Hydrodynamic descriptions of nuclear collisions – successes and limitations

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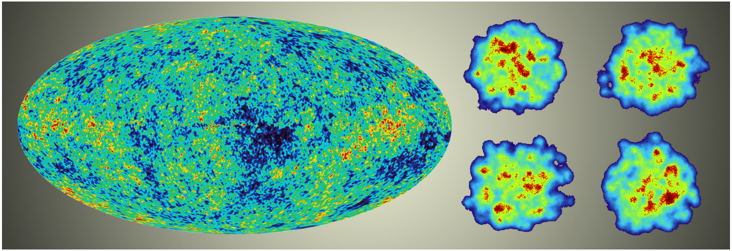
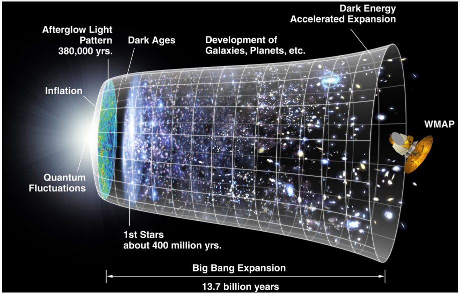
The Big Bang and the little bangs

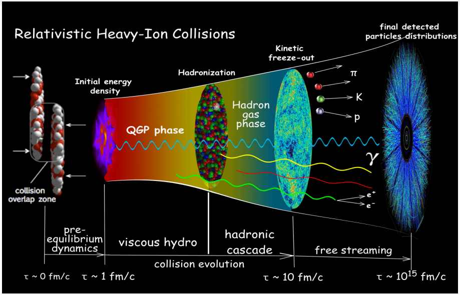


TH Institute, CERN, August 15-26, 2016

Overview

Big Bang vs. Little Bang



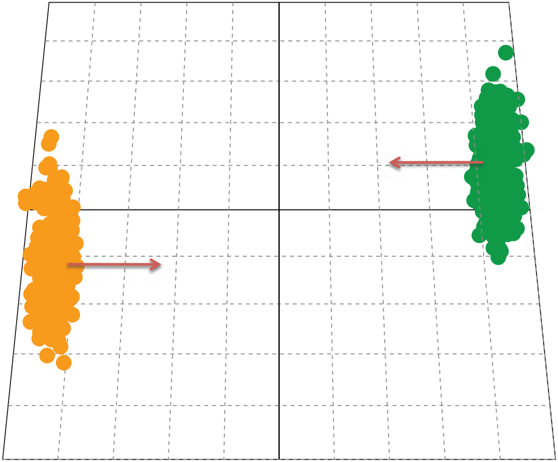


Similarities: Hubble-like expansion, expansion-driven dynamical freeze-out
chemical freeze-out (nucleo-/hadrosynthesis) before thermal freeze-out (CMB, hadron p_T-spectra)
initial-state quantum fluctuations imprinted on final state

Differences: Expansion rates differ by 18 orders of magnitude
Expansion in 3d, not 4d; driven by pressure gradients, not gravity
Time scales measured in fm$/c$ rather than billions of years
Distances measured in fm rather than light years
"Heavy-Ion Standard Model" still under construction

Relativistic Nucleus-Nucleus Collisions

Animation: P. Sorensen



Collision of two Lorentz contracted gold nuclei

Relativistic Nucleus-Nucleus Collisions

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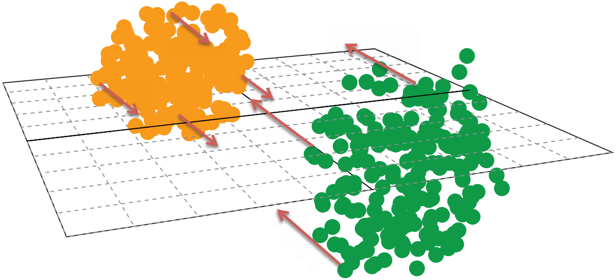

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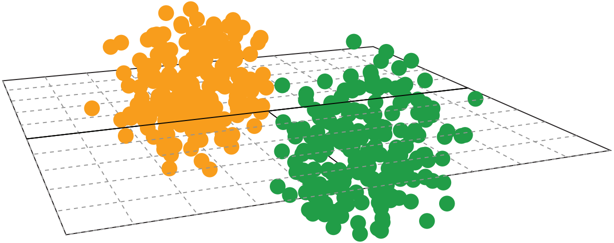

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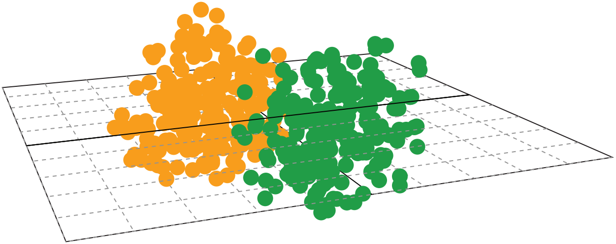

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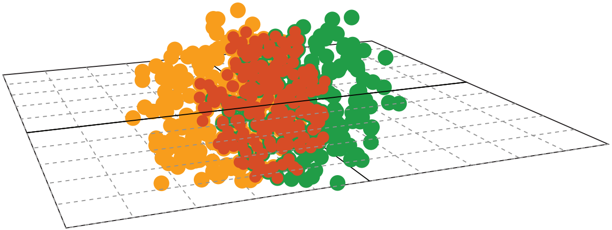

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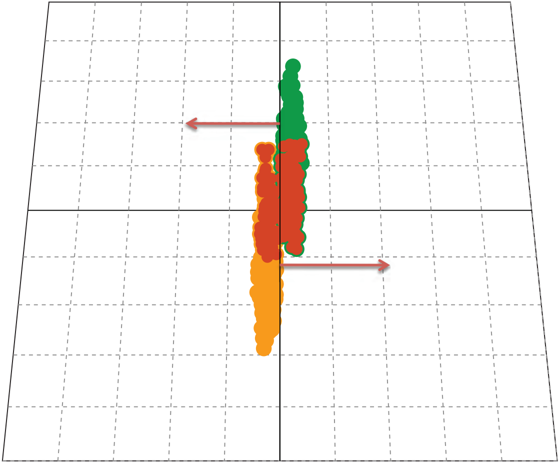

Collision of two Lorentz contracted gold nuclei

Relativistic Nucleus-Nucleus Collisions

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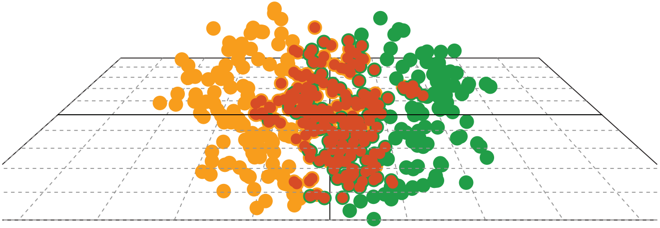
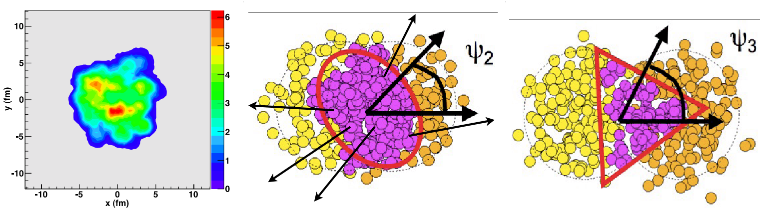


Produced fireball is ~10^{-14} meters across and lives for ~$5\text{x}10^{-23}$ seconds

Collision of two Lorentz contracted gold nuclei

Event-by-event shape and flow fluctuations rule!

(Alver and Roland, PRC81 (2010) 054905)

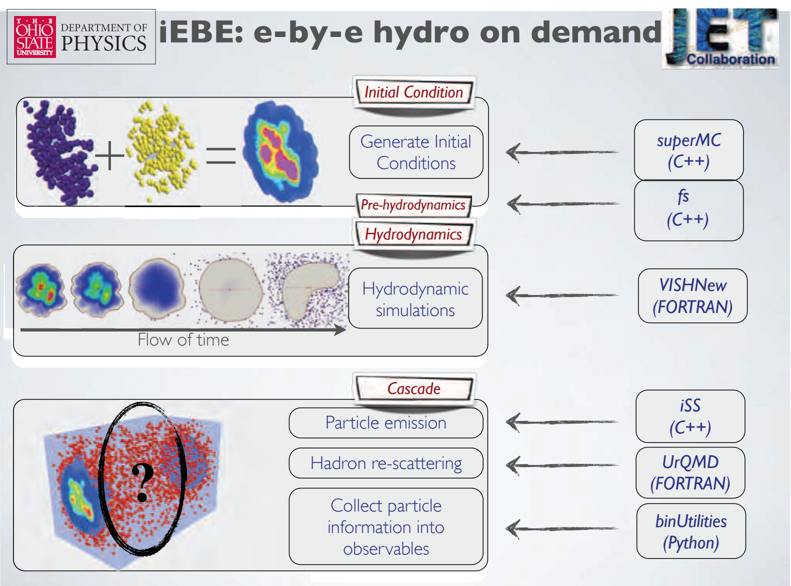


- Each event has a different initial shape and density distribution, characterized by different set of harmonic eccentricity coefficients ε_n
- Each event develops its individual hydrodynamic flow, characterized by a set of harmonic flow coefficients v_n and flow angles ψ_n
- At small impact parameters fluctuations ("hot spots") dominate over geometric overlap effects (Alver & Roland, PRC81 (2010) 054905; Qin, Petersen, Bass, Müller, PRC82 (2010) 064903)

Definition of flow coefficients:

$$\frac{dN^{(i)}}{dy\, p_T dp_T\, d\phi_p}(b) = \frac{dN^{(i)}}{dy\, p_T dp_T}(b) \left(1 + 2\sum_{n=1}^{\infty} \boldsymbol{v_n^{(i)}(y, p_T; b)} \cos(\phi_p - \Psi_n^{(i)})\right).$$

https://u.osu.edu/vishnu: A product of the JET Collaboration



Viscous relativistic hydrodynamics (Israel & Stewart 1979)

Include shear viscosity η, neglect bulk viscosity (massless partons) and heat conduction ($\mu_B \approx 0$); solve

$$\partial_\mu T^{\mu\nu} = 0$$

with modified energy momentum tensor

$$T^{\mu\nu}(x) = \big(e(x){+}p(x)\big)u^\mu(x)u^\nu(x) - g^{\mu\nu}p(x) + \pi^{\mu\nu}.$$

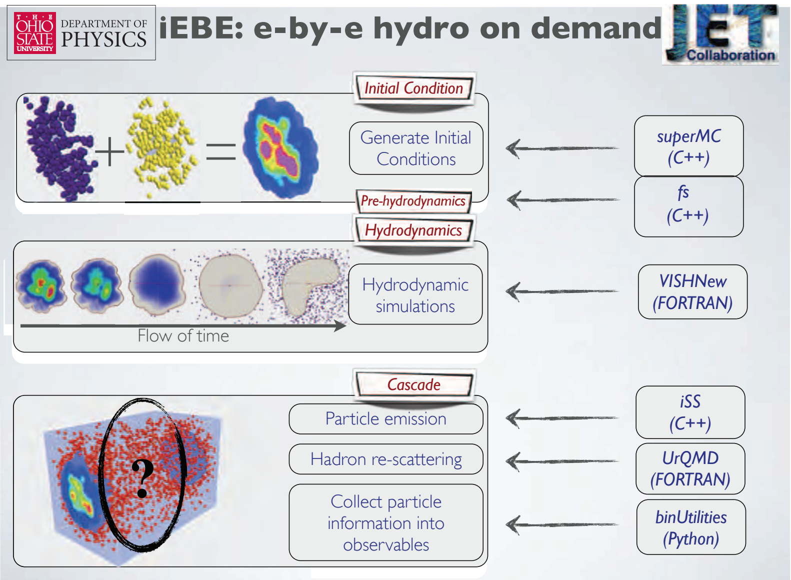
$\pi^{\mu\nu}$ = traceless viscous pressure tensor which relaxes locally to 2η times the shear tensor $\nabla^{\langle\mu}u^{\nu\rangle}$ on a microscopic kinetic time scale τ_π:

$$D\pi^{\mu\nu} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - 2\eta\nabla^{\langle\mu}u^{\nu\rangle}\right) + \dots$$

where $D \equiv u^\mu\partial_\mu$ is the time derivative in the local rest frame.

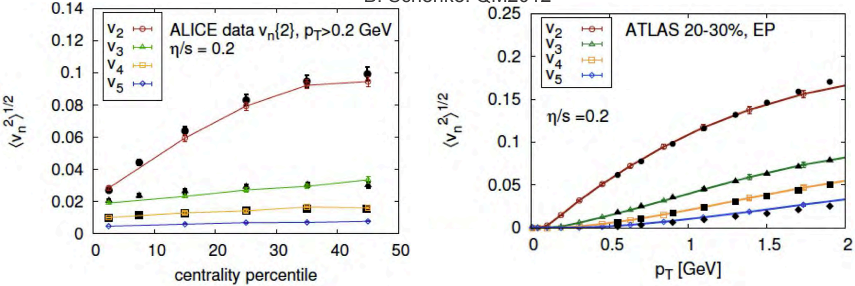
Kinetic theory relates η and τ_π, but for a strongly coupled QGP neither η nor this relation are known $\Longrightarrow$ treat η and τ_π as independent phenomenological parameters. For consistency: $\tau_\pi\theta \ll 1$ ($\theta = \partial^\mu u_\mu =$ local expansion rate).

https://u.osu.edu/vishnu: A product of the JET Collaboration



Towards a Standard Model of the Little Bang

B. Schenke: QM2012



Schenke, Tribedy, Venugopalan, Phys.Rev.Lett. 108:25231 (2012)

With inclusion of sub-nucleonic quantum fluctuations and pre-equilbrium dynamics of gluon fields:

→ outstanding agreement between data and model

Rapid convergence on a standard model of the Little Bang!

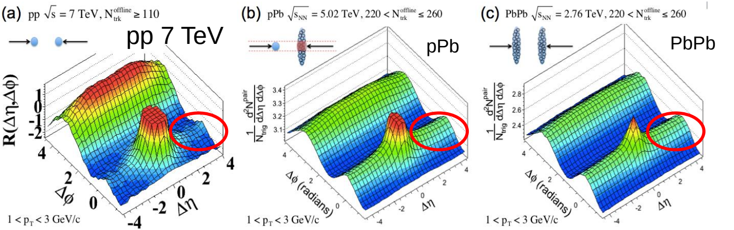
Perfect liquidity reveals in the final state initial-state gluon field correlations of size $1/Q_s$ (sub-hadronic)!

The big question

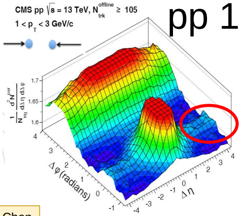
- Flow-like signatures of similar characteristics as those in AA collisions were also seen in pA and high-multiplicity pp.
- Seen in both single-particle observables ("radial flow") and two-particle correlations ("anisotropic flow").
- Initial-state momentum correlations can also manifest themselves as "anisotropic flow" in the final state, especially in small collision systems where they may survive final-state interactions.
- **What is the true origin of these flow-like signatures? How can we separate initial-state from final-state effects, in particular in small systems?**
- **What is the internal phase-space structure of a proton?**

Overview

Ridge in pp, pPb and PbPb



NEW



pp 13 TeV

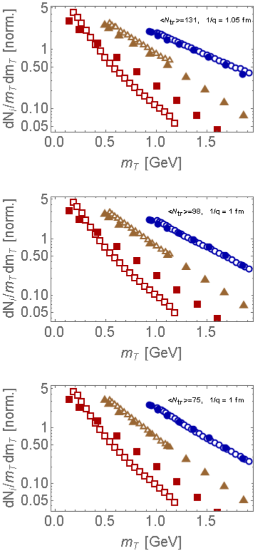
Ridge observed in high multiplicity pp collisions at 13 TeV !

13 TeV vs. 7 TeV?

Zhenyu Chen

CMS-FSQ-15-002

Flow in small systems?



Kalaydzhyan & Shuryak PRC91 (2015) 054913

Open symbols: CMS data;
filled symbols: Glubser flow

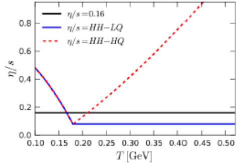
K-*p* mass splitting of m_T-slopes increases with *pp* multiplicity

Radial flow in pp?

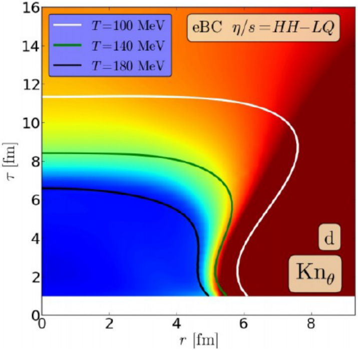
Validity of viscous hydro: Knudsen number check

Niemi & Denicol, arXiv:1404.7327

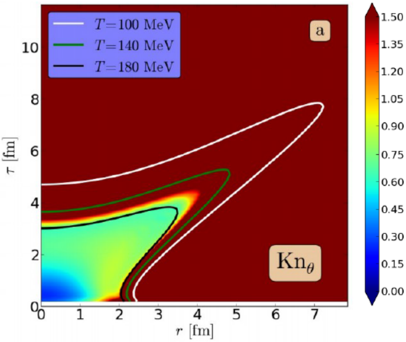
$$\mathrm{Kn} = \tau_{\mathrm{micro}}\, \theta = \tau_{\mathrm{micro}} / \tau_{\mathrm{macro}}$$



Pb+Pb



p+Pb



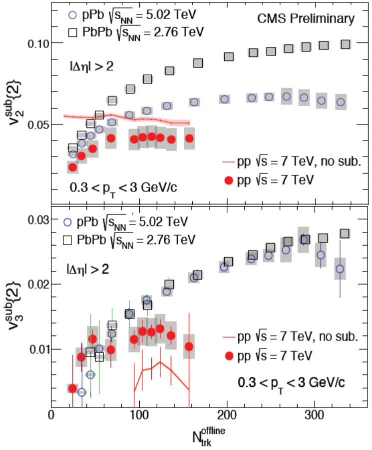
Predicts freeze-out at higher temperature in p+Pb than in Pb+Pb

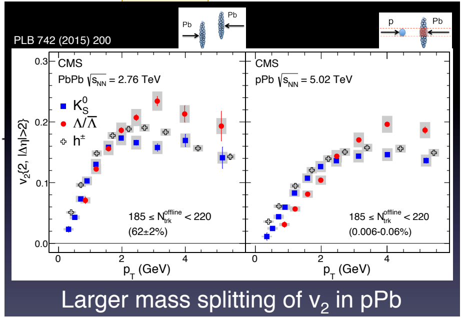
Flow in small systems?

Long-range correlations in high-mult. pp

Z. Chen CMS-HIN-15-009

Flow parameter analysis





- $v_2(\mathrm{pp}) < v_2(\mathrm{pPb}) < v_2(\mathrm{PbPb})$
- $v_3(\mathrm{pp}) \approx v_3(\mathrm{pPb}) \approx v_3(\mathrm{PbPb})$, but $v_3(\mathrm{pp})$ deviates for $N_{trk}^{offline} \gtrsim 90$
- Mass ordering for $v_2^{sub}\{2\}$ at low p_T

Long-range correlations in high-mult. pp

Z. Chen   CMS-HIN-15-009

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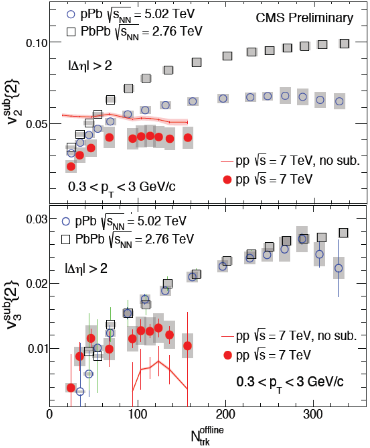

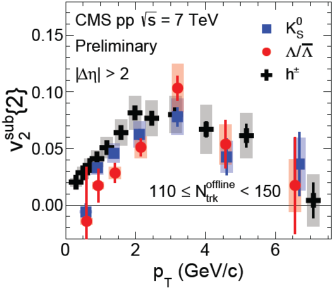

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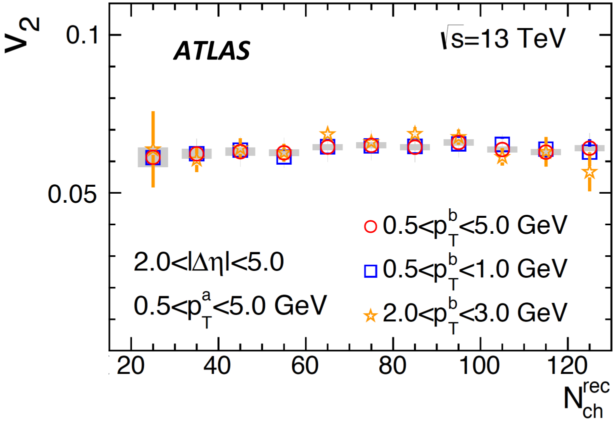
- Mass ordering for $v_2^{sub}\{2\}$ at low p_T

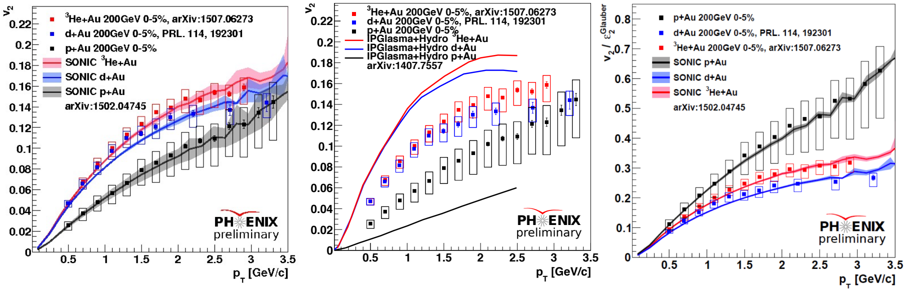
Byungsik Hong   Quark Matter 2015, Kobe   12

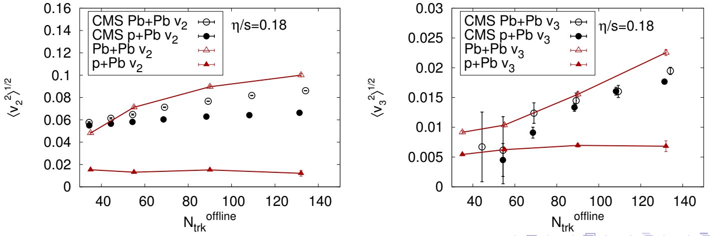
Flow in small systems?



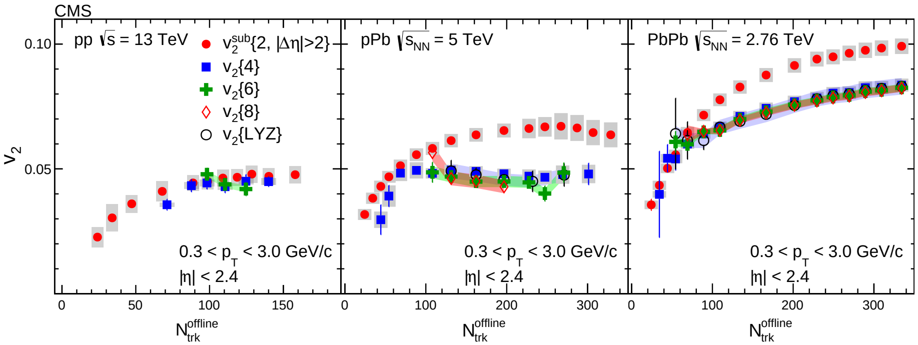
No centrality dependence of elliptic flow in pp?!
Flow not just in high-multiplicity pp?!
Not flow but something else?

Do small systems behave hydrodynamically?





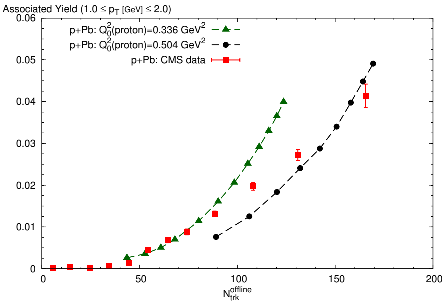
Collectivity in small systems!

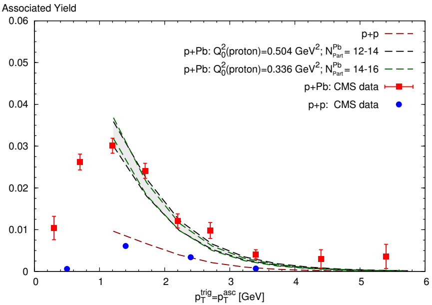


Whatever its origin, the "flow signal" represents a collective response (to what?) of all particles!

Initial-state momentum correlations?

Dusling and Venugopalan, PRD87 (2013) 054014

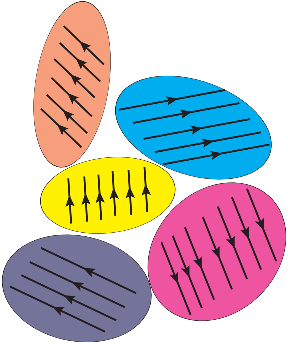


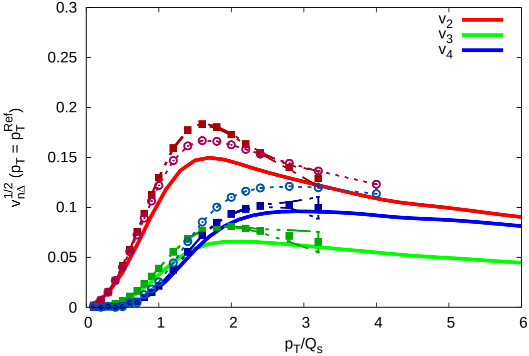


Initial-state momentum (anti-)correlations from "Glasma graphs" qualitatively explain the multiplicity dependence and p_T-dependence at high p_T of the ridge yields in pPb and high-multiplicity pp collisions

Initial-state momentum correlations?

Lappi, Schenke, Schlichting, Venugopalan, JHEP 2016 (arXiv:1509.03499)





Spatial inhomogeneity of CGC and spatial deformation of CGC regions of homogeneity generate momentum anisotropies among the initially produced partons, corresponding to non-zero v_n for all n, with "reasonable-looking" p_T dependence.

Overview

What is needed to resolve this ambiguity?

- Initial conditions for the phase-space distribution of the produced matter,

$$f_{\rm matter}(x_\perp,\phi_s;p_\perp,\phi_p;y_p{-}\eta_s;\tau_0)$$

which depends on the

- phase-space (Wigner) distribution of the glue inside the nucleons bound into small nuclei:

$$f_{\rm glue}(x_\perp,\phi_s;k_\perp,\phi_k;y_k{-}\eta_s;\tau_0)$$

- From $f_{\rm matter}$ we obtain the initial energy-momentum tensor

$$T^{\mu\nu}(x_\perp,\eta_s,\tau_0)=\frac{\nu_{\rm dof}}{(2\pi)^3}\int dy_p d^2p_\perp p^\mu p^\nu f_{\rm matter}(x_\perp,\phi_s;p_\perp,\phi_p;y_p{-}\eta_s;\tau_0)$$

What is needed to resolve this ambiguity?

- Once the initial $T^{\mu\nu}(x)$ is known, we can evolve it for some time $\tau_{\rm eq}-\tau_0$ with a pre-equilibrium model, match it to viscous hydrodynamic form,

$$T^{\mu\nu} = e u^\mu u^\nu - \big(P(e)+\Pi\big)\Delta^{\mu\nu} + \pi^{\mu\nu},$$

  run it through viscous hydrodynamics plus hadronic afterburner, and compare its output with experiment.
- To account for event-by-event quantum fluctuations in the initial $T^{\mu\nu}(x)$, and for thermal noise during the evolution, the dynamical evolution must be performed many times before taking ensemble averages as done in experiment.

What is missing in present calculations?

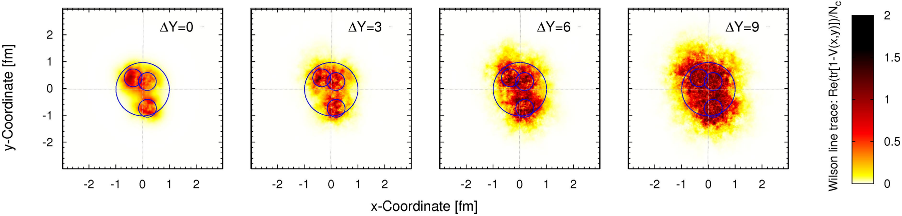
Present modeling uses simplified assumptions for the initial phase-space distrib'n:

- Few models account for the initial momentum structure of the medium; most ignore it completely. $\Longrightarrow$ incorrect/unreliable initial conditions for $\Pi, \pi^{\mu\nu}$
- While granularity of the initial spatial density distribution is accounted for at the nucleon length scale, by Monte-Carlo sampling the nucleon positions from a smooth Woods-Saxon probability distribution before allowing them to collide and lose energy to create lower-rapidity secondary matter, quantum fluctuations on sub-nucleonic length scales are poorly controlled and mostly ignored. IP-Glasma includes sub-nucleonic gluon field fluctuations, but appears to get them wrong, yielding spatial gluon distributions inside protons that are too compact.
- Most approaches (e.g. PHOBOS Glauber Monte Carlo) use disk-like nucleons for computing the collision probability. More realistic collision detection using Gaussian nucleons is implemented in GLISSANDO and iEBE-VISHNU.
- Most approaches ignore quantum fluctuations in the amount of beam energy lost to lower rapidities in a NN collision. Without these, the measured KNO-like multiplicity distributions in pp collisions are not reproduced, and pp collisions produce zero ϵ_3 by symmetry. GLISSANDO and iEBE-VISHNU include pp multiplicity fluctuations, creating non-zero triangularity in pp, even without sub-nucleonic structure.

Overview

"Three quarks for Muster Mark!"

Schlichting, Schenke, PLB739 (2014) 313



- 3 valence quarks act as large-x color sources of the low-x gluon fields.
- Spatial positions of quarks at the instant of collision fluctuate from event to event and generate a lumpy color distribution at large x.
- This lumpiness is tracked by the quarks' gluon clouds, becoming more diffuse at smaller $x \Longrightarrow$ triune lumpiness of the gluon fields inside the nucleon when viewed through midrapidity particle production, with an intrinsic length scale ("gluonic radius of a quark") that appears to grow with collision energy.
- $\Longrightarrow$ Protons have just as much intrinsic triangularity as ^{3}He nuclei, just on a shorter length scale. But in p+A ***all*** particle production occurs on a smaller length scale than in ^{3}He+A! This affects mostly radial flow, though.

Modeling quark substructure of the nucleon I

K. Welsh, J. Singer, UH, PRC, in press (arXiv:1605.09418)

- The gluon field density inside the proton is the sum of three 3-d Gaussians of norm $\frac{1}{3}$ and width σ_g (representing the gluon clouds around the valence quarks). Default value: $\sigma_g = 0.3\,\mathrm{fm}$ (best fit of pPb mult. dist. at LHC)
- The quark positions (centers of the gluon clouds) are sampled from a 3-d Gaussian with width σ_q around the center of the nucleon, requiring their center of mass to coincide with the nucleon center.
- The widths are constrained by $\sigma_g^2 + \frac{2}{3}\sigma_q^2 = B$ such that the average proton density is a normalized Gaussian

$$\langle \rho_p(\boldsymbol{r}) \rangle = \frac{e^{-\frac{\boldsymbol{r}^2}{2B}}}{(2\pi B)^{2/3}}$$

with $\sqrt{s}$-dependent width $B(\sqrt{s}) = \frac{\sigma_{\mathrm{NN}}^{\mathrm{inel}}(\sqrt{s})}{8\pi}$, to reproduce the measured inelastic NN cross section.

Modeling quark substructure of the nucleon II

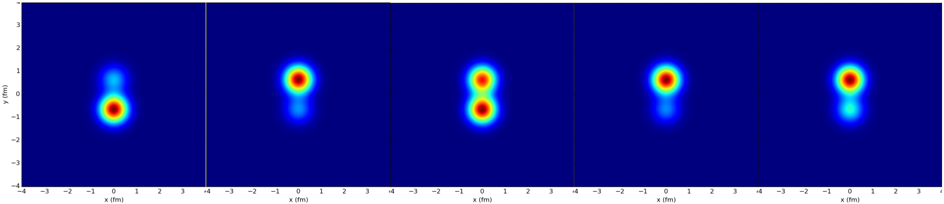
- Projecting ρ_p along z gives the nucleon thickness function $T_N(\boldsymbol{r}_\perp)$ in the transverse plane.
- Folding two nucleon thickness functions yields the nucleon-nucleon overlap function $T_{NN}(\boldsymbol{b})$ at impact parameter $\boldsymbol{b}$ (which actually depends on all 6 quark positions), from which the probability for each of the two nucleons to get wounded in the collision is computed as

$$P_{ij}(\boldsymbol{r}_{\perp i}-\boldsymbol{r}_{\perp j}) = 1 - \exp\left[-\sigma_{gg}\, T_{NN}(\boldsymbol{r}_{\perp i}-\boldsymbol{r}_{\perp j})\right]$$

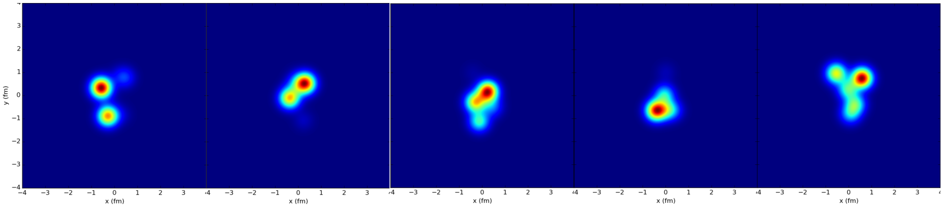
  where i and j are from projectile and target, respectively. The gluon-gluon cross section σ_{gg} is determined by the normalization of P_{ij} to the inelastic NN cross section.
- For each wounded nucleon, all three quarks are assumed to contribute to energy production at midrapidity, with a Gaussian density profile of width σ_g and independently fluctuating (Γ-distributed) normalization, with variance adjusted to reproduce measured pp multiplicity distributions.

Initial entropy density in b=1.3 fm pp collisions

smooth Gaussian protons:

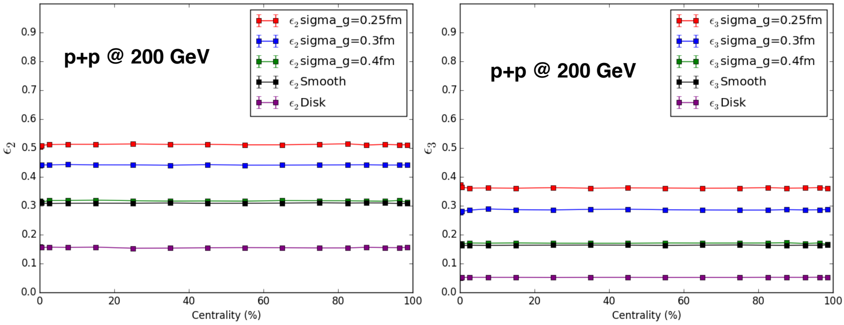


protons with fluctuating quark substructure ($\sigma_g = 0.3\,$fm):



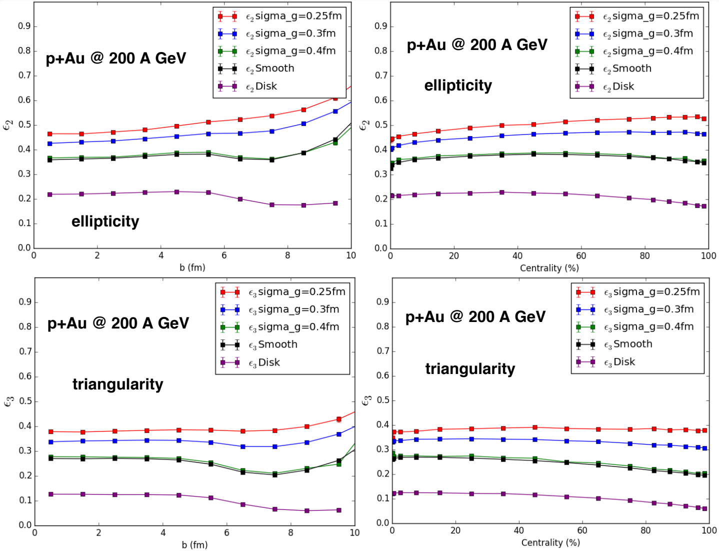
For protons with quark substructure the Gaussian collision criterium appears to favor somewhat more compact distributions of produced entropy density

$\varepsilon_{2,3}$ vs. centrality: pp @ $\sqrt{s}$=200 A GeV

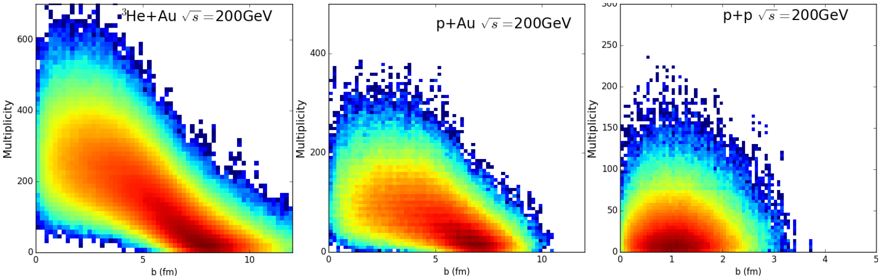


- Ellipticity and triangularity show strong sensitivity to σ_g.
- Since $\sqrt{B} = 0.408$ fm at $\sqrt{s} = 200$ GeV, quark subdivision with $\sigma_g = 0.4$ fm is almost indistinguishable from a smooth Gaussian proton.
- Disk-like collision detection gives smallest eccentricities.

$\varepsilon_{2,3}$ vs. centrality: p+Au @ $\sqrt{s}$=200 A GeV



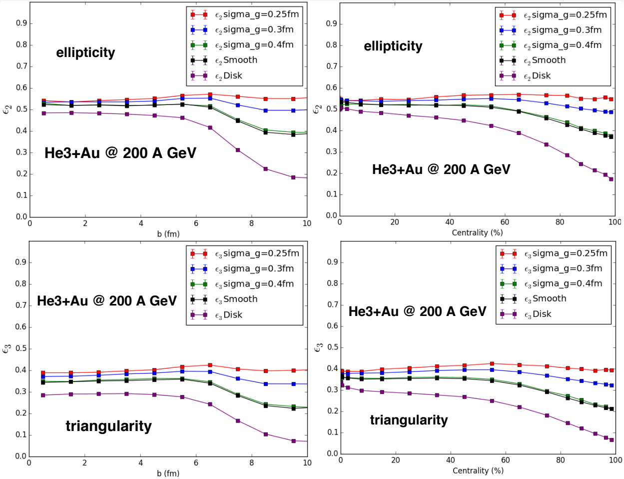
In p+p and light+heavy "centrality" does not measure b!



pp multiplicity fluctuations destroy strong anticorrelation between multiplicity and impact parameter seen in Au+Au and Pb+Pb

$\Longrightarrow$ "centrality" measured by multiplicity is a misnomer in collisions involving light projectiles

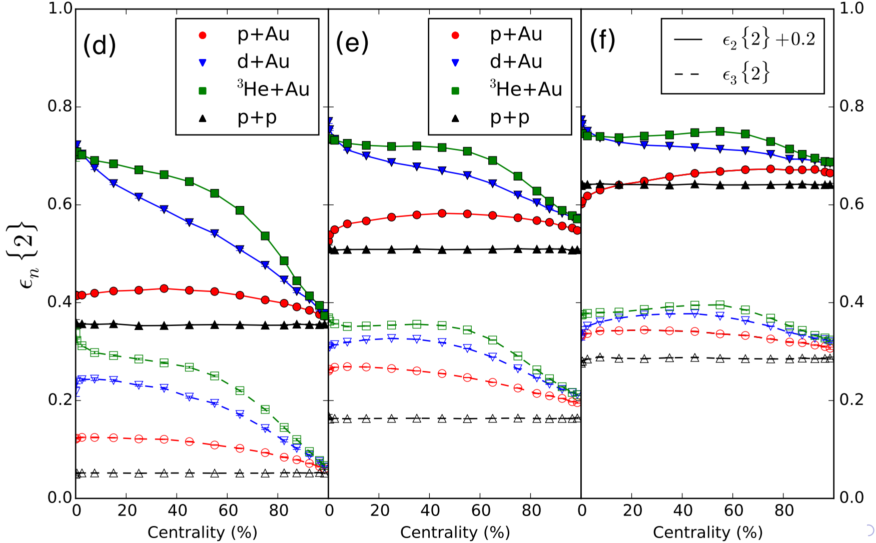
$\varepsilon_{2,3}$ vs. centrality: ^{3}He+Au @ $\sqrt{s}$=200 A GeV



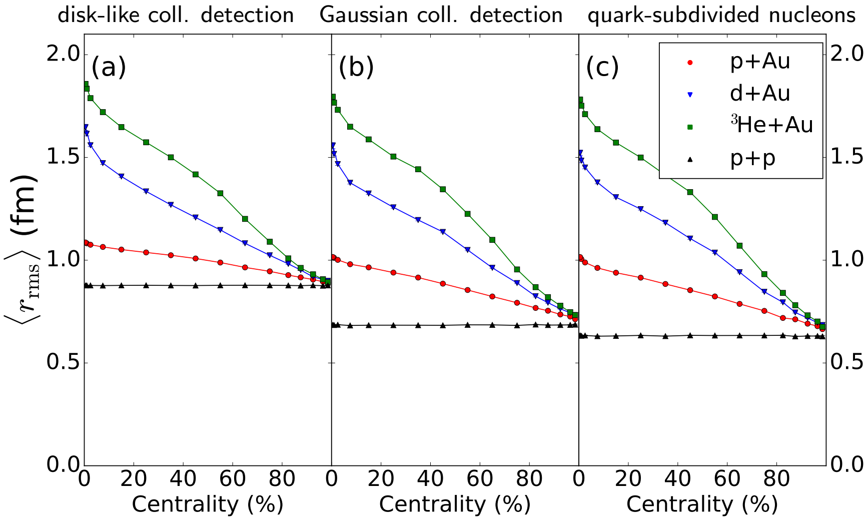
Reduced sensitivity to p-substructure and σ_g for larger projectiles, except in peripheral events

$\varepsilon_{2,3}$ vs. "centrality" for different collision systems

disk-like coll. detection Gaussian coll. detection quark-subdivided nucleons



Initial radius vs. "centrality" for different collision systems



Overview

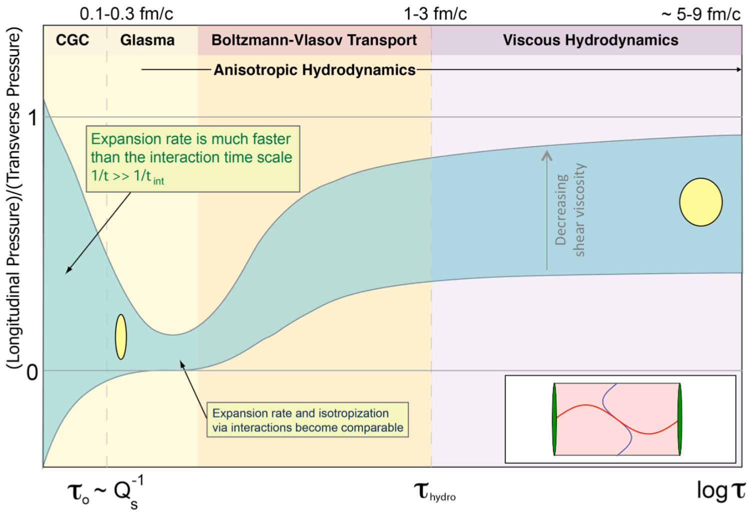
Back to the big question: do pp and pA collisions create droplets of flowing QGP?

- Hydrodynamics is an effective field theory that describes the macroscopic effects of the microscopic transport dynamics
- Gerry Brown: **“Some EFTs are more effective than others!”**
- Israel-Stewart theory cannot handle the rapid, very anisotropic expansion in pp and pA, and fails similarly during the earliest stages in AA collisions
- **Welcome the “more effective” anisotropic hydrodynamics framework** (Strickland, Martinez, Florkowski, Bazow, UH, et al.)
- vaHydro minimizes second-order viscous hydro effects by resumming large first-order corrections at leading order

Testing different hydrodynamic approximation schemes

- Relativistic viscous hydrodynamics is an effective macroscopic description based on coarse-graining (gradient expansion) of the microscopic dynamics.
- Its systematic construction is still a matter of debate, complicated by the existence of a complex hierarchy of micro- and macroscopic time scales that are not well separated in relativistic heavy-ion collisions.
- Exact solutions of the highly nonlinear microscopic dynamics can serve as a testbed for macroscopic hydrodynamic approximation schemes, but are hard to come by.
- Exact solutions have been found for weakly interacting systems with highly symmetric flow patterns and density distributions:
  Bjorken and Gubser flow (RTA),
  FLRW universe (full Boltzmann collision term)
- Can be used to test different hydrodynamic expansion schemes for the Boltzmann equation

Longitudinal-transverse pressure anisotropy in heavy-ion collisions



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Kinetic theory vs. hydrodynamics

Both simultaneously valid if weakly coupled and small pressure gradients.

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Boltzmann Equation in Relaxation Time Approximation (RTA):

$$p^\mu \partial_\mu f(x,p) = C(x,p) = \frac{p\cdot u(x)}{\tau_{\rm rel}(x)}\Big(f_{\rm eq}(x,p) - f(x,p)\Big)$$

For conformal systems $\tau_{\rm rel}(x) = c/T(x) = 5\eta/(\mathcal{S}\,T) \equiv 5\bar{\eta}/T(x)$.

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Macroscopic currents:

$$j^\mu(x) = \int_p p^\mu\, f(x,p) \equiv \langle p^\mu \rangle; \quad T^{\mu\nu}(x) = \int_p p^\mu\, p^\nu\, f(x,p) \equiv \langle p^\mu p^\nu \rangle$$

where $$\int_p \cdots \equiv \frac{g}{(2\pi)^3}\int \frac{d^3p}{E_p}\cdots \equiv \langle \dots \rangle$$

Hydrodynamics from kinetic theory (I):

Expand the solution $f(x,p)$ of the Boltzmann equation as

$$f(x,p) = f_0(x,p) + \delta f(x,p) \qquad \Big(\big|\delta f / f_0\big| \ll 1\Big)$$

where f_0 is parametrized through macroscopic observables as

$$f_0(x,p) = f_0\left(\frac{\sqrt{p_\mu \Xi^{\mu\nu}(x) p_\nu} - \tilde{\mu}(x)}{\tilde{T}(x)}\right)$$

where $\quad \Xi^{\mu\nu}(x) = u^\mu(x) u^\nu(x) - \Phi(x)\Delta^{\mu\nu}(x) + \xi^{\mu\nu}(x).$

$u^\mu(x)$ defines the local fluid rest frame (LRF).
$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ is the spatial projector in the LRF.
$\tilde{T}(x)$, $\tilde{\mu}(x)$ are the effective temperature and chem. potential in the LRF.
$\Phi(x)$ partially accounts for bulk viscous effects in expanding systems.
$\xi^{\mu\nu}(x)$ describes deviations from local momentum isotropy in anisotropically expanding systems due to shear viscosity.

Hydrodynamics from kinetic theory (II):

$u^\mu(x)$, $\tilde{T}(x)$, $\tilde{\mu}(x)$ are fixed by the Landau matching conditions:

$$T^\mu_{\ \nu} u^\nu = \mathcal{E}(\tilde{T}, \tilde{\mu}; \xi, \Phi) u^\mu; \qquad \Big\langle u\cdot p \Big\rangle_{\delta f} = \Big\langle (u\cdot p)^2 \Big\rangle_{\delta f} = 0$$

$\mathcal{E}$ is the LRF energy density. We introduce the true local temperature $T(\tilde{T}, \tilde{\mu}; \xi, \Phi)$ and chemical potential $\mu(\tilde{T}, \tilde{\mu}; \xi, \Phi)$ by demanding $\mathcal{E}(\tilde{T}, \tilde{\mu}; \xi, \Phi) = \mathcal{E}_{\rm eq}(T, \mu)$ and $\mathcal{N}(\tilde{T}, \tilde{\mu}; \xi, \Phi) \equiv \langle u\cdot p \rangle_{f_0} = \mathcal{R}_0(\xi, \Phi) \mathcal{N}_{\rm eq}(T, \mu)$ (see cited literature for $\mathcal{R}$ functions).
Writing

$$T^{\mu\nu} = T_0^{\mu\nu} + \delta T^{\mu\nu} \equiv T_0^{\mu\nu} + \Pi^{\mu\nu}, \qquad j^\mu = j_0^\mu + \delta j^\mu \equiv j_0^\mu + V^\mu,$$

the conservation laws

$$\partial_\mu T^{\mu\nu}(x) = 0, \qquad \partial_\mu j^\mu(x) = \frac{\mathcal{N}(x) - \mathcal{N}_{\rm eq}(x)}{\tau_{\rm rel}(x)}$$

are sufficient to determine $u^\mu(x)$, $T(x)$, $\mu(x)$, but not the dissipative corrections $\xi^{\mu\nu}$, Φ, $\Pi^{\mu\nu}$, and V^μ whose evolution is controlled by microscopic physics.

Hydrodynamics from kinetic theory (III):

Different hydrodynamic approaches can be characterized by the different assumptions they make about the dissipative corrections and/or the different approximations they use to derive their dynamics from the underlying Boltzmann equation:

- **Ideal hydro:** local momentum isotropy ($\xi^{\mu\nu} = 0$), $\Phi = \Pi^{\mu\nu} = V^{\mu} = 0$.

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- **Viscous anisotropic hydrodynamics (vaHydro):** improved **aHydro** that additionally evolves residual dissipative flows $\Pi^{\mu\nu}$, V^μ with **IS** or **DNMR theory**.

BE for systems with highly symmetric flows: I. Bjorken flow

- Longitudinal boost invariance, transverse homogeneity ("physics on the light cone", no transverse flow) $\Longrightarrow$ $u^\mu = (1,0,0,0)$ in Milne coordinates (τ, r, ϕ, η) where $\tau = (t^2 - z^2)^{1/2}$ and $\eta = \frac{1}{2}\ln[(t-z)/(t+z)]$ $\Longrightarrow$ $v_z = z/t$

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$$f(x,p) = f(\tau; p_\perp, w) \qquad \text{where} \qquad w = t p_z - zE = \tau m_\perp \sinh(y - \eta).$$

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where $$D(\tau_2, \tau_1) = \exp\left(-\int_{\tau_1}^{\tau_2} \frac{d\tau''}{\tau_{\mathrm{rel}}(\tau'')}\right).$$

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$$\frac{\partial}{\partial\rho} f(\rho;\hat{p}_\Omega^2,\hat{p}_\varsigma) = -\frac{\hat{T}(\rho)}{c}\left[f\left(\rho;\hat{p}_\Omega^2,\hat{p}_\varsigma\right) - f_{\mathrm{eq}}\left(\hat{p}^\rho/\hat{T}(\rho)\right)\right].$$

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- The exact solution for f can be worked out for any "initial" condition $f_0(\hat{p}^2_\Omega, w) \equiv f(\rho_0; \hat{p}^2_\Omega, w)$. We here use equilibrium initial conditions, $f_0 = f_{\mathrm{eq}}$.

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- This exact solution of the BE can be compared to solutions of the various hydrodynamic equations in de Sitter coordinates, using identical initial conditions.
  - **Ideal:** $\hat{T}_{\rm ideal}(\rho) = \frac{\hat{T}_0}{\cosh^{2/3}(\rho)}$
  - **NS:** $\frac{1}{\hat{T}}\frac{d\hat{T}}{d\rho} + \frac{2}{3}\tanh\rho = \frac{1}{3}\bar{\pi}^\eta_\eta(\rho)\tanh\rho$ (viscous T-evolution) with $\bar{\pi}^\eta_\eta \equiv \hat{\pi}^\eta_\eta/(\hat{T}\hat{\mathcal{S}})$ and $\hat{\pi}^{\eta\eta}_{NS} = \frac{4}{3}\hat{\eta}\tanh\rho$ where $\frac{\hat{\eta}}{\hat{\mathcal{S}}} \equiv \bar{\eta} = \frac{1}{5}\hat{T}\hat{\tau}_{\rm rel}$
  - **IS:** $\frac{d\bar{\pi}^\eta_\eta}{d\rho} + \frac{4}{3}\left(\bar{\pi}^\eta_\eta\right)^2\tanh\rho + \frac{\bar{\pi}^\eta_\eta}{\hat{\tau}_{\rm rel}} = \frac{4}{15}\tanh\rho$
  - **DNMR:** $\frac{d\bar{\pi}^\eta_\eta}{d\rho} + \frac{4}{3}\left(\bar{\pi}^\eta_\eta\right)^2\tanh\rho + \frac{\bar{\pi}^\eta_\eta}{\hat{\tau}_{\rm rel}} = \frac{4}{15}\tanh\rho + \frac{10}{21}\bar{\pi}^\eta_\eta\tanh\rho$
  - **aHydro:** see M. Nopoush et al., PRD 91 (2015) 045007
  - **vaHydro:** working on it ...

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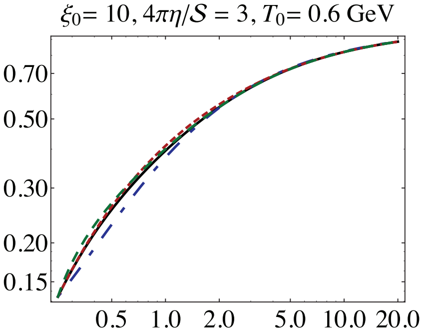
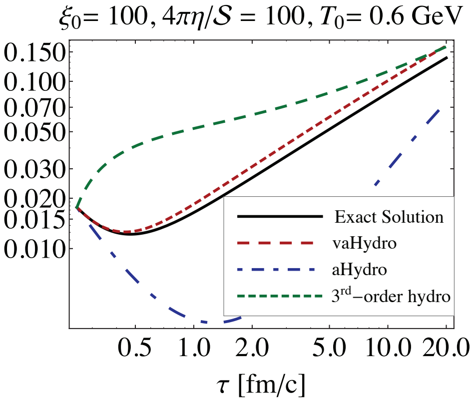
Bjorken flow (I)

Pressure anisotropy P_L/P_T vs. τ:

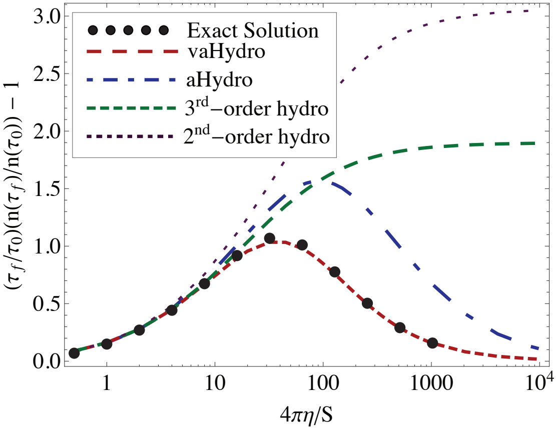




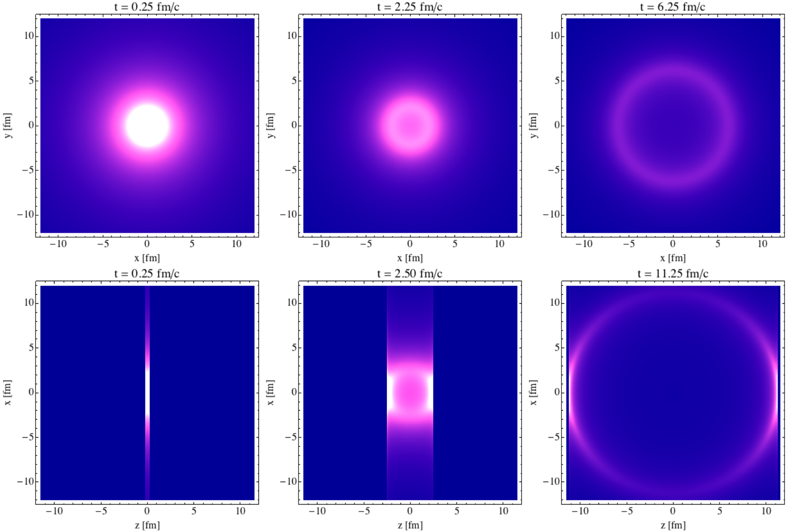
In the right plot, IS theory yields negative $P_L/P_T < 0$!

Bjorken flow (II)

Total entropy (particle) production $\frac{n(\tau_f)\cdot\tau_f}{n(\tau_0)\cdot\tau_0} - 1$

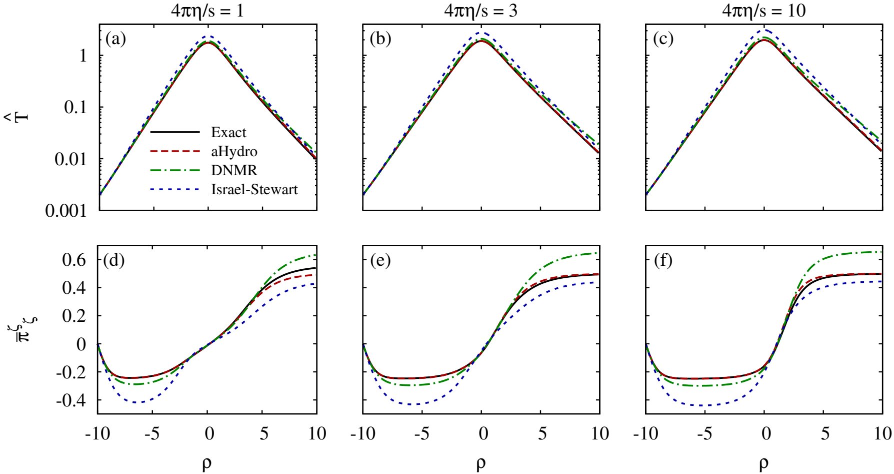


Gubser flow: temperature profile in (x, y) and (x, z)



Gubser flow in aHydro: ρ-evolution of T and shear stress

M. Nopoush, R. Ryblewski, M. Strickland, PRD 91 (2015) 045007



Thermal equil. initial conditions at $\rho_0 \to -\infty$. aHydro works better than IS & DNMR

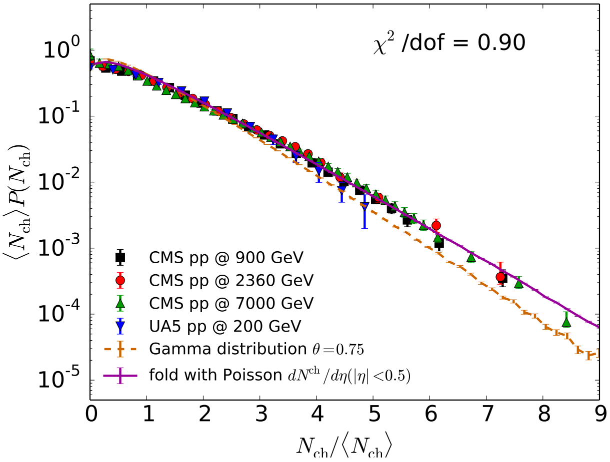
Conclusions

- Signs of hydrodynamic behavior are pervasive in heavy-ion collisions, from high to relatively low energies and from large to small collision systems.
- A new exact solution of the RTA Boltzmann equation for systems undergoing Gubser flow enables **tests of hydrodynamic approximation schemes in situations that resemble heavy-ion collisions** where the created matter undergoes simultaneous longitudinal and transverse expansion.
- When compared with the exact solution, second-order viscous hydrodynamics (IS and DNMR) works better than NS theory, anisotropic hydrodynamics (aHydro) works better than hydrodynamic schemes based on an expansion around local mometum isotropy (IS and DNMR), and viscous anisotropic hydrodynamic (vaHydro) (which treats small viscous corrections as IS or DNMR but resums the largest viscous terms) outperforms aHydro.
  Performance hierarchy: vaHydro $>$ aHydro $>$ DNMR $\sim$ IS $>$ NS $>$ ideal fluid.
- Improved hydro versions describe the macroscopic results of microscopic kinetics even in far-from-equilibrium situations much more accurately than previously thought.
- **Still unresolved: Where does the hydrodynamic approach really break down?**

Thank You!

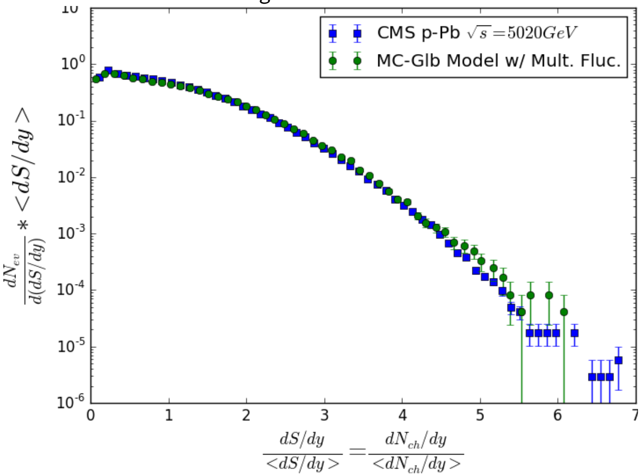
pp multiplicity distribution

Same for smooth Gaussian and quark-subdivided protons, after rescaling of the Γ-distribution:

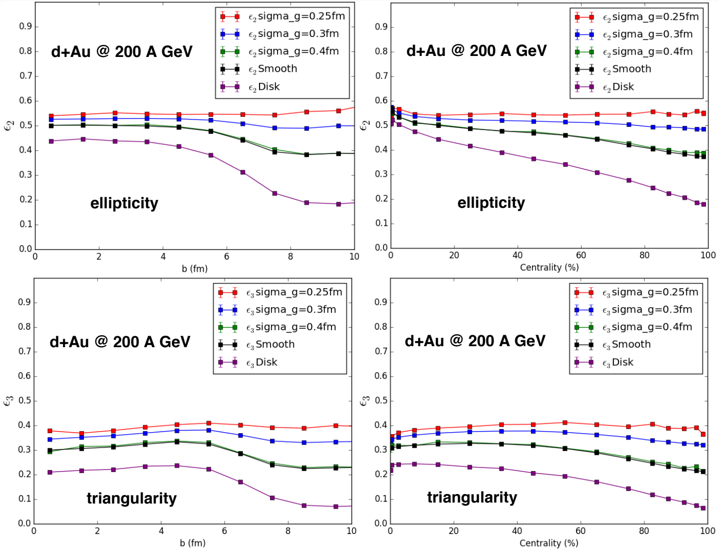


pPb multiplicity distribution

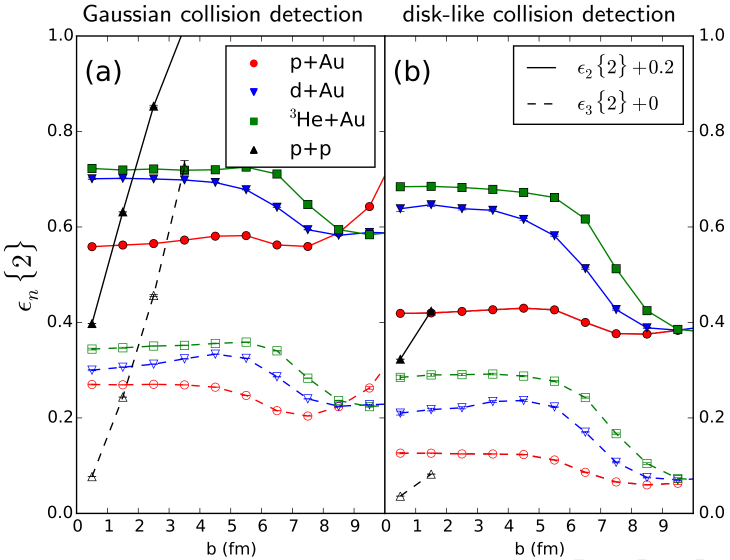
$\sigma_g = 0.3\,\text{fm}$:



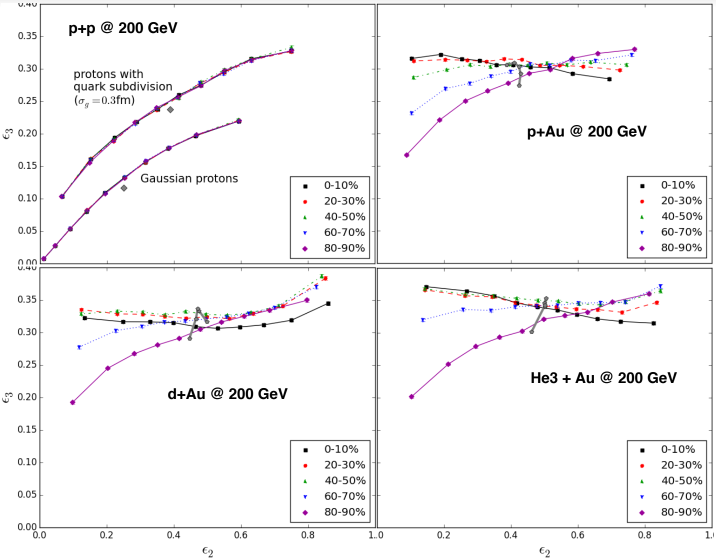
$\varepsilon_{2,3}$ vs. centrality: d+Au @ $\sqrt{s}$=200 A GeV



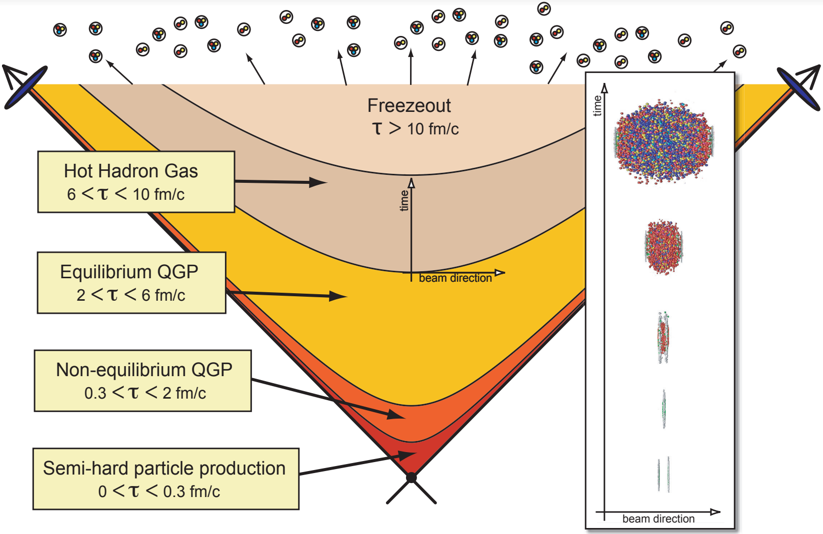
$\varepsilon_{2,3}$ vs. impact parameter for different collision systems



ε_2-ε_3 correlations: pp & light-heavy collisions, $\sigma_g = 0.3\,\text{fm}$

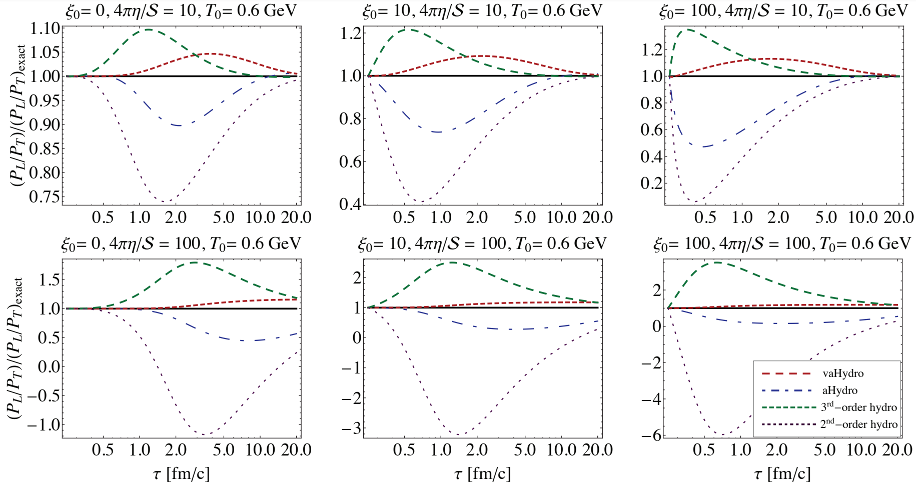


Motivation



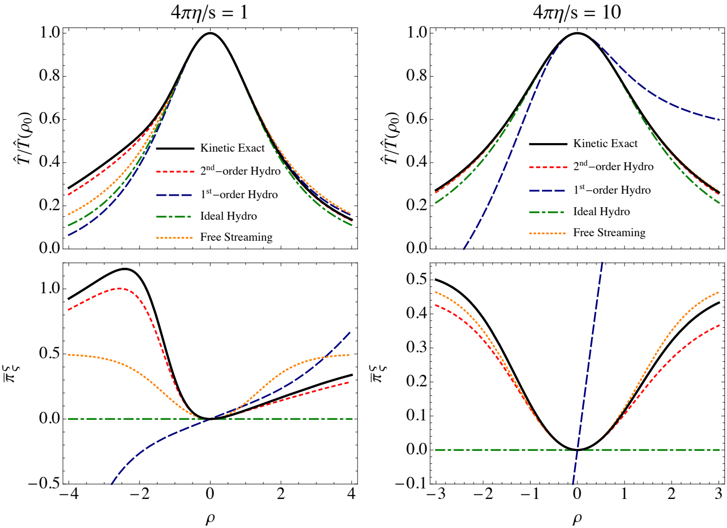
(From M. Strickland, arXiv:1410.5786)

Bjorken flow (II)



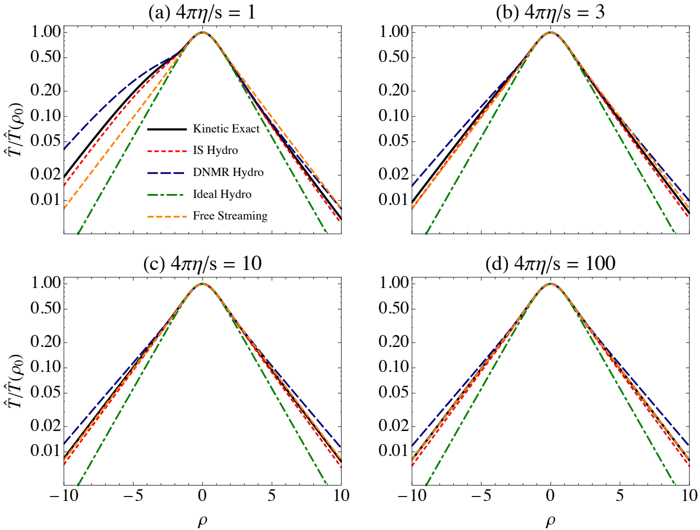
vaHydro agrees within a few % with exact result, even for very large $\eta/\mathcal{S}$!

Gubser flow II: ρ-evolution of temperature and shear stress



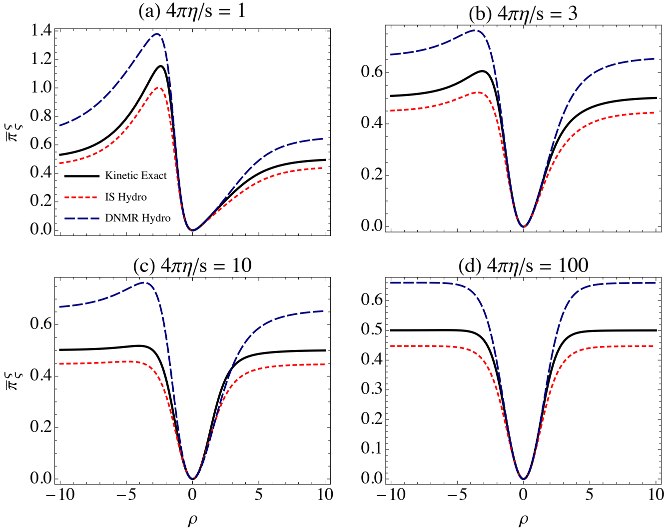
Note: $\bar{\pi}^{\varsigma}_{\varsigma} \equiv \bar{\pi}^{\eta}_{\eta}$! Thermal equil. initial conditions at $\rho_0 = 0$.

Gubser flow III: temperature evolution in de Sitter time ρ



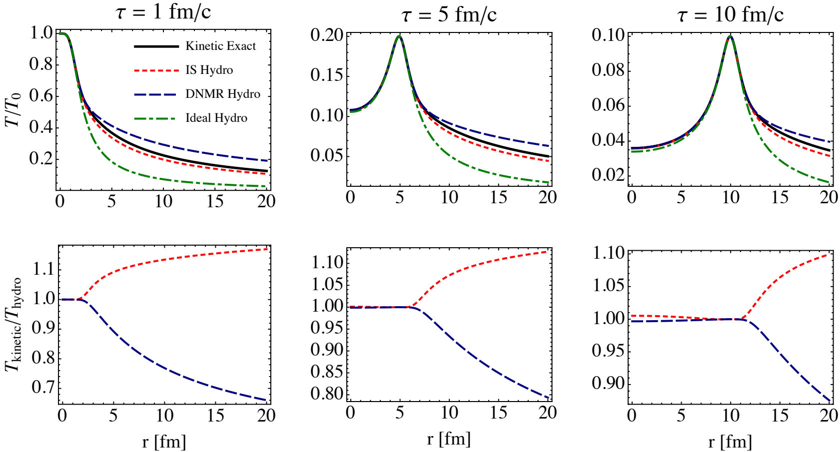
IS seems to work better than DNMR (!?)

Gubser flow IV: shear stress evolution in de Sitter time ρ



IS seems to work better than DNMR (!?)

Gubser flow V: temperature evolution in Minkowski space



IS seems to work better than DNMR (!?)

Both seem to have problems at large r ↔ large negative ρ