The QCD Critical Point and Heavy-Ion Collisions

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Outline

1. Introduction.
   - Critical point. History.
   - QCD Critical point
   - Heavy-Ion Collisions vs Cosmology

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   - Critical fluctuations
   - Mapping to QCD and observables
   - Intriguing data from RHIC BES I

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4. Summary

History

Cagniard de la Tour (1822): discovered continuos transition from liquid to vapour by heating alcohol, water, etc. in a gun barrel, glass tubes.

Name

Faraday (1844) – liquefying gases:

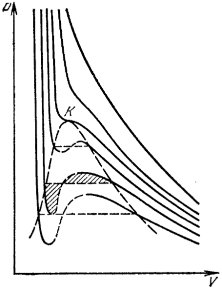
"Cagniard de la Tour made an experiment some years ago which gave me occasion to want a new word."

Mendeleev (1860) – measured vanishing of liquid-vapour surface tension: "Absolute boiling temperature".

Andrews (1869) – systematic studies of many substances established continuity of vapour-liquid phases. Coined the name "critical point".

Theory

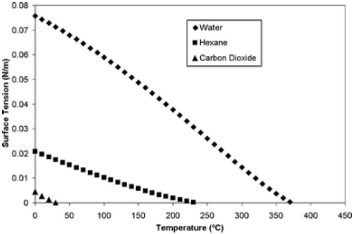
van der Waals (1879) –
in “On the continuity of the gas and liquid state”
(PhD thesis) wrote e.o.s. with a critical point.



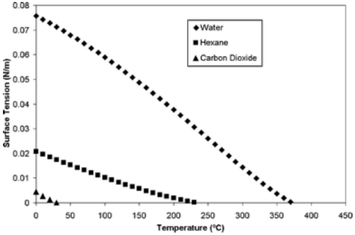
Smoluchowski, Einstein (1908,1910) – explained critical opalescence.

Landau – classical theory of critical phenomena

Fisher, Kadanoff, Wilson – scaling, full fluctuation theory based on RG.



Water
Hexane
Carbon Dioxide
Surface Tension (N/m)
Temperature (°C)
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
50
100
150
200
250
300
350
400
450



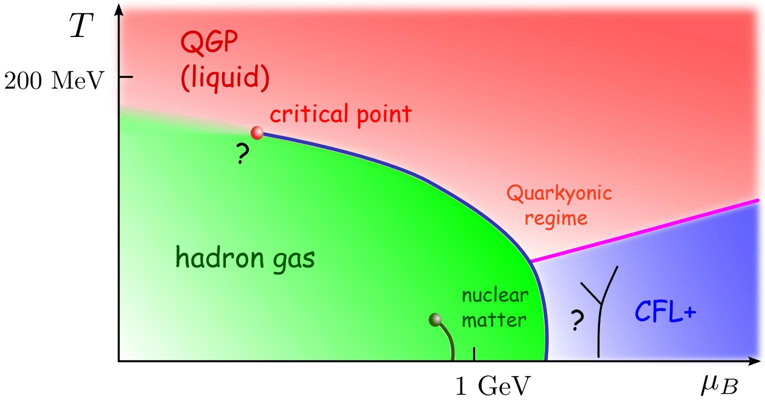
Among applications: integrated circuit manufacturing – deposition, cleaning, etc. (efficient and environmentally friendly).

Substance[13][14]	Critical temperature	Critical pressure (absolute)
Argon	−122.4 °C (150.8 K)	48.1 atm (4,870 kPa)
Ammonia[15]	132.4 °C (405.5 K)	111.3 atm (11,280 kPa)
Bromine	310.8 °C (584.0 K)	102 atm (10,300 kPa)
Caesium	1,664.85 °C (1,938.00 K)	94 atm (9,500 kPa)
Chlorine	143.8 °C (416.9 K)	76.0 atm (7,700 kPa)
Ethanol	241 °C (514 K)	62.18 atm (6,300 kPa)
Fluorine	−128.85 °C (144.30 K)	51.5 atm (5,220 kPa)
Helium	−267.96 °C (5.19 K)	2.24 atm (227 kPa)
Hydrogen	−239.95 °C (33.20 K)	12.8 atm (1,300 kPa)
Krypton	−63.8 °C (209.3 K)	54.3 atm (5,500 kPa)
CH_4 (methane)	−82.3 °C (190.8 K)	45.79 atm (4,640 kPa)
Neon	−228.75 °C (44.40 K)	27.2 atm (2,760 kPa)
Nitrogen	−146.9 °C (126.2 K)	33.5 atm (3,390 kPa)
Oxygen	−118.6 °C (154.6 K)	49.8 atm (5,050 kPa)
CO_2	31.04 °C (304.19 K)	72.8 atm (7,380 kPa)
N_2O	36.4 °C (309.5 K)	71.5 atm (7,240 kPa)
H_2SO_4	654 °C (927 K)	45.4 atm (4,600 kPa)
Xenon	16.6 °C (289.8 K)	57.6 atm (5,840 kPa)
Lithium	2,950 °C (3,220 K)	652 atm (66,100 kPa)
Mercury	1,476.9 °C (1,750.1 K)	1,720 atm (174,000 kPa)
Sulfur	1,040.85 °C (1,314.00 K)	207 atm (21,000 kPa)
Iron	8,227 °C (8,500 K)	
Gold	6,977 °C (7,250 K)	5,000 atm (510,000 kPa)
Water[2][16]	373.946 °C (647.096 K)	217.7 atm (22.06 MPa)

Critical point is a ubiquitous phenomenon

Critical point between the QGP and hadron gas phases?

- QCD is a relativistic theory of a fundamental force.
- CP is a singularity of EOS, anchors the 1st order transition.



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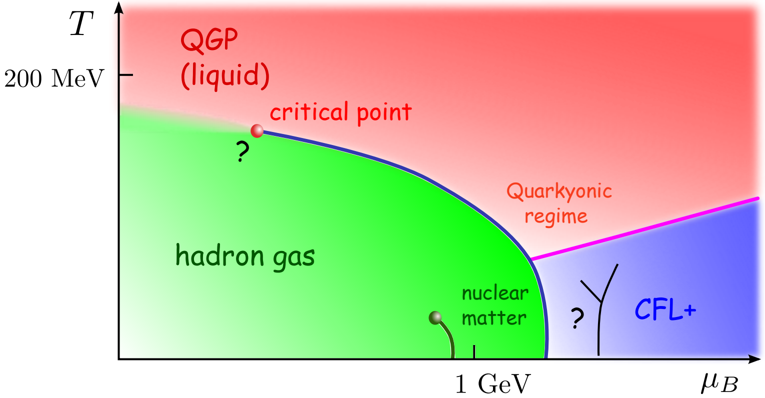


Lattice QCD at $\mu_B \lesssim 2T$ – a crossover.

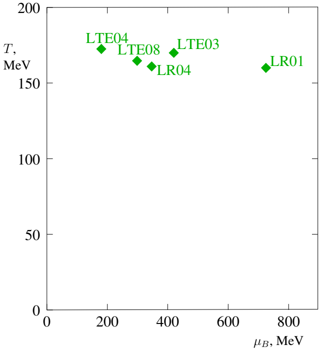
C.P. is ubiquitous in models (NJL, RM, Holog., Strong coupl. LQCD, . . .)

Essentially two approaches to discovering the QCD critical point. Each with its own challenges.

- Lattice simulations.

  The *sign problem* restricts reliable lattice calculations to $\mu_B = 0$.

  Under different assumptions one can estimate the position of the critical point, assuming it exists, by extrapolation from $\mu = 0$.



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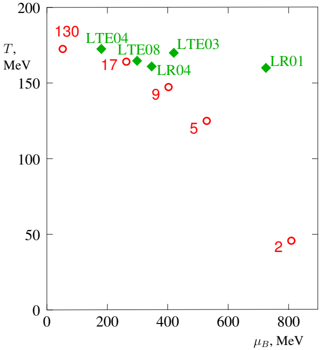

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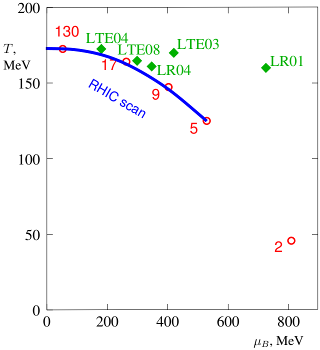

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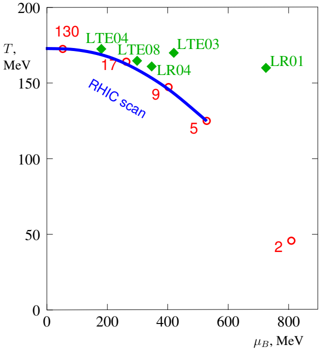
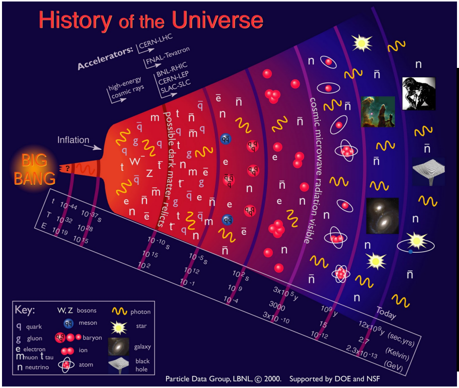
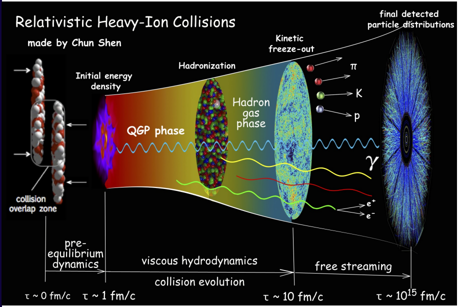


- Heavy-ion collisions. *Non-equilibrium.*

Big Bang vs little bangs

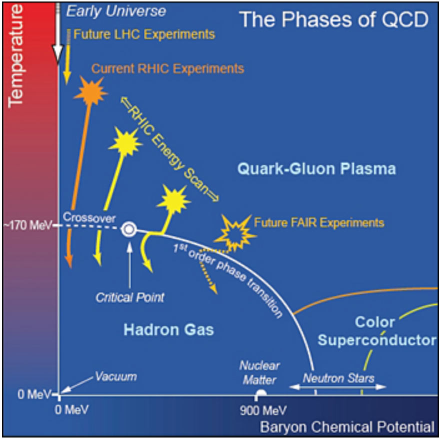


Little Bang



Expanding systems. Difference: space not expanding.

Difference: One Event vs many events (cosmic variance vs e.b.e. fluctuations)



- Expansion accompanied by cooling, followed by freezeout.

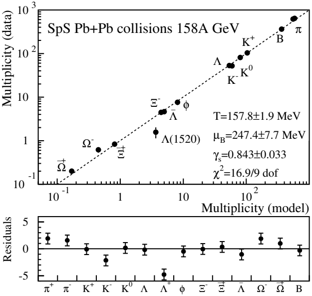
  Difference: tunable parameter μ_B via $\sqrt{s}$.

- Critical slowing down near CP determines ξ via KZ mechanism.

Heavy-Ion Collisions. Thermalization.

"Little Bang"

- The final state looks thermal.
- Similar to CMB.



(Becattini et al)

- Flow – looks hydrodynamic. Initial anisotropy fluctuations are propagated to final state hydrodynamically.
- Why and when this thermalization occurs – an open question.

Assumption for this talk

H.I.C. are sufficiently close to equilibrium that we can study thermodynamics at freezeout T and μ_B — as a first approximation.

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Why fluctuations are large at a critical point?

- The key equation:

$$P(X) \sim e^{S(X)} \qquad \textit{(Einstein 1910)}$$

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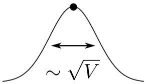

- For an extensive quantity $\langle X \rangle \sim V$:

$$\langle (\delta X)^2 \rangle_c = -\left(S''\right)^{-1} = VT\chi$$

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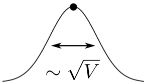

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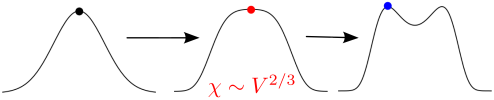

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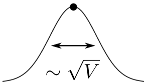

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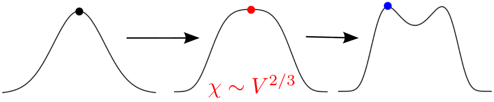
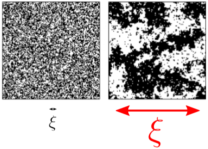




CLT? X is not a sum of ∞ many *uncorrelated* contributions: $\xi \to \infty$

Fluctuations of order parameter and ξ

- Fluctuations at CP – conformal field theory.

  Parameter-free $\to$ universality. Only one scale $\xi = m_\sigma^{-1} < \infty$,

$$P[\sigma] \sim \exp\left\{-\Omega[\sigma]/T\right\},$$

$$\Omega = \int d^3x \left[\frac{1}{2}(\boldsymbol{\nabla}\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \dots\right].$$

  - Width/shape of $P(\sigma_0 \equiv \int_{\boldsymbol{x}}\sigma)$ best expressed via cumulants:

- Higher cumulants (shape of $P(\sigma_0)$) depend stronger on ξ.

  Universal: $\boxed{\langle\sigma_0^k\rangle_c \sim V\xi^p}$, $p = k(3-[\sigma]) - 3$, $[\sigma] = \beta/\nu \approx 1/2$.

  E.g., $p \approx 2$ for $k = 2$, but $p \approx 7$ for $k = 4$.

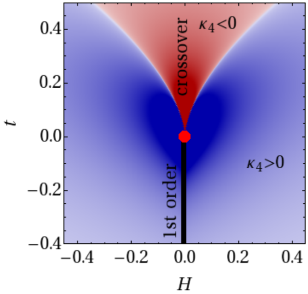
Sign

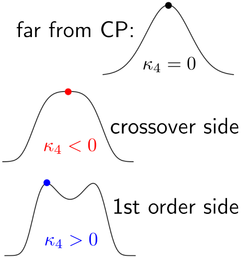
- Higher moments also depend on which side of the CP we are

$$\kappa_3[\sigma] = 2VT^{3/2}\,\tilde{\lambda}_3\,\xi^{4.5}\,; \quad \kappa_4[\sigma] = 6VT^2\,[\,2(\tilde{\lambda}_3)^2 - \tilde{\lambda}_4\,]\,\xi^7\,.$$

This dependence is also universal.

- 2 relevant directions/parameters. Using Ising model variables:





Experiments do not measure σ.

Experimental observables: simple model

Consider statistical fluctuations in a gas of particles without interaction:

$$\langle(\delta n_{\boldsymbol{p}}^{\text{free}})^2\rangle = \langle n_{\boldsymbol{p}}\rangle$$

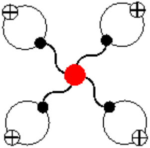
Think of a collective mode described by field σ such that $m = m(\sigma)$:

$$\delta n_{\boldsymbol{p}} = \delta n_{\boldsymbol{p}}^{\text{free}} + \frac{\partial\langle n_{\boldsymbol{p}}\rangle}{\partial\sigma} \times \delta\sigma$$

- The cumulants of multiplicity $M \equiv \int_{\boldsymbol{p}} n_{\boldsymbol{p}}$:

$$\kappa_k[M] = \underbrace{\langle M\rangle}_{\text{Poisson}} + \kappa_k[\sigma_0] \times g^k \left(\bullet\!\!\bigcirc\!\!\otimes \right)^k + \ldots,$$

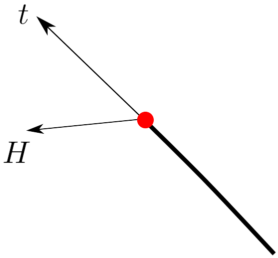
g – coupling of the critical mode ($g = dm/d\sigma$).



(diagramatically: PRD65(2002)096008)

Mapping Ising to QCD phase diagram

T vs μ_B:



- In QCD $(t, H) \to (\mu - \mu_{\text{CP}}, T - T_{\text{CP}})$

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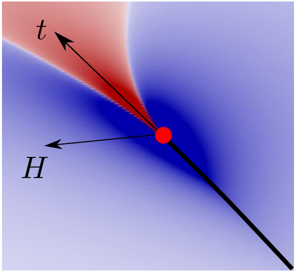

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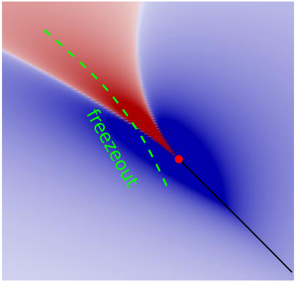
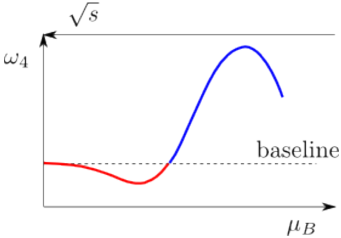
What should we see in the BES?





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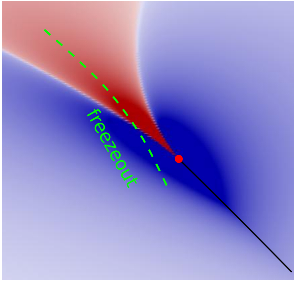
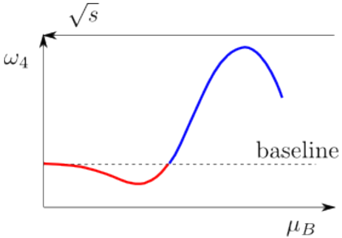
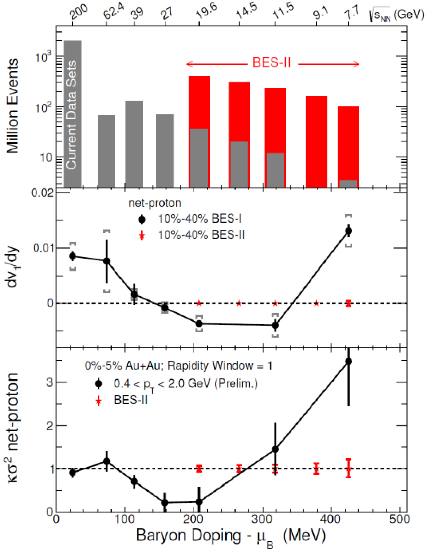
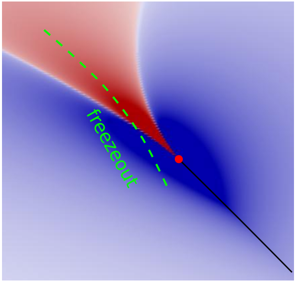
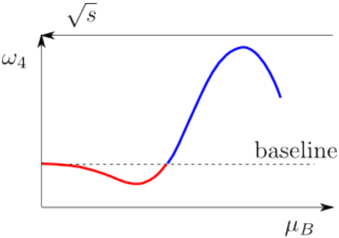
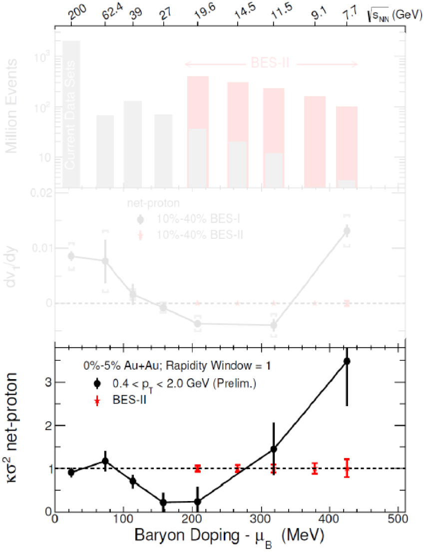







freezeout


$\sqrt{s}$
ω_4
baseline
μ_B


200
62.4
39
27
19.6
14.5
11.5
9.1
7.7
$\sqrt{s_{NN}}$ (GeV)
Million Events
Current Data Sets
BES-II
net-proton
10%-40% BES-I
10%-40% BES-II
dv_1/dy
0%-5% Au+Au; Rapidity Window = 1
0.4 < p_T < 2.0 GeV (Prelim.)
BES-II
$\kappa\sigma^2$ net-proton
Baryon Doping - μ_B (MeV)

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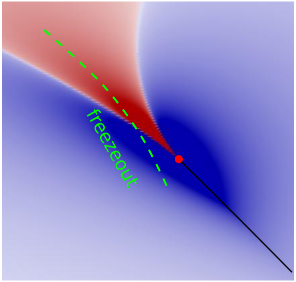

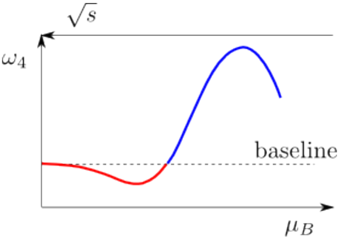

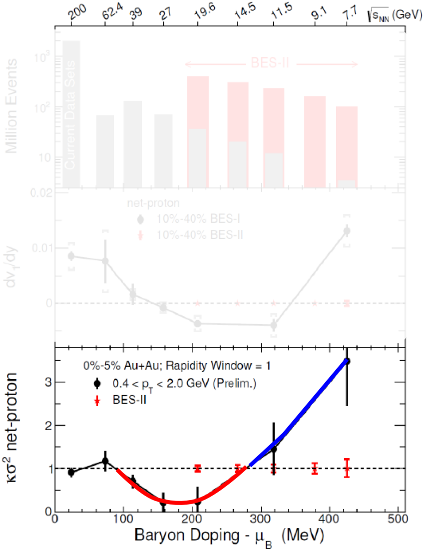

“intriguing hint” (2015 LRPNS)

Outline

Why ξ is finite

System expands and is *out of equilibrium*

Kibble-Zurek mechanism.

Critical slowing down means $\tau_{\rm relax} \sim \xi^z$.
Given $\tau_{\rm relax} \lesssim \tau$ (expansion time scale):

$$\xi \lesssim \tau^{1/z},$$

$z \approx 3$ (universal).

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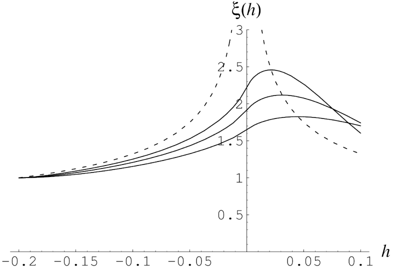
Estimates: $\xi \sim 2 - 3$ fm
(Berdnikov-Rajagopal)

KZ scaling for $\xi(t)$
and cumulants
(Mukherjee-Venugopalan-Yin)



Lessons

$$\kappa_n \sim \xi^p \quad \text{and} \quad \xi_{\max} \sim \tau^{1/z}$$

- Therefore, the magnitude of fluctuation signals is determined by non-equilibrium physics.
- Higher moments are more sensitive to ξ – good for detecting critical point. But harder to predict for the same reason.

Time evolution of cumulants (memory)

Mukherjee-Venugopalan-Yin

Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$

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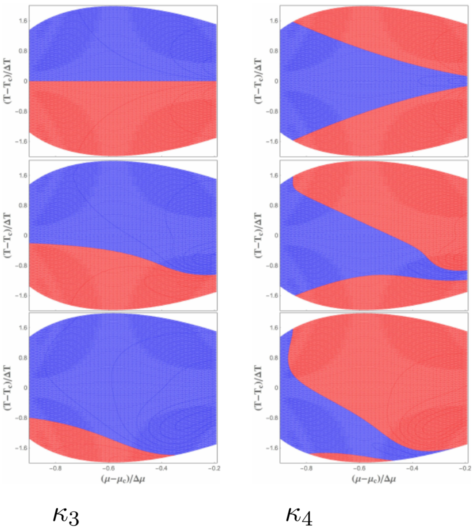

Signs of cumulants also depend on off-equilibrium dynamics.

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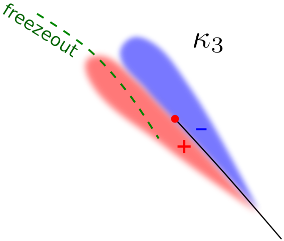
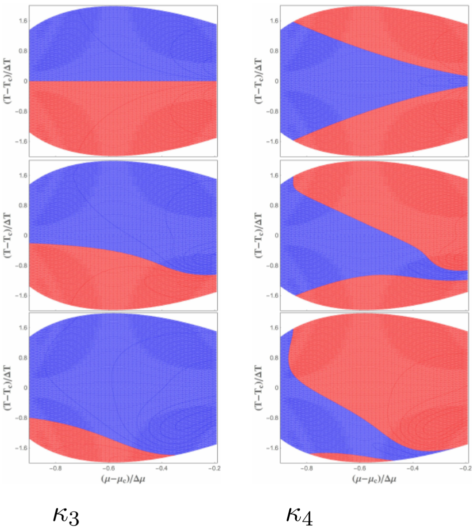




Signs of cumulants also depend on off-equilibrium dynamics.

Time evolution of cumulants (memory)

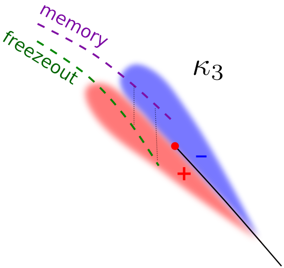
Mukherjee-Venugopalan-Yin

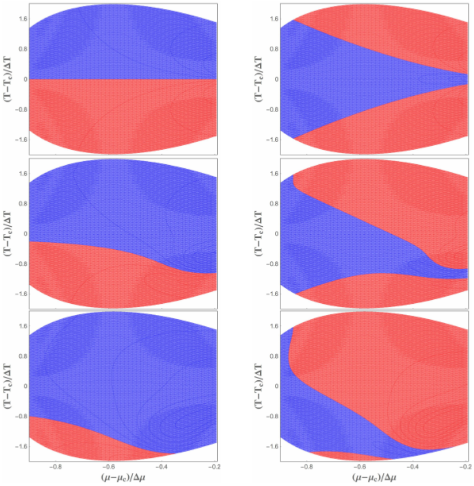
Relaxation to equilibrium

$$\frac{dP(\sigma_0)}{d\tau} = \mathcal{F}[P(\sigma_0)]$$

$$\Downarrow$$

$$\frac{d\kappa_n}{d\tau} = L[\kappa_n, \kappa_{n-1}, \ldots]$$





κ_3 κ_4

Signs of cumulants also depend on off-equilibrium dynamics.

Experiments do not measure σ.

Kinetic theory with critical mode

- Boltzmann equation, with collisions and noise:

$$\frac{p^\mu}{M}\frac{\partial f}{\partial x^\mu} + \partial^\mu M \frac{\partial f}{\partial p^\mu} + \mathcal{C}[f] = \xi\,,$$

  (Fox-Uhlenbeck) + field equation:

$$\partial^2\sigma + dU/d\sigma + (dM/d\sigma)\int_{\boldsymbol{p}} f/\gamma + \Gamma_0\dot{\sigma} = \eta.$$

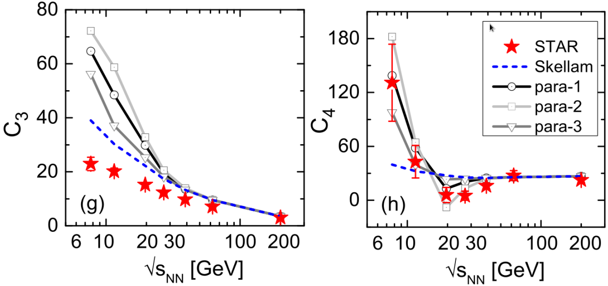
- Noise is fixed by fluctuation-dissipation relations.

  Fluctuations in equilibrium are reproduced correctly.

  We can now study non-equilibrium evolution of fluctuations.

  E.g., memory effects can be described (PRD81:054012,2010)

Recent (limited) application of this model in Jiang-Song (2015)



Spatial dependence at freezeout is considered but not *time*-dependence. So, no memory effects accounted for, yet.

Critical slowing down and bulk viscosity

Bulk viscosity is the effect of system taking time to adjust to local equilibrium.

$$p_{\text{hydro}} = p_{\text{equilibrium}} - \zeta\, \boldsymbol{\nabla} \cdot \boldsymbol{v}$$

$\boldsymbol{\nabla} \cdot \boldsymbol{v}$ – expansion rate

$\zeta \sim \tau_{\text{relaxation}} \sim \xi^z$

(Onuki, Moore-Saremi, Monnai-Mukherjee-Yin)

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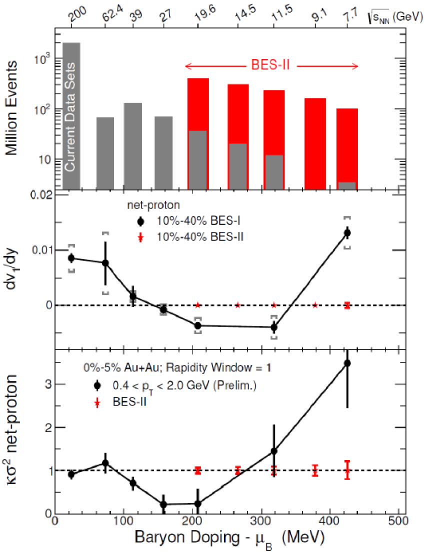

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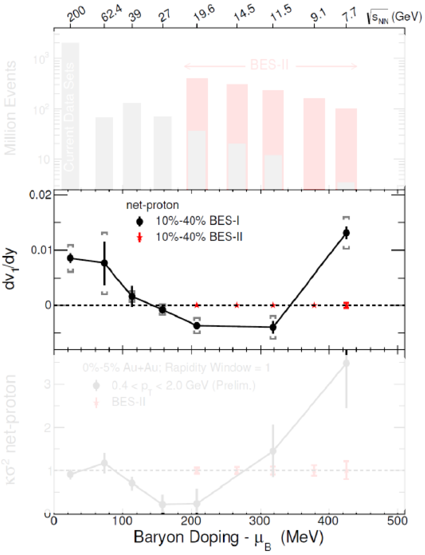

Stochastic hydrodynamics

$$\frac{\partial}{\partial t}\textbf{(variables)} = \boldsymbol{\nabla} \cdot [\,\textbf{(Flux)} + \textbf{(noise)}\,]$$ (Landau-Lifshits)

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- Linearized version has been considered and applied to heavy-ion collisions (Kapusta-Muller-MS, Kapusta-Torres-Rincon, ...)
- Non-linear case contains interesting challenges. E.g., multiplicative noise.
- Critical slowing down suggests adding additional slow, but not hydrodynamics mode.

Summary

- Is there a critical point between QGP and hadron gas phases?

  Heavy-Ion collision experiments may answer.

  The quest for the QCD critical point challenges us to creatively apply existing concepts and develop new ideas.

- Large (non-gaussian) fluctuations – universal signature of a critical point.

- In H.I.C., the magnitude of the signatures is controlled by non-equilibrium effects. The interplay of critical phenomena and non-equilibrium dynamics opens interesting questions.