Astroparticles

ESIPAP 2016

Tutorials

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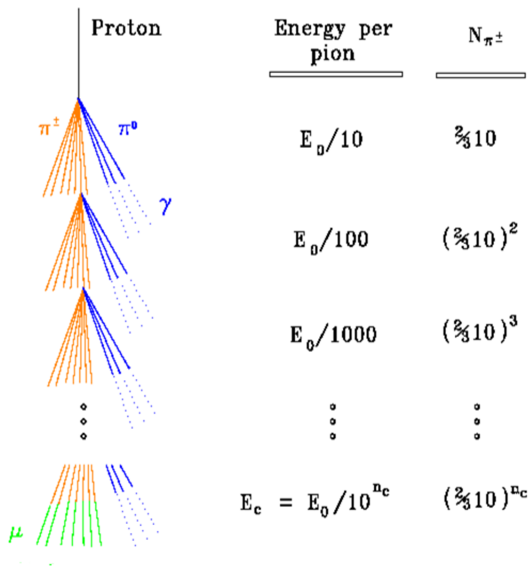
Menu:

First a classical problem on the Heitler model of shower development followed by the short answer questions and a problem that were proposed at last year's exam.

1 Heitler model for EM showers

Using a simplified model à la Heitler, we want to model the EM and the muonic component of shower induced by a proton with an energy $E_0 = 10^{18}$eV. One gives the critical energy to be (fixed) $\varepsilon_\pi = 100$ GeV and $\lambda_\pi = 100$ g/cm^2 is the pion interaction length. We will do the following hypotheses:



1. At each fixed interaction length λ, all the non decayed hadrons with energy $> \varepsilon_\pi$, interact with a nucleus of the atmosphere.

2. Each interaction produces secondary hadrons (only pions) with multiplicity m, $2/3$ of them are $\pi^\pm$ and 1/3 are π^0. One assume that the multiplicity is fixed and its value is $m = 10$.

3. Each of the m pions produced takes away a fraction of the parent energy $1/m$.

4. π^0 decay immediately in two γ that will feed the EM component.

5. When charged pions reach the critical energy ε_π, they all decay and produce each one muon that propagates to the ground. We will assume that the muons takes away 1/2 of the critical energy.

Try to answer the following questions:

1. Give the number of interaction length before reaching the critical energy and give an approximate value using the given parameters.

$$n_c = \log_{10}\left(\frac{E_0}{E_c}\right)$$

With $E_0 = 10^{18}$ eV and $E_c = 100$ GeV $= 10^{11}$ eV one immediately finds $n_c = 7$.

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2. Compute what is the fraction of the total energy transferred to the EM component.

$$\begin{aligned} E_{\text{EM}} &= \sum_{n=1}^{n_c}\left(\frac{E_0}{10^n}\right)\left(\frac{2}{3}\times 10\right)^{n-1}\left(\frac{1}{3}\times 10\right) = E_0\sum_{n=1}^{n_c}\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^n \\ &= E_0\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{1-(2/3)^7}{1-2/3}\right) = 1 - 5.85\times 10^{-2} = 94\% \end{aligned}$$

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3. Compute the number of muons produced at the end of the development. What fraction of energy do they take away.

The number of muons is equal to the number of pions at the critical energy:

$$\begin{aligned} N_\mu &\approx N^c_{\pi^\pm} = \left(\frac{2}{3}\times 10\right)^{n_c} \\ \log_{10} N_\mu &\approx \left(1+\log_{10}\left(\frac{2}{3}\right)\right) n_c \\ &= 0.82\log_{10}(E_0/E_c) \\ \Rightarrow N_\mu &\approx \left(\frac{E_0}{E_c}\right)^{0.82} \end{aligned}$$

The energy of each muon is assumed to be $E_c/2$ and hence:

$$E_\mu = \frac{1}{2}E_c\left(\frac{2}{3}\times 10\right)^{n_c} = \frac{1}{2}E_0\left(\frac{2}{3}\right)^{n_c} = \frac{1}{2}\times 5.85\times 10^{-2} \approx 3\%$$

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4. Compare the EM, muonic and initial energy. Do you observed missing energy? What is taking this missing energy away? (this will help you with

Obviously when imposing half of the pion energy is transferred to the muon, one takes into account the energy transferred to neutrinos ! The approximatly 3% of missing energy is the energy carried by neutrinos.

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5. Suppose now that the incident particle is an iron nucleus with the same total energy than the proton. Assuming superposition principle holds, what is now the muons energy fraction?

Superposition principle: a nucleus ${}^{A}N$ is equivalent to A protons.

Given:

$$N_\mu \approx \left(\frac{E_0}{E_c}\right)^{0.82}$$

at equivalent total energy, the superposition principle says:

$$N_\mu^A(E) \propto A\left(\frac{E_0}{A}\right)^{0.82}$$

Thus:

$$\begin{aligned} N_\mu^A(E) &\approx A^{(1-0.82)} \times N_\mu^p(E) \approx A^{0.18} \times N_\mu^p(E) \\ N_\mu^{Fe} &\approx 2 \times N_\mu^p(E) \end{aligned}$$

In fact one observes 80% more muons
for a ${}^{56}Fe$ primary compared to a proton with the same total energy.

Conclude if this muons energy fraction can be used to deduce the nature of the incident particle.

Measuring total energy or sum of muon + EM energy to the energy of the muonic component, one obtains a rather reliable measurement of the $\log_{10}(A)$ of the primary nuclei.

A few useful numerical values:
$(2/3)^7 \approx 5.85 \times 10^{-2}$
$\log_{10}(2/3) \approx -0.18$
$\log_{10}(56) \approx 1.75$
$(56)^{-0.18} \approx 2$