Inflation with Dissipation and Metastability

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And earlier work with: T. Biswas, F. di Marco, I. Masina

Outline

Inflation

- In FLRW metric, expansion described by $a(t)$

$$ds^2 = -dt^2 + a^2(t)d\vec{x}^2$$

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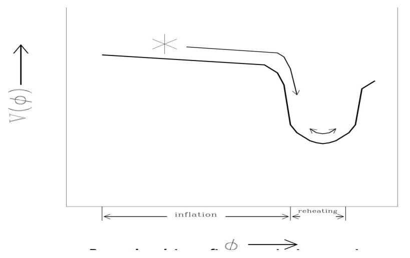
- $\Rightarrow$ Slowly rolls for $\left(\frac{a_I}{a_F}\right) = e^N \gtrsim e^{60}$

Slow-roll



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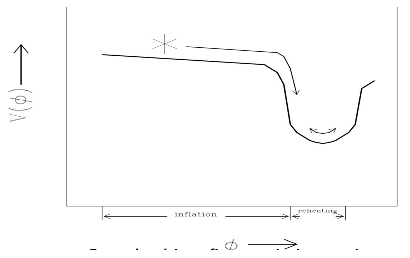

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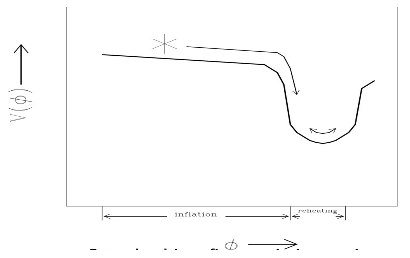

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- May be also “multi-field"

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- Unknown V
- Unusually flat V
- Unknown couplings and Reheating process
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- Difficult to test:
  - Usually at very high-energy
  - Thermalization

Alternatives

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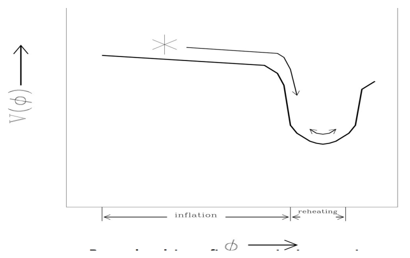
- Can Inflation end with a phase transition? (additional signatures)
- Can we link to Standard Model and experiments?

Dissipation: the Rehating case

- Qualitatively reheating is described by:

$$\ddot{\phi} + 3H\dot{\phi} + V_{\phi}(\phi) + \Gamma\dot{\phi} = 0$$

with $\Gamma \gg H$ coming from decay into other particles: $\phi \to 2\psi$



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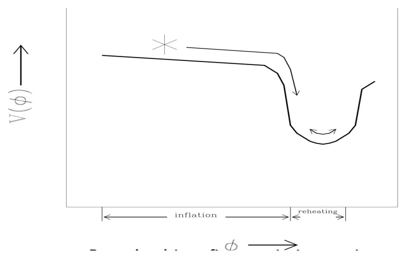

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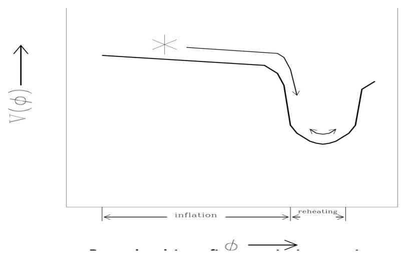

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- Is it possible already during inflation?

Dissipation during Inflation?

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- If it works: decay product must be massless
- $\mathcal{L}_{int} = \lambda\phi\psi^2 \;\; \to m_\psi = \lambda\langle\phi\rangle + m_\psi^0$ does NOT work

A concrete realization

- ϕ coupled to U(1) gauge fields:

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\phi) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\phi}{4f_\gamma}F_{\mu\nu}\tilde{F}^{\mu\nu}\right)$$

(Sorbo and Anber '2009)

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- Photons are massless (ϕ coupled derivatively)

A concrete realization

- (Sorbo & Anber '09): in a time dependent ϕ and in FLRW (with conformal time $a d\tau = dt$):

$$A''_{\pm} + \left(k^2 \mp \frac{k\phi'}{f_\gamma}\right) A_{\pm} = 0\,,$$

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- $\phi' = a\dot{\phi} \neq 0$ (and $\pm$ positive (negative) helicity)
- One helicity is unstable: $\langle F\tilde{F}\rangle$ becomes quickly large
- Assumed that its backreaction slows down ϕ:

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi}(\phi) + \frac{\langle F\tilde{F}\rangle}{f_\gamma} = 0\,,$$

- Found inflationary solution

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$$A_{-}^{an} = \frac{1}{\sqrt{2k}} \left(\frac{k}{2\xi aH} \right)^{1/4} e^{\pi\xi - 2\sqrt{2\xi k/(aH)}} \qquad (4)$$

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($\xi \equiv \frac{\dot{\phi}}{2f_{\gamma}H}$)

- "Almost" going to vacuum fluctuations $A_k = e^{ik\tau}\sqrt{2k}$ at $\tau = -\infty$

A concrete realization

- (Sorbo & Anber '09) estimated:

$$\frac{\langle F\tilde{F}\rangle}{4} = \frac{1}{2a^4}\int \frac{d^3k}{(2\pi)^3}\, k\frac{d\left[|A_+|^2 - |A_-|^2\right]}{d\tau},$$

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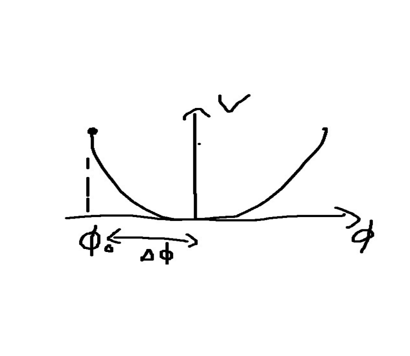
- $\frac{d\phi}{dN} \equiv \frac{d\phi}{Hdt} \approx f_\gamma \to N_{TOT} \approx \frac{\Delta\phi}{f_\gamma}$
- f_γ much smaller than the field excursion $\Delta\phi$



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- Do we actually get $\dot{\phi} \approx const$ and $H \approx const$?
- Cosmological Perturbations?

Our work

- We solved numerically for $A_{\pm}$ from an initial condition:

$$\begin{aligned} A(0) &= \frac{1}{\sqrt{2k}}\,, \qquad \dot{A}(0) = \frac{ik}{\sqrt{2k}}\,, \\ \phi(0) &= \phi_0\,, \qquad \dot{\phi}(0) = 0\,. \end{aligned} \tag{5}$$

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- For $k < f_\gamma$ (cutoff of the effective theory)

Flat spacetime case

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Flat spacetime case: Results

Higgs & Inflation

Introduction

Dissipation

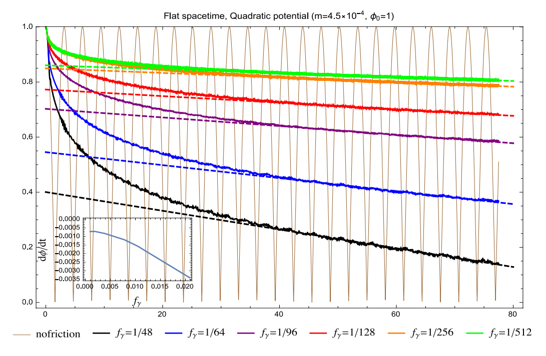
Metastability

Metastability from Higgs potential?

- The A_{-} and $\langle\tilde{F}F\rangle$ grow faster than the free evolution if $f_\gamma \ll \Delta\phi$
- Independently on $V(\phi)$

Flat spacetime case: Results

ϕ evolution



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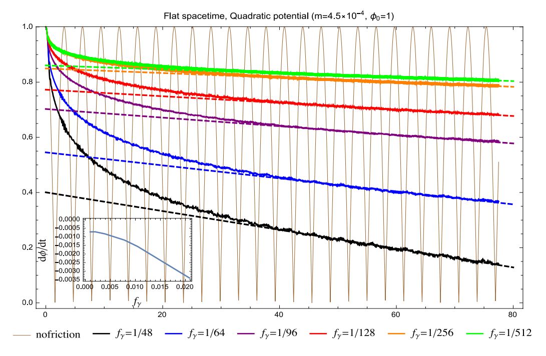

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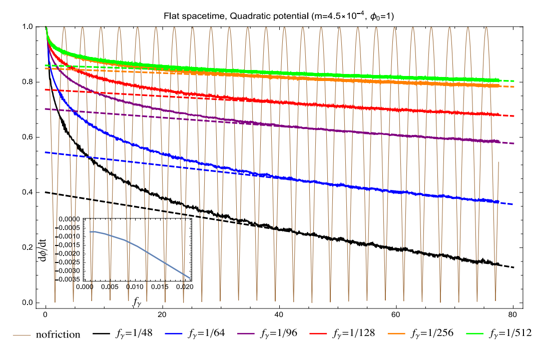

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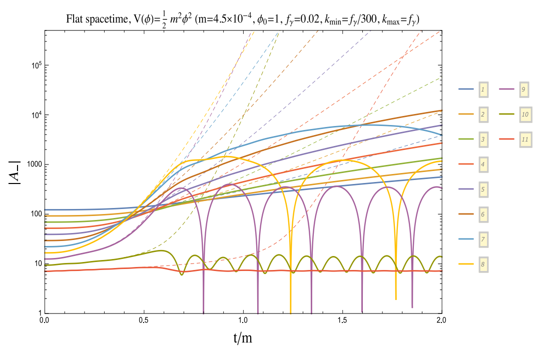
Plus oscillations

Flat spacetime case: Results

A_- Mode evolution



Flat spacetime case: Results

Spectra for: $\langle \tilde{F} F \rangle = E \cdot B$, $\rho_R = \frac{E^2 + B^2}{2}$

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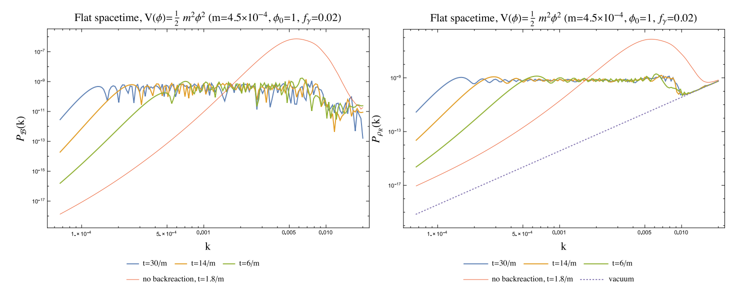

FLRW case: Results

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- If the scale of the potential $V_\phi \gg H^3$ and for a time $t \ll 1/H$, we can neglect the expansion
- So we are back to the previous case:
- The A_- and $\langle \tilde{F}F \rangle$ grow faster than the free evolution if $f_\gamma \ll \Delta\phi$
- Independently on $V(\phi)$

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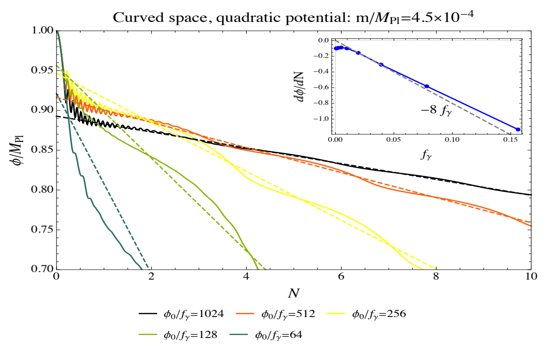
- So, we confirm $N_{TOT} \approx \frac{\Delta\phi}{f_\gamma}$

FLRW case: Results

ϕ evolution



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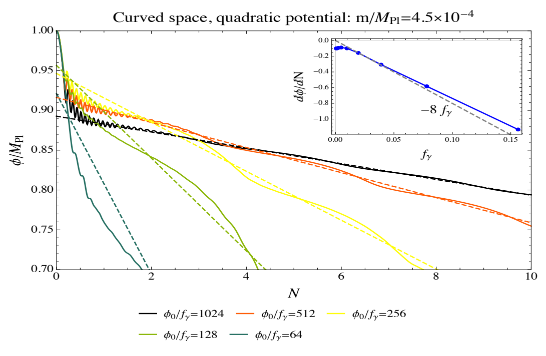

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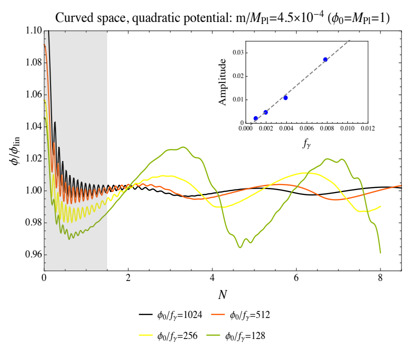
Oscillations in ϕ:



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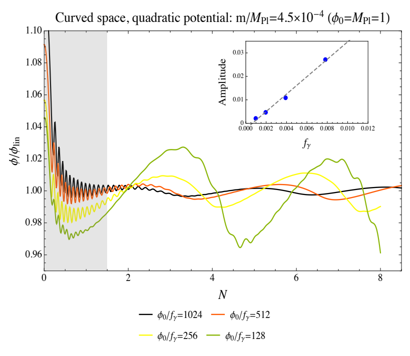

- Period of about 4 efolds
- Amplitude $\propto f_\gamma$

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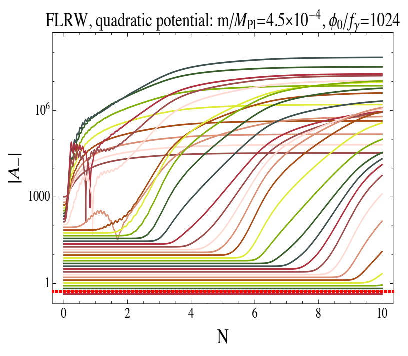

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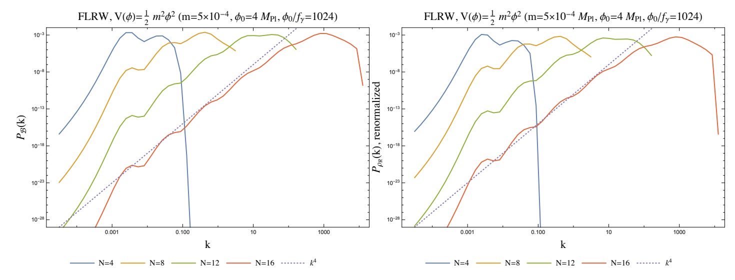
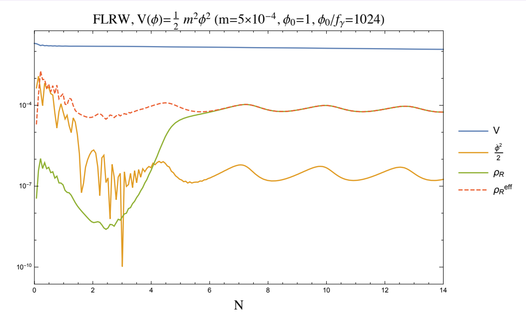


k^4 behavior (suppression on large scales)

FLRW case:Results

Energy densities:



$$V \gg \rho_R \gg \dot{\phi}^2/2$$

Reheating

- There is no separation between reheating and inflation
- Model independent reheating

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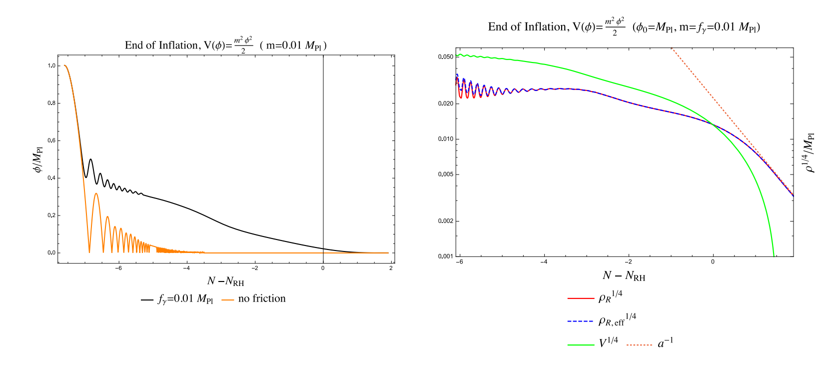

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- Found too large density fluctuations
  (unless a large number N of gauge fields is invoked)

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- Assumed $(\tilde{F}F)_p = \frac{\partial<\tilde{F}F>}{\partial\dot{\phi}}\delta\dot{\phi}_p + (\tilde{F}F)_p|_{\delta\phi=0}$
- A dissipative term and a source term

Perturbations??

So they wrote:

- $\delta\ddot{\phi}_p + (3H - \frac{1}{f_\gamma}\frac{\partial < \tilde{F}F >}{\partial\dot{\phi}})\delta\dot{\phi}_p + \frac{p^2}{a^2}\delta\phi_p = \frac{1}{f_\gamma}(\tilde{F}F)_p|_{\delta\phi=0}$
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  - On small scales: a term $\nabla\delta\phi \wedge \mathbf{A}'$ is present

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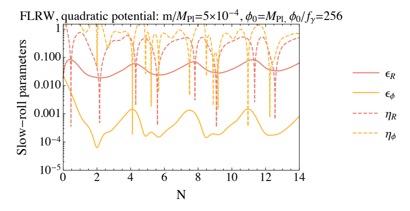

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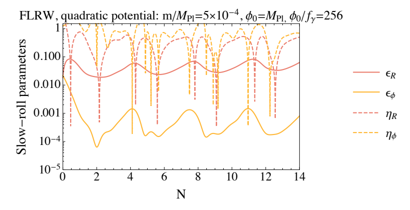

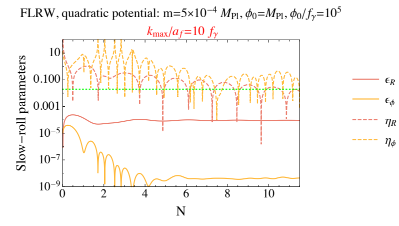

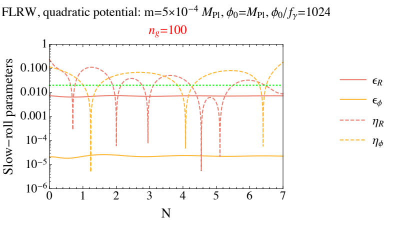

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- It is possible: independent parameters. $\frac{1}{f_\gamma}$ does not induce corrections to $\frac{1}{f_G}$ (would break the shift symmetry)

Inflation in a False Minimum

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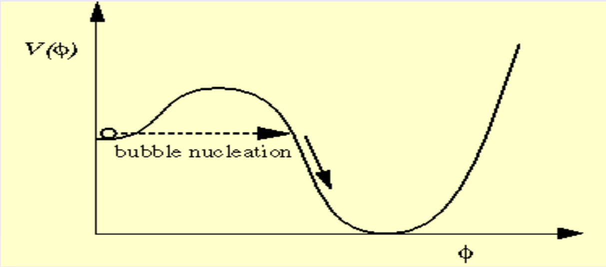
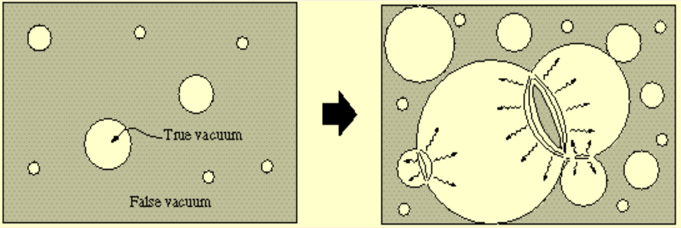
- Old Inflation does not have Graceful Exit: non-successful Bubble Nucleation

Nucleation of Bubbles





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Way-out:

- Start with $\Gamma \ll H^4$
- And then $\Gamma \simeq H^4$

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- Two possibilities
  - Make H variable (couple ϕ to gravity)
  - Make Γ variable (couple ϕ to χ)

Variable H

- If H decreases with time
- At some point $\Gamma^{1/4} = H$

Modifying $\mathcal{L}_G$...

Adding a Non-Minimally coupled scalar field (ϕ)

$\Rightarrow$ Graceful exit

F.Di Marco & A.N., Phys.Rev.D '05,
T. Biswas & A.N. Phys.Rev.D '06

An initial Lagrangian

- As a starting point we take the action

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- Assume $U(\phi)$ to be negligible ($U \lesssim V(\chi_0)$) in the Early Universe.

Early Time Evolution

$$\boxed{S_1 = \int d^4x\sqrt{-g}\left[\frac{1}{2}M^2R + \frac{1}{2}\gamma\frac{\phi^n}{M^{n-2}}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + V(\chi_0)\right],}$$

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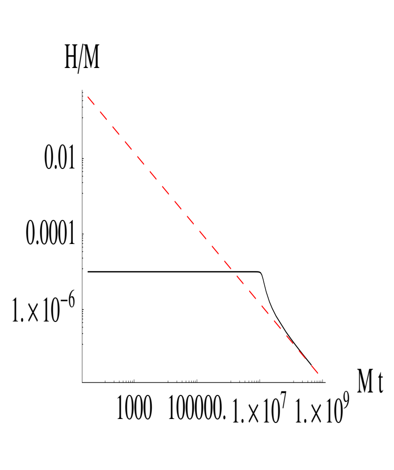
4. $H \propto \frac{1}{t}$ and when $H = \Gamma^{1/4} \Rightarrow$ Graceful Exit.

Evolution of H



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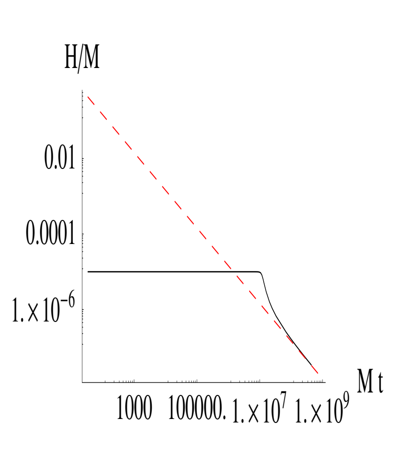

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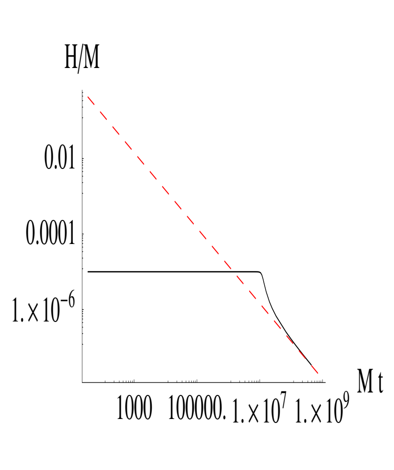

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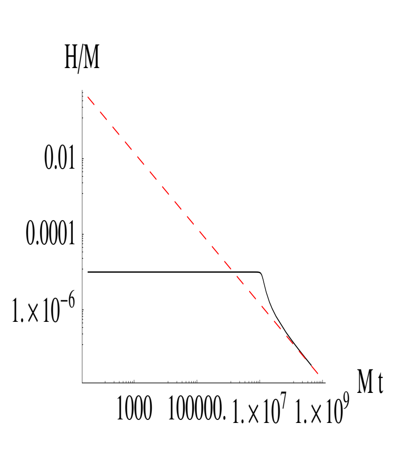

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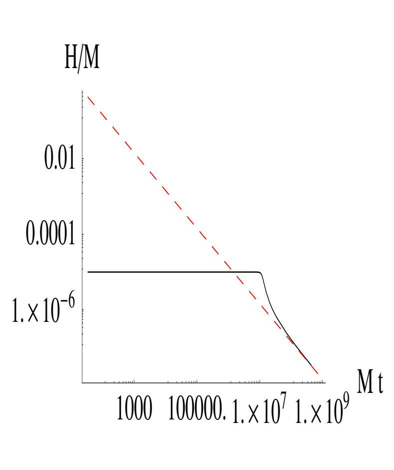

Crucially

- *Phase I*: Perturbations that we see
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- When $H \simeq \Gamma^{1/4} \Rightarrow$ Graceful Exit

General Lagrangian

- If we regard M as the fundamental scale of the theory
- The full theory has operators like

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- Without knowing exactly $f(\phi)$ (...an infinite number of couplings)!

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- During radiation ϕ slows down:

$$R = 6(2H^2 + \dot{H}) \approx 0\,.$$

Stabilization of ϕ

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- ...We can reintroduce the potential $U(\phi)$ in the original Lagrangian
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- Any potential which drives ϕ to a minimum ($\phi \to 0$) is good

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- In Einstein frame there is a potential $\Rightarrow$ almost flat spectrum

$$\bar{V} \approx V(\chi_0)\left[1 - 2\gamma\left(\frac{\phi}{M}\right)^n\right].$$

$$\begin{cases} n_S - 1 = 2\eta - 6\epsilon \\ \Delta_R^2 = \left(\frac{\bar{H}_I}{M}\right)^2 \frac{1}{8\pi^2\epsilon}\Big|_{\phi=\phi(\bar{\mathcal{N}}\approx\bar{\mathcal{N}}_{3000h^{-1}Mpc})} \end{cases}$$

Parameter values (quadratic term absent)

- We predicted the spectral index:

$$n_S \simeq 1 + 2\eta \simeq 1 - \frac{2}{\bar{\mathcal{N}}_{3000h^{-1}\mathrm{Mpc}}}\left(\frac{n-1}{n-2}\right) = 0.95 - \frac{0.04}{n-2}$$

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$$\left\{ \begin{array}{rl} \text{Expected value at peak} & \Omega_{GW} h^2 \approx 10^{-7} \\ \text{LISA sensitivity} & \Omega_{GW} h^2 \approx 10^{-11} \end{array} \right.$$

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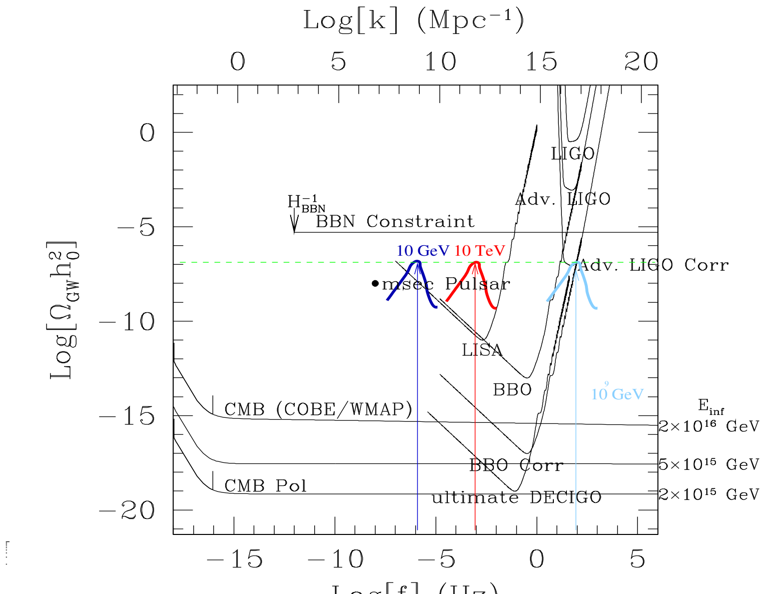
GW detectors



Higgs Potential?

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- Link to Standard Model and experiments

Close look to Higgs potential

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- RGE equations $\frac{d\lambda}{d\log(\mu)} = \ldots$

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$$
\begin{aligned}
\frac{d}{dt}\lambda(t) &= \beta_\lambda(\lambda, h, g_1, g_2, g_3)\,, \\
\frac{d}{dt}h(t) &= \beta_h(\lambda, h, g_1, g_2, g_3)\,, \\
\frac{d}{dt}g_1(t) &= \beta_{g_1}(\lambda, h, g_1, g_2, g_3)\,, \\
\frac{d}{dt}g_2(t) &= \beta_{g_2}(\lambda, h, g_1, g_2, g_3)\,, \\
\frac{d}{dt}g_3(t) &= \beta_{g_3}(\lambda, h, g_1, g_2, g_3)
\end{aligned} \tag{9}
$$

$$t \equiv log(\mu/m_Z) \qquad \mu \simeq \chi$$

Running

- $\lambda(0)$ given by $m_H^2 \propto \lambda(0)v^2$

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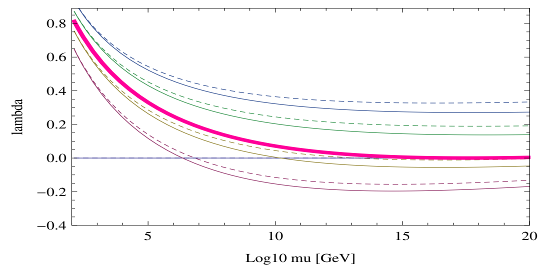
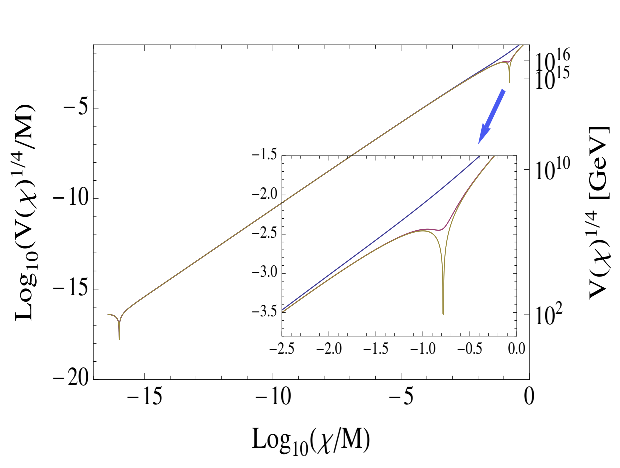


Instability & False Vacuum



- $m_H = 125.2, 125.158, 125.157663$ GeV, $m_t = 171.8$ GeV

Higgs False vacuum Inflation?

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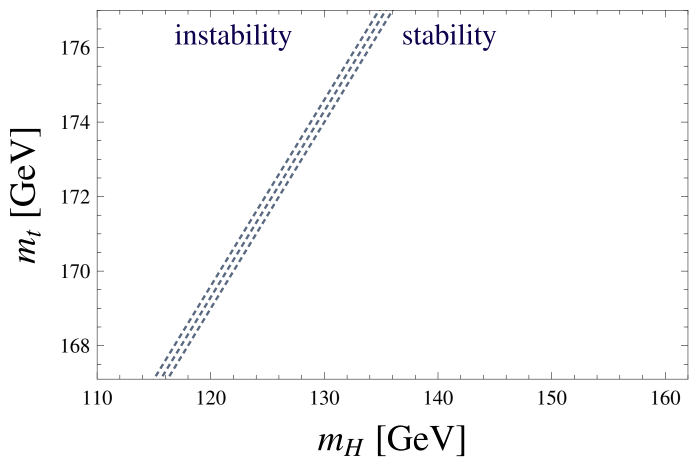
- "Minimal"? Need another scalar!
- Experiments can decide!

Requirements on Higgs and top mass

- Having a metastable vacuum $\Rightarrow$
- Fixes a line in the plane $m_{top} - m_H$



A.N., I.Masina, PRL, PRD, 2012

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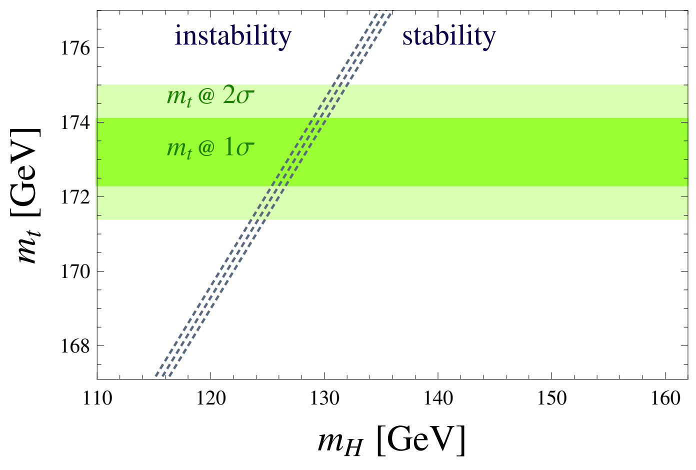
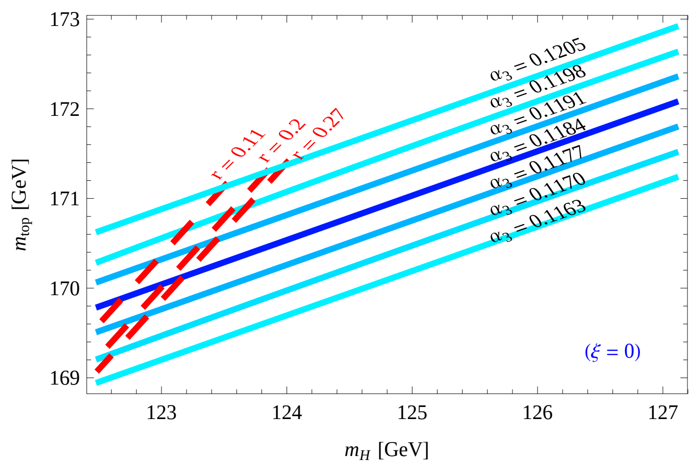


Requirements on Higgs, α_s and top mass

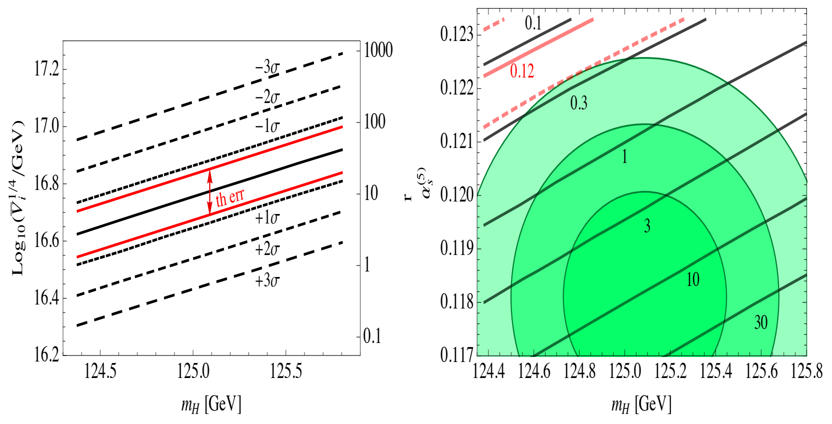
- Having the right Amplitude of tensors r



A.N., PRD, 2015

Requirements on Higgs, α_s and top mass

- With updated (doubled) error on world average of α_s:



G.Iacobellis and A.Masina, arXiv:1604.06046

Requirements on Higgs and top mass

- From Bezrukov et al. JHEP '2012:

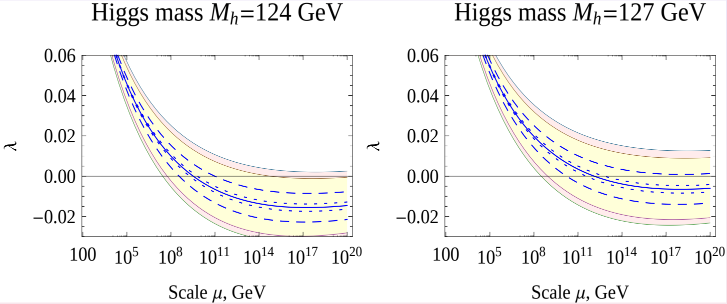


Figure: Dashed (dotted) line corresponding to the experimental uncertainty in the top mass M_t (strong coupling constant), and the shaded yellow (pink) regions correspond to the total experimental error and theoretical uncertainty, with the latter estimated as $[1.2]GeV$ ($[2.5]GeV$)

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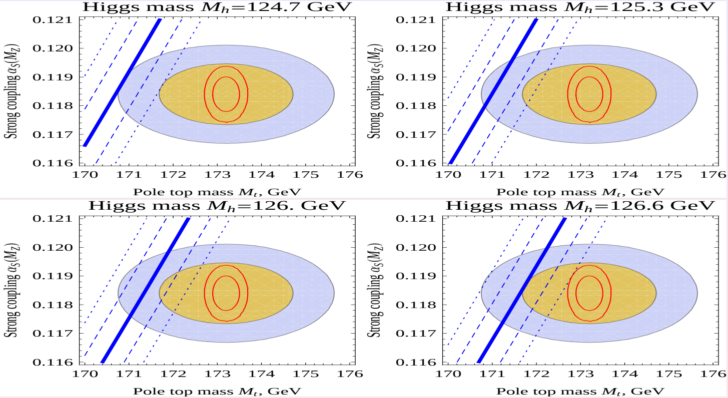

Figure: Tevatron 68% and 95% experimentally allowed regions for and M_t are given by shaded areas. The dashed (dotted) lines correspond to [1]GeV ([2]GeV) uncertainty in the M_{min} theoretical determination. Red lines in the center: expected precision from a e^+e^- collider.

Requirements on Higgs and top mass

From Bezrukov et al. JHEP '2012:

$$M_{\min} = \left[128.95 + \frac{M_t - [172.9]GeV}{[1.1]GeV} \times 2.2 - \frac{-0.1184}{0.0007} \times 0.56\right] GeV.$$

Source of uncertainty	*Nature of estimate*	$\Delta M_{\min}$, GeV
3-loop matching λ	Sensitivity to μ	1.0
3-loop matching y_t	Sensitivity to μ	0.2
4-loop to y_t	educated guess	0.4
confinement, y_t	educated guess $\sim \Lambda_{QCD}$	0.5
4-loop running $M_W \to M_P$	educated guess	< 0.2
total uncertainty	sum of squares	1.2
total uncertainty	linear sum	2.3

Table: Theoretical uncertainties in the present $M_{\min}$ evaluation.

Requirements on Higgs and top mass

- From Degrassi et al. JHEP '2012:

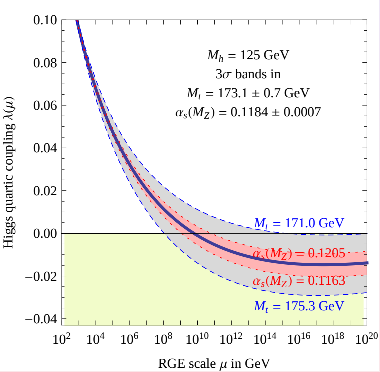

Figure: *RG evolution of λ varying M_t and α_s by $\pm 3\sigma$.*

Requirements on Higgs and top mass

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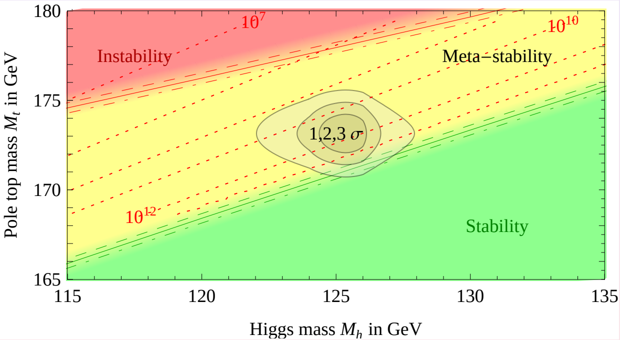

Figure: *The three boundaries lines correspond to* $\alpha_s(M_Z) = 0.1184 \pm 0.0007$.

Requirements on Higgs and top mass

- From Degrassi et al. JHEP '2012:

$$M_h\ [\mathrm{GeV}] > 129.4 + 1.4\left(\frac{M_t\ [\mathrm{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\mathrm{th}}\ .$$

Type of error	Estimate of the error	Impact on M
M_t	experimental uncertainty in M_t	± 1.4 GeV
α_s	experimental uncertainty in α_s	± 0.5 GeV
Experiment	**Total combined in quadrature**	± 1.5 GeV
λ	scale variation in λ	± 0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\mathrm{QCD}})$ correction to M_t	± 0.6 GeV
y_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	± 0.2 GeV
Theory	**Total combined in quadrature**	± 1.0 GeV

Requirements on Higgs and top mass

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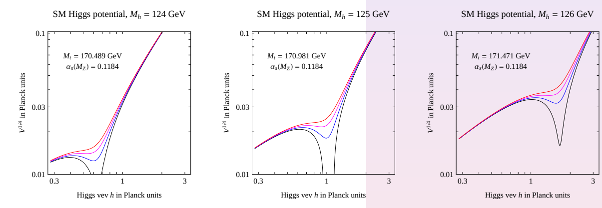

Figure: *Higgs potential around the critical top mass. The various curves correspond to variations in M_t by 0.1.*

Flat potential

- Require: almost flat potential
- Otherwise too suppressed tunneling rate

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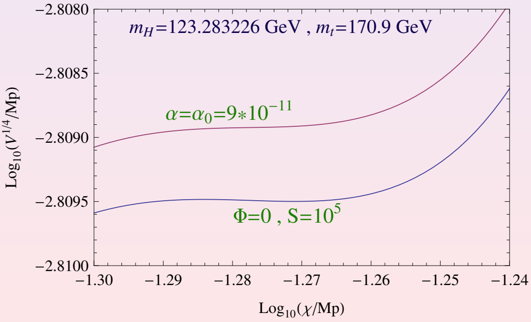
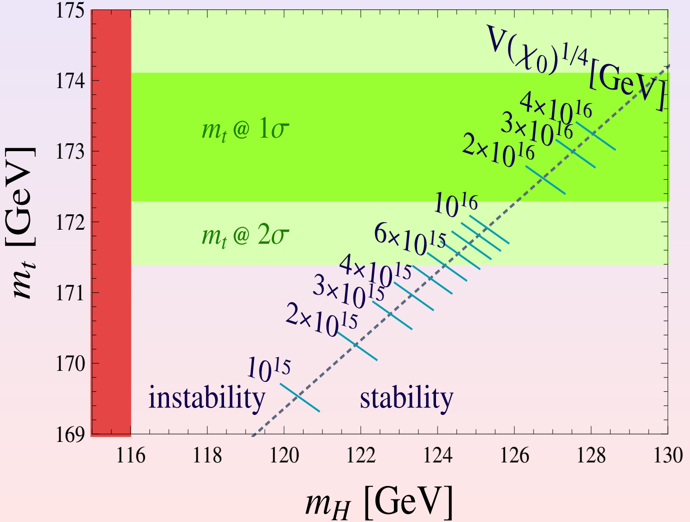


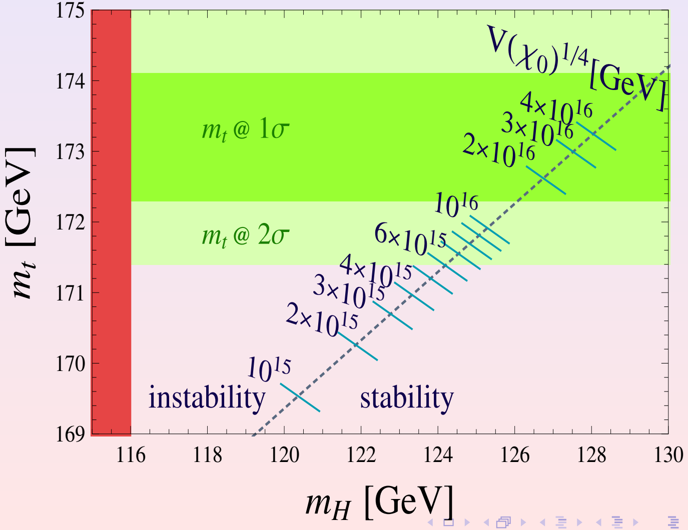
Potential Height

- Fixes V



Potential Height

- In any case: strictly an upper bound



Tensors from Inflation

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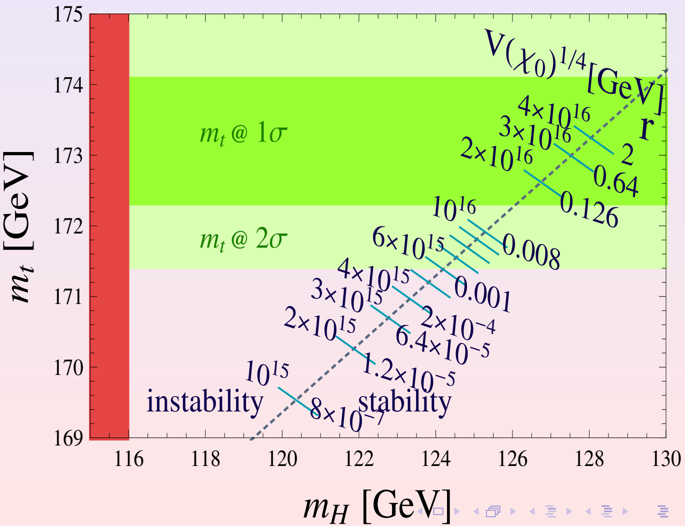
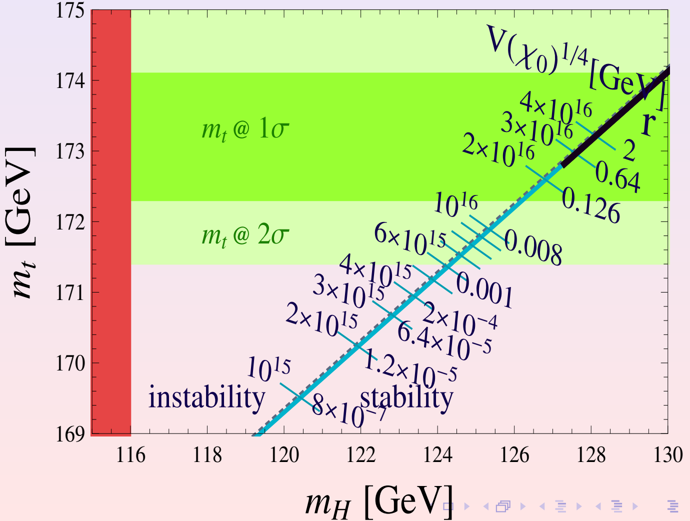


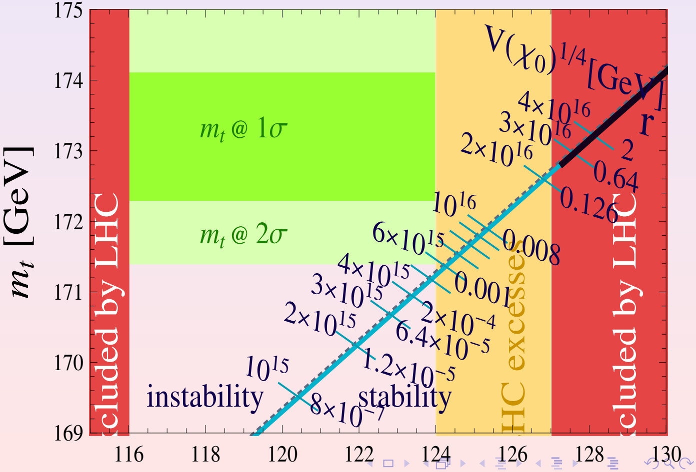
Bound on $m_H - m_t$

- $r \lesssim 0.2$ WMAP, $r \lesssim 0.11$ Planck



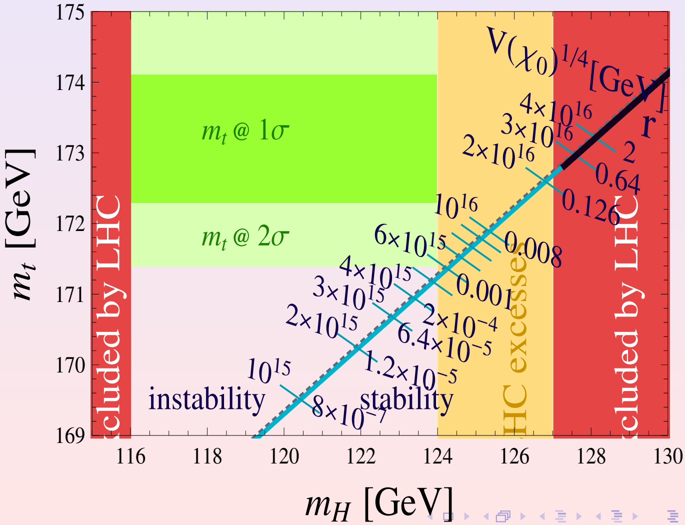
Correlating $m_H - m_t - r$

$m_H = 125.3 \pm 0.4(stat) \pm 0.5(sys) GeV$ (CMS) ,
$m_H = 126.0 \pm 0.4(stat) \pm 0.4(sys) GeV$ (ATLAS)



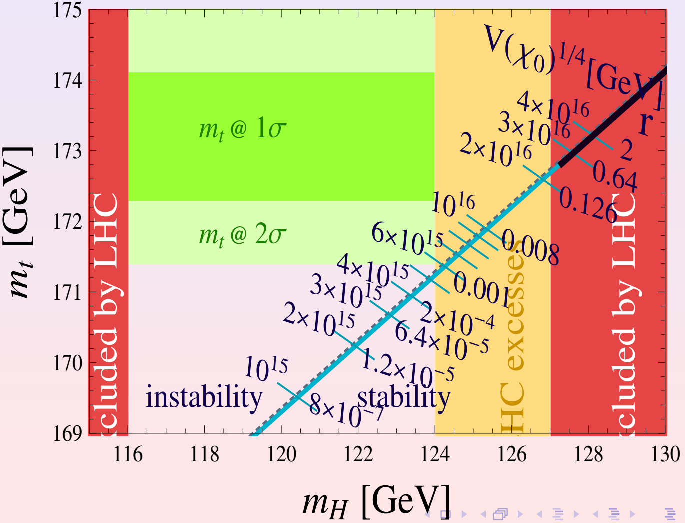
Correlating $m_H - m_t - r$

- Theoretical errors: $\Delta m_H \approx 3 GeV$, $\Delta m_t \approx 1 GeV$ (?).



Correlating $m_H - m_t - r$

- Improved errors: $\Delta m_H \approx 1 GeV$, $\Delta m_t \approx 1 GeV$.



Requirements on Higgs and top mass

[3]

Figure: *arxiv:1403.5244 based on De Grassi et al.*

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[3] arxiv:1403.5244

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Shallow barrier

- Requirement:
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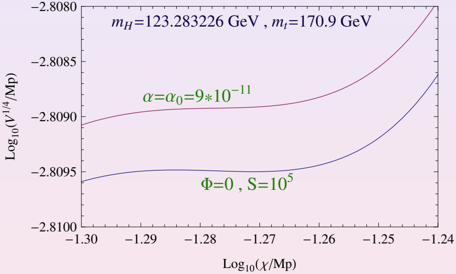
  - $\Rightarrow$ $S \lesssim 380$

Small action

- $S \lesssim 380$ only if potential almost flat



- Strong tuning!
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and the false vacuum energy, in this frame

$$-S_0 = \int d^4x\, \sqrt{-\bar{g}}\frac{V(\chi_0)}{f^2(\phi)} \equiv \int d^4x \sqrt{\bar{g}}\, \bar{V}(\chi_0, \phi)\,.$$

becomes a potential (but it disappears after tunneling)

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- it looks like slow roll on top of a hill

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- When ϕ of order M: end of slow-roll

Phase II: $\phi \gg M$

- In this phase:

$$K(\phi) \equiv \frac{2f(\phi) + 3M^2 f'^2(\phi)}{2f^2(\phi)} \approx \frac{3M^2}{2} \left(\frac{f'}{f} \right)^2 , \qquad \bar{V}(\phi) \approx \frac{V(\chi_0)}{f(\phi)^2} .$$

Phase II: $\phi \gg M$

- In this phase:

$$K(\phi) \equiv \frac{2f(\phi) + 3M^2 f'^2(\phi)}{2f^2(\phi)} \approx \frac{3M^2}{2}\left(\frac{f'}{f}\right)^2 , \qquad \bar{V}(\phi) \approx \frac{V(\chi_0)}{f(\phi)^2} .$$

- So we introduce a canonical variable via

$$\Phi \equiv \sqrt{\frac{3}{2}} M \ln f(\phi) ,$$

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- So we introduce a canonical variable via

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- The kinetic term is canonical and the potential becomes:

$$\bar{V}(\Phi) = V(\chi_0) \exp\left(-2\sqrt{\frac{2}{3}}\frac{\Phi}{M}\right) .$$

Phase II: $\phi \gg M$

- The exponential potential is well-known to lead to power-law expansion

$$\bar{a} \sim \bar{t}^p \quad \text{with} \quad p = \frac{3}{4} \, .$$

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- The end of this phase when

$$\bar{H}^2 \simeq \bar{\Gamma}^{1/2}$$