Scalar field damping & transport coefficients[1]

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weakly coupled scalar field

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examples

inflaton, reheating

axion

moduli field

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interaction with other fields ‘matter’ $\Rightarrow$ dissipation, damping

$$(\Box + m^2)\varphi = -2\gamma\dot{\varphi}$$

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When is this equation valid?

weakly coupled scalar field

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- axion
- moduli field

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When is this equation valid?

What is γ?

Outline

effective equation of motion, damping rate

motivation: moduli decay in early Universe

explicit computation, relation to bulk viscosity

Damping rate

When and why local equation of motion?

$\varphi = \varphi(t, \mathbf{x})$ non-equilbrium problem

non-local in space an time

memory effects, ...

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in general

$$(\Box + m^2)\varphi = \text{ something complicated}$$

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in general

$$(\Box + m^2)\varphi = \text{ something complicated}$$

when is the effective equation of motion simple and local?

When and why local equation of motion?

needs separation of scales

φ varies slowly, time scale t_φ

'matter' fields equilibrate fast, time scale $t_{\rm eq}$

$$t_\varphi \gg t_{\rm eq}$$

Leading order in the ratio of scales

fast fields

'see' $\varphi \simeq$ constant

equilibrate at fixed φ

equilibrium state at $(t, \mathbf{x})$ only depends on $\varphi(t, \mathbf{x})$

thermal corrections to φ-potential, $m^2 \to m^2 + \delta m^2(T) = m^2_{\text{eff}}(T)$

Beyond leading order

equilibration not instantaneous

corrections = higher derivative terms in φ-equation of motion

suppressed by

ration of chararcteristic time scales = $t_{\mathrm{eq}}/t_{\varphi}$

coupling of φ to matter

$$\gamma\dot{\varphi}, \quad \kappa(\nabla\varphi)^2, \ldots$$

$$\gamma \sim t_{\mathrm{eq}}, \quad \kappa \sim t_{\mathrm{eq}}^2$$

NLO:

$$(\Box + m_{\mathrm{eff}}^2)\varphi = -2\gamma\dot{\varphi}$$

Damping rate

for sufficiently small φ:

$$\gamma = \gamma(g, T, \mu), \qquad g = \text{ matter self-coupling}$$

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when $g \to 0 \qquad \Rightarrow \qquad t_{\rm eq} \to \infty, \qquad \gamma \to \infty$

$$\gamma \sim \frac{1}{g^n}$$

non-trivial calculation!

γ from QFT

small oscillations in the 'effective field theory'

$$\ddot{\varphi} + m_{\text{eff}}^2 \varphi = -2\gamma\dot{\varphi}, \qquad \varphi \propto e^{-i\omega t}, \qquad \omega = m_{\text{eff}} - i\gamma$$

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microscopic theory: linear response $\langle\varphi(t)\rangle = \int d^4x' \Delta_{\text{ret}}(x-x')J(t')$

$$\Delta_{\text{ret}}(\omega, \mathbf{p}) = \Delta(\omega + i\varepsilon, \mathbf{p}), \quad \Delta(p) = \frac{-1}{p^2 - m^2 - \Pi(p)}$$

pole at $\omega \simeq m_{\text{eff}} + \frac{i}{2m_{\text{eff}}} \text{Im}\ \Pi(m_{\text{eff}} + i\varepsilon, \mathbf{0}), \ \mathbf{p} = \mathbf{0}$

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damping rate $\boxed{\gamma = -\frac{1}{2m_{\rm eff}} {\rm Im}\ \Pi(m_{\rm eff} + i\epsilon, \mathbf{0})}$

γ from QFT

separation of time scales $\Rightarrow$ at LO in φ-matter interactions

$$\gamma = -\lim_{\omega \to 0} \frac{1}{2\omega} \mathrm{Im}\Pi(\omega + i\epsilon, \mathbf{0})$$

for interaction $\mathcal{L}_{\rm int} = \varphi A$, lowest order in $\mathcal{L}_{\rm int}$

$$\Pi^{>}(x) = \langle A(x) A(0) \rangle$$

$$\gamma = -\frac{1}{4T} \lim_{\omega \to 0} \Pi^{>}(\omega, \mathbf{0})$$

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similar to Kubo relations for transport coefficients

Kubo relations

electric conductivity

$$\sigma = \lim_{\omega \to 0} \frac{1}{3T} \int d^4x \, e^{i\omega t} \langle J_i(x) J_i(0) \rangle$$

bulk viscosity

$$\zeta = \frac{1}{18T} \lim_{\omega \to 0} \int d^4x \, e^{i\omega t} \langle \Theta(x) \Theta(0) \rangle \quad (\Theta = T^{\mu}{}_{\mu})$$

Microscopic scales

in general: coupled dynamics of scalar and other fields

local effective theory when scalar field "slow"

"microscopic" time scales for weak coupling g

T	de Broglie wavelength, "hard"
gT	electrostatic and dynamical screening
$g^2 T$	magnetic screening (non-abelian),
	mean free path for small angle scattering
$g^4 T$	mean free path for large angle scattering

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"macroscopic" time scale $t_\varphi \gg \left(g^4 T\right)^{-1}$

requires weakly coupled and light φ

Moduli problem in cosmology

Moduli fields

SUSY $\Rightarrow$ flat directions in field space "moduli fields"

interactions are Planck scale suppressed, e.g.

$$\frac{\varphi}{M} F^{\mu\nu} F_{\mu\nu}$$

SUSY breaking $\Rightarrow$ degeneracy is lifted, moduli fields acquire mass

SUSY breaking scale $\sim 1\mathrm{TeV} \quad \Rightarrow \quad m_\varphi \sim 1\mathrm{TeV}$
moduli fields are light and decouple at low energy

Moduli fields in cosmology

$$\ddot{\varphi} + 3H\dot{\varphi} + m^2\varphi = 0$$

over-damped when $H \gg m$

oscillations when $H \lesssim m \quad \Leftrightarrow \quad T \lesssim T_\mathrm{i} \sim \sqrt{mM}$

$m \sim 1\text{TeV} : \; T_\mathrm{i} \sim 10^{11}$ GeV

oscillations contribute to ρ like NR matter

start dominating over radiation for

$$T \sim T_\mathrm{i} \left(\frac{\varphi_\mathrm{i}}{M}\right)^2$$

typically $\varphi_\mathrm{i} \sim M$

Moduli problem

decay rate in vacuum

$$\Gamma \sim \frac{m^3}{M^2}$$

moduli disappear when $\Gamma \gtrsim H$

sudden decay approximation $H_{\mathrm{before}} = H_{\mathrm{after}}$

$$T_{\mathrm{reheat}} \sim 5 \left(\frac{m}{1\mathrm{TeV}} \right)^{3/2} \mathrm{keV}$$

problem: BBN requires radiation domination at $T \sim 1\mathrm{MeV}$

Moduli damping rate

Planck scale suppressed interactions like

$$\frac{\varphi}{M} F^{\mu\nu} F_{\mu\nu}$$

$$\Pi \propto \frac{1}{M^2}$$

damping rate

$$\gamma = f(g) \frac{T^3}{M^2}$$

compare with Hubble rate $H \sim T^2/M$

$$\frac{\gamma}{H} \sim f \frac{T}{M}$$

can thermal effects solve the moduli problem ??? [2]

[2] M. Drewes, J. U. Kang, 1305.0267   M. Drewes, 1406.6243.

Computing the damping rate γ

Explicit calculation of γ

toy model with massless scalar 'matter' field χ

$$
\begin{aligned}
\mathcal{L} = & \frac{1}{2}(\partial\varphi)^2 - \frac{m^2}{2}\varphi^2 \\
& + \frac{1}{2}(\partial\chi)^2 - \frac{\lambda}{4!}\imath^4 \\
& + \frac{1}{2M}\varphi(\partial\chi)^2
\end{aligned}
$$

$$
\Pi^{>}(x) = -\frac{1}{4M^2}\left\langle \left(\partial\chi\right)^2(x)\ \left(\partial\chi\right)^2(0) \right\rangle + O(M^{-4})
$$

Trace anomaly

$T^{\mu\nu} \equiv$ energy momentum tensor of χ only

trace anomaly

$$\Theta \equiv T^{\mu}{}_{\mu} = -\frac{\beta(\lambda)}{4!}\chi^4 + \cdots = -\frac{\beta(\lambda)}{4\lambda}(\partial\chi)^2 + \cdots$$

$$(\cdots) \quad \sim \quad \Box\chi^2 \quad + \quad \chi \cdot \text{ EOM}$$

$$\beta(\lambda) = \frac{3\lambda^2}{16\pi^2} + O(\lambda^3) \qquad \Rightarrow \qquad (\partial\chi)^2 = -\frac{(8\pi)^2}{3\lambda}\Theta$$

$$\gamma = \frac{1}{8M^2}\frac{8\pi^4}{9\lambda^2}\frac{1}{2T}\lim_{\omega\to 0}\int d^4x\, e^{i\omega t}\Big\langle\Theta(x)\Theta(0)\Big\rangle$$

Relation to bulk viscosity

matter bulk viscosity $$\zeta = \frac{1}{9}\frac{1}{2T}\lim_{\omega\to 0}\int d^4x e^{i\omega t}\left\langle \Theta(x)\Theta(0)\right\rangle$$

$$\boxed{\gamma = \frac{\pi^4}{M^2\lambda^2}\,\zeta}$$

result for $\lambda\chi^4$ theory:[3]

$$\zeta = \frac{b}{6(32\pi)^4}\lambda \ln^2(\xi\lambda)\,T^3, \qquad b = 5\times 10^4, \xi \simeq 0.06$$

$$\boxed{\gamma = 4.5\;\frac{\ln^2(\xi\lambda)}{\lambda}\,\frac{T^3}{M^2}}$$

[3] S. Jeon, L. G. Yaffe, [hep-ph/9512263].

Questions

generalization to other interactions?

transport coefficients from different operators?

application: reheating after inflaction,
maximal temperature of the Universe

Nebenrechnung

$$\Delta^{>}(\omega) \simeq \frac{T}{\omega}\rho(\omega)$$

$$\rho(\omega) = 2\mathrm{Im}\Pi(\omega + i0^{+})$$

$$\mathrm{Im}\frac{\Pi}{\omega} = \frac{1}{2\omega}\rho = \frac{1}{2\omega}\frac{\omega}{T}\Delta^{>} = \frac{1}{2T}\Delta^{>}$$