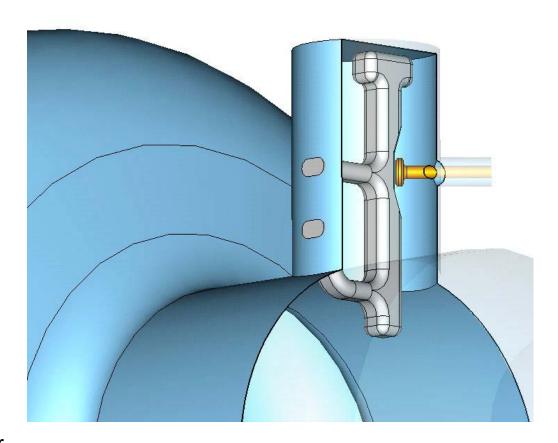
# Resonant coaxial transmission line dampers: Technical solution 1/3



LHC HOM coupler



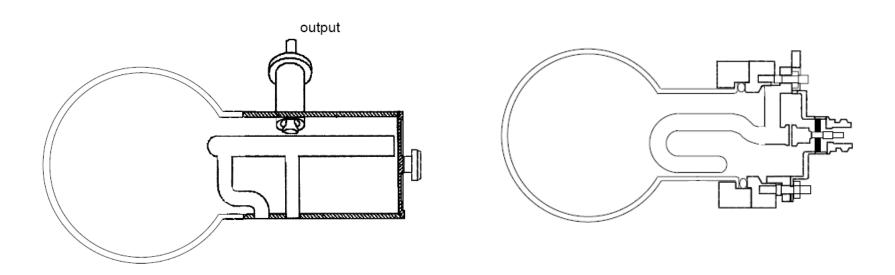
# Resonant coaxial transmission line dampers: Technical solution 2/3



#### SNS HOM coupler



# Resonant coaxial transmission line dampers: Technical solution 3/3



#### TESLA HOM coupler

#### The frequency tuner

The frequency of the cavity must be tuned to the harmonic spectral line of the bunched beam => need to develop a frequency tuner. Slater's theorem states that

$$\frac{\Delta f}{f} = \frac{1}{4U} \int \left( \varepsilon_0 E^2 - \mu_0 H^2 \right) V$$

$$U = \frac{1}{4} \int_{V} \left( \varepsilon_0 E^2 + \mu_0 H^2 \right) V$$
plunger
Multipacting !!!

#### Mechanical oscillations

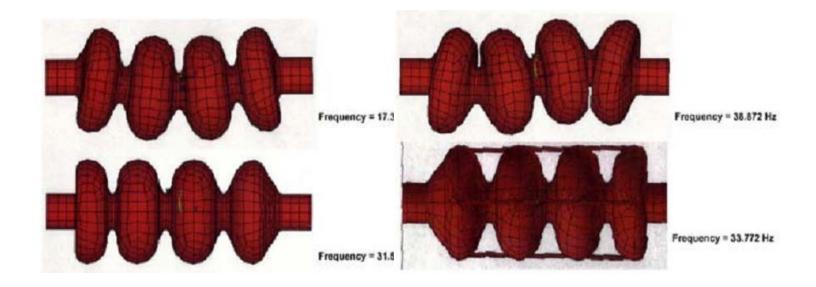
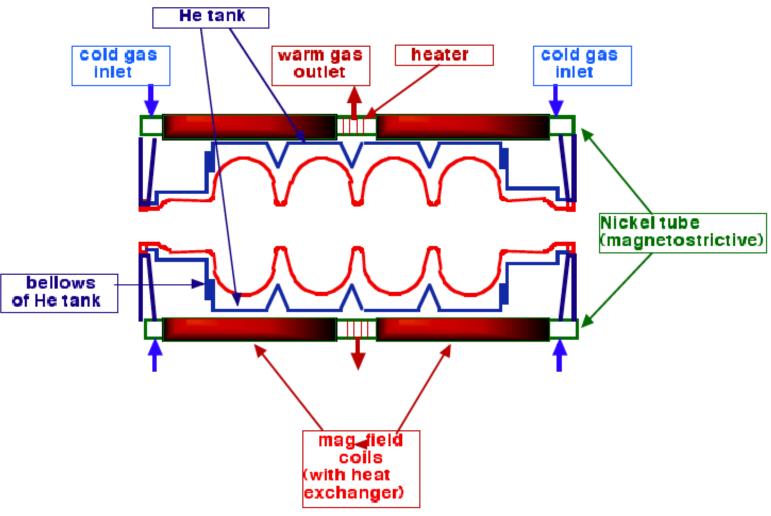


Fig. 20 Mechanical resonant modes of a 4-cell, 200 MHz cavity with 8 mm wall thickness. The low resonant frequencies spell trouble in the form of microphonics. Reducing the number of cells or stiffening is essential.

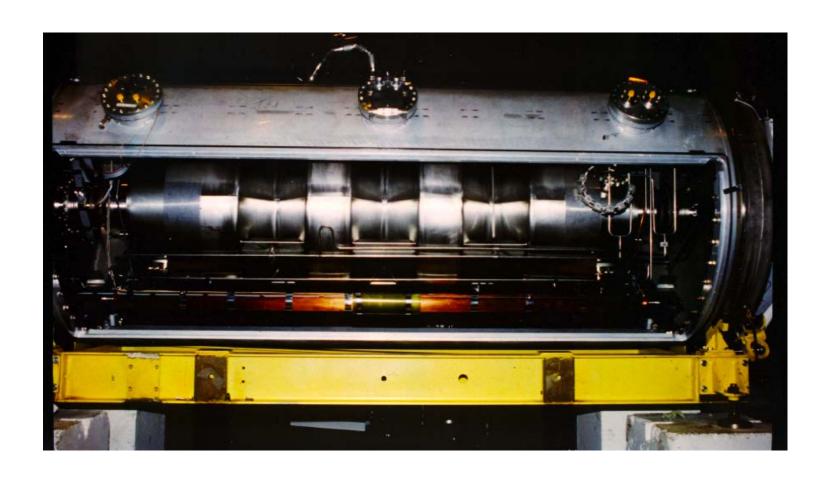
For example: at LEP, radiation pressure on the cavity walls of about 1000N in total possible

#### The LEP solution

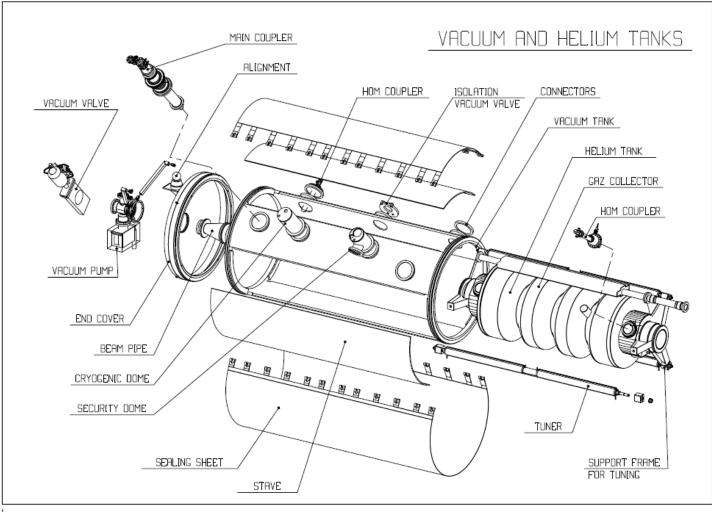


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# Integration into LEP cryostat 1/2



#### Integration into LEP cryostat 2/2



#### Comments:

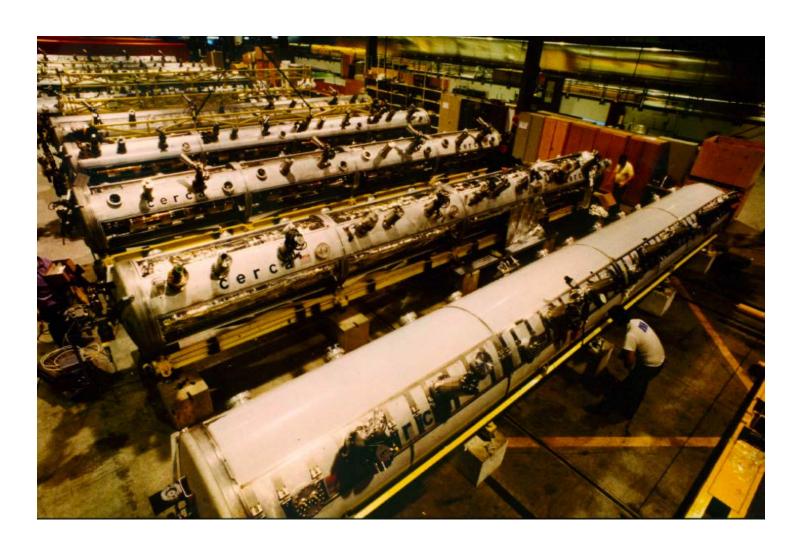
The LEP cryostat could reliably be operated under CW conditions with beam and in pulsed conditions without beam in the present LHC tunnel environment (1.4 % slope).

It is worth noting that the lHe tank, the gas openings, and gHe collector were relatively small.

**Pulsed operation**: The thermal diffusivity  $\kappa = \lambda/(c \cdot \rho)$  is such that it takes  $\sim 1$  ms before the temperature pulse arrives at the niobium helium interface => advantage compared to CW operation.

This cryostat was tested under pulsed conditions with beam in the CERN SPS.

## Cryomodules 1/2



# Cryomodules 2/2

• installed in LEP tunnel



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# Summary

- A lumped network circuit diagram allows an analytical description of the interaction of the RF cavity with the beam
- The cavity is designed to minimize the reflected RF power (which would be wasted anyhow in a load) by eliminating the « reactive beam loading » through tuning the frequency of the cavity and by matching the external Q to the nominal beam current.
- The beam consists of bunches passing the cavity in fractions of milliseconds<sup>1</sup> that may excite higher order modes (HOMs) of the cavity to high voltages, if not sufficiently damped by HOM couplers.
- Frequency tuners are in addition needed to damp frequency shifts from mechanical resonances excited by external noise sources (microphonics) or the interaction of the electromagnetic pressure with the cavity wall (Lorentz force detuning).

<sup>1</sup> for large storage rings such as LEP

# Technological issues

- Cryogenics
- Anomalous losses:
  - Residual losses\magnetic shielding
  - Electron field emission
  - Electro polishing
  - Electron Multipacting (dust free assembly)
  - Heat removal (Quench the role of large thermal conductivity, Coating a copper cavity with a thin niobium film)
  - Quality assurance and stochastic parameters
- Cavity production
- Improvement of cavity performance
- Summary

### **Basic Cryogenics**

### First law of thermodynamics:

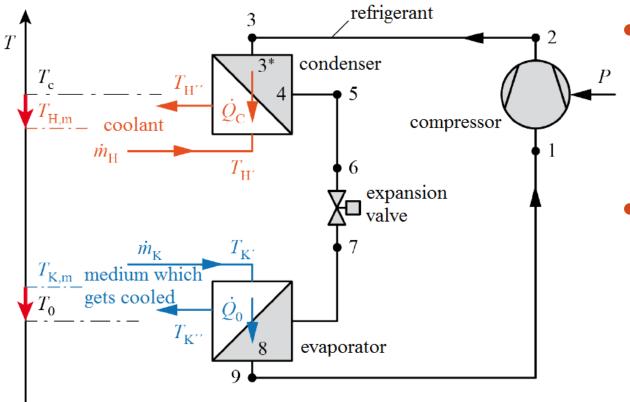
The energy of a closed system stays constant.

 $dU = \delta Q + \delta W = 0$ 

### Second law of thermodynamics:

The entropy of a closed system can not decrease.

 $\delta Q = TdS$ 

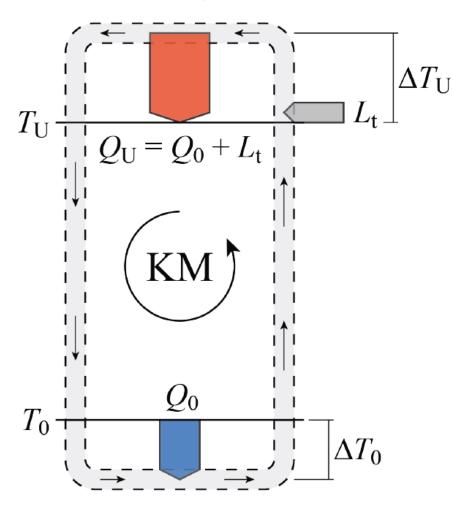


- Heat always flows from a warm region to a less warm region
- The most common cooling principle is decompression (throtteling)

Schematic of cooling cycle, Kaeltetechnik A, S. Grohmann (ITTK, KIT)

### **Basic Cryogenics**

### Carnot efficiency



- In the ideal case the  $\Delta T$ 's are zero.
- With the 1st law of thermodynamics the work of the compressor is given by

$$L_{t} = Q_{U} - Q_{0} = Q_{0}(Q_{U}/Q_{0} - 1)$$
$$= Q_{0}(T_{U}/T_{0} - 1)$$

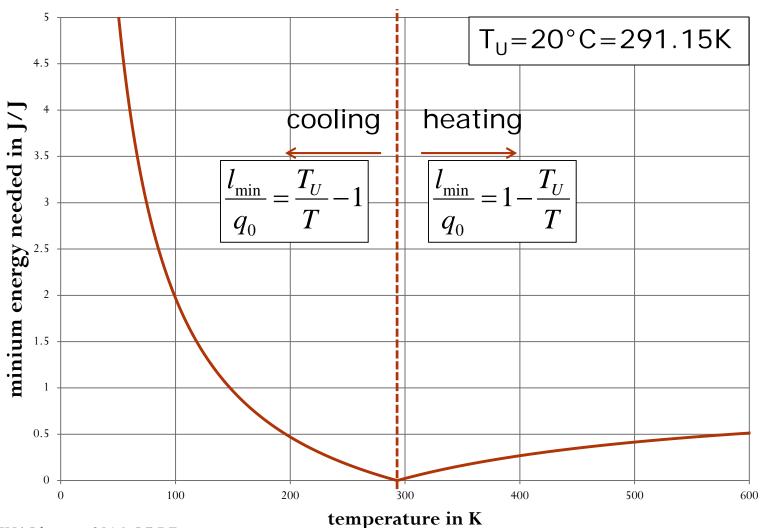
• The Carnot efficiency  $\mu_{C}$  for a refrigerator is defined as

$$\mu_C = Q_0 / L_t = T_0 / (T_U - T_0)$$

Schematic of cooling cycle, Kaeltetechnik A, S. Grohmann (ITTK, KIT)

### **Basic Cryogenics**

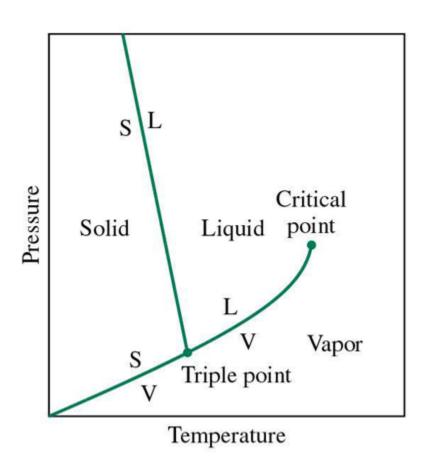
### minimum energy for cooling and heating



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### Basic Cryogenics cont.

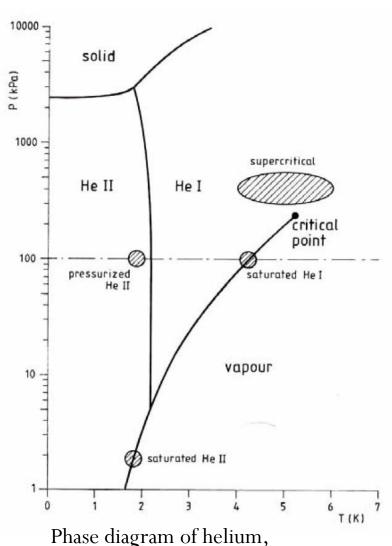
### pT-curve, Material properties



Schematic of a phase diagram,

http://moodle.zhaw.ch/mod/book/tool/print/index.php?id=63256

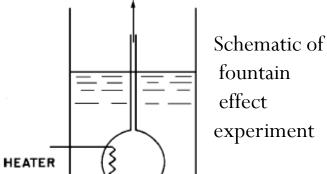
JUAS lecture 2016: SC RF cavities Caspers/Weingarten/Junginger/Aull/Peters/Bartel/Wendt



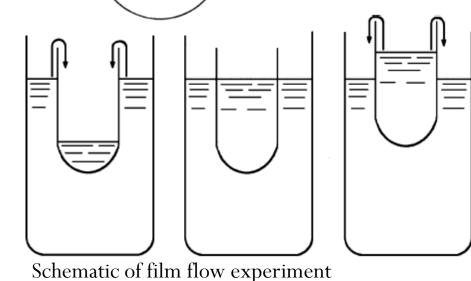
Phase diagram of helium,

CAS 2002, G. Vandoni

### Basic Cryogenics cont.



- **He II and superfluidity** One of the few macroscopic quantum phenomena
  - Special effects
  - Extremely low viscosity in thin channels
  - Very high heat transport capability
  - Can be described by two-fluid theory



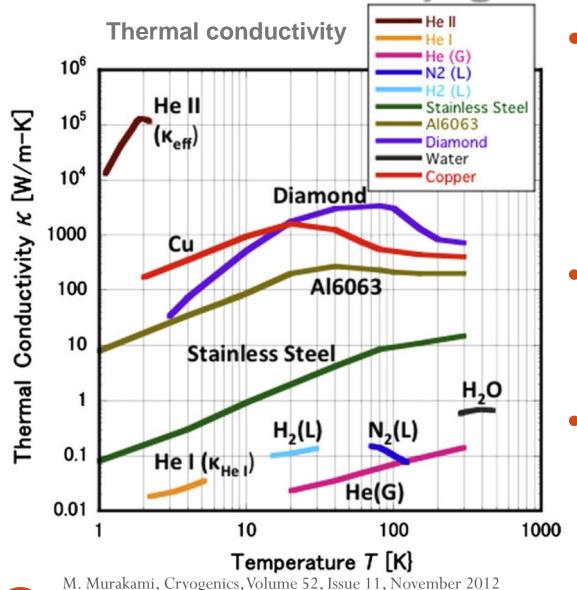
**PLUG** 

NORMALIZED DENSITY 0.5 2.0 T(K)

Ratio of normal and superfluid densities of He II

All pictures on this slide by: S. V. Van Sciver, Helium Cryogenics

### Basic Cryogenics cont.



- The change in thermal conductivity in helium increases seven orders of magnitude at the lambda point
- At low temperatures many material properties can change very rapidly with temperature.
- e.g. heat capacity, electrical conductivity, vapour pressure

M. Murakami, Cryogenics, Volume 52, Issue 11, November 2012 JUAS lecture 2016: SC RF cavities
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### SC vs. NC

For  $T_1 = 300$  K and  $T_2 = 4.2$  K,  $\eta_c = 1/70$ . The 'thermodynamic efficiency'

$$\eta_{\rm td} = \dot{W}_{\rm c} / \dot{W} \tag{4}$$

is the ratio of the power  $\dot{W}_{\rm c}$  needed to operate the compressor in the ideal case to the 'real' power  $\dot{W}$ . The total cryogenic efficiency is

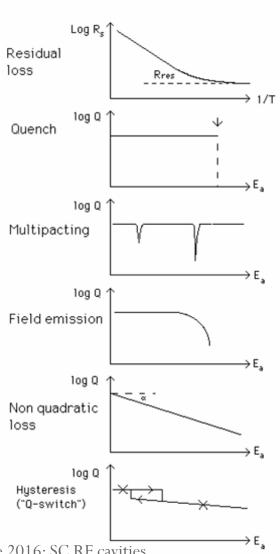
$$\eta_{\rm cr} = Q_2 / \dot{W} = (Q_2 / \dot{W}_{\rm c})(\dot{W}_{\rm c} / \dot{W}) = \eta_{\rm c} \eta_{\rm td} .$$
(5)

With  $\eta_{\rm td} \approx 0.3$  for large units the total cryogenic efficiency is  $\eta_{\rm cr} = 4.5 \times 10^{-3}$ . Unavoidably, in an sc accelerator some power  $P_{\rm cr}$  flows into the liquid He, even in the absence of RF (standby heat load of cryostat). The efficiency  $\eta$  for a sc accelerator of RF-to-beam power conversion is then

$$\eta = \left[1 + (P_{c} + P_{cr}) / (P_{b} \eta_{cr})\right]^{-1}. \tag{6}$$

As an example, for the sc cavity and cryostat for LEP with  $P_c = 50 \text{ W}$ ,  $P_b = 50 \text{ kW}$  and  $P_{cr} = 25 \text{ W}$ , we obtain a total efficiency of  $\eta = 0.75$ , which is larger by a factor of 5 than for a conventional RF system (Table 1).

### **Anomalous losses**



So-called « anomalous losses » account for all contributions to the RF losses that are not described by the intrinsic parameters of the superconducting material (critical temperature, critical field, BCS (or two fluid ) surface resistance  $R_s$ , etc.).

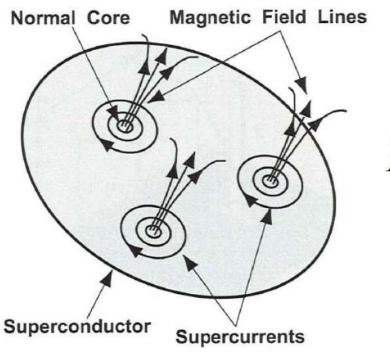
These anomalous losses show up as heat and are visible in the  $R_s$  (T) and  $Q_0(E_a)$  plots, as well as in the « temperature maps ».

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### Magnetic shielding

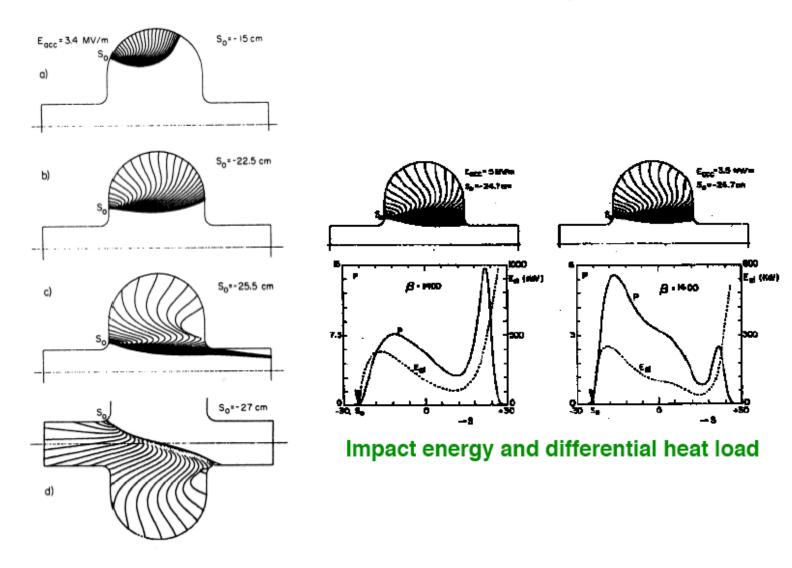
• Why do we need a magnetic shielding?



$$R_{mag} = \frac{H_{ext}}{2H_{c2}} R_n$$

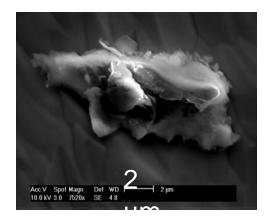
$$R_{mag}[n\Omega] = 3H_{ext}[\mu T]\sqrt{f[GHz]}$$

### Electron field emission 1/4

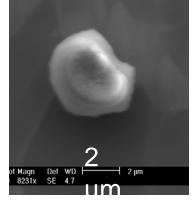


### Electron field emission 2/4

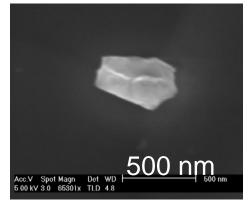
• Typical particulate emitters containing impurities



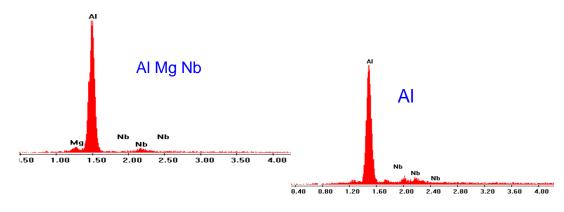
$$E_{on}(2nA) = 140 \text{ MV/m}$$
  
 $\beta = 31, S = 6.8 \cdot 10^{-6} \, \mu\text{m}^2$ 

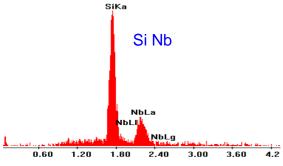


$$E_{on}(2nA) = 132 \text{ MV/m}$$
  
 $\beta = 27, S = 7 \cdot 10^{-5} \mu \text{m}^2$ 



$$E_{on}(2nA) > 120 \text{ MV/m}$$
  
 $\beta = 46, S = 6.10^{-7} \mu\text{m}^2$ 



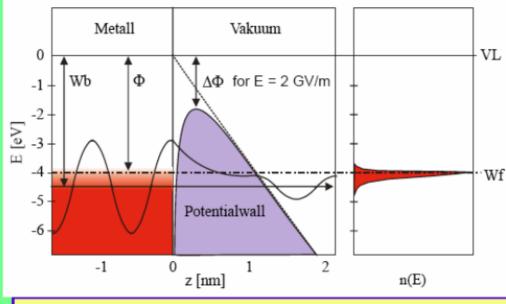


### Electron field emission 3/4

Fowler Nordheim theory

### Field emission of electrons from flat metal surfaces

Electron waves of bound states in a metal can tunnel through the potential barrier V(z) at the solid surface into vacuum by means of the quantum mechanical tunnelling effect



 $V(z) = -e \cdot E \cdot z - e^2/(16\pi\epsilon_0 \cdot z)$ work function Φ of metal applied field E on surface image charge correction  $\Delta \Phi = (e^3E/4\pi\epsilon_0)^{3/2}$ 

Calculation of the current density j(E) within the Fowler-Nordheim theory results in

$$j(E) = \frac{AE^2}{\Phi t^2(y)} \exp\left(-\frac{B\Phi^{3/2}v(y)}{E}\right)$$

with constants A=154 and B=6830 and slight correction functions t(y) and v(y)

 $\Phi$ =4eV at E=2000 MV/m  $\Rightarrow$  j= 1nA/ $\mu$ m<sup>2</sup>

G. Müller, 15.11.2006





CARE06, Frascati

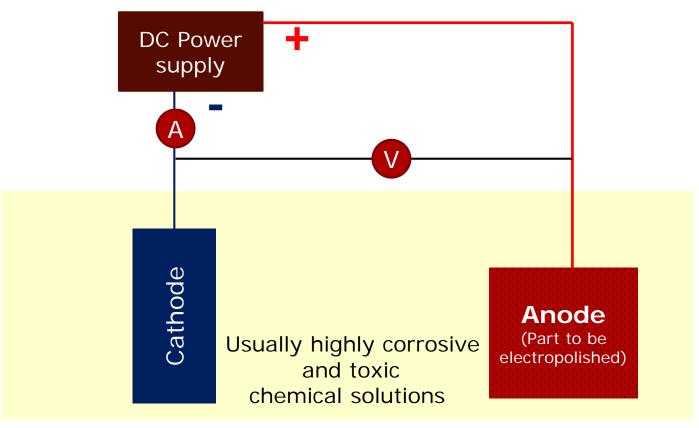
### Electron field emission 4/4

Clean room preparation mandatory



### Electropolishing: How it works

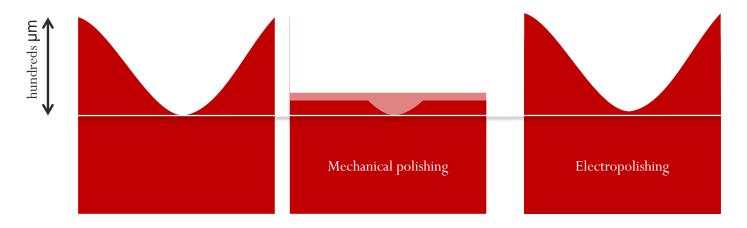
• The metal is immersed in an electrolyte and subjected to direct current. The metal part to be treated is made anodic and under certain conditions, a controlled dissolution of the metal is achieved.



# Disadvantages

### Electropolishing vs Mechanical based polishing

• Final roughness is function of initial surface finishing and removed thickness



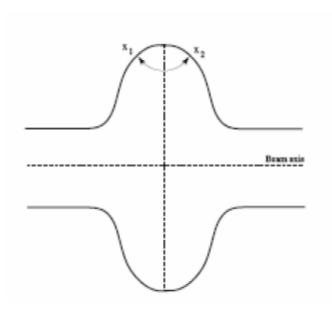
- Usually highly corrosive and/or toxic solutions
  - Handling;
  - Process equipment;
  - Installation to process extracted fumes;
  - Installation to process waste water.

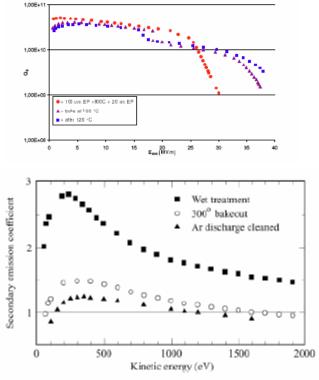
### **Electron multipacting**

Localized heating by multiple impact from electron current due to secondary emission in resonance with RF field.

Historically this phenomenon was a severe limitation for the performance of sc cavities.

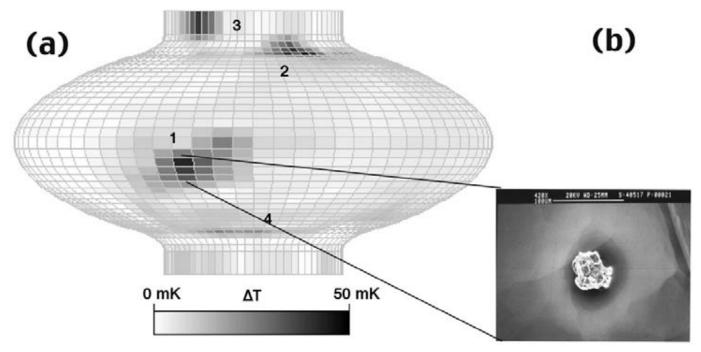
The invention of the "circular" shape opened up the avenue for higher gradients.





### Field limitations - thermal breakdown

- Occurs at sub mm size defects with high resistance
- RF currents flow through the defects
- Defects heat up due to ohmic losses
- Area surrounding the defect is heated as well
- $\bullet$   $\;$  Thermal breakdown occurs if the surrounding area is heated above  $T_{_{\rm C}}$



### Heat removal

• Thermal Improvement of thermal conductivity for Niobium sheets

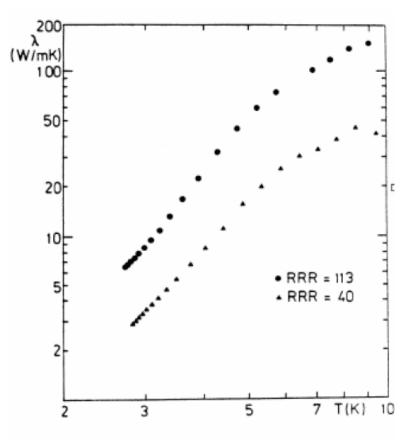
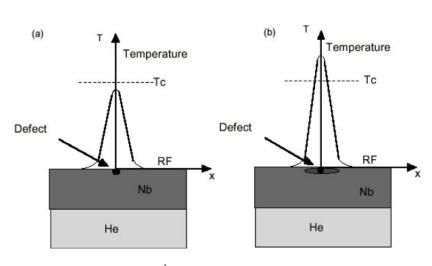


Fig. 1: Thermal conductivity of reactor grade niobium (RRR=40) and niobium of higher purity (RRR=113)

Cause for "quench":



 $H_{max} \approx \sqrt{(4(T_c - T_B)\lambda/(R_n r))}$ .

RRR.... residual resistance ratio

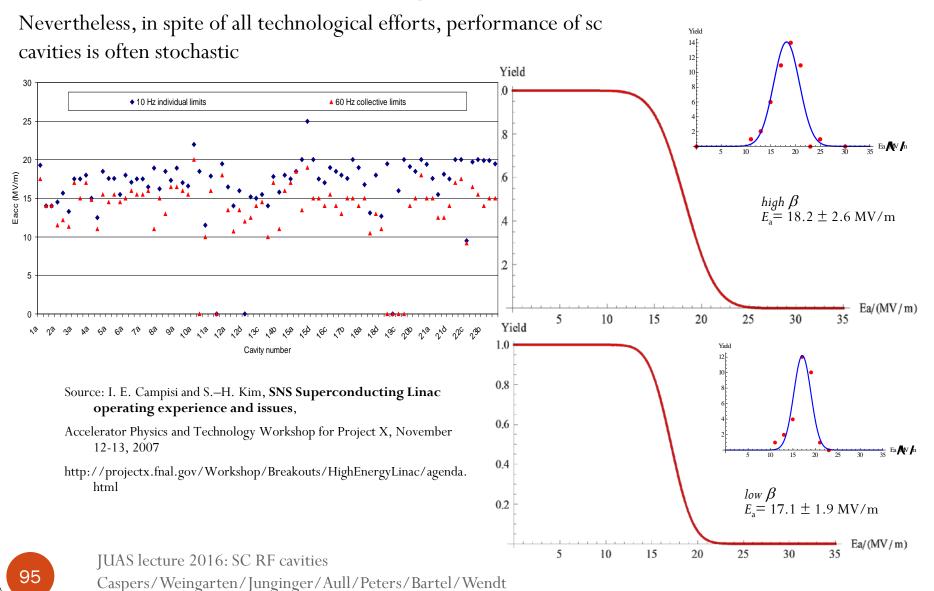
# Thin film Nb coating

- Coating a copper cavity with a thin Nb film
- Important role of high thermal conductivity substrate (Nb/Cu cavity)



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# Improvement of quality assurance efforts: ORNL/JLAB results



### Stochastic parameters

Influencing quantity	Impact quantity	Physical explanation	Cure
Field emission sites (foreign particles sticking to the surface, size, density)	Q – value / acc. gradient γ radiation HOM coupler quench	Modified Fowler- Nordheim-theory	Electro-polishing Assembling in dust-free air Rinsing with ultrapure water (control of resistivity and particulate content of outlet water) and alcohol High pressure ultrapure water rinsing (ditto) "He- processing" Heat treatment @ 800 – 1400 °C
Secondary emission coefficient $\delta$	Electron-multipacting	Theory of secondary electron emission	Rounded shape of cavity Rinsing with ultrapure water Bake-out RF - Processing
Unknown	Q – slope / Q-drop (Q – value / acc. gradient)	Unknown	Annealing 150 °C Electro-polishing
Metallic normal- conducting inclusions in Nb	Acc. gradient	Local heating up till critical temperature of Nb	Inspection of Nb sheets (eddy current or SQUID scanning) Removal of defects ( $\approx$ 1 $\mu m$ ) Sufficiently large thermal conductivity (30 - 40 [W/(mK)])
Residual surface resistance	Q – value / acc. gradient	Unknown to large extent	Quality assurance control of a multitude of parameters

Forming	WHY	COMMENTS		
EB Welding	Clean welding	Nb = getter material.  If RRR/ 10 @ welding => $Q_0/10$		
Ti p <mark>urificat</mark> ion	RRR enhancement	RRR 300-400 now commercially available		
Chemical etching 100-200 µm EP	Remove contamination and damage layer	Limitation : BCP ~ 30MV/m; EP => >40 mV/m but lack of reproducibility		
Annealing 800°C, 2h (or 600°C, 10h)	Get rid of hydrogen	Source of H: wet processes H segregates near surface in form of hydrides (= bad SC)		
Chemical etching 5-20 μm	Remove diffusion layer (O, C, N)	Diffusion layer < ~1µm in bulk, a little higher at Grain Boundaries		
Spe <mark>cific rin</mark> sing	e.g. remove S particles due to EP	Under evaluation HF, $H_2O_2$ , ethanol, degreasing,		
High pressure rinsing (HPR)	Get rid of dust particles	Not always enough (recontamination during assembly)		
Assembling	Ancillaries : antennas, couplers , vacuum ports	In clean room, but recontamination still possible		
Baking, 120°C, 48h	Decrease high field losses (Q-drop)	Unknown mechanism, first 10 nm of the surface in concern.		
Post processing	Get rid of "re-contamination"?	Under evaluation: dry ice cleaning, plasma		
Test RF	Cavity's performance	First naked cavity in vertical cryostat, then dressed in horizontal cryostat/accelerating facility		
He processing, HPP  JUAS lecture 2016: SC RF caviti	•	RF power with/ without He to destroy field emitters (dust particles) NB field emission : principal practical problem in accelerators		
Caspers/Wolngarten/Junginger Aull/Peters Parte Antonie (Practical) Limitations & possible solutions, CAS 2013				

### Improvement of cavity performance

Lilje & Schmueser

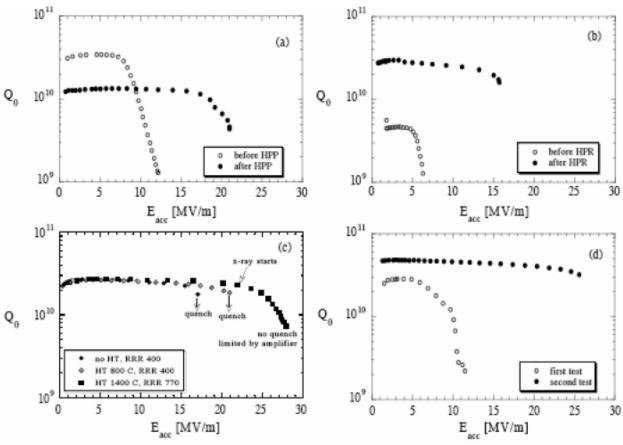


Figure 27: Improvement in cavity performance due to various treatments: (a) high power processing, (b) high pressure water rinsing, (c) successive application of 800°C and 1400°C heat treatment, (d) removal of surface defects or titanium in grain boundaries by additional BCP. All tests were done at 1.8 K [Aune et al. 2000].

### Summary

- The choice of the technology (normal conducting vs. superconducting) depends on a variety of parameters: mass of accelerated particle, beam energy, beam current, mains power consumption, etc.
- If superconducting, the typical interval of RF frequencies is between 300 MHz and 3 GHz.
- The technically most suitable superconducting material being niobium, choosing lower frequencies allows operation at 4.2-4.5 K, the boiling temperature of lHe, higher frequencies request operation at 1.8-2 K. However, the cryogenic installation is much more demanding.
- The production of sc cavities requests careful application of quality control measures during the whole cycle of assembly in order to avoid the degradation of performance by « anomalous losses ».
- The « anomalous losses » contribute to an extra heat load, which is expensive to cool and which may limit the performance.

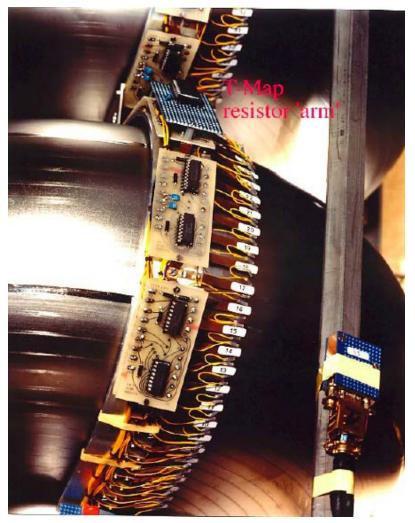
# Diagnostics 1/8

- Many features of the cavity can be tested by RFmeasurements.
- But losses, which occur in the form of localized heat can only be detected by additional diagnostics.
- The classical approach is temperature mapping.



# Diagnostics 2/8

• Temperature mapping equipment (~ 1980)



### Diagnostics 3/8

Temperature mapping results (today)

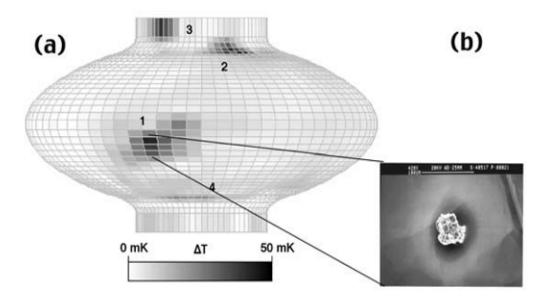


Fig. 16 (Left) Temperature map at 400 Oe of a 1.5 GHz, single cell cavity showing heating at a defect site, labelled #1 and field emission sites labelled #2, 3, and 4. (b) SEM micrograph of the RF surface taken at site #1 showing a copper particle [5].

From H. Padamsee: CERN -2004 - 008

### Diagnostics 4/8

• T-mapping for the diagnosis of anomalous losses

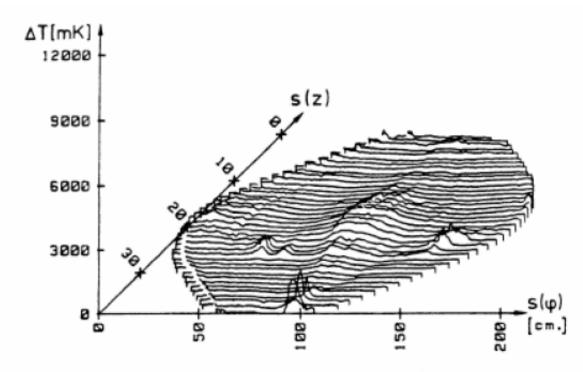
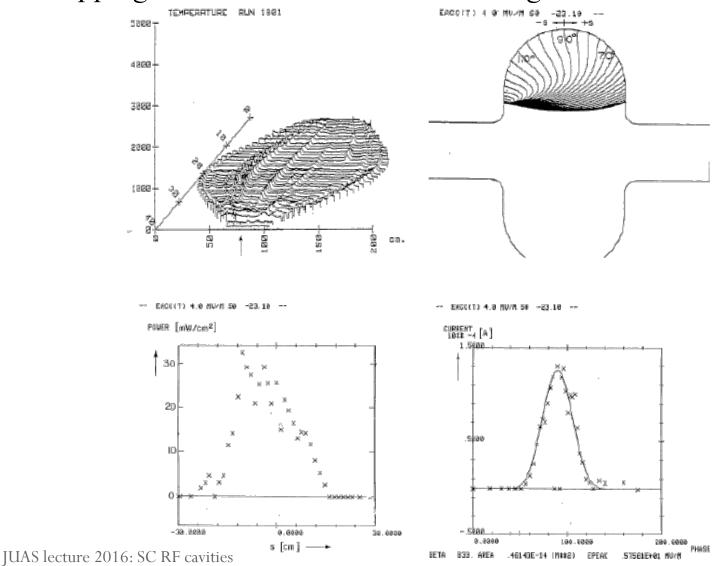


Fig. 4: Temperature map of a 500 MHz cavity at E<sub>acc</sub> = 12.5 MV/m. The temperature increase ΔT of the outer cavity surface is plotted against the surface coordinates s(z) and s(φ)(z=length in arbitrary units along a meridian, φ = azimuthal location)

# Diagnostics 5/8

• T-mapping for electron field emission diagnosis

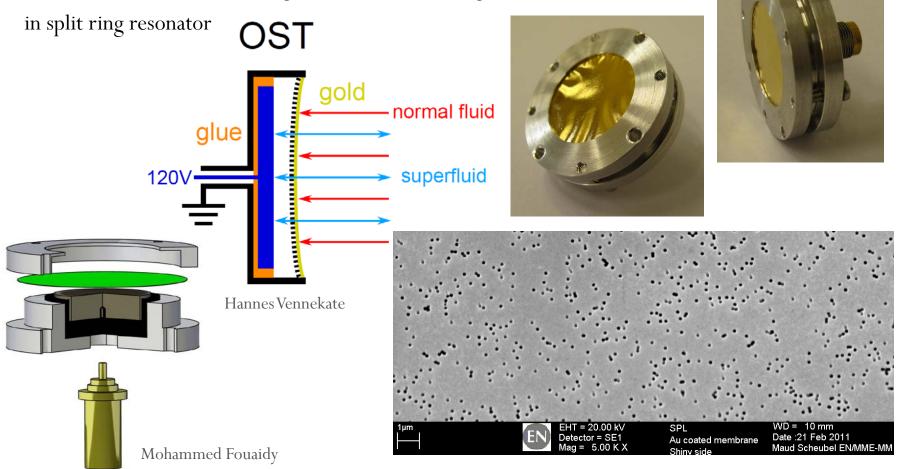
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### Diagnostics 6/8 an Introduction to OSTs

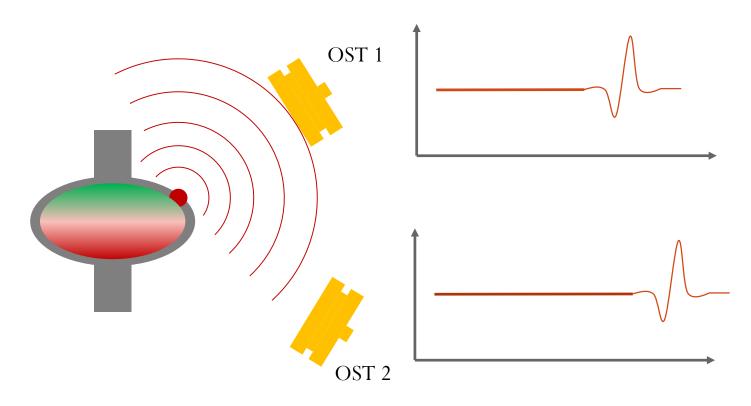
• Second sound in superfluid helium

First used by K. Shepard at Argonne NL for detecting the quench location



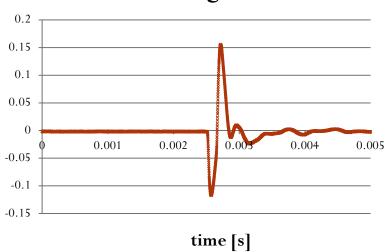
### Diagnostics 7/8

- Detection and localisation of quenches on superconducting RF cavities by the measurement of the second sound with OSTs
- The localisation of a quench can be done with a relatively small number of sensors



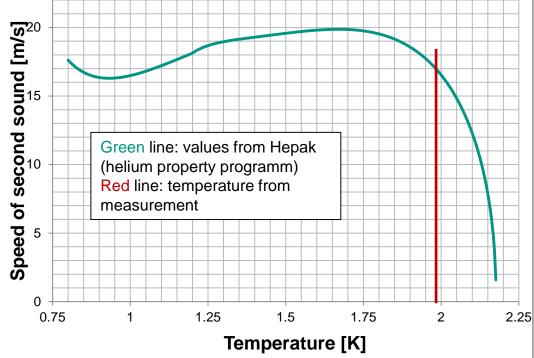
### Diagnostics 8/8

### **OST** signal



- This measurement was done at 1.977 K
- $v_2(1.977 \text{ K})=17.14 \text{ m/s}$
- Signal of the measurement at t=2.52 ms
- Distance to heater  $v_2t=4.32$  cm





signal of the measurement at t-2.52 if

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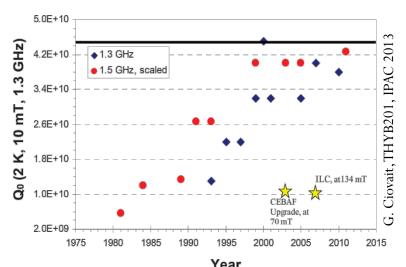
### State of the art SRF research

- Reaching ultimate performance with bulk Nb cavities
  - Maximizing the quality factor Q<sub>0</sub>
  - Reaching high accelerating gradients E<sub>acc</sub>
- Beyond Niobium: New materials
  - High temperature superconductors
  - Low temperature superconductors: Nb based materials

# High Q versus high E<sub>acc</sub>

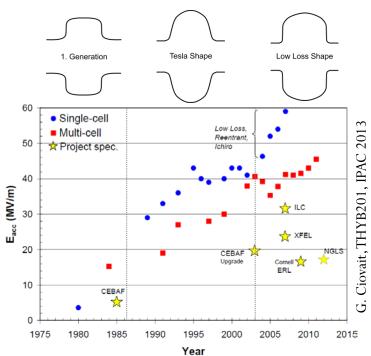
- High Q is crucial for cw applications (e.g. light sources)
  - moderate  $E_{acc}$  (12 20 MV/m)
  - Cryogenics is cost driver
  - High Q reduces cryogenic load

$$(P_{\rm diss} \sim \frac{E^2acc}{Q})$$



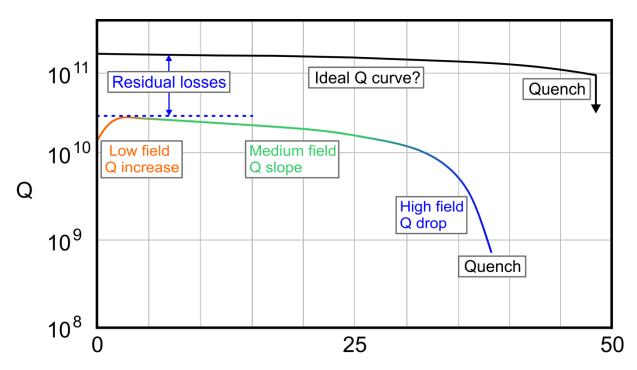
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- High E<sub>acc</sub> is crucial for pulsed applications (e.g. particle physics)
  - Machine size is cost driver.



## Maximizing Q: Ideal and Reality

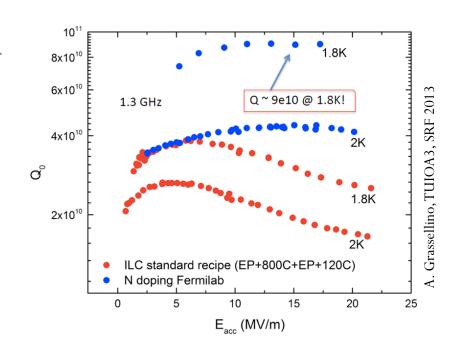
- In the ideal case: Is Q constant up to the theoretical limit?
- Recent theoretical calculations yield increasing Q for increasing rf field.[See B.P. Xiao et al., Physica C 490 (2013)]



Accelerating Gradient [MV/m]

# Maximizing Q: Improving treatments

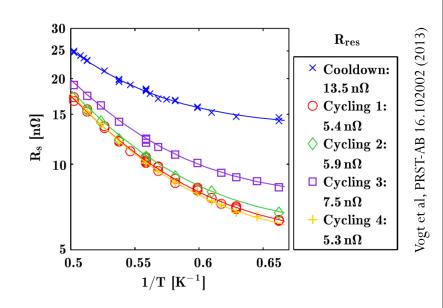
- Baking at 800°C with injection of N<sub>2</sub> degrades cavity performance.
- After the removal of several µm by EP, the performance increases and exceeds baseline.
- Q slope reverses to "anti Q slope".
- Comparison with Argon suggest interstitial effect instead of NbN formation.



• Experimental data in good agreement with B.P. Xiao's field dependent model.

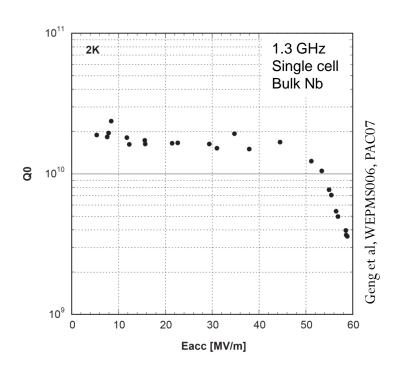
#### Maximizing Q: Improving cooling procedures

- Cooling speed through T<sub>c</sub> and spatial temperature gradients impact the residual resistance.
- Measurements of the ambient field suggest that changes in flux trapping and the creation of thermal currents cause changes in R<sub>res</sub>.



# Achieving maximal gradients

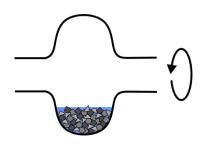
• World record  $E_{acc} = 59 \text{ MV/m}$  $(Q = 4 \cdot 10^9)$ 



- High performing cavities are limited by field emission or quench
- Avoiding emission sites by
  - Centrifugal barrel polishing: grinds larger defects
  - Improved Electro-polishing: smoothens surface on subµm scale
  - Cleaner handling: avoid (re-) contamination

#### **Surface Preparation**

- Centrifugal barrel polishing:
  - Mechanical removal and smoothening of the surface with abrasive "stones".

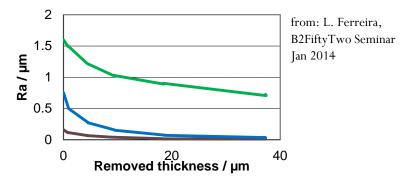




P. Michelato,
SRFTutorials, 201

 Produces a new damage layer that need to be etched.

- Electro-Polishing:
  - *Best* surface finish for cavities
  - Final roughness depends on initial surface finish

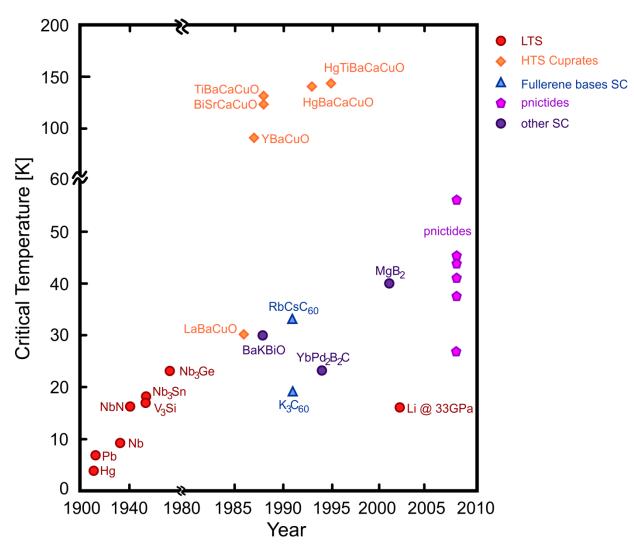


- Clean handling:
  - Any preparation of the surface and the final assembly needs to be done in a clean room (ISO 4).
  - Re-contamination has to be avoided.

#### Materials beyond Nb: Potential Benefits

- Higher Q due to lower BCS surface resistance:
  - Reduces cryogenic dynamic losses (operation costs)
  - Allows operation at higher temperature (reduces cryogenic static losses)
- Higher accelerating gradients:
  - Reduces installation costs due to more compact accelerators
- Reduced materials costs
  - Inexpensive materials, well formable, high thermal conductivity
- Simplified fabrication and assembly
  - Separating cavity shape from rf surface (Coatings)
  - More flexibility in design of cryomodules

#### Zoo of Superconductors



#### Materials beyond Niobium: Requirements

- High critical temperature T<sub>c</sub>:
- Small penetration depth  $\lambda$ :
- High critical field H<sub>c</sub>:
- High thermal conductivity:

- $R_{\rm BCS} \propto e^{\left(-\Delta/k_T\right)}; T_{\rm c} \propto \Delta$
- $R_{\rm BCS} \propto \lambda^3$
- Operation at high gradient

prevent quenches

- Compound phase should be stable over a broad composition range
- Compound phase needs to be stable from 2 − 300 K
- Material should be inert and formable

#### Classes of superconductors

	Nb	Low Temp. SC	$MgB_2$	YBCO
T <sub>c</sub> [K]	9.2	10 - 20	39	> 90
λ[nm]	40	60-180	140	150-1000
H <sub>c</sub> [mT]	200	200-600	430	1400
К	0.8	20-130	40	100
remarks			2 sc gaps	Ceramic, anisotropic

- High temperature superconductors are not suitable for srf applications
- Not all parameters are known for all potential candidates.

## LTS: A15 & B1 compounds



#### A15 structure A<sub>3</sub>B

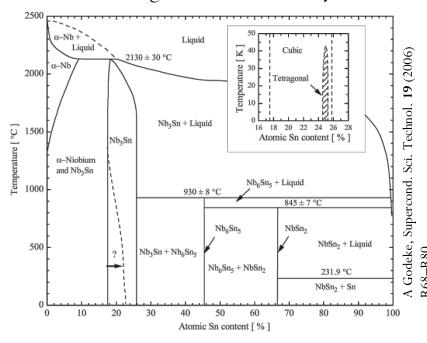
- A atoms: transition elements
- B atoms: non transition or transition elements
- Stable and high T<sub>c</sub>: Nb<sub>3</sub>Sn, V<sub>3</sub>Ga, V<sub>3</sub>Si,
   Mo<sub>3</sub>Re
- A15 compounds are not formable due to extreme brittleness



#### **B1** structure **AB**

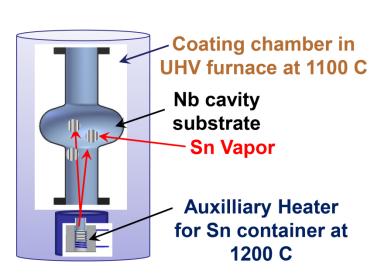
- A atoms: metallic
- B atoms: non-metallic
- Stable and high T<sub>c</sub>: NbC, NbN

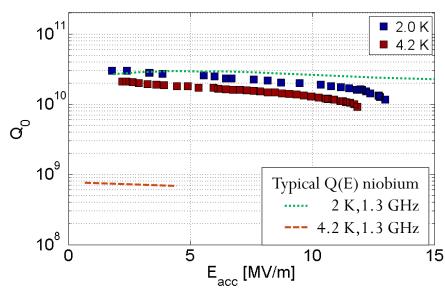
#### Phase diagram of the Nb-Sn system



# A15 compounds: Nb<sub>3</sub>Sn

- Low thermal conductivity would favor coating a copper cavity with Nb<sub>3</sub>Sn.
- Only successful fabrication so far: Sn vapor diffusion into Nb cavity, alloying Nb<sub>3</sub>Sn (Wuppertal 1985, Cornell 2013)





Both pictures from: *Nb3Sn for SRF Application* M. Liepe (Cornell), WEIOA04, SRF 2013

## Summary

- For now, only high Q at moderate  $E_{acc}$  or high  $E_{acc}$  at moderate Q can be achieved.
- The high Q research tries to understand loss mechanisms and develops new recipes to minimize the residual resistance.
- Maximum  $E_{acc}$  can only be achieved by high-end surface preparation. Improving polishing and cleaning procedures is mandatory for multicell cavities and serial production.
- New materials have lower BCS surface resistance (higher Q) and/or higher critical field (higher  $E_{\rm acc}$ ).
- HTS are not suitable for rf applications.
- Nb<sub>3</sub>Sn is the most promising alternative material so far.

#### **ACKNOWLEDGMENT**

- A lot of 'material' from CERN will be used but this does not mean that 'material' from other institutions is considered inferior.
- Other material was taken from contributions to the CERN Accelerator Schools, CAS <sup>1</sup> (CERN-2006-002, CERN-2005-03, CERN-2005-04, CERN-2004-08, CERN-1996-03, CERN-1992-03, as well as earlier ones and other sources mentioned).
  - R. P. Feynman et al., Lectures on Physics Vol. II
  - P. Schmüser et al.; The Superconducting TESLA Cavities; Phys. Rev. Special Topics AB 3 (9) 092001
  - A. W. Chao & M. Tigner, Handbook of Accelerator Physics and Engineering, World Scientific
  - Alexey Ustinov, Lecture on superconductivity, University Erlangen-Nürnberg (from which I used some slides w.r.t. Superconductivity)

(<a href="http://www.pi.uni-karlsruhe.de/ustinov/group-hp/fluxon.physik.uni-erlangen.de/pages/lectures/WS-0708/Superconductivity-2007-01.pdf">http://www.pi.uni-karlsruhe.de/ustinov/group-hp/fluxon.physik.uni-erlangen.de/pages/lectures/WS-0708/Superconductivity-2007-01.pdf</a>)

• and an excellent textbook of the field: H. Padamsee, J. Knobloch, and T. Hays, RF Superconductivity for accelerators & H. Padamsee, RF Superconductivity, Weinheim 2008, resp.

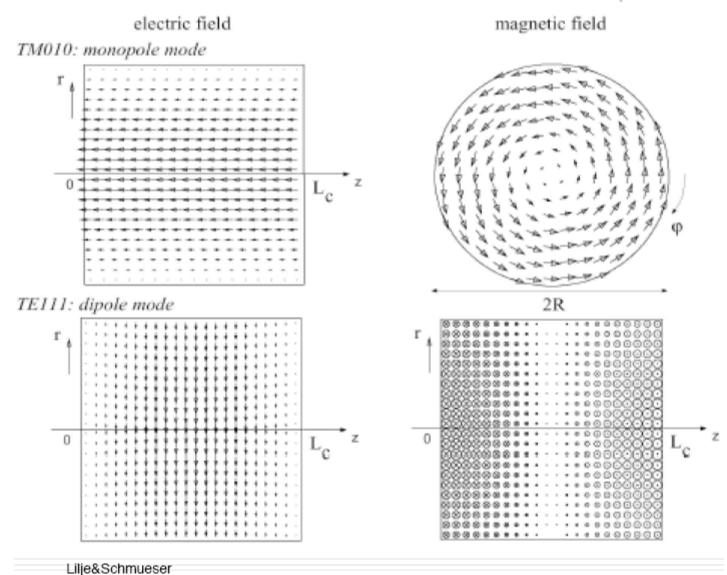
<sup>1</sup> http://cdsweb.cern.ch/collection/CERN%20Yellow%20Reports

Last but not least I thank my colleagues E. Haebel and J. Tuckmantel for many discussions, clarifications, and presentation material.



#### **BACKUP SLIDES**

# Different mode families 1/2



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# Different mode families 2/2

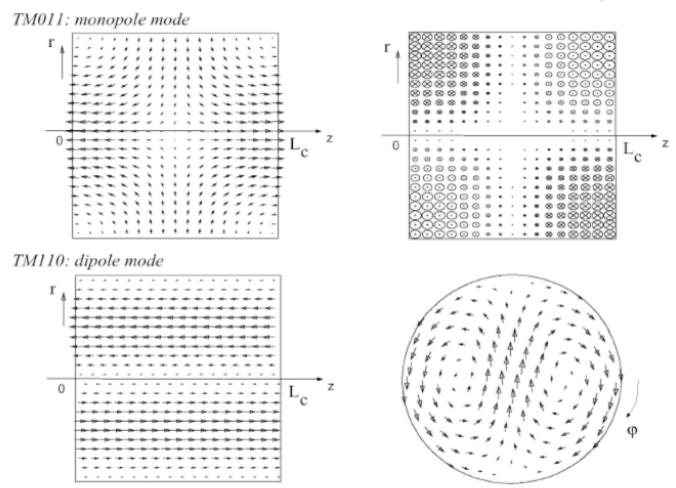


Figure 1: Electric and magnetic field in a pillbox cavity for several resonant modes (Courtesy of M. Liepe).

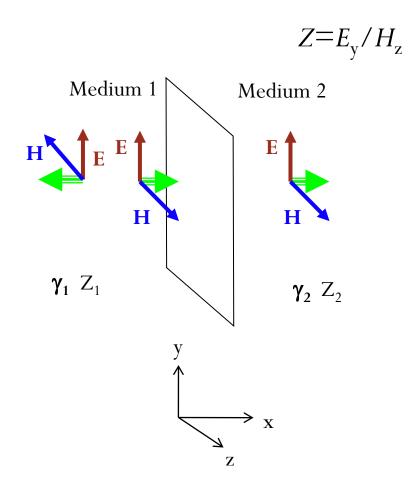
Lilje&Schmueser

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#### Transmission line 1/2

Introduction of the notion of reflection and transmission factors.



$$E_{yi} = \hat{E}e^{-\gamma_1 x}e^{i\omega t}$$

$$H_{zi} = \frac{\hat{E}}{Z_1} e^{-\gamma_1 x} e^{i\omega t}$$

$$E_{yr} = \rho \hat{E} e^{\gamma_1 x} e^{i\omega t}$$

$$H_{zr} = -\rho \frac{\hat{E}}{Z_1} e^{\gamma_1 x} e^{i\omega t}$$

$$E_{yt} = \tau \hat{E} e^{-\gamma_2 x} e^{i\omega t}$$

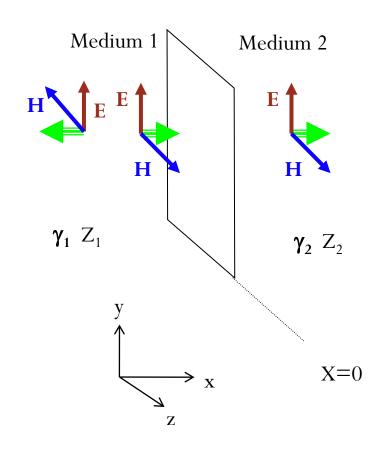
$$H_{zt} = \tau \frac{\hat{E}}{Z_2} e^{-\gamma_2 x} e^{i\omega t}$$

#### Transmission line 2/2

From continuity at the interface:

$$E_{yi} + E_{yr} = E_{yt}$$

$$H_{zi} + H_{zr} = H_{zt}$$



$$1+\Gamma=\tau$$

$$\frac{1}{Z_1}-\Gamma\frac{1}{Z_1}=\tau\frac{1}{Z_2}$$

$$\frac{Z_2}{Z_1}(1-\Gamma)=\tau$$

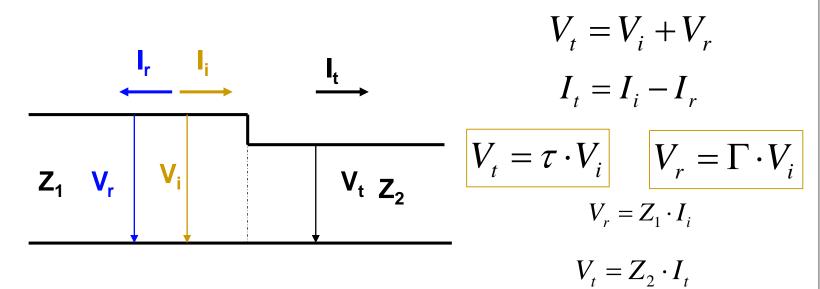
$$1 + \Gamma = \frac{Z_2}{Z_1} (1 - \Gamma)$$

$$\Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\tau = \frac{2Z_2}{Z_2 + Z_1}$$

#### Response of a cavity to RF 1/5

• Apply transmission line theory (to a one-port impedance):



Reflexion factor  $\rho$ 

Transmission factor  $\tau$ 

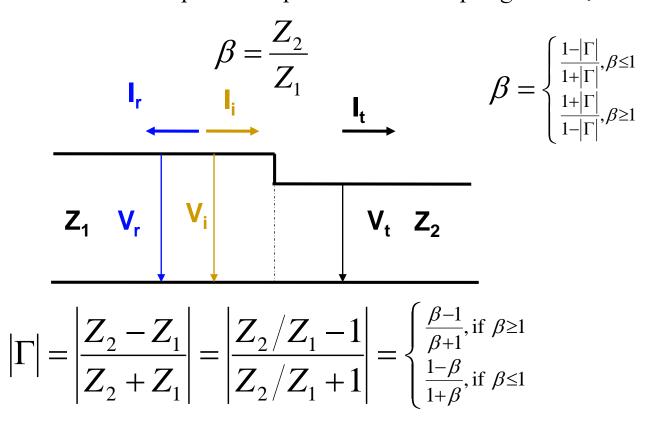
$$\Rightarrow \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} \Rightarrow \tau = \frac{2 \cdot Z_2}{Z_2 + Z_1}$$

$$\tau - \Gamma = \frac{2 \cdot Z_2 - (Z_2 - Z_1)}{Z_1 + Z_2} = 1$$

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#### Response of a cavity to RF 2/5

Reflexion factor  $\Gamma$  depends on position, the coupling factor  $\beta$  does not:



$$|\tau| = |1 + \Gamma| = 1 + |\Gamma| = 1 + \frac{\beta - 1}{\beta + 1} = \frac{2\beta}{1 + \beta} \qquad |\tau| = |1 + \Gamma| = 1 - \frac{1 - \beta}{1 + \beta} = \frac{2\beta}{1 + \beta}$$

#### Response of a cavity to RF 3/5

• Determination of Q<sub>0</sub> and accelerating voltage/gradient

$$V_{t} = \tau \cdot V_{i} = \frac{2\beta}{1+\beta} V_{i} \Rightarrow V_{t}^{2} = \frac{4\beta^{2}}{\left(1+\beta\right)^{2}} V_{i}^{2} = \frac{8\beta^{2}}{\left(1+\beta\right)^{2}} \cdot \underbrace{\frac{Z_{1}}{(R/Q) \cdot Q_{0}/\beta}} \cdot \underbrace{\frac{V_{i}^{2}}{2Z_{1}}}_{P_{i}} \qquad 1)$$

$$\Rightarrow V_{t} = \sqrt{\frac{8\beta}{\left(1+\beta\right)^{2}} \cdot \left(R/Q\right) \cdot Q_{0} \cdot P_{i}}$$

$$\frac{1}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{ext}} = \frac{1 + \frac{Q_0}{Q_{ext}}}{Q_0} \Rightarrow Q_0 = \left(1 + \frac{Q_0}{Q_{ext}}\right) \cdot Q_L = (1 + \beta) \cdot \underbrace{Q_L}_{\omega\tau}$$

$$\beta = \begin{cases} \frac{1-|\Gamma|}{1+|\Gamma|}, \beta \le 1\\ \frac{1+|\Gamma|}{1-|\Gamma|}, \beta \ge 1 \end{cases}$$

$$E_{acc} = V_t / L$$

L = nominal cavity length: only cells, cutoff excluded

<sup>1</sup>) Remember

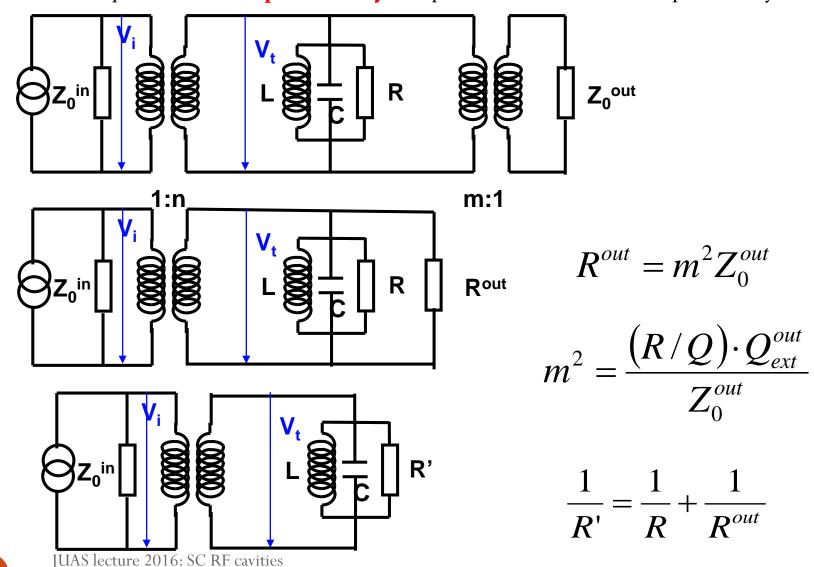
$$Z_{1} = \underbrace{\frac{Z_{1}}{Z_{2}}}_{1/\beta} \cdot Z_{2} = \frac{Z_{2}}{\beta} = \frac{R}{\beta} = \frac{1}{\beta} \cdot \underbrace{\frac{R}{V_{c}^{2}/2}}_{P_{c}} \cdot \omega U \cdot \underbrace{\frac{V_{c}^{2}}{2\omega U}}_{P_{c}} = \frac{1}{\beta} \cdot \underbrace{\frac{\omega U}{P_{c}}}_{Q_{0}} \cdot \underbrace{\frac{V_{c}^{2}}{2\omega U}}_{R/Q} = \underbrace{\frac{(R/Q) \cdot Q_{0}}{\beta}}_{R}$$

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#### Response of a cavity to RF 4/5

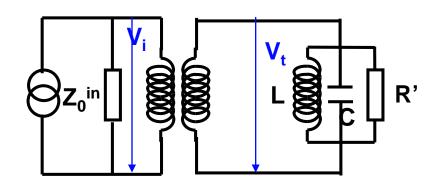
• The response of a **two port cavity** is equivalent to that of a one-port cavity



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## Response of a cavity to RF 5/5



$$V_{t} = \tau \cdot V_{i} = \frac{2\beta'}{1+\beta'} V_{i} \Rightarrow V_{t}^{2} = \frac{4\beta'^{2}}{(1+\beta')^{2}} V_{i}^{2} = \frac{8\beta'^{2}}{(1+\beta')^{2}} \cdot \underbrace{Z_{0}^{in}}_{(R/Q)Q_{0}'/\beta'} \cdot \underbrace{\frac{V_{i}^{2}}{2Z_{0}^{in}}}_{P_{i}}$$

$$\Rightarrow V_{t} = \sqrt{\frac{8\beta'}{(1+\beta')^{2}} \cdot (R/Q) \cdot Q_{0}' \cdot P_{i}}$$

$$\Rightarrow V_{t} = \sqrt{\frac{8\beta'}{\left(1+\beta'\right)^{2}} \cdot \left(R/Q\right) \cdot Q_{0}' \cdot P_{i}}$$

$$Q_{0}' = \left(1+\beta'\right) \cdot Q_{L}$$

$$E_{acc} = V_{t}/L$$

$$\beta' = \begin{cases} \frac{1-|\Gamma|}{1+|\Gamma|}, \beta' \leq 1 \\ \frac{1+|\Gamma|}{1-|\Gamma|}, \beta' \geq 1 \end{cases}$$

$$\frac{1}{Q_0} = \frac{1}{Q_0'} - \frac{1}{Q_{ext}^{out}} \qquad Q_{ext}^{out} = \frac{\omega U}{P_{out}} = \frac{V^2}{2 \cdot (R/Q) \cdot P_{out}}$$

# Transient Response 1/2

(1)

1. Apply Kirchhoff's current law at node (1)

$$\frac{V}{R_i} + \frac{1}{L} \int V(t) + C \frac{dV}{dt} + \frac{V}{R} = I_{g0} \cdot \cos \omega t$$

 Differentiate and transform lumped circuit elements into cavity parameters by using preceding "Table 7"

$$\frac{d^2V}{dt^2} + \frac{\omega_0}{Q_L} \frac{dV}{dt} + \omega_0^2 V = -I_{g0} \cdot \left(\frac{R}{Q}\right) \cdot \omega \cdot \omega_0 \cdot \sin \omega t$$

3. Find the general solution of the homogeneous differential equation

$$V(t) = e^{-\frac{\omega_0 t}{2Q_L}} \cdot \left( c_1 \cdot e^{i\sqrt{1 - 1/(2Q_L)^2} \cdot \omega_0 t} + c_2 \cdot e^{-i\sqrt{1 - 1/(2Q_L)^2} \cdot \omega_0 t} \right)$$

4. Find the solution of the inhomogeneous differential equation

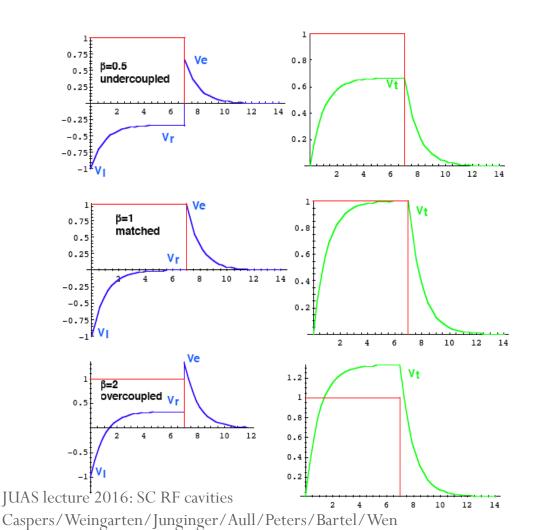
$$V(t) = V_0 \cdot \cos \omega_0 t \cdot \left\{ 1 - e^{-\frac{\omega_0 t}{2Q_L}} \right\}$$

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# Transient response 2/2

- Determination of  $Q_0$  and accelerating voltage/gradient (2)
- Oscilloscope signal for voltage measurement



$$V_t = V_i + V_r$$

Remember:  $V_e = V_i + V_r$ 

$$|\rho| = \left| \frac{V_r}{V_i} \right| \Rightarrow 1^{\text{st}} \text{ method}$$

$$\beta = \frac{1 - \left| \frac{V_r}{V_i} \right|}{1 + \left| \frac{V_r}{V} \right|} = \begin{cases} \frac{|V_i| - |V_r|}{|V_i| + |V_r|}; \beta \le 1 \\ \frac{|V_i| + |V_r|}{|V_i| - |V_r|}; \beta \ge 1 \end{cases}$$

$$2^{\text{nd}} \text{ method}: \beta = \frac{1 - \left| \frac{V_r}{V_i} \right|}{1 + \left| \frac{V_r}{V_i} \right|} = \frac{1 - \left| \frac{V_i - V_e}{V_i} \right|}{1 + \left| \frac{V_i - V_e}{V_i} \right|}$$

$$= \frac{|V_e|}{2 \cdot |V_i| - |V_e|} = \frac{1}{2 \cdot \left| \frac{V_i}{V_e} \right| - 1}$$

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