```
!------------- 1B -------------

!  1. Make a simple FODO cell of L cell = 100 m. Each quad is
!  L quad = 5 m long. Put the start of the first quadrupole at the
!  start of the sequence. Each quad has a focal length of
!  f = 200 m (K1   L quad = 1/f in thin lens approximation).

!  2. Define a proton beam at E tot = 2 GeV. Activate the
!  sequence, try to find the periodic solution and plot the
!  β-functions. If you found β max ≈ 460 m you succeeded.

!  3. Using the plot you obtained can you estimate the phase
!  advance of the cell? Compare with the tunes obtained
!  from the TWISS.
!  --- sin(mu/2) = (b_max - b_min)/(b_max + b_min)
!  --- cos(mu) = 1 - L^2 / 2f^2   (L is half cell here)

!  4. Try with E tot = 0.7 GeV: what is the MADX error message?
!  --- The beam energy needs to be at least the rest mass

!  5. Try with f = 20 m: what is the MADX error message?
!  --- The lattice is now unstable


////////////////////////////
f = 200;
Lq = 5;
QF: QUADRUPOLE, L=Lq, K1=1/f/Lq;
QD: QUADRUPOLE, L=Lq, K1=-1/f/Lq;

JUAS: SEQUENCE, REFER=entry, L=100;
qf1: QF, at=0;
qd1: QD, at=50;
ENDSEQUENCE;

beam, particle=proton, energy=2;
!beam, particle=proton, energy=0.7;

use, sequence=JUAS;

twiss, file="juas.twi";
plot, HAXIS=s, VAXIS=betx,bety, colour=100, interpolate;

quit;
////////////////////////////
```

```
!------------- 2A -------------

!  Consider the FODO cell of tutorial 1 (L cell = 100 m,
!  L quad = 5 m and f = 200 m).
!  I Define the beam (proton at E tot = 7 TeV), activate the
!  sequence and try to twiss it powering the quads to obtain
!  Δμ ≈ 90 deg phase advance in the cell using the thin lens
!  approximation (use Fig. 1). What is the actual phase
!  advance computed by MADX?

!  --- from figure: K1 L Lq ≈ 2.8 --> K1 = 0.0056
!  --- mux_mad = 0.236 2pi rad = 84.96

////////////////////////////
f = 200;
Lq = 5;
QF: QUADRUPOLE, L=Lq, K1=0.0056;
QD: QUADRUPOLE, L=Lq, K1=-0.0056;

JUAS: SEQUENCE, REFER=entry, L=100;
qf1: QF, at=0;
qd1: QD, at=50;
ENDSEQUENCE;

beam, particle=proton, energy=7000;

use, sequence=JUAS;

twiss, file="juas.twi";
plot, HAXIS=s, VAXIS=betx,bety, colour=100, interpolate;

quit;
//////////////////////////


!------------ 2B -------------

!  What is the β max ? Compare with the thin lens
!  approximation (Fig. 2). Compute the maximum beam σ
!  assuming en =3 mrad mm, E tot = 7 TeV?

!  --- bx_madx = 160.6 m
!  --- sigma = sqrt(en/g * beta) = sqrt(3e-6/7460 * 160) = 0.25 mm

!  Halve the focusing strength of the quadrupoles, what is
!  the effect of it on the β max , β min and on the Δμ? Compare
!  with the parametric plots in Fig. 1 and Fig. 2.
```

```
!------------- 3A -------------

!  Consider now that in the cell of Tutorial 2 there are 4 sector
!  dipoles of 10 m (assume 5 m of drift space between
!  magnets). In the ring there are a total of 736 dipoles with
!  equal bending angles. Install the four dipoles in the FODO
!  cell. Do the dipoles (weak focusing) affect on the β max and
!  the dispersion? Compute the relative variation on the β max
!  on the two planes.

!  --- betx_max = 160.5m -> it was 160.6 m before. y stays the same
!  --- rel = (160.6-160.5)/160.6 = 0.06 %

!  From the phase advance of the FODO cell compute the
!  horizontal and vertical tune of the machine?

!  --- phase adv of a cell = 0.24
!  --- total ph adv = 0.24 * 736/4 = 44.16

//////////////////////////
f = 200;
Lq = 5;
Ld = 10;
nBend=736;

QF: QUADRUPOLE, L=Lq, K1=0.0056;
QD: QUADRUPOLE, L=Lq, K1=-0.0056;
BM: SBEND, L=Ld, angle=2*pi/nBend;

JUAS: SEQUENCE, REFER=entry, L=100;
qf: QF, at=0;
b1:  BM, at=10;
b2:  BM, at=25;
qd: QD, at=50;
b3:  BM, at=60;
b4:  BM, at=75;
ENDSEQUENCE;

beam, particle=proton, energy=7000;

use, sequence=JUAS;

MATCH, SEQUENCE=juas;
GLOBAL, Q1=60.2/8/23; //H-tune
GLOBAL, Q2=67.2/8/23; //V-tune
VARY, NAME= qf.K1, STEP=0.00001;
VARY, NAME= qd.K1, STEP=0.00001;
```

```
LMDIF, CALLS=50, TOLERANCE=1e-6;//method adopted
ENDMATCH;


twiss, file="juas.twi";
plot, HAXIS=s, VAXIS=betx,bety,dx,dy, colour=100, interpolate;

quit;
//////////////////////////
```

!------------- 3B -------------

```
!  Change the beam to E tot = 3.5 TeV. What is the new tune of
!  the machine? Why?

!  --- new ph adv = 0.24 = previous (magnets are scaled)

!  Suppose you want to set a tune of (60.2, 67.2), match the
!  FODO to get it. What is the maximum tune that you can
!  reach with 23 cells/octant and 8 octants? (HINT: what it
!  the maximum phase advance per FODO cell in thin
!  approximation?...)

!  --- 180  per fodo cell * 23*8 cells.
```

```
!------------- 4A -------------
!  Chromaticity and sextupoles

!  1. After the definition of the sequence, convert it in thin
!  lenses with the commands:
!  MAKETHIN, SEQUENCE = MY_SEQUENCE;
!  use, sequence = MY_SEQUENCE;
!  This step is required to allow particle tracking in MAD-X.

!  2. With a matching block adjust the tunes of the cell to 0.25.

!  3. Using the chromaticitities obtained from the twiss,
!  compute the tunes for Δp/p = 10e-3 .

!  --- ΔQ = dq1 * Δp/p = -0.318*8*23 * 1e-3 = -0.059

!  4. Track a particle with initial coordinates
!  x, y, px, py = (1, 1, 0, 0) mm in 100 cells. Plot the x-px phase
!  space (use gnuplot). How does the particle move in the
!  phase space, cell after cell? Do you see the tunes?

!  --- The particle always comes back to the same four points.

!  5. Track a particle with initial coordinates
!  x, y, px, py = (100, 100, 0, 0) mm in 100 cells. Plot the x-px
!  phase space. Does something change with respect to the
!  previous case? Why?

!  --- Same plot as before, but with larger amplitude.
!  --- OK: it’s a linear machine!

!  6. Repeat point 4 adding DELTAP = 0.01 to the track
!  command. How does the phase space look now? Is the
!  tune still the same? It may help to look only at the first few
!  (4) turns, to get a clearer picture.

!  --- The particle has now a smaller tune and slips behind.


!------------- 4B -------------
!  Non-linearities and large amplitude oscillations.

!  7. Add 0.5 m long sextupoles attached to the two
!  quadrupoles. With a matching block adjust the vertical and
!  horizontal chromaticity of the cell (global parameters dq1,
!  dq2) to zero, by powering the two sextupoles (K2_1 and K2_2 ).
```

```
!  8. using the obtained K2 1 and K2 2 , β-function and dispersion
!  at sextupoles location, evaluate using the formulas the
!  sextupolar effect on the Q1 for a particle at DELTAP = 1e-3.
!  Compare the results with the value obtained in point 1.

!  --- ΔQ = Sum over the sexts of (beta * K2L * D * Δp/p)
!         =     8*23/4/pi*(147.5 * 0.0165 * 2.417 * 1e-3 -
!                          34.44 * 0.0301 * 1.173 * 1e-3) = 0.068
!  --- The discrepancy comes from the different off-momentum beta

!  9. Repeat point 4 adding DELTAP = 0.01 to the track
!  command. Did you manage to recover the original tune
!  for the off-momentum particle?

!  --- Yep, the tune is now close to 0.25 again

!  10. Repeat point 5. What is going on now?

!  --- With a larger initial amplitude we are loosing the particle
!  --- We are outside the dynamic aperture

!  11. Move the tunes to (0.23, 0.23) and repeat the previous
!  point. Is now the particle stable?

!  --- The particle looks more stable than before. Moving away
!  --- from the resonance we improved the dynamic aperture


/////////////////////////////
!! General parameters
Lcell = 100;
nBend=736;

!! Dipole Parameters
Ld = 15;
Ad = 2*pi/nBend;

!! Quadrupole Parameters
K1F = 5e-3;
K1D = -K1F;
Lq = 5;

!! Sextupole Parameters
K2F = 0.0;
K2D = -K2F;
Ls = 0.5;
```

```
QF: QUADRUPOLE, L:=Lq, K1:=K1F;
QD: QUADRUPOLE, L:=Lq, K1:=K1D;
BM: SBEND, L=Ld, angle:=Ad;
SF: SEXTUPOLE, L:=Ls, K2:=K2F;
SD: SEXTUPOLE, L:=Ls, K2:=K2D;

JUAS: SEQUENCE, REFER=entry, L=100;
qf: QF, at=0;
S1 : SF, at=5;
b1:  BM, at=10;
b2:  BM, at=30;
qd: QD, at=50;
S2 : SD, at=55;
b3:  BM, at=60;
b4:  BM, at=80;
ENDSEQUENCE;

beam, particle=proton, energy=7000;

MAKETHIN, SEQUENCE=JUAS;
use, sequence=JUAS;

//**************************************************//
!   MATCHING OF THE TUNES
//**************************************************//

match, sequence=JUAS;
!! Variables
vary,name=K1F,step=0.0001;
vary,name=K1D,step=0.0001;
!! Constraints
global, Q1=0.25;
global, Q2=0.25;
!! The next line ask MAD-X to do the matching itself
LMDIF, calls = 1000, tolerance=1E-12;
endmatch;

//**************************************************//
!   MATCHING OF THE CHROMATICITY
//**************************************************//

!match, sequence=JUAS;
!!! Variables
!vary,name=K2F,step=0.0001;
!vary,name=K2D,step=0.0001;
!!! Constraints
!global, dq1=0.0;
```

```
!global, dq2=0.0;
!!! The next line ask MAD-X to do the matching itself
!LMDIF, calls = 1000, tolerance=1E-12;
!endmatch;


//*************************************************//
!   TWISS
//*************************************************//

SELECT,FLAG=TWISS, column=name,s,betx,bety,dx,K1L,K2L;
twiss,file="juas.twi";
plot, HAXIS=s, VAXIS=betx,bety,dx,dy,colour=100, interpolate;


//*************************************************//
!   TRACKING
//*************************************************//

track,dump, DELTAP=0.01;
start, x= 1e-3, px=0, y= 1e-3, py=0;
start, x= 1e-1, px=0, y= 1e-1, py=0;
run,turns=4;
endtrack;

plot, file="MAD_track",table=track,haxis=x,vaxis=px,
particle=1,2, colour=100;
plot, file="MAD_track",table=track,haxis=y,vaxis=py,
particle=1,2, colour=100;

quit;
/////////////////////////////
```

!------------ 5A ---------------

! Build a transfer line of 10 m with 4 quads of L=0.4 m
! (centered at 2, 4, 6, and 8 m). With K1 respectively of 0.1,
! 0.1 , 0.1 , 0.1 m −2 . Can you find a periodic solution?

! --- no, the lattice is not stable in either planes

! Can you find a Initial Condition solution starting from
! (β x , α x , β y , α y) = (1, 0, 2, 0)?

! --- Sure!

! What is the final optical condition (β x end , α x end , β y end , α y end)?

! --- x: 50.05527223 m -2.04007934
! --- y: 98.13871917 m -13.56203145

!------------ 5B ---------------

! Starting from (β x , α x , β y , α y) = (1 m, 0, 2 m, 0) match the
! line to (β x , α x , β y , α y) = (2, 0, 1, 0) at the end.

! Starting from (β x , α x , β y , α y) = (1 m, 0, 2 m, 0) and the
! gradient obtained with the previous matching, match to
! (β x end , α x end , β y end , α y end). Can you find back K1 respectively of
! 0.1, 0.1 , 0.1 , 0.1 m −2 ?

! --- it converges to a different solution

! Consider that the quadrupoles have an excitation current
! factor of 100 A/m 2 and an excitation magnetic factor of 100
! T/m/A and aperture of 40 mm diameter. Compute the
! magnetic field at the poles of the four quads after matching
! (HINT: assume linear regime and use a dimensional
! approach).

! --- Current = 100 A/m^2 * K1 = 10 A (with K=0.1)
! --- Field = 100 T/m/A * Current * Aperture = 40 T (with 10 A)

```
////////////////////////////////////////
Kq = 0.1;
Lq = 0.4;

QUAD: QUADRUPOLE, L=Lq, K1=Kq;

JUAS: SEQUENCE, REFER=centre, L=10;
Q1: QUAD, at=2;
Q2: QUAD, at=4;
Q3: QUAD, at=6;
Q4: QUAD, at=8;
ENDSEQUENCE;

beam, particle=proton, energy=7000;

use, sequence=JUAS;

MATCH, SEQUENCE=JUAS, betx=1, bety=2;
constraint, betx=50.055, range=#e;
constraint, alfx=-2.0401, range=#e;
constraint, bety=98.139, range=#e;
constraint, alfy=-13.5620, range=#e;
VARY, NAME= kq1, STEP=0.00001;
VARY, NAME= kq2, STEP=0.00001;
VARY, NAME= kq3, STEP=0.00001;
VARY, NAME= kq4, STEP=0.00001;
LMDIF, CALLS=500, TOLERANCE=1e-6;//method adopted
ENDMATCH;

twiss, betx=1, alfx=0, bety=2, alfy=0, file="juas.twi";
plot, HAXIS=s, VAXIS=betx,bety,colour=100, interpolate;

quit;
////////////////////////////////////////
```

```
!------------ 6 ---------------

!  Retrieve the LHC injection optics from the repository.
!  Download the LHC Run 1 protons, injection optics from
!  http://lhc-optics.web.cern.ch/lhc-optics/www/
!
!  Build a the MADX scripts to call the file and to twiss the
!  machine.
!  What is the LHC length? What is the s-position of IP1 and
!  IP5? and the β-functions there?
!  What are the beam1 and beam2 tunes at injections?
!  Are the two beams colliding in IP1 at injection?

!  Retrieve the collision optics.
!  Is the crossing of the two beams vertical or horizontal in
!  IP1 at collision?
!  What are the beta function at the IPs at collision energy?
!  Why do we inject with a higher β-function at the IPs?


!  --- For instance:
!  wget http://lhc-optics.web.cern.ch/lhc-
optics/www/opt2015/inj/lhc_opt2015_inj.seq
!  wget http://lhc-optics.web.cern.ch/lhc-
optics/www/opt2015/coll400/lhc_opt2015_coll400.seq

/////////////////////
Option, -echo,warn,-info;

call, file="lhc_opt2015_inj.seq";
!call, file="lhc_opt2015_coll400.seq";

use, sequence=lhcb1;
!use, sequence=lhcb2;

SELECT,FLAG=TWISS, column=keyword,name,s,betx,bety,x,y,dx,dy;
twiss, file="lhc.twi";
plot, HAXIS=s, VAXIS=betx,bety,dx,dy,colour=100;
plot, HAXIS=s, VAXIS=x,y,colour=100;
/////////////////////

! --- then look in the plots and in the Twiss table
```