Thermalization of soft modes in qgp's.

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kinetics of massless particles - relevance to cosmology and astrophysics.

Some work done in cosmology have been inspiring (Misha - Tkachev, BEC, etc ...)

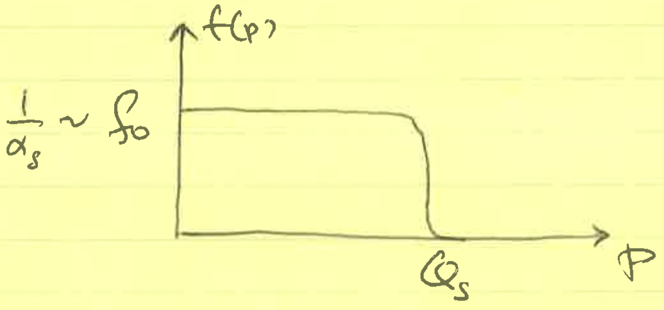
Equations similar to those used in some astro applis (Compton scattering, preserve particle nbre, gas of photons interacting with dilute gas of electrons Kompaneets equation ...)

main focus is QGP and heavy ions.

a few words on initial conditions

- at high energy nuclei can be thought of as made of gluons. (small x gluons)
- gluons distribution is characterized by a single scale Q_s
  $k_\perp > Q_s$ dilute, $k_\perp < Q_s$ dense, saturated.
- gluons in w.f. with $k_\perp \lesssim Q_s$ are freed on a time scale of order Q_s^{-1}.

initial gluon distribution



$$n_0 = f_0 \frac{Q_s^3}{6\pi^2}$$

$$\mathcal{E}_0 = f_0 \frac{Q_s^4}{8\pi^2}$$

$$n_0 \mathcal{E}_0^{-3/4} \sim f_0^{1/4} \sim \frac{1}{\alpha^{1/4}}$$

in thermal equilibrium

$$n_0 \mathcal{E}_0^{-3/4} \sim 1$$

at weak coupling ($\alpha \ll 1$)

$\Rightarrow$ There are more gluons initially than can be accommodated in a thermal distribution.

Question = how does such a system thermalize?
i.e. how does it get rid of the excess gluons?

Complex pb. many issues

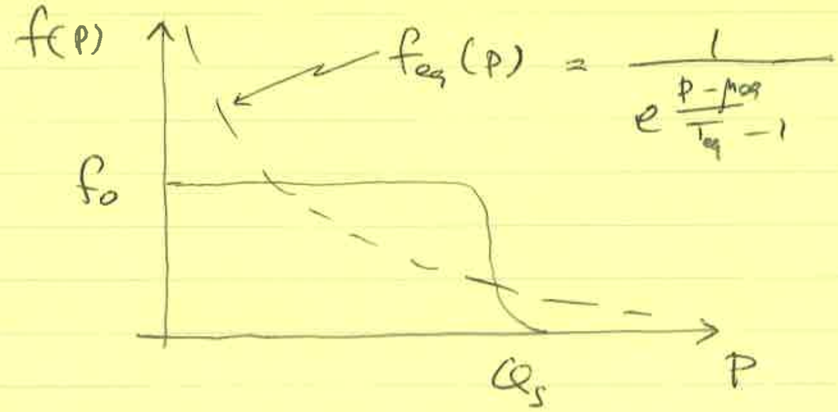
Simplify ... to the point when we can formulate simple and relevant questions that can be answered.

- System spatially uniform, non expanding, isotropic in momentum space
- Use kinetic theory

goal = get analytic control on robust behaviors, and on the major factors that determine the time scales.

deal with simple distribution $f(p)$

interested in "tiny" region of phase space



Questions = how do we go from $f_{ini}(p)$ to $f_{eq}(p)$?
where do the excess gluons go?

Use simple kinetic equations with elastic and inelastic eqn.

$$\frac{\partial f(p)}{\partial t} = C_{el}[f] + C_{inel}[f]$$

Conservation laws

	energy ε_0	$\longrightarrow$	$\varepsilon_f = \varepsilon_0$
$C_{inel} = 0$	nbre of gluons n_0		$n_f = n_0$

$\varepsilon_0, n_0 \longrightarrow T_{eq}, \mu_{eq}$

note $n_0 \leftrightarrow f_0$
$\exists f_{0c} \simeq 0.15$
not large!

Underpopulation $\mu < 0$
Overpopulation $\mu = 0$, T, n_0 — excess particles condense

$C_{inel} \neq 0$ $\mu = 0$

$\varepsilon_0 \longrightarrow \varepsilon_f = \varepsilon_0$
n_0 $n_f \neq n_0$

Inelastic $\underbrace{\text{collisions eliminate the excess}}$ particles. How fast?

Time scales are important — Transient BEC?
Also nature of (dominant) inelastic processes in QCD

$\boxed{\text{Elastic}}$

Small angle scattering

Boltzmann eq. $\to$ Fokker-Planck eqn

$$\frac{\partial f}{\partial t} = -\,\mathrm{div}\, J = -\frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 J(p)\right]$$

$$J(p) = -36\pi\alpha^2\mathcal{L}\left[I_a\,\partial_p f + I_b\, f(1+f)\right]$$

$$I_a = \int_p f(1+f)$$

$$I_b = \int_p \frac{2f}{p}$$

note thermal fixed point

$$T^* \equiv \frac{I_a}{I_b}$$

rescale time

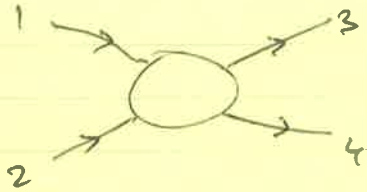
$$\tau = 36\pi\alpha^2\mathcal{L}\, t \qquad \mathcal{L} = \int\frac{dq}{q}$$

$\tau \simeq 10\, t$ (in units of Q_s^{-1})

With rescaled time, universal equation

$$-J(p) = I_a \partial_p f(p) + I_b f(1+f) = I_a \left[\partial_p f + \frac{f(1+f)}{T^*} \right]$$

"diffusion" "drag"

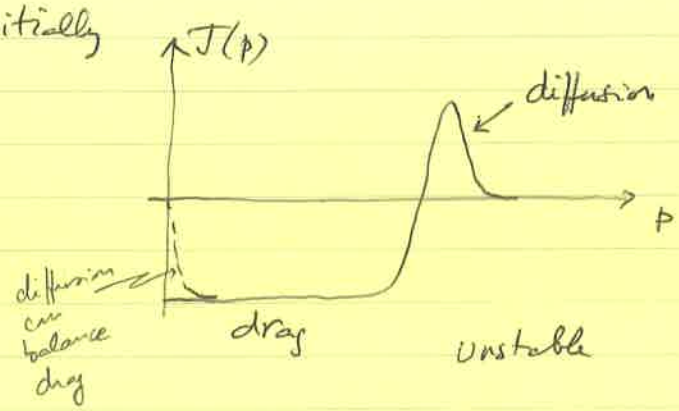


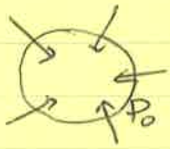
$p_3 = p_1 + q$

$p_4 = p_2 - q$

Stat factor $\rightarrow$ $\underbrace{-f_2(1+f_2)}_{\to I_a\ (\hat{q})} \vec{q}\cdot\vec{\nabla} f_1 + f_1(1+f_1) \underbrace{\vec{q}\cdot\vec{\nabla} f_2}_{I_b\ (m_D)}$

initially





$$\frac{dN}{d\tau} = \int^{p_0} dp\, p^2 \dot{f}(0) \sim p_0^2 J(p_0)$$

$$\simeq p_0^3 \dot{f}(0)$$

$$\Longrightarrow J(p_0) \sim p_0$$

Question: why does the system not condense immediately?
or how does the kinetic equation know about over/underpopulation?

Through balance of drag and diffusion

First it tries to equilibrate, and it does so instantly.

$$\underset{0}{\underset{\Downarrow}{f(p,\tau=0^+)}} \simeq \frac{T^*}{p-\mu^*} \qquad T^* \equiv \frac{I_a}{I_b}$$

$$\mu^* \leftrightarrow f(0)$$

Then it takes time for the system to know about over/under population.

Remarkably FP eqn has two solutions

$f_0 < f_{0c}$ evolution smooth towards BEC

$f_0 > f_{0c}$ $f(p) \sim \frac{c_{-1}}{p} + c_0 + c_1 p + \dots$ $f(0)$ finite

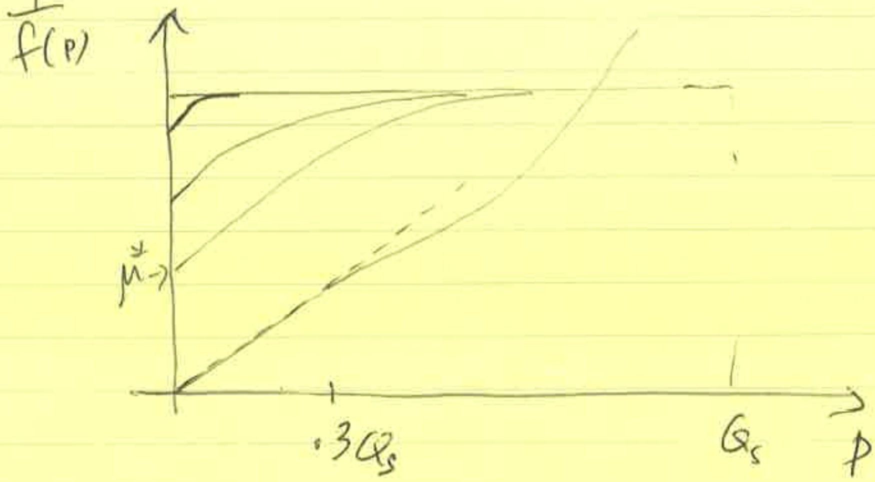
$\frac{c_{-1}}{p}$ allows for flow through $p=0$ since $c_{-1} \neq T^*$

diff $\sim \frac{c_{-1}}{p^2}$

drag $\sim \frac{T^*}{p^2}$

But in case $f_0 > f_{0c}$ condensation is not reached immediately, but after finite time τ_c.

for $f_0 \gtrsim 1$ $\tau_c \simeq \frac{2}{f_0(1+f_0)}$ ↖ comes from I_a



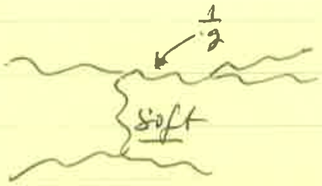
The "entire" soft region is thermalized.

Until onset particle density is constant.
At onset excess gluons flow into condensate.

Inelastic. Then $\mu_{eq} = 0$. However does it preclude the existence of a transient condensate? Or accumulation of soft modes? No

[Tkachev cosmology] ← cf scalar example. Details matter a priori. However for the kind of inelastic process of QCD, one can give a rather simple and robust answer

typical inelastic processes $2 \to 3$



a priori $\ni$ order g^3 but IR div. soft collinear processes $\to$ cutoff $\sim \frac{1}{g} \Rightarrow g^2$

analogy with medium induced ~~[illegible]~~ and radiation - jet physics - LPM effect can be included.

Same magnitude as elastic

<u>Simplify</u> i) $(2 \to 3) \to (1 \to 2)$ (collinear splitting dominate)

TSVD

ii) Simplify splitting kernel

$$\partial_\tau f(p,\tau) = \frac{R I_a}{p^3}\left[T^* - p f(p)\right]$$

does not thermalize to BE but to T^*/p

$(R = 1.83)$

$$T^* = \frac{I_a}{I_b}$$

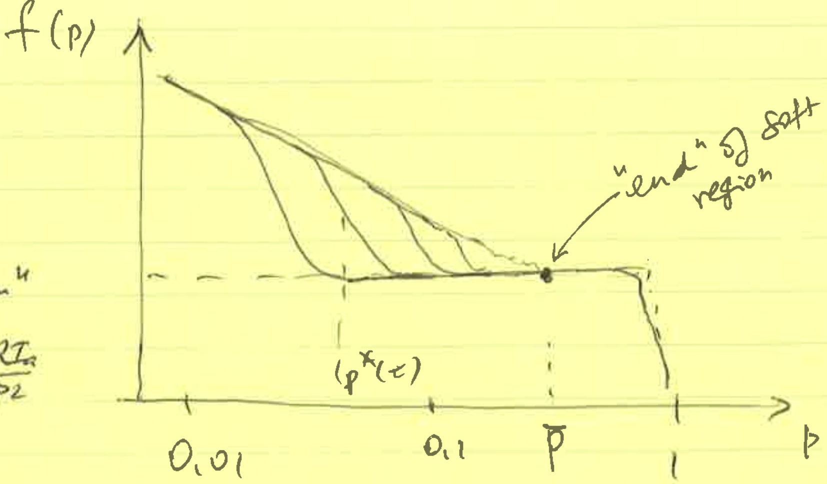
<u>Note</u>: in doing simplifications, we have not lost the most singular behavior at small p.

$\boxed{\text{Inelastic alone}}$ a bit artificial but instructive

Simple solution if one ignores time dependence of T^* and I_a:

$$f(p,t) = \frac{T^*}{p} + \left(f_0(p) - \frac{T^*}{p}\right) e^{-p_*^2(\tau)/p^2}$$

$$p_*^2(\tau) = R I_a (\tau - \tau_0)$$



$p_*(\tau) \sim \sqrt{\tau}$ mimics "diffusion"

also $e^{-\frac{p_*^2}{p^2}} \sim e^{-\tau \frac{R I_a}{p^2}}$

$\tau_p \sim \frac{1}{p^2}$

fast relaxation towards $\frac{T^*}{p}$ at small p.

$$\frac{T^*}{\bar{p}} = f_0$$

$$\bar{\tau} = \frac{\bar{p}^2}{R I_a} \approx 1.6 \quad (f_0 = 1)$$

$$\sim \frac{3.2}{f_0(1+f_0)}$$

very small nbr of particles are "created" in the soft region

Interestingly, particle nbr. stays approximately constant until soft region is thermalized. Like in elastic. Time scale is (approximately) the same. Again, it takes time before the system can decide what to do = create or destroy part!

Kind of "bottom up", in fact "perfect bottom up" (without however "details" of the bottom up scenario).

Note however that "equilibrium" of soft region is different in elastic ($\mu \neq 0$) and inelastic ($\mu = 0$)

Instantaneous thermalization at $\tau = 0^+$

el. $f(p) = \dfrac{T^*}{p - \mu^*}$ $\qquad - \mu^* = \dfrac{T^*}{f(0)}$

$T^* = \dfrac{I_a}{I_b}$

inel. $f(p) = \dfrac{T^*}{p}$ $\qquad \mu^* = 0$

Evolution of soft region (= thermalization)

el $\quad \mu^* \to 0 \quad T^* \searrow$

inel $\quad T^* \nearrow$

) ends at $\tau_c \sim \dfrac{1}{f_0^2} Q_s^{-1}$

Note: important time scale

Recall $\tau \sim 10 t$

or $t \sim \dfrac{\tau}{10}$

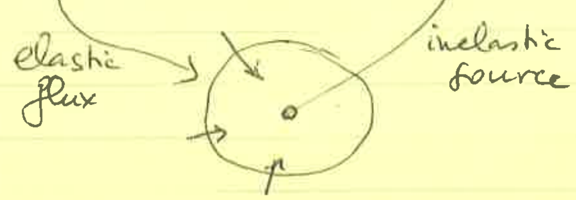
$t \sim \dfrac{\tau}{10 f_0^2} Q_s^{-1}$

$f_0 \sim 10^{-1}$ $\quad t \sim 10 Q_s^{-1}$

Elastic and inelastic competing.

$$p^2 C[f] = I_a \frac{\partial}{\partial p}\left[p^2 \frac{\partial f}{\partial p} + \frac{f^2}{T^*}\right] + I_a R\left[\frac{T^*}{p} - f(p)\right]$$

$$\frac{\partial n_{p_0}}{\partial \tau} = -I_a F(p_0) + I_a R \int_0^{p_0} dp\left[\frac{T^*}{p} - f(p)\right]$$



i) At small p, small τ, inelastic <u>wins</u>.

i.e. $f(p, \tau \to 0^+) = \frac{T^*}{p}$ $\quad T^* = \frac{I_a}{I_b}$

But for this distribution the elastic flux exactly cancels. No condensation.

Entire collision term vanishes – Instant thermalization.

ii) Deviations (small) from equilibrium distrib.

$$f(p,\tau) = \frac{T^*}{p} + A p^\alpha$$

$$\cancel{p^2 C[f] = I_a A(\alpha^2+3\alpha+2-R)p^\alpha}$$

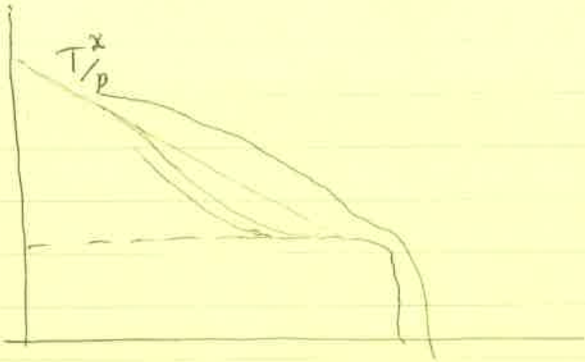
Inelastic source $\quad -I_a R A \frac{p_0^{\alpha+1}}{\alpha+1}$

Elastic current $\quad J(p_0) = -I_a A(\alpha+2) p^{\alpha-1}$

$$F(p_0) = -I_a A(\alpha+2) p^{\alpha+1}$$

At small times, $A<0$: inelastic dominates and sends particles in sphere p_0

But if $A<0$, the flux is <u>positive</u> = elastic collisions counteract inelastic source and try to remove particles from the sphere.



iii) Until the soft region is fully thermalized, the system cannot decide whether it should continue to produce (a small amount of) particles or destroy particles.

Then A changes sign as inelastic events to eliminate particles

Overpopulated case = would condense in absence of inelastic collisions - i.e. a flux of particles would appear at $p=0$.

This cannot take place here - BUT the elastic collisions continue to generate a strong drag current towards the IR. where inelastic collisions (through merging) eliminate the excess particles.

As a result an excess of soft particles remain for some time in the soft region.

Full treatment requires taking into account variations of I_a, T^*, etc.. and therefore treating correctly the ~~[illegible]~~ ultraviolet.

Figures and details will be available soon.