Real-time dynamics of fermion production in gauge theories

Schwinger pair production & string breaking

Florian Hebenstreit

Albert-Einstein-Center for Fundamental Physics, University of Bern

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CERN TH institute 'The big bang and the little bangs'

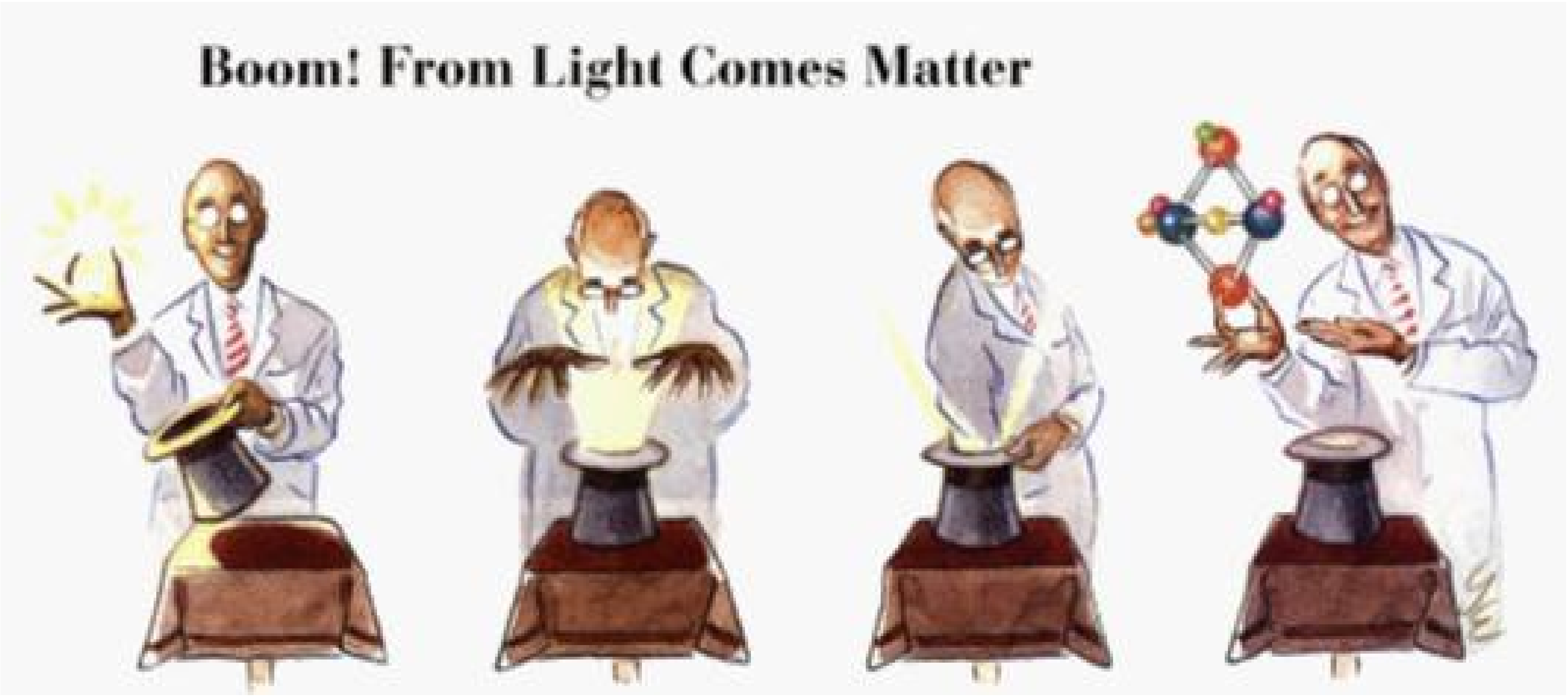
based on: FH, J. Berges, and D. Gelfand, **PRD 87** (2013); **PRL 111** (2013)

V. Kasper, FH, and J. Berges, **PRD 90** (2014)

FH and J. Berges, **PRD 90** (2014)

N. Mueller, FH, and J. Berges, **PRL 117** (2016)

Motivation: Fermion production



Non-linear Breit-Wheeler (SLAC E-144; perturbative):

$$\omega + n\omega_L \to e^+e^- \qquad (\omega_L \sim eV, n \sim 5, \omega \sim 30GeV)$$

Schwinger effect (future HIL; non-perturbative):

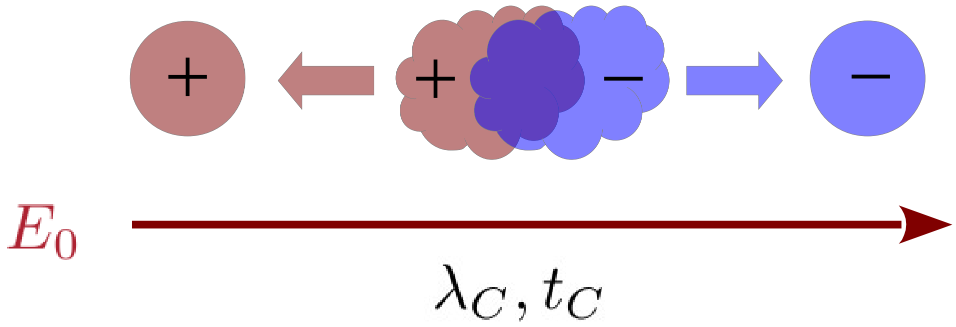
$$n\omega_L \to e^+e^- \qquad (\omega_L \to 0, n \to \infty)$$

Motivation: Schwinger effect

- QED vacuum is unstable in the presence of external fields

  vacuum: no particles
  vacuum + electric field: preferred to create particles

- electron-positron pair creation ⟶ delocalization of charges



$$eE_0\lambda_C \sim m$$
$$\Rightarrow E_0 \sim m^2/e$$

- analytic solution for vacuum decay rate in a static electric field

$$\mathcal{P}[vac] = \frac{(eE_0)^2}{4\pi^3} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp\left(-\frac{n\pi m^2}{eE_0}\right)$$

[Schwinger, PR 82 (1951)]

Evolution for dynamical gauge fields?!

Outline

- classical-statistical gauge theory with fermions
- Schwinger effect and plasma oscillations (QED 1+1 / 3+1)
- A model for QCD dynamics & string breaking (QED 1+1)
- Axial charge production out-of-equilibrium (QED 3+1)
- summary

Quantum electrodynamics

- theory of the interaction of matter (electrons) with light (photons)

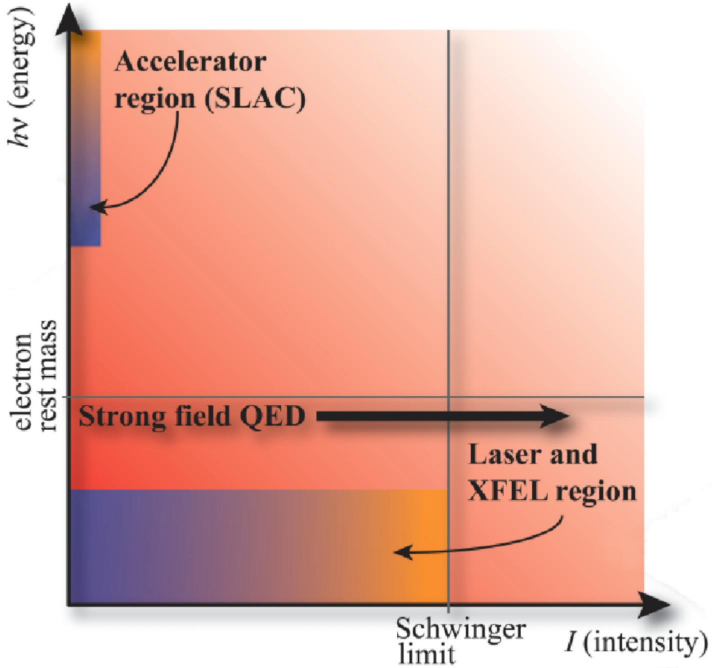
$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - g\bar{\psi}A\!\!\!/\psi$$

- perturbative QED

  low intensity – high energy: accelerator physics

- strong field QED

  high intensity – low energy: laser physics



[for non-Abelian gauge theories, see talks by: J. Berges, S. Schlichting...]

Non-equilibrium quantum field theory

time evolution: initial value problem in QFT

$$Z = \int \mathcal{D}\varphi \, e^{iS[\varphi]}$$

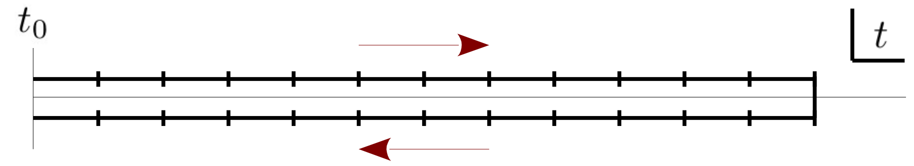
no probability measure

(sign problem!)

$$Z[J,\eta,\bar{\eta}] = \mathrm{Tr}\left[\hat{\rho}(t_0) T_{\mathcal{C}} \exp\left(i \int_{\mathcal{C}} J^{\mu} A_{\mu} + \bar{\psi}\,\eta + \bar{\eta}\,\psi \right)\right]$$

density matrix at initial time

time-ordering on the Schwinger-Keldysh contour



[Kasper, FH, Berges, PRD 90 (2014)]

Outline of the derivation

- functional integral representation:

$$Z = \int [\mathcal{D}A] \int [\mathcal{D}\psi \mathcal{D}\bar{\psi}]\, \rho_0(\psi, \bar{\psi}, A) \exp\left(i \int_{\mathcal{C}} \mathcal{L}_G[A] + \mathcal{L}_F[\psi, \bar{\psi}, A] \right)$$

- integrate out fermions (non-linear effective theory):

$$Z = \int [\mathcal{D}A]\, \rho_G(A) \exp\left(\mathrm{Tr}_{\mathcal{C}} \log \Delta[A]^{-1} + i \int_{\mathcal{C}} \mathcal{L}_G[A] \right)$$

- Keldysh rotation $A^{\pm} = \bar{A} \pm \widetilde{A}/2$ and expansion in $\widetilde{A}$:

$$\mathrm{Tr}_{\mathcal{C}} \log \Delta[A]^{-1} = \mathrm{Tr}_{\mathcal{C}} \log \Delta[\bar{A}]^{-1} + \frac{g}{2} \mathrm{Tr}_{\mathcal{C}} \left\{ \Delta[\bar{A}]\, \mathrm{sig}_{\mathcal{C}}\, \widetilde{A} \right\} + \mathcal{O}(\widetilde{A}^2)$$

That's the approximation!

[Kasper, FH, Berges, PRD 90 (2014)]

Outline of the derivation

- classical-statistical approximation of the generating functional:

$$Z = \int [\mathcal{D}\bar{A}][\mathcal{D}\widetilde{A}]\, \rho_G(A) \exp\left(i \int_{t_0}^{t_f} \int_{\mathbf{x}} \widetilde{A}^{\nu} \left\{ \partial^{\mu} \bar{F}_{\mu\nu} + \frac{g}{2} \operatorname{tr}[\Delta_K \gamma_{\nu}] \right\} \right)$$

$$\Delta_K(x,y) \equiv \left\langle [\psi(x), \bar{\psi}(y)] \right\rangle_{\bar{A}}$$

sampling over initial conditions

classical equations of motion

$$(i\partial\!\!\!/_x - e\bar{A\!\!\!/}(x) - m)\Delta_K(x,y) = 0$$

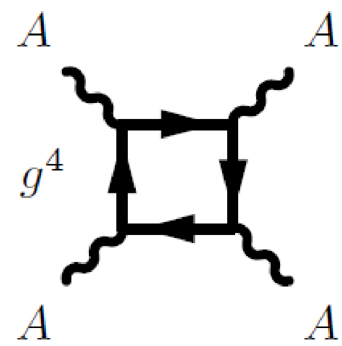
$$\partial^{\mu} \bar{F}_{\mu\nu}(x) = -\frac{g}{2} \operatorname{tr}[\Delta_K(x,x)\gamma_{\nu}]$$

- observables in classical-statistical approximation:

$$\langle O \rangle_{\mathrm{cl}} = \int [\mathcal{D}\bar{A}][\mathcal{D}\Pi_0]\, \rho_W[\bar{A}_0, \Pi_0] O[\bar{A}] \delta[\mathrm{E.o.M.}]$$

Diagrammatic understanding

$\mathrm{Tr}_{\mathcal{C}} \log \Delta[A]^{-1} = \quad g^2$ 

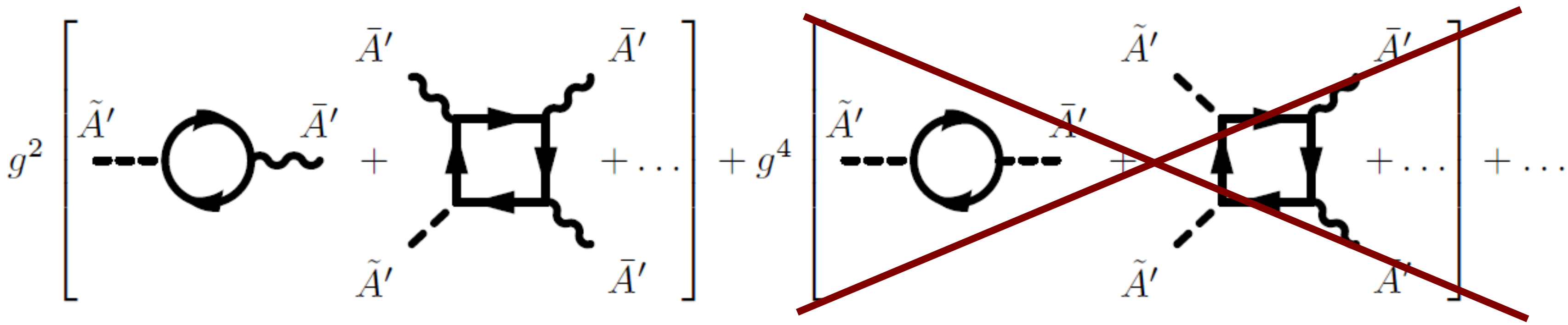
$+ \quad g^4$ 

$+ \quad \ldots$

- Keldysh rotation and field rescaling: $\bar{A} = g^{-1} \bar{A}'$ $\quad \tilde{A} = g \tilde{A}'$

[Kasper, FH, Berges, PRD 90 (2014)]

Diagrammatic understanding



- Keldysh rotation and field rescaling: $\bar{A} = g^{-1}\bar{A}'$ $\tilde{A} = g\tilde{A}'$
- series expansion: $g \ll 1$
- exact to quadratic order: g^2
- valid for: large coherent fields or high occupation numbers

$$\bar{A} \sim \mathcal{O}(1/g) \qquad n(\mathbf{p}) \sim \mathcal{O}(1/g^2)$$

g^0	g^2	g^4
bosonic	fermionic	bosonic/fermionic

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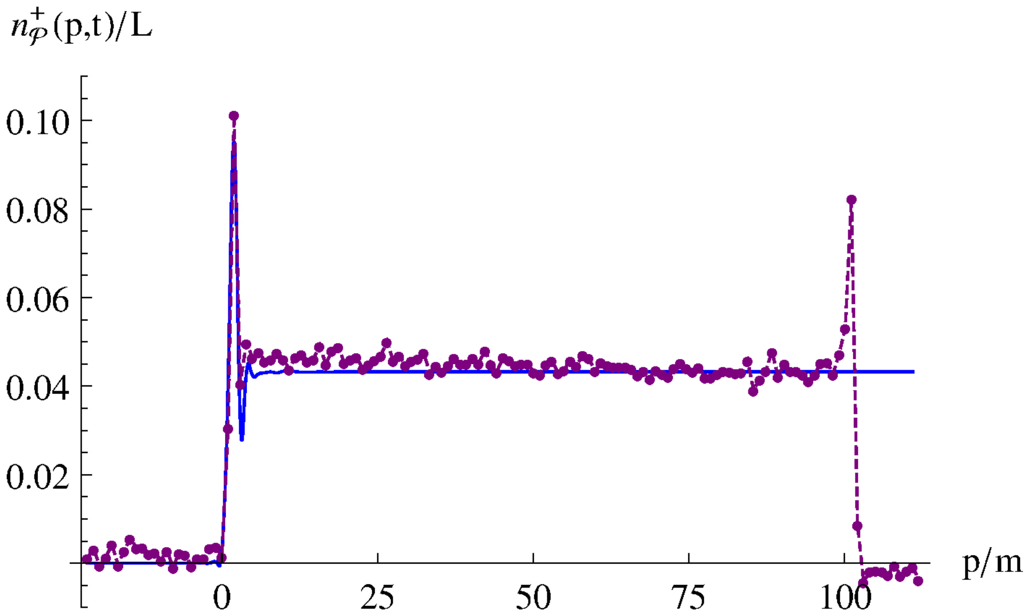
[FH, Berges, Gelfand, PRD 87 (2013)]

Schwinger formula on the lattice

static electric field without backreaction in QED (1+1)



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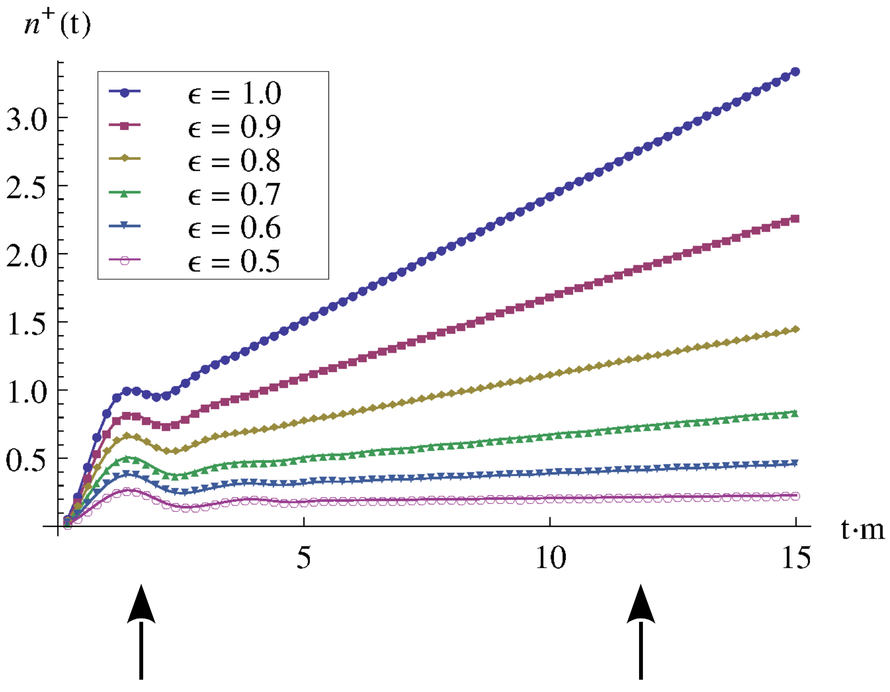
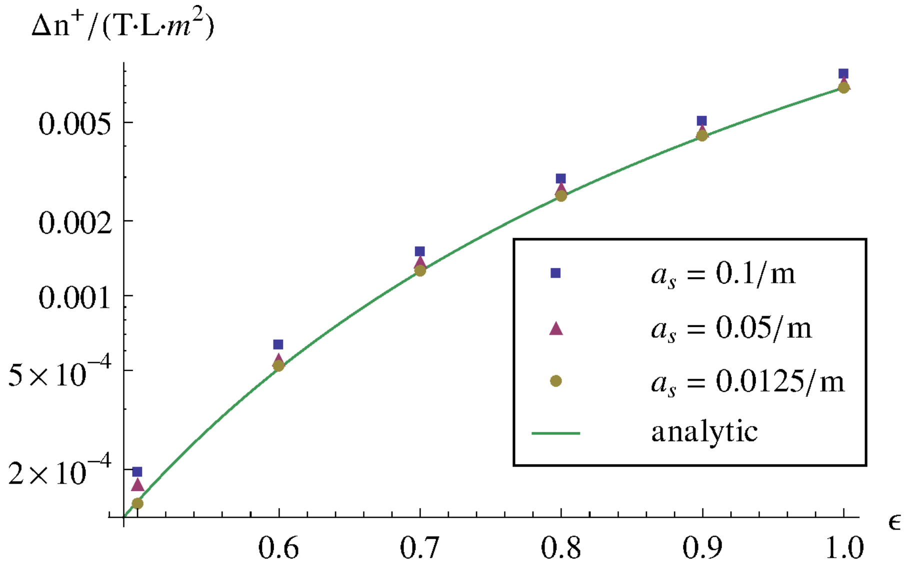




transient effect

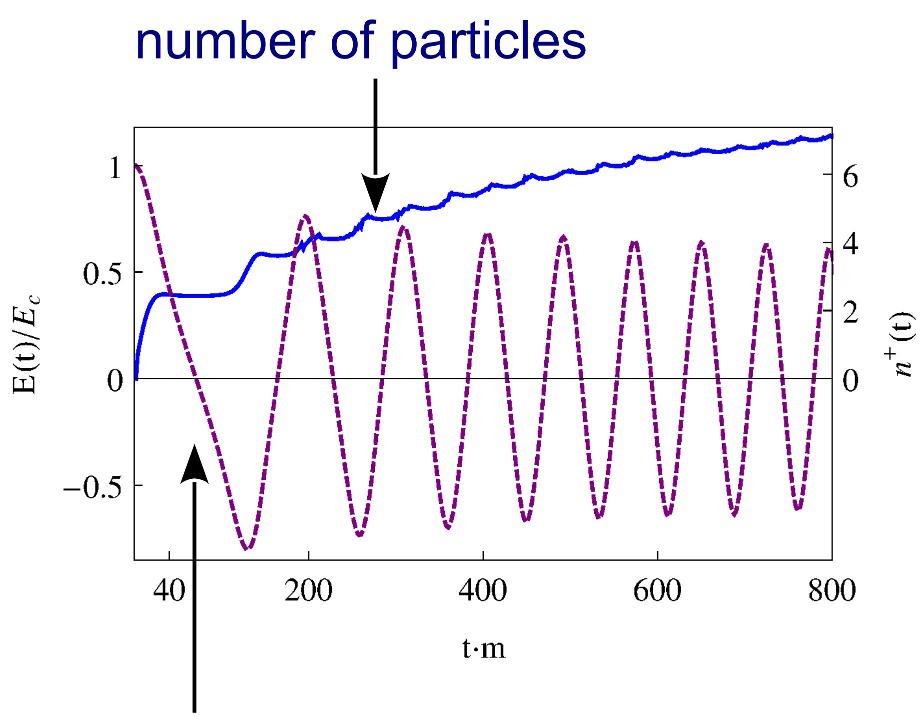
linear growth

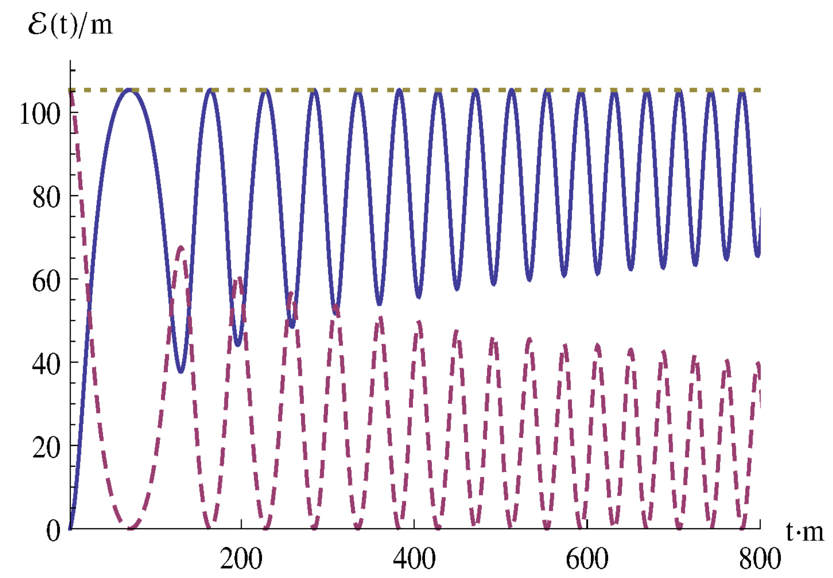
$$\frac{\Delta n^+}{TLm^2} = \frac{\epsilon}{2\pi} \exp\left(-\frac{\pi}{\epsilon}\right)$$

[FH, Berges, Gelfand, PRD 87 (2013)]

Plasma oscillations

static electric field incl. backreaction in QED (1+1)



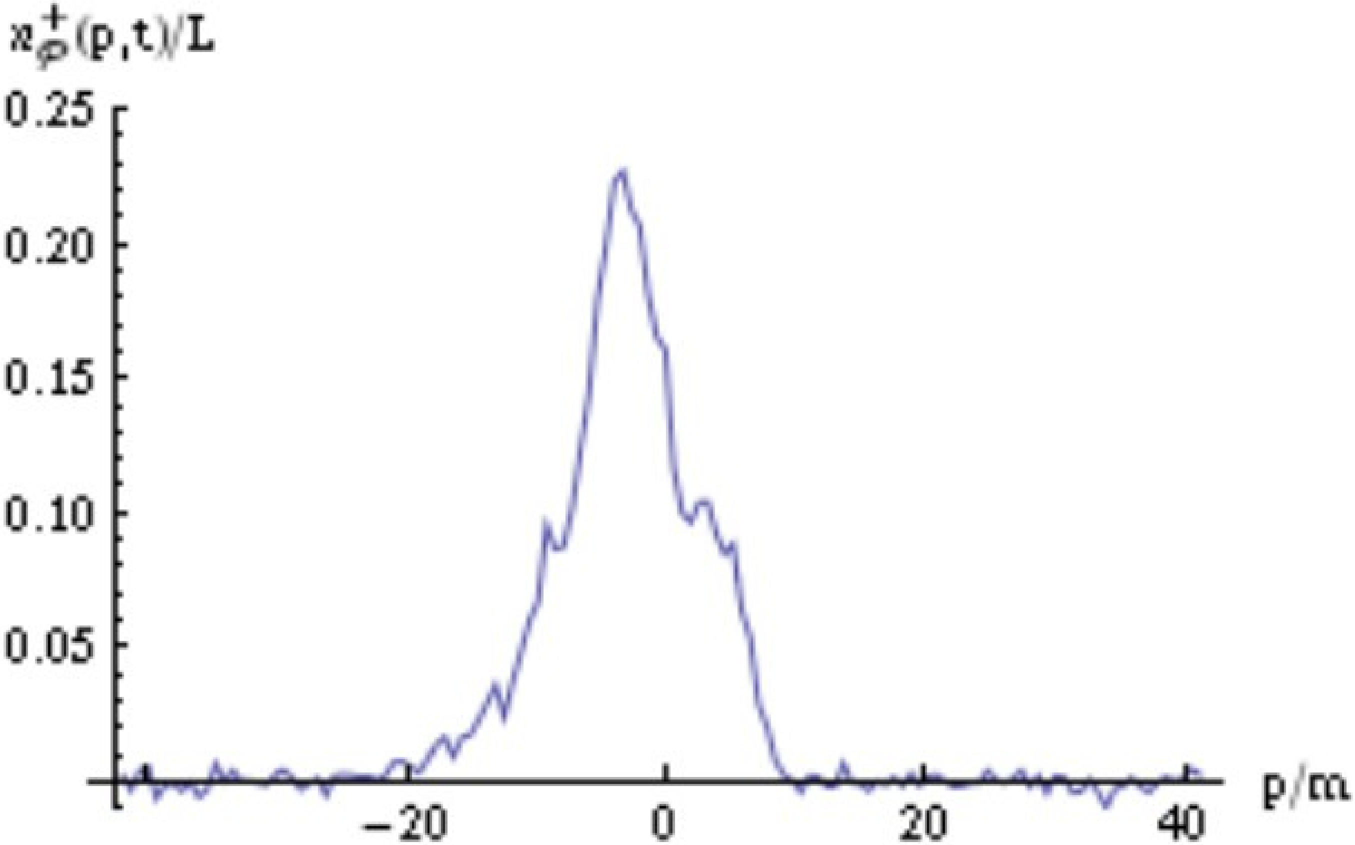


energy conservation

[FH, Berges, Gelfand, PRD 87 (2013)]

Plasma oscillations

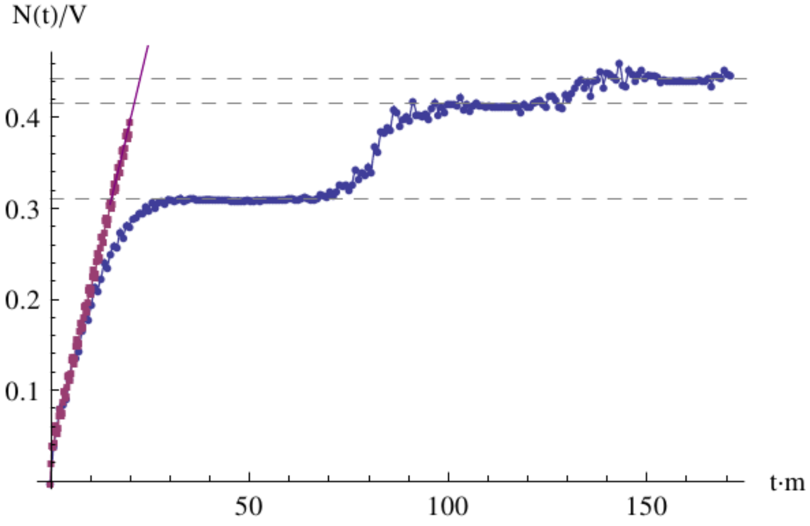
static electric field incl. backreaction in QED (1+1)



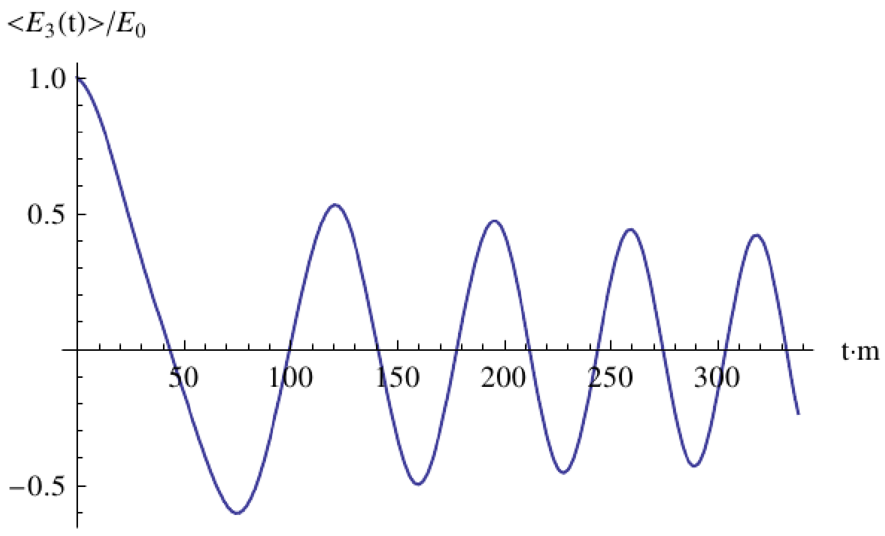
[Kasper, FH, Berges, PRD 90 (2014)]

Plasma oscillations

static electric field incl. backreaction in QED (3+1)



number of particles



field strength

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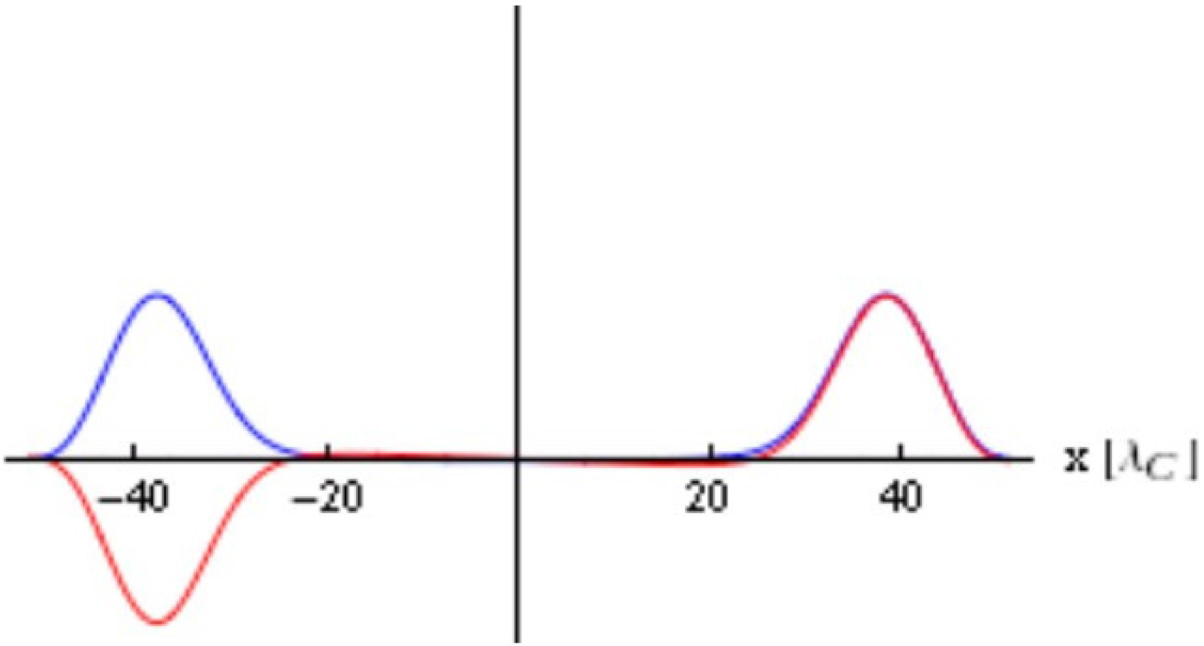
[FH, Alkofer, Gies, PRL 107 (2011)]

Bunching in space-time pulse

single electric pulse in space and time in QED (1+1)

$$E(x,t) = E_0 \, \mathrm{sech}^2(\omega t) \exp(-x^2/2\lambda^2)$$



fermion density

charge density

electric field

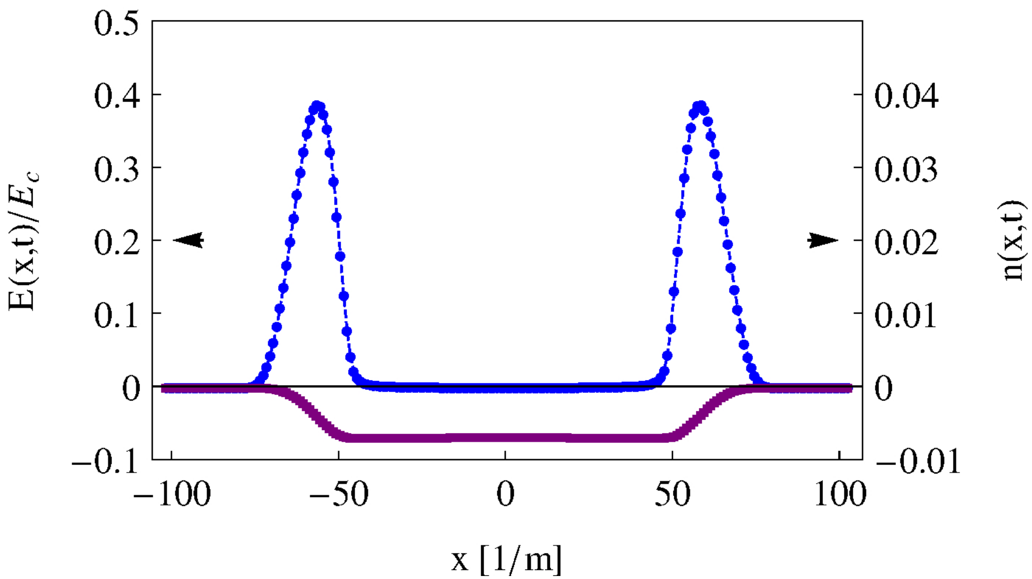
[FH, Berges, Gelfand, PRL 111 (2013)]

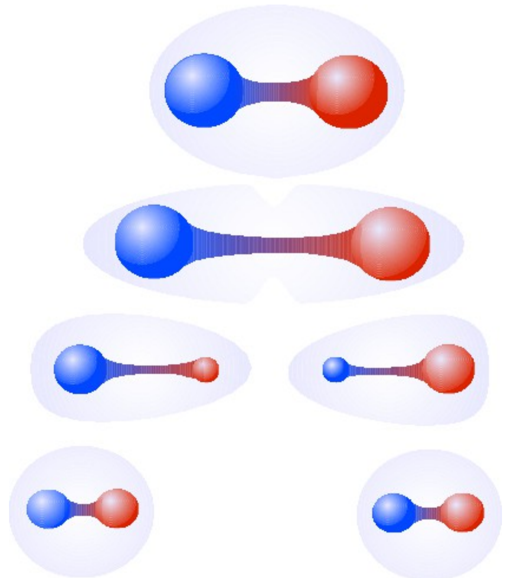
The string breaking analogue

- fermion bunches act as capacitor

  1+1 dimensional geometry: Coulomb potential = linear potential

  cf. QCD string breaking: linear potential due to strong interaction



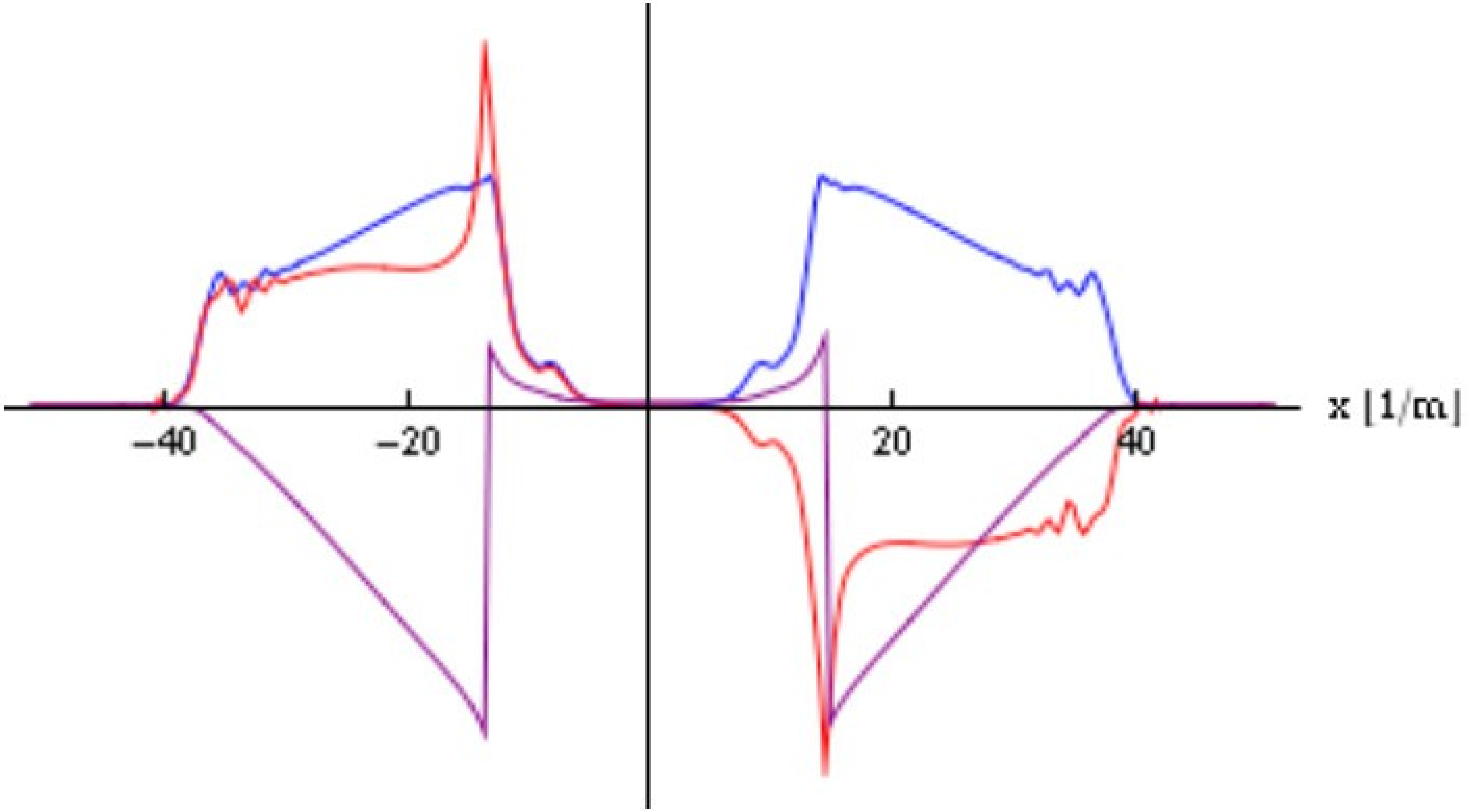


Can we learn something about the dynamics of string breaking?

[FH, Berges, Gelfand, PRL 111 (2013)]

Dynamics of string breaking

two static charges separated by distance d_C



[FH, Berges, Gelfand, PRL 111 (2013)]

Dynamics of string breaking

two static charges separated by distance d_C

- two-stage process (different scales)
  - ⟶ creation on top of each other
  - ⟶ separation of charges

- Naïve estimate for critical distance

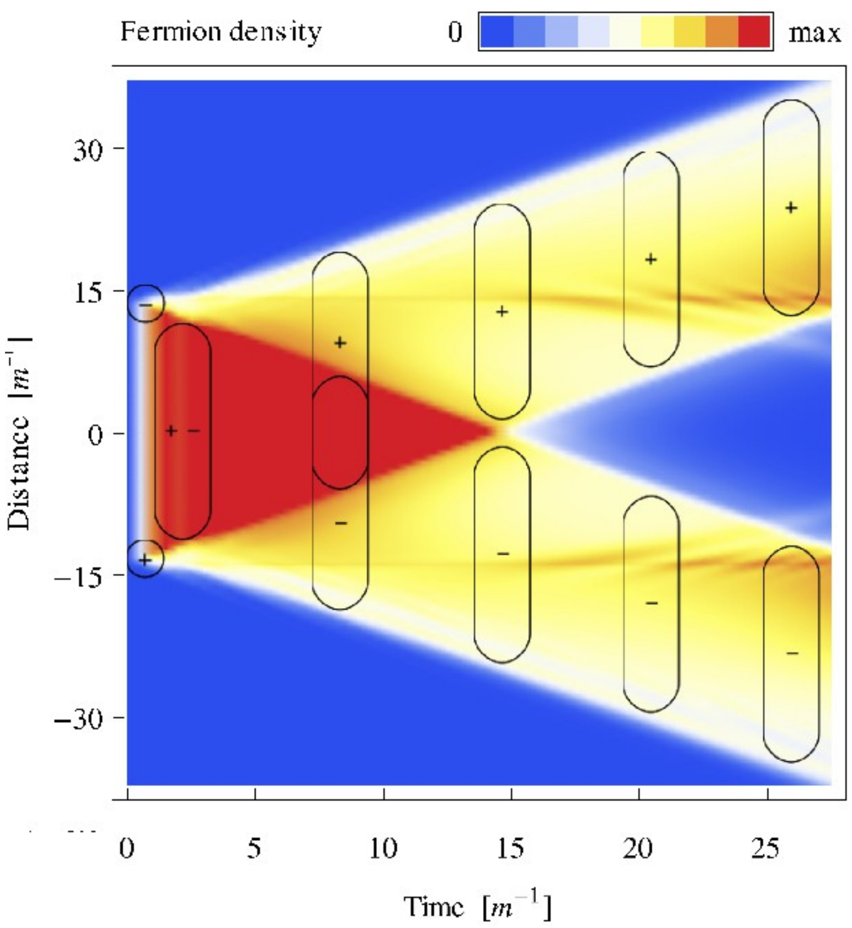
$$V_{\mathrm{str}}[d_C] = 2m$$

modified ↓

$$V_{\mathrm{str}}[d_C] = 2m + W[d_C]$$

- very substantial work contribution

$$W[d_C] > 2m$$



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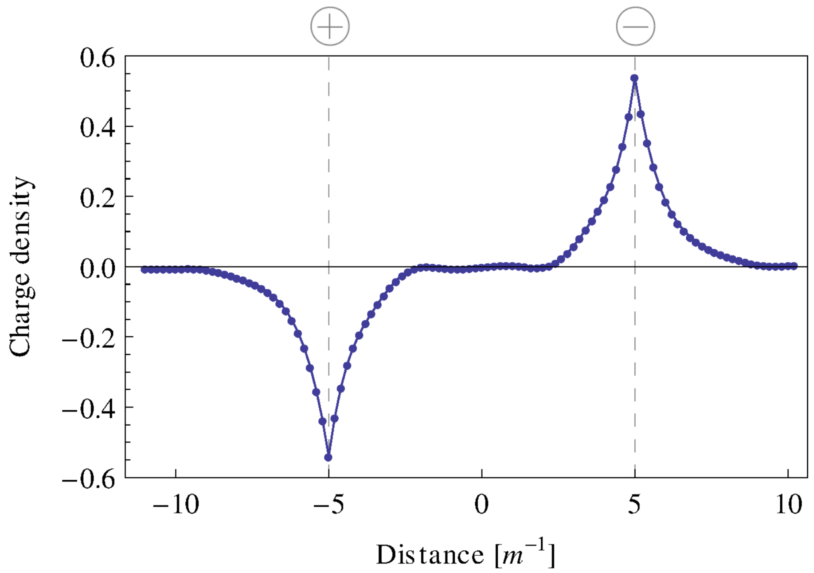

asymptotic screening

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[Mueller, FH, Berges, PRL 117 (2016)]

Axial anomaly in non-equilibrium QED

- QED axial anomaly equation

$$\partial_\mu j_5^\mu = 2im\langle\bar{\psi}\gamma_5\psi\rangle - \frac{e^2}{8\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu}$$

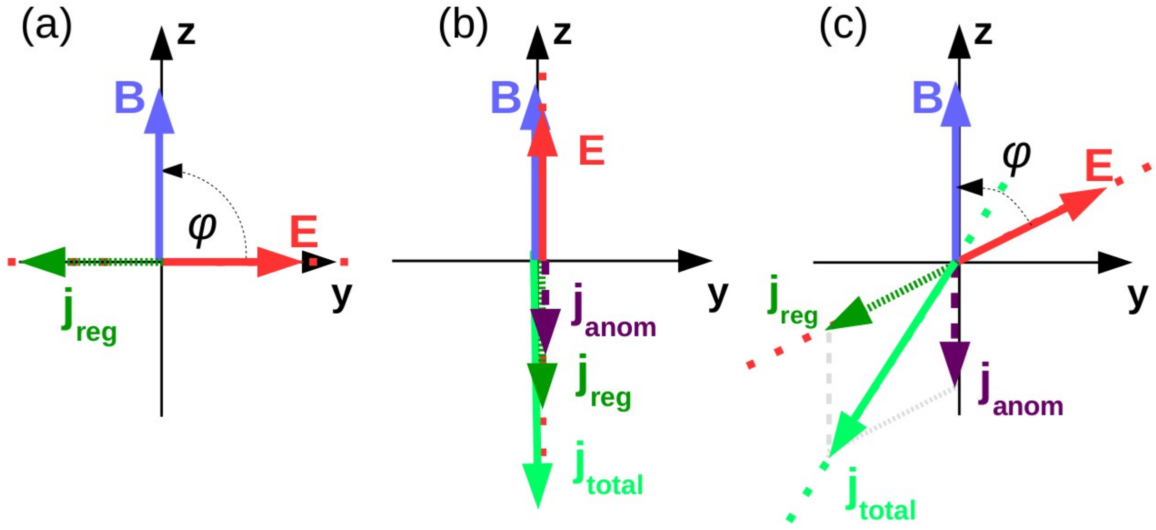
- Vacuum of QED is trivial (no theta-term)

Net effects vanish in vacuum or thermal equilibrium

- What about non-equilibrium: Schwinger + chiral magnetic effect

$$\mathbf{j}_{reg} \sim \mathbf{E}$$

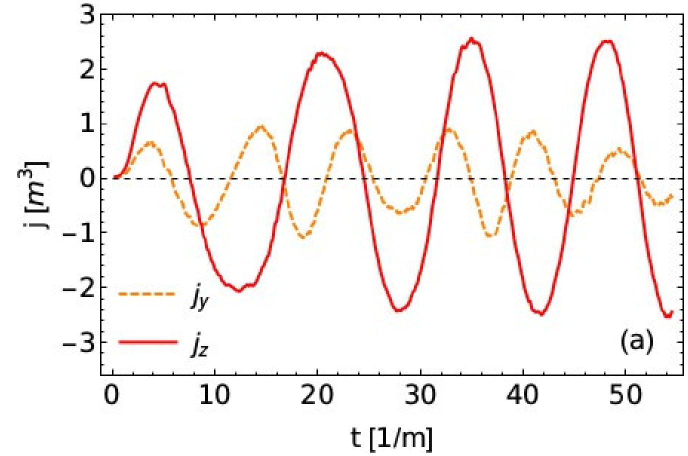
$$\mathbf{j}_{anom} \sim \mathbf{B}$$

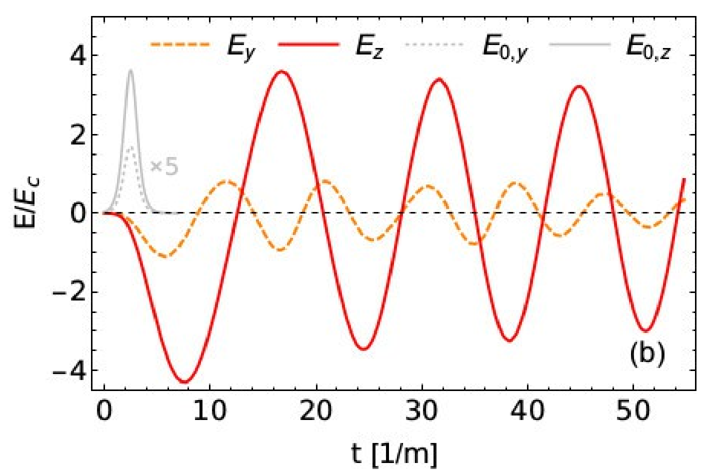


[Mueller, FH, Berges, PRL 117 (2016)]

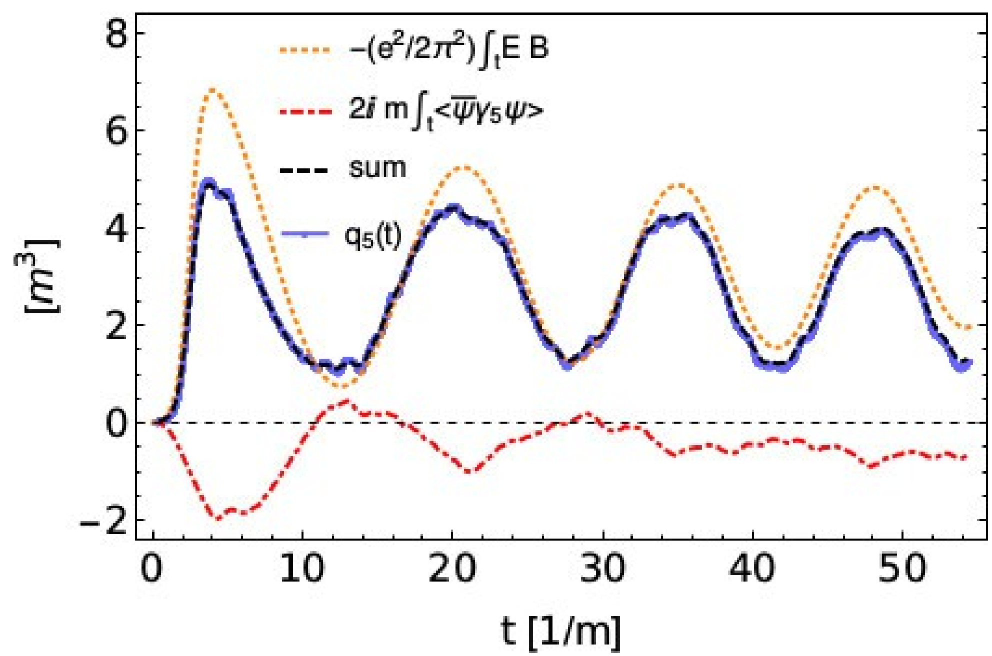
Plasma oscillations & anomaly equation

Out-of phase plasma oscillations





anomaly equation satisfied

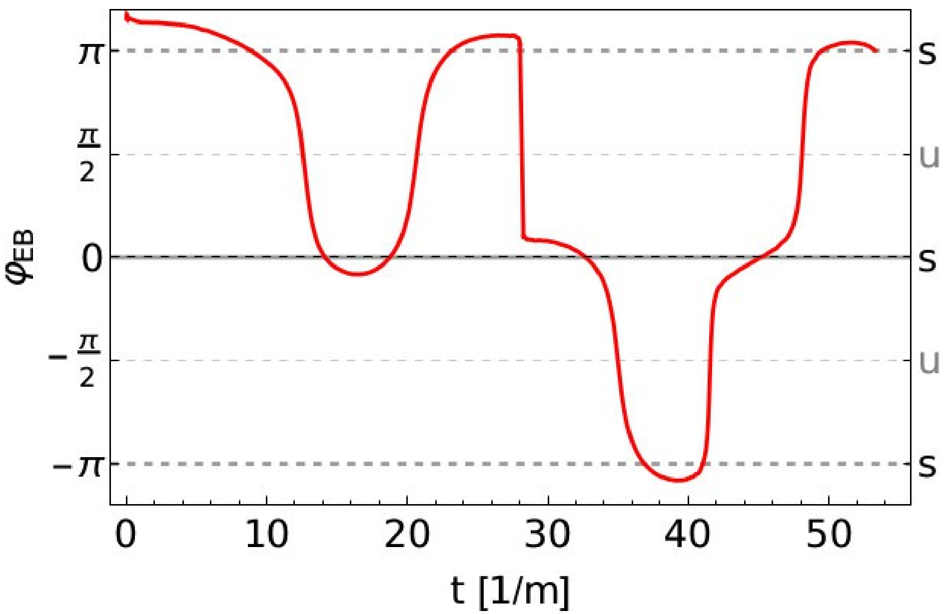


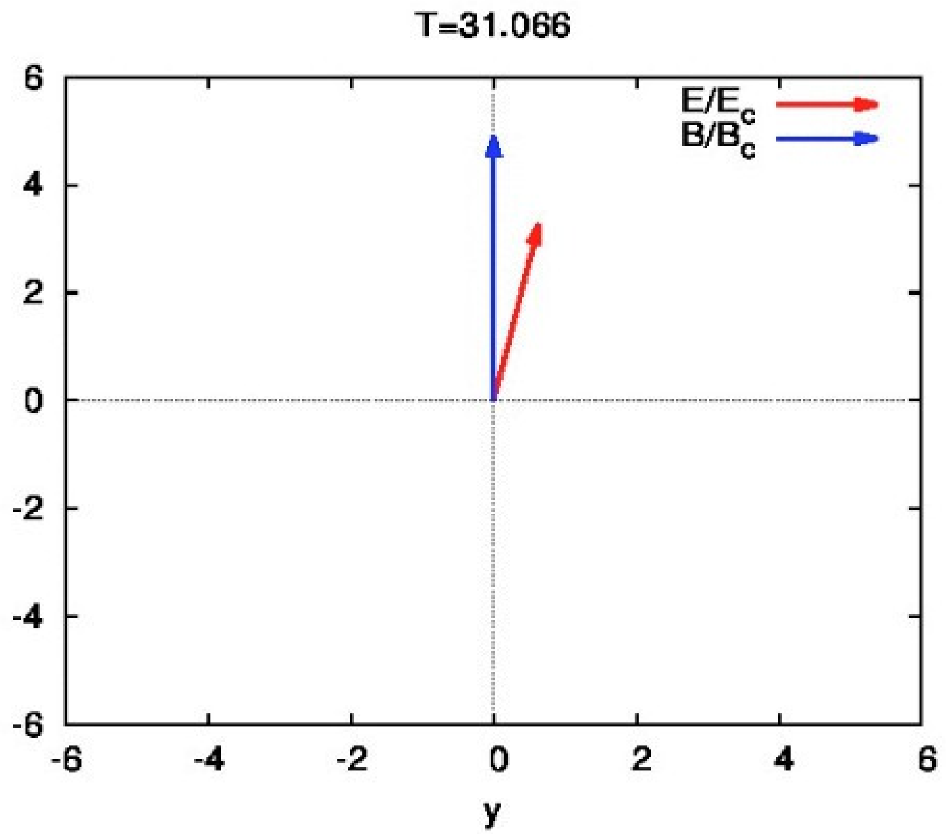
[cf. also talk by S. Schlichting]

[Mueller, FH, Berges, PRL 117 (2016)]

Anomaly-induced dynamical refringence

tracking behavior: longest time near collinear configurations

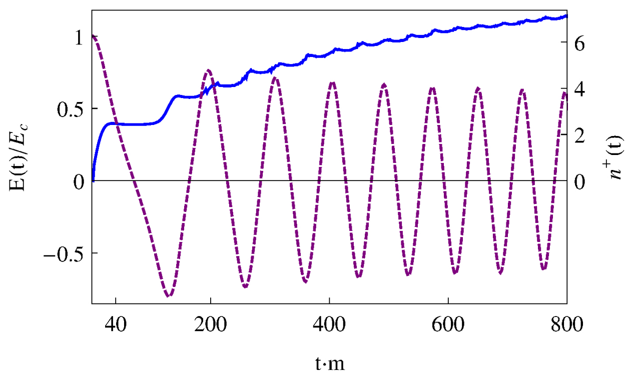




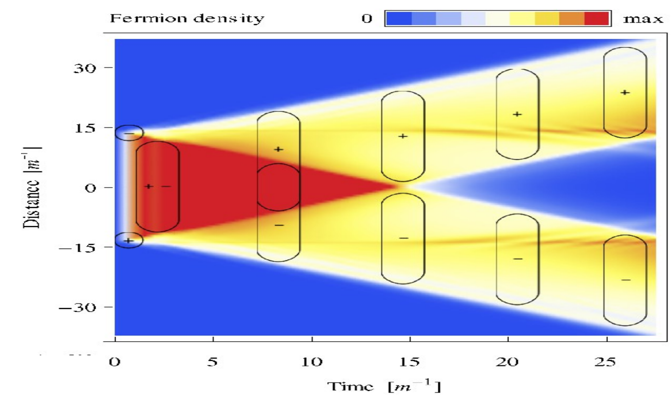
system tries to align the electric and magnetic field directions

Summary (take-home lessons)

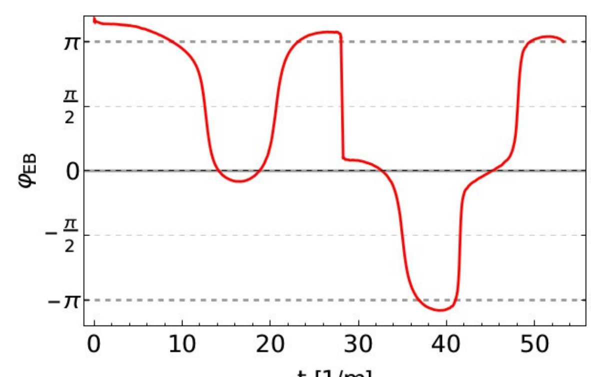
Schwinger effect and plasma oscillations



QED (1+1) string breaking = two-stage process



axial anomaly in QED leads to an anomalous rotation



Thank you!