IR and UV effects in the evolution of the Large Scale Structure

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The Big Bang And The Little Bangs - Non-Equilibrium Phenomena In Cosmology And In Heavy-Ion Collisions - Cern, 19/8/2016

Outline

- ✤ The coarse-grained Vlasov equation
- ✤ Why beyond standard PT (SPT)
- ✤ IR effects and safe resummations
- ✤ The UV failure of SPT and effective/mixed approaches

The LSS challenge(s)

Learn fundamental physics from Large Scale Structure measurements

Initial metric perturbations: spectra, non-gaussianity Properties of the DM (cold, mixed, warm, fuzzy, …) Neutrino masses GR constraints Properties of DE

Data!

Data!

O(10^9) photometric redshifts of galaxies, full sky (LSST) at z< 1.5

ARK ENFRO

Photometric redshifts

z and beyond beyond beyond a start of

条

EUCI

O(10^7) spectroscopic redshifts of galaxies, $O(10^{19})$ photometric redshifts of galaxies, full sky $\frac{\text{det}}{\text{d}t}$ z< 1.5 $rac{1}{2}$

Data!

Linear and non-linear scales

linear Power Spectrum @z=0, ΛCDM

Vlasov Equation

Liouville theorem+ neglect non-gravitational interactions:

$$
\frac{d}{d\tau} f_{mic} = \left[\frac{\partial}{\partial \tau} + \frac{p^i}{am} \frac{\partial}{\partial x^i} - am \nabla_x^i \phi(\mathbf{x}, \tau) \right] f_{mic}(\mathbf{x}, \mathbf{p}, \tau) = 0
$$

moments:

…

$$
n_{mic}(\mathbf{x}, \tau) = \int d^3p f_{mic}(\mathbf{x}, \mathbf{p}, \tau)
$$
 density

$$
\mathbf{v}_{mic}(\mathbf{x}, \tau) = \frac{1}{n_{mic}(\mathbf{x}, \tau)} \int d^3p \, \frac{\mathbf{p}}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau) \quad \text{velocity}
$$

$$
\sigma_{mic}^{ij}(\mathbf{x},\tau) = \frac{1}{n_{mic}(\mathbf{x},\tau)} \int d^3p \; \frac{p^i}{am} \frac{p^j}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau) - v_{mic}^i(\mathbf{x}, \tau) v_{mic}^j(\mathbf{x}, \tau) \; \text{ (respectly) } \; \text{ (respectively) } \; \
$$

From particles to fluids

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 . Manzotti, Peloso, M.P., Viel, Villaescusa Navarro, 1407.1342, Hulemann, Kopp, 1407.4810 … Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10

 $f_{mic}(x, p, \tau) = \sum \delta_D(x - x_n(\tau)) \delta_D(p - p_n(\tau))$ Satisfies the "Vlasov *n* eq."

From particles to fluids

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 $f(x, p, \tau) \equiv$ 1 *V* Z $d^3y\mathcal{W}(y/L_{UV})f_{mic}(x+y,p,\tau)$

Coarse-grained Vlasov equation

$$
\left[\frac{\partial}{\partial \tau} + \frac{p^i}{am} \frac{\partial}{\partial x^i} - am\nabla_x^i \phi(\mathbf{x}, \tau) \frac{\partial}{\partial p^i}\right] f(\mathbf{x}, \mathbf{p}, \tau) =
$$

am
$$
\left[\langle \frac{\partial}{\partial p^i} f_{mic} \nabla^i \phi_{mic} \rangle_{L_{UV}}(\mathbf{x}, \mathbf{p}, \tau) - \frac{\partial}{\partial p^i} f(\mathbf{x}, \mathbf{p}, \tau) \nabla_x^i \phi(\mathbf{x}, \tau)\right]
$$

short scales

large scales

$$
\langle g \rangle_{L_{UV}}(\mathbf{x}) \equiv \frac{1}{V_{UV}} \int d^3 y \mathcal{W}(y/L_{UV}) g(\mathbf{x} + \mathbf{y})
$$

\n
$$
\phi = \langle \phi_{mic} \rangle_{L_{UV}}
$$

\n
$$
f = \langle f_{mic} \rangle_{L_{UV}}
$$

Vlasov equation in the $L_{\text{uv}} \rightarrow 0$ limit!

Taking moments…

Exact large scale dynamics for density and velocity fields

$$
\frac{\partial}{\partial \tau} \delta(\mathbf{x}) + \frac{\partial}{\partial x^i} \big[(1 + \delta(\mathbf{x})) v^i(\mathbf{x}) \big] = 0
$$

 $\left\{ \right.$

$$
\frac{\partial}{\partial \tau} v^{i}(\mathbf{x}) + \mathcal{H}v^{i}(\mathbf{x}) + v^{k}(\mathbf{x}) \frac{\partial}{\partial x^{k}} v^{i}(\mathbf{x}) = -\nabla_{x}^{i} \phi(\mathbf{x}) - J_{\sigma}^{i}(\mathbf{x}) - J_{1}^{i}(\mathbf{x})
$$
\n
$$
\nabla^{2} \phi(\mathbf{x}) = \frac{3}{2} \Omega_{M} \mathcal{H}^{2} \delta(\mathbf{x})
$$
\n
$$
n(\mathbf{x}) = n_{0} (1 + \delta(\mathbf{x})) = n_{0} (1 + \langle \delta_{mic} \rangle(\mathbf{x}))
$$
\n
$$
v^{i}(\mathbf{x}) = \frac{\langle (1 + \delta_{mic}) v_{mic}^{i} \rangle(\mathbf{x})}{1 + \delta(\mathbf{x})}
$$

external input on UV-physics needed to close the system

 Ω

$$
J_{\sigma}^{i}(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} \frac{\partial}{\partial x^{k}} (n(\mathbf{x}) \sigma^{ki}(\mathbf{x}))
$$

$$
J_{1}^{i}(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} (\langle n_{mic} \nabla^{i} \phi_{mic} \rangle(\mathbf{x}) - n(\mathbf{x}) \nabla^{i} \phi(\mathbf{x}))
$$

Single stream regime

Set
$$
\sigma^{ij} = \omega^{ijk} = \cdots = \nabla \times \mathbf{v} = 0
$$

$$
\left(J^i_\sigma=J^i_1=0\right)
$$

$$
f(\mathbf{x}, \mathbf{p}, \tau) = n(\mathbf{x}, \tau) \delta_D(\mathbf{p} - a m \mathbf{v}(\mathbf{x}, \tau))
$$

$$
System described by \delta(\mathbf{x}, \tau), \ \theta(\mathbf{x}, \tau) \equiv \nabla \cdot \mathbf{v}(\mathbf{x}, \tau)
$$

 $\partial \mathbf{v}$ $\partial \tau$ $+ \mathcal{H} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = - \nabla \phi$ $\nabla^2 \phi =$ 3 2 Ω_M \mathcal{H}^2 δ continuity Euler Poisson $\partial \delta$ $\frac{\partial \phi}{\partial \tau} + \nabla \cdot ((1 + \delta) \mathbf{v})$

Single stream regime

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 $\partial \mathbf{v}$ $\partial \tau$ $+ \mathcal{H} \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{v} = - \nabla \phi$ $\nabla^2 \phi =$ 3 2 Ω_M \mathcal{H}^2 δ continuity Euler Poisson warning: self-consistent … but ultimately wrong! $\partial \delta$ $\frac{\partial \phi}{\partial \tau} + \nabla \cdot ((1 + \delta) \mathbf{v})$

Linear Perturbation Theory

$$
\frac{\partial \delta}{\partial \tau} + \theta = 0
$$
\n
$$
\frac{\partial \theta}{\partial \tau} + \mathcal{H}\theta = -\nabla^2 \phi
$$
\n
$$
\delta + \mathcal{H}\dot{\delta} = \frac{3}{2}\Omega_M \mathcal{H}^2 \delta
$$
\n
$$
\nabla^2 \phi = \frac{3}{2}\Omega_M \mathcal{H}^2 \delta
$$
\nlinear GR equation for $k \gg \mathcal{H}$

Solution:
$$
\delta^{(1)}(\mathbf{k}, \tau) = -\frac{\theta^{(1)}(\mathbf{k}, \tau)}{\mathcal{H}f_{\pm}} = \delta(\mathbf{k}, \tau_{in}) D_{\pm}(\tau)
$$
 $\frac{(D_{\pm}(\tau_{in}) = 1)}{\int f_{\pm} = \frac{d \ln D_{\pm}}{d \ln \sigma}}$

growing/decaying mode

 $\sqrt{2}$

 $f_{\pm} \equiv$

 $\frac{d \ln D_{\pm}}{2}$

◆

d ln *a*

For EdS (
$$
\Omega_M=1
$$
): $D_{\pm} = \left(\frac{a(\tau)}{a(\tau_{in})}\right)^{f_{\pm}}$ $f_{+} = 1, f_{-} = -3/2$

Standard Perturbation Theory

It is an expansion of the density and velocity fields in terms of the initial conditions

Compact notation: $\eta = \log(a/a_{in})$

The continuity+Euler+Poisson system reads:

 $(\delta_{ab}\partial_{\eta} + \Omega_{ab}(\eta)) \varphi_b(\mathbf{k}, \eta) = e^{\eta}$ $\int d^3q$ $(2\pi)^3$ d^3p $\frac{d^2 P}{(2\pi)^3} \delta_D({\bf k} - {\bf q} - {\bf p}) \gamma_{abc}({\bf k},{\bf q},{\bf p}) \varphi_b({\bf q},\eta) \varphi_c({\bf p},\eta)$

nonlinear

$$
\Omega_{ab}(\eta) = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(\eta) & 2 + \frac{d\log H}{d\eta} \end{pmatrix}
$$

$$
\gamma_{112}(\mathbf{k}, \mathbf{q}, \mathbf{p}) = \gamma_{121}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \frac{\mathbf{k} \cdot \mathbf{p}}{2p^2}
$$

$$
\gamma_{222}(\mathbf{k}, \mathbf{q}, \mathbf{p}) = \frac{k^2 \mathbf{q} \cdot \mathbf{p}}{2 q^2 p^2}
$$

Standard Perturbation Theory

It is an expansion of the density and velocity fields in terms of the initial conditions

$$
Compact notation: \left(\begin{array}{c} \varphi_1(\eta, \mathbf{k}) \\ \varphi_2(\eta, \mathbf{k}) \end{array} \right)
$$

$$
\frac{\varphi_1(\eta, \mathbf{k})}{\varphi_2(\eta, \mathbf{k})} = e^{-\eta} \begin{pmatrix} \delta(\eta, \mathbf{k}) \\ -\theta(\eta, \mathbf{k})/\mathcal{H} \end{pmatrix} \qquad \eta = \log(a/a_{in})
$$

The continuity+Euler+Poisson system reads:

$$
(\delta_{ab}\partial_{\eta} + \Omega_{ab}(\eta))\varphi_b(\mathbf{k}, \eta) = e^{\eta} \int \frac{d^3q}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \delta_D(\mathbf{k} - \mathbf{q} - \mathbf{p}) \gamma_{abc}(\mathbf{k}, \mathbf{q}, \mathbf{p}) \varphi_b(\mathbf{q}, \eta) \varphi_c(\mathbf{p}, \eta)
$$

linear
nonlinear

$$
\Omega_{ab}(\eta) = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(\eta) & 2 + \frac{d\log H}{d\eta} \end{pmatrix}
$$

$$
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$$

$$
\gamma_{222}(\mathbf{k}, \mathbf{q}, \mathbf{p}) = \frac{k^2 \mathbf{q} \cdot \mathbf{p}}{2 q^2 p^2}
$$

Iterative solution (EdS)

 $\varphi^{(1)}_a(\mathbf k, \eta) = g_{ab}(\eta) \varphi^{in}_b(\mathbf k)$ $g_{ab}(\eta) = \begin{bmatrix} 3/5 & 2/5 \\ 3/5 & 2/5 \end{bmatrix}$ 3*/*5 2*/*5 ◆ $+ e^{-5/2 \eta}$ $\binom{2/5}{2}$ -2/5 3*/*5 3*/*5 $\int \Theta(\eta)$ linear propagator $\varphi^{(2)}_a(\mathbf k, \eta) = \int^\eta$ 0 $ds \, g_{ab}(\eta-s) \, e^s \, I_{\mathbf{k},\mathbf{q},\mathbf{p}} \gamma_{bcd}(\mathbf{k},\mathbf{q},\mathbf{p}) \varphi^{(1)}_c(\mathbf{q},s) \varphi^{(1)}_d(\mathbf{p},s)$ linear solution 2nd order solution nth order solution $\varphi_{a}^{(n)}(\mathbf k, \eta) = I_{\mathbf k, \mathbf q_1, \cdots, \mathbf q_n}$ $F^{(n)}_{a,b_1,\cdots,b_n}(\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n};\eta)\varphi_{b_1}^{in}(\mathbf{q_1})\cdots\varphi_{b_n}^{in}(\mathbf{q_n})$

$$
\left(I_{\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n}} \equiv \int \frac{d^3q_1}{(2\pi)^3} \cdots \frac{d^3q_n}{(2\pi)^3} \delta_D(\mathbf{k} - \sum_{i=1}^n \mathbf{q_i})\right)
$$

IODE MODE COUPLING

Correlators

If the initial conditions are gaussian, then only correlators involving an even number of fields are non-vanishing tree-level

Power spectrum:
$$
\langle \varphi_a(\mathbf{k}, \eta) \varphi_b(\mathbf{k}', \eta) \rangle = \langle \varphi_a^{(1)}(\mathbf{k}, \eta) \varphi_b^{(1)}(\mathbf{k}', \eta) \rangle
$$

\n
$$
\longrightarrow + \langle \varphi_a^{(1)}(\mathbf{k}, \eta) \varphi_b^{(3)}(\mathbf{k}', \eta) \rangle + \langle \varphi_a^{(3)}(\mathbf{k}, \eta) \varphi_b^{(1)}(\mathbf{k}', \eta) \rangle
$$
\none-loop
\n
$$
\longrightarrow + \langle \varphi_a^{(2)}(\mathbf{k}, \eta) \varphi_b^{(2)}(\mathbf{k}', \eta) \rangle + O((\varphi^{in})^6)
$$

Bispectrum: $\langle \varphi_a(\mathbf{k}, \eta) \varphi_b(\mathbf{k}', \eta) \varphi_c(\mathbf{k}'', \eta) \rangle = \langle \varphi_a^{(2)}(\mathbf{k}, \eta) \varphi_b^{(1)}(\mathbf{k}', \eta) \varphi_c^{(1)}(\mathbf{k}'', \eta) \rangle$ tree-level $+2$ permutations $+ O((\varphi^{in})^6)$

Diagrammar

propagator (linear growth factor): $-i g_{ab}(\eta_a, \eta_b)$ $\text{linear power spectrum:} \qquad P_{ab}^L(\eta_a, \eta_b; \mathbf{k})$

interaction vertex: $-i\,e^\eta\,\gamma_{abc}({\bf k_a,\,k_b,\,k_c})$

Diagrammar

propagator (linear growth factor): $-i g_{ab}(\eta_a, \eta_b)$ $\text{linear power spectrum:} \qquad P_{ab}^L(\eta_a, \eta_b; \mathbf{k})$ interaction vertex: $-i\,e^\eta\,\gamma_{abc}({\bf k_a,\,k_b,\,k_c})$

Example: 1-loop correction to the density power spectrum:

PT in the BAO range

the PT series blows up in the BAO range

... but it can be resummed

`coherence momentum' damping in the BAO range! $k_{ch} = (\sigma e^{\eta})^{-1} \simeq 0.15 \, \mathrm{h} \, \mathrm{Mpc}^{-1}$

... but it can be resummed

`coherence momentum' damping in the BAO range! $k_{ch} = (\sigma e^{\eta})^{-1} \simeq 0.15 \, \mathrm{h} \, \mathrm{Mpc}^{-1}$

RPT: use G, and not g, as the linear propagator

Physical meaning of the IR resummation

$$
\bar{\delta}_{\alpha}(\mathbf{x},\tau) = \delta_{\alpha}(\mathbf{x} - \mathbf{D}_{\alpha}(\mathbf{x},\tau),\tau). \quad \mathbf{D}_{\alpha}(\mathbf{x},\tau) \equiv \int_{\tau_{in}} d\tau' \mathbf{v}_{\alpha,\text{long}}(\mathbf{x},\tau') \simeq \mathbf{D}_{\alpha}(\tau)
$$

$$
\langle \delta_{\alpha}(\mathbf{k}, \tau) \delta_{\alpha}(\mathbf{k}', \tau') \rangle = \langle \bar{\delta}_{\alpha}(\mathbf{k}, \tau) \bar{\delta}_{\alpha}(\mathbf{k}', \tau') \rangle \langle e^{-i\mathbf{k} \cdot (\mathbf{D}_{\alpha}(\tau) - \mathbf{D}_{\alpha}(\tau'))} \rangle \n= \langle \bar{\delta}_{\alpha}(\mathbf{k}, \tau) \bar{\delta}_{\alpha}(\mathbf{k}', \tau') \rangle e^{\frac{-k^2 \sigma_v^2 (D(\tau) - D(\tau'))^2}{2}} \n\sigma_v^2 = -\frac{1}{3\mathcal{H}^2 f^2} \int^{\Lambda} d^3 q \langle v_{long}^i(q) v_{long}^i(q) \rangle' = \frac{1}{3} \int^{\Lambda} d^3 q \frac{P^0(q)}{q^2}
$$

 $3H^2f^2$

PT at any finite order truncates the full exponential behavior (P_{13}, P_{15}, \ldots) (IR) Resummations take into account the large scale bulk motions at all PT orders

3

 q^2

Zel'dovich and beyond

PMC/Plin PMC1loop/Plin PMC-TRG-1loop-sig/Plin $PMC-TRG-\sigma-gPhi/Plin$ PMC/Plin (Nbody) 1.5 2.0 Interpolation built in the equation! Small k limit: 1-№0p $\bf 5$ Large k limit: Zel'dovich

Large scale flows and BAO's

O(6 Mpc/h)

displacements

reconstruction

Seo et al, 0910.5005, Padmanabhan et al 1202.0090, Tassev, Zaldarriaga 1203.6066, ... , 0910.5005, Padmanabhan et al 1202.0090, Tassey, Zaldarriaga 1205. of the black points from the centroid of the blue points is shown in the inset. (*top right*) We evolve the particles to the present day, here

Effect on the Correlation *The e*↵ Function *ect of massive neutrinos on the BAO peak* ¹²

(simplified) Zel'dovich approximation α information is basically confined to the *G*²(*k*; *z*)*Plin*(*k*) term which, after Fourier d proximation has been studied that been studied that the recent literature a μ α **P** α **P** computation involves momentum integrals in which two or more linear PS evaluated at di \mathbf{I}_c are convolved. Therefore, as we will demonstrate below, the BAO \mathbf{I}_c information is basically confined to the *G*²(*k*; *z*)*Plin*(*k*) term which, after Fourier approximation, a The propagator in the propagator in the propagator in the recent literature \mathcal{I}_1 is the recent literature \mathcal{I}_2 , \mathcal{I}_3 $\sqrt{3}$ *P*(2)(*k, z*) = *Plin*(*k, z*) + *P*13(*k, z*) + *P*22(*k, z*) + *O*(2 loop)*.* (9) annroximation to higher loop orders, possibly including also a better treatment of the set of the set of the s $\frac{1}{2}$ the short modes along the e $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{2}$

computation involves momentum integrals integrals in \mathcal{F} evaluated two or more linear PS evaluated in \mathcal{F}

$$
G^{Zeld}(k, z) = e^{-\frac{k^2 \sigma_v^2(z)}{2}}
$$

\n
$$
P_{11}^P(k, z) = e^{-\frac{k^2 \sigma_v^2(z)}{2}} P^{lin}(k, z)
$$

\n
$$
\sigma_v^2(z) = \frac{1}{3} \int \frac{d^3q}{(2\pi)^3} \frac{P^{lin}(q, z)}{q^2}
$$

In Zel'dovich approximation it is given by (see Appendix B)

although computationally more and more demanding.

In Zelf down and the Linux B) is given by (see Appendix B) is given by (see Appendix B) is given by (see Appendix B)
In the see Appendix B) is given by (see Appendix B) is given by (see Appendix B) is given by (see Appendi

$$
P_{11}^{P}(k,z) = e^{-\frac{k^2 \sigma_v^2(z)}{2}} P^{lin}(k;z)
$$

v,⇤(*z*) ' 2 ⌃0*,*⇤(*k, z*), we see

linear velocity dispersion: In the following, we will most like the following, we will most \mathbb{R}^n obtained in \mathbb{R}^n 3 (2⇡)³ mical velocity *v ocity* $\mathbf{1}$: $\mathbf{1}$ $\mathbf{2}$ $\mathbf{3}$ $\mathbf{4}$ $\mathbf{5}$ $\mathbf{5}$ $\mathbf{6}$ $\mathbf{7}$ $\mathbf{8}$ $\mathbf{1}$ $\mathbf{5}$ $\mathbf{1}$ $\mathbf{6}$ $\mathbf{1}$ $\mathbf{5}$ $\mathbf{1}$ $\mathbf{5}$ $\mathbf{1}$ $\mathbf{6}$ $\mathbf{1}$ $\mathbf{5}$ $\mathbf{1}$ $\mathbf{5}$ \math \overline{a} mear veroeny and persion. *Plin*(*q*)

contains information on inteal r 5, growth factor,... $\overline{}$ contains information on linear PS, growth factor,… (*R*) tormation *d d d d d d ed ez ed e ^q*² *,* (14) the CF changes by contains information on

$$
\delta \xi(R) = \frac{1}{2\pi^2} \int dq \, q^2 \, \delta P^{lin}(q) \left(\frac{\sin(qR)}{qR} \, e^{-q^2 \sigma_v^2} - \frac{1}{3} \frac{\xi_2(R)}{q^2 R^2} \right)
$$

$$
\xi_n(R) \equiv \frac{1}{2\pi^2 R} \int_0^\infty dq \, q \, (qR)^n \sin(qR) \, P(q)
$$

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477 *A <i>R*_(*R*) *R*₂</sup> *R*_{*R*}₂ *R*_{*R*}₂ *R*_{*R*}_{*R*}_{*R*}_{*R*}_{*R*}^{*R*}_{*R*}^{*R*}_{*R*}^{*R*}_{*R*}^{*R*}_{*R*}^{*R*}_{*R*}^{*R*} *P***eloso, MP**, Viel, Villaescusa-Navarro, 1505.07477

Redshift ratios The black solid (dashed) line at small *R*²⇠ values in the left panel is the di↵erence (5)

 $\mathcal{F}_{\mathcal{A}}$ is real space, for mass $\mathcal{F}_{\mathcal{A}}$ in real space, and at redshift in r

also rescaled by *R*². The black solid line in the right panel is the FrankenEmu CF,

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Effect of Massive neutrinos on BAO peak

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477 with diagonal masses. The first, second, and the first, second, and third row show the CF for form α

Massive neutrinos

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477 S_0 and S_0 and S_1 and S_2 and S_3 are S_4 are T_1 and T_2 and T_3 and T_4 and T_5 and T_5 and T_6 and T_7 and T_8 and T_7 and T_8 and T_9 and T_9 and T_9 and T_9 and T_9 and T $\frac{1}{2}$ eloso, $\frac{1}{2}$ $t_{\rm H}$

Mode coupling-Response functions INFN, Sezione di Padova, via Marzolo 8, I-35131, Padova, Italy Abstract. The Response function

The nonlinear PS is a functional of the initial one (in a given cosmology and assuming no PNG): $P_{ab}[P^0](\mathbf{k}, \eta)$

SPT is an expansion around $P^0(q)=0$ density and velocity divergence, the nonlinear PS at time α is a functional of the linear PS at time α $P^0(q) = 0$, that is, an expansion around $P^0(q) = 0$

$$
P_{ab}[P^{0}](\mathbf{k};\eta) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^{3}q_{1} \cdots d^{3}q_{n} \frac{\delta^{n} P_{ab}[P^{0}](\mathbf{k};\eta)}{\delta P^{0}(\mathbf{q}_{1}) \cdots \delta P^{0}(\mathbf{q}_{n})}\Big|_{P^{0}=0} P^{0}(\mathbf{q}_{1}) \cdots P^{0}(\mathbf{q}_{n})
$$

n=1 linear order (= "0-loop") $n=2$ "1-loop"

ab [*P*⁰](k; ⌘)

 $a, \dots, d = 1$ density \dots *P*0(*p*) *P*0(*p*) *P*0(*p*) *P*⁰ $a, \cdots, d = 2$ velocity div.

Mode coupling-Response functions *Response function* 2 centered around a non-vanishing linear PS, *P*¯⁰(q), namely, *Response function* 2

We can instead expand around a reference PS: $P^0(q) = \bar{P}^0(q)$ centered around a non-vanishing linear PS, *P*¯⁰(q), namely, $(q) = \bar{P}^{0}(q)$ \vec{P} around a reference PS, $P^0(a) - \bar{P}^0(a)$

$$
P_{ab}[P^0](\mathbf{k};\eta) = P_{ab}[\bar{P}^0](\mathbf{k};\eta)
$$

+
$$
\sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots \delta P^0(\mathbf{q}_n)} \right|_{P^0=\bar{P}^0} \delta P^0(\mathbf{q}_1)\cdots \delta P^0(\mathbf{q}_n),
$$

=
$$
P_{ab}[\bar{P}^0](\mathbf{k};\eta) + \int \frac{dq}{q} K_{ab}(k,q;\eta) \delta P^0(q) + \cdots, \qquad \delta P^0(\mathbf{q}) \equiv P^0(\mathbf{q}) - \bar{P}^0(\mathbf{q})
$$

 I n the following, we will consider a di I erent expansion, and I

^Kab(*k, q*; ⌘) ⌘ *^q*³ \mathfrak{c} $\Omega_{ab}(\kappa, q; \eta) = q$ *f* $\delta P^{\circ}(\mathbf{q})$ $|_{P^0 = \bar{P}^0}$ $K_{ab}(k, q; \eta) \equiv q^3$ Z $d\Omega_{\bf q}$ $\delta P_{ab}[P^0]({\bf k};\eta)$ $\delta P^0({\bf q})$ $\overline{1}$ $\overline{}$ $\overline{}$ $|P^0 = \bar{P}^0$ Linear response function:

Non porturbotive (cote contributions from e¹¹ CDT endoug) their permittent (get communities in the district) $\text{tr}\left(\mathbf{r}\right)$ formally write an expansion for the LRF in terms of the PT kernels therefore contributions from all SPT orders formally write an expansion for the LRF in terms of the PT kernels Non-perturbative (gets contributions from all SPT orders)

 P _{*P*} *P*_p^{*p*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}_{*P*}^{*P*}*P*_{*P*}^{*P*} or mor *Pab*[*P*⁰](k; ⌘) *FILCIEI* \overline{a} \mathbf{r} *P*₁**1**</sup> Key object for more efficient interpolators ?

$$
K_{ab}(k, q; \eta) = q \, \delta_D(k - q) \, G_{ac}(k; \eta, \eta_{in}) u_c \, G_{bd}(k; \eta, \eta_{in}) u_d - \frac{1}{2} \frac{q^3}{(2\pi)^3} \int d\Omega_{\mathbf{q}} \, \langle \varphi_a(\mathbf{k}; \eta) \chi_c(-\mathbf{q}; \eta_{in}) \chi_d(\mathbf{q}; \eta_{in}) \varphi_b(-\mathbf{k}; \eta) \rangle_c' \, u_c u_d,
$$

 $\delta \varphi_b(\mathbf{k}, \eta_{in})'$ eq. *methods from Matarrese, MP, '07* $G_{ab}(k;\eta,\eta_{in}) = \langle \frac{\delta \varphi_a(\mathbf{k},\eta)}{\delta \varphi_a(\mathbf{k},\eta)} \rangle$ $\delta\varphi_{b}(\mathbf{k},\eta_{in})$ $\langle \rangle' = -i \langle \varphi_a(\mathbf{k},\eta) \chi_b(-\mathbf{k},\eta_{in}) \rangle'$

IR consistency relations

M. Peloso, M.P. 1302.0223/1310.7915 A. Kehagias, A. Riotto et al 1302.0130 Creminelli et al. 1309.3557 P. Valageas 1311.1236

the effect of a long wavelength (time dependent) velocity mode can be exactly reabsorbed by a change of coordinates in the uniform limit

exact relations between N and N+1 point functions

No 1/q dependence for soft q

 $K_{ab}(k,q) \propto q^3$ for $q \ll k$

IR screening regularization schemes employed in analytical models. \Box in screening duce a well-defined kernel function and investigate it at

ment of the kernel structure from cosmological N-body

IR screening regularization schemes employed in analytical models. \Box in screening duce a well-defined kernel function and investigate it at

ment of the kernel structure from cosmological N-body

Resummations and IR sensitivity If we assume growing mode initial conditions then '*in* ¹ (q) = '*in* ² (q), and therefore ¹¹ (*q*) = *Pin* ¹² (*q*) = *Pin* ²² (*q*) = *Plin*(*q, zin*). Moreover, if we are interested in the density PS we have to take *a* = *b* = 1. This gives exactly the expression in (1) with *^G*(*k, z*) ⌘ *^G*11(*k, z*) + *^G*12(*k, z*)*, PMC*(*k, z*) ⌘ *^P MC* ¹¹ (*k, z*)*.* (A.7) *^G*(*k, z*) ⌘ *^G*11(*k, z*) + *^G*12(*k, z*)*, PMC*(*k, z*) ⌘ *^P MC* ¹¹ (*k, z*)*.* (A.7) *dqmin* = *dq P*11(*k*; ⌘*, qmin*) *s* and IR se ⁼ *P*11(*k*; ⌘*, qmin*) *dP*11(*k*; ⌘*, qmin*) $\overline{}$ $\sqrt{ }$ s and IR sensitivity *P*⁰(*qmin*) *^P*⁰ (*qmin*)*,* (C.11)

*dq P*11(*k*; ⌘*, qmin*)

*P*⁰(*qmin*) *^P*⁰

¹¹ (*k, z*)*.* (A.7)

⁼ *P*11(*k*; ⌘*, qmin*)

*P*⁰(*qmin*) *^P*⁰

10 m Lagrangian Hame PS in Lagrangian framework $TC: L. L. (17)$ *^K*(*k*; *^q*) = *^P NL*(*k*) PS in Lagrangian framework*P*000 *, (CO) , (CO*

Appendix B. Zel'dovich approximation approximation approximation approximation approximation approximation appr

To consider the construction of the construction \mathcal{A}

In this appendix we discuss the relation between formula (4) and the relation between formula (4) and the Zel'
In this appendix we discuss the Zel'dovich formula (4) and the Zel'dovich formula (4) and the Zel'dovich formu $\mathbf{F}(\mathbf{q}) = \mathbf{x} - \mathbf{q}$ as given the following equations, which we omit in the following equations, which we obtain \mathbf{r} and \mathbf{r} and \mathbf{r} are following to \mathbf{r} and \mathbf{r} are following to \mathbf{r} a $\mathbf{\Psi}(\mathbf{q}) = \mathbf{x} - \mathbf{q}$ displacement field $\bf{H}(\bf{q}) = 2$ *^ei*k*·* (q) ¹ *,* (B.1)

the linear response function defined in \mathcal{L}_1 and \mathcal{L}_2 is \mathcal{L}_3 . In function defined in \mathcal{L}_3

particles have moved from the the initial (Lagrangian) positions q by the dispacement $f(1 + o(\mathbf{X})) a^{\circ} x = a^{\circ} q$ mass conservation $(1 + \delta(\mathbf{x})) d^3x = d^3q$ mass conservation $\frac{1}{2}$ mass conser

In this appendix we discuss the relation between formula (4) and the Zel'dovich

่
Va mass conservation *q ei*k*·*^q

^G(*k, z*) ⌘ *^G*11(*k, z*) + *^G*12(*k, z*)*, PMC*(*k, z*) ⌘ *^P MC*

If we assume growing mode initial conditions then '*in*

$$
\delta(\mathbf{k}) = \int d^3x \, \delta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} = \int d^3q \, e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1\right)
$$

$$
P(k; \tau, \tau') = \int d^3 r \ e^{-i \mathbf{k} \cdot \mathbf{r}} \left(\langle e^{-i \mathbf{k} \cdot \Delta \Psi} \rangle - 1 \right) \qquad \begin{aligned} \Delta \Psi &= \Psi(\mathbf{q}, \tau) - \Psi(\mathbf{q}', \tau'), \\ \mathbf{r} &= \mathbf{q} - \mathbf{q}' \end{aligned}
$$

Zel'dovich approximation: displacement field from linear PT $Zol'dovich approximation: dienlecompact field from linear PT$ Zei dovien approxime The universe Political associations as a very control to the universe Political times Political as Z el' dovich approximate

)*,* r = q q⁰

$$
\Psi_Z(\mathbf{q},\tau) = \int_0^{\tau} d\tau'' \mathbf{v}(\mathbf{q},\tau'') = i D_{+}(\tau) \int \frac{d^3k}{(2\pi)^3} e^{i\mathbf{q}\cdot\mathbf{k}} \frac{\mathbf{k}}{k^2} \delta_l(\mathbf{k},0), \quad \text{(gaussian field)}
$$

(2⇡)³ ^e*ⁱ* ^q*·*^k ^k

$$
\langle \mathrm{e}^{-i\mathbf{k}\cdot\Delta\mathbf{\Psi}_Z} \rangle = \mathrm{e}^{-\frac{1}{2}k^ik^j\langle \Delta\Psi^i_Z \Delta\Psi^j_Z \rangle}
$$

1

em

$$
P_Z(k; \tau, \tau') = \int d^3r \cos(\mathbf{k} \cdot \mathbf{r}) \left[e^{-\frac{k^2 \sigma^2}{2} (D - D')^2} e^{-DD' (k^2 \sigma^2 - I(k, \mathbf{r}))} - 1 \right]
$$

(*p*)*, I*(k*,* 0) = *k*²

(qmin *·* k)²

,
2010

i
iliyoo

(*p*)*, I*(k*,* 0) = *k*²

*^k*² *l*(k*,* 0)*,* (D.3)

² *.* (D.6)

, (1)

² (*DD*⁰

 $D \equiv D_{+}(\tau), D' \equiv D_{+}(\tau')$ linear growth factor $D \equiv D_1(\tau), D' \equiv D_1(\tau')$ linear growth factor $I(\tau), D' \equiv D_+(\tau')$ linear growth factor

) = ^Z

which, after some standard manipulation gives the control of the Zellowich PS as a standard manipulation gives the control of the co

$$
I(\mathbf{k}, \mathbf{r}) = \int \frac{d^3p}{(2\pi)^3} e^{i \mathbf{p} \cdot \mathbf{r}} \frac{(\mathbf{p} \cdot \mathbf{k})^2}{p^4} P^0(p), \qquad I(\mathbf{k}, 0) = k^2 \sigma^2
$$

*^p*⁴ *^P*⁰

.
Men

which, after some standard manipulation, after some standard manipulation gives the Zellin gives the Zellin PS assembly as a standard manipulation gives the α

)2 \overline{f} $\frac{1}{2}$ DPT , good testing gro [·] resummati $\frac{1}{2}$ $\$ *a*ll orders in SPT good \mathfrak{a}^{\dagger} *d*⌦*^q*min rot *d* for resur *k*²(*D D*⁰ 2 $\frac{1}{2}$ (2⇡)³ cos(^p *·* ^r) *^pⁱ ^p*⁴ *^P*⁰ *^r*² *^I*2(*r*)*,* (4) All orders in SPT, good testing ground for resummations

 $e^{-DD'(k^2\sigma^2 - I(k,\mathbf{r}))}$

insensitive to IR velocity modes with momentum p $p < O(1/r) \simeq O(Min[k, 1/l_{BAO}])$

SPT:

$$
e^{-DD'(k^2\sigma^2 - I(k, \mathbf{r}))} = \sum_{n=0}^{N} \frac{(-1)^n}{n!} \left(DD'(k^2\sigma^2 - I(k, \mathbf{r})) \right)^n
$$

No spurious IR dependence

but truncated "propagator"
$$
e^{-\frac{k^2\sigma^2}{2}(D-D')^2} = \sum_{n=0}^{N} \frac{(-1)^n}{n!} \left(\frac{k^2\sigma^2}{2}(D-D')^2\right)^n
$$

$$
\text{RPT & Co.:} \quad e^{-DD'(k^2\sigma^2 - I(\mathbf{k}, \mathbf{r}))} \simeq e^{-DD'k^2\sigma^2} \sum_{n=0}^N \frac{1}{n!} \left(DD'I(\mathbf{k}, \mathbf{r}) \right)^n
$$

Spurious IR dependence at order N+1

intact "propagator"
$$
e^{-\frac{k^2\sigma^2}{2}}(D-D')^2
$$

How much does it matter? Function *ect of massive neutrinos on the BAO peak* ¹²

IR safe resummations

General idea: define
$$
\sigma^2(\bar{p}) = \frac{1}{3} \int \frac{d^3p}{(2\pi)^3} f\left(\frac{p}{\bar{p}}\right) \frac{P^0(p)}{p^2}
$$
 ex: $f(x) = e^{-x^2}$

$$
e^{-DD'(k^2\sigma^2 - I(\mathbf{k}, \mathbf{r}))} \simeq e^{-DD'k^2(\sigma^2 - \sigma^2(\bar{p}))} \sum_{n=0}^{N} \frac{(DD')^n}{n!} \left(I(\mathbf{k}, \mathbf{r}) - k^2 \sigma^2(\bar{p}) \right)^n
$$

Properties:

- 1) approaches SPT as $\bar{p} \to \infty$;
- 2) for any finite \bar{p} is IR safe;

3) RPT corresponds to $\bar{p} = 0$ strictly. "Singular" non IR safe point.

Restoring the IR behaviour

M. Peloso, MP (in preparation)

UV screening regularization schemes employed in analytical models. Designation and methodology. The methodology of the methodology of the methodology of the methodology. The methodology of the methodology. The methodology of the methodology of the methodology. The methodology of the metho duce a well-defined kernel function and investigate it at

ment of the kernel structure from cosmological N-body

UV screening regularization schemes employed in analytical models. Designation and methodology. The methodology of the methodology of the methodology of the methodology. The methodology of the methodology. The methodology of the methodology of the methodology. The methodology of the metho duce a well-defined kernel function and investigate it at

ment of the kernel structure from cosmological N-body

UV screening

The effect of virialized structures on larger scales is screened (Peebles '80, Baumann et al 1004.2488, Blas et al 1408.2995).

However, the departure from the PT predictions starts at small q's: is it only a virialization effect?

NL propagator and LRF

UV lessons

- ✤ SPT fails when loop momenta become higher than the nonlinear scale (q ≥ 0.4 h/Mpc)
- ✤ The real response to modifications in the UV regime is mild
- ✤ Most of the cosmology dependence is on intermediate scales

Dealing with the UV

✤ General idea: take the UV physics from N-body simulations and use (IR resummed) PT only for the large and intermediate scales

sources

Physics at k must be independent on L, L_uv ("Wilsonian approach")

Expansion in sources:

$$
\langle \delta \delta \rangle_J = \langle \delta \delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta J J \delta \rangle_{J=0} + \cdots
$$

computed in PT measured from
with cutoff at 1/L simultaneously

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203 Manzotti, Peloso, MP, Villaescusa-Navarro, Viel, 1407.1342

$\frac{\partial}{\partial \tau}\delta({\bf x}) + \frac{\partial}{\partial x^i}$ $[(1 + \delta(\mathbf{x}))v^i(\mathbf{x})] = 0$ Exact large scale dynamics for density and velocity fields

$$
\frac{\partial}{\partial \tau} v^{i}(\mathbf{x}) + \mathcal{H}v^{i}(\mathbf{x}) + v^{k}(\mathbf{x}) \frac{\partial}{\partial x^{k}} v^{i}(\mathbf{x}) = -\nabla_{x}^{i} \phi(\mathbf{x}) - J_{\sigma}^{i}(\mathbf{x}) - J_{1}^{i}(\mathbf{x})
$$
\n
$$
\nabla^{2} \phi(\mathbf{x}) = \frac{3}{2} \Omega_{M} \mathcal{H}^{2} \delta(\mathbf{x})
$$
\n
$$
n(\mathbf{x}) = n_{0} (1 + \delta(\mathbf{x})) = n_{0} (1 + \langle \delta_{mic} \rangle(\mathbf{x}))
$$
\n
$$
v^{i}(\mathbf{x}) = \frac{\langle (1 + \delta_{mic}) v_{mic}^{i} \rangle(\mathbf{x})}{1 + \delta(\mathbf{x})}
$$

external input on UV-physics JV-physics {

$$
J^i_{\sigma}(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} \frac{\partial}{\partial x^k} (n(\mathbf{x}) \sigma^{ki}(\mathbf{x}))
$$

$$
J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})} \left(\langle n_{mic} \nabla^i \phi_{mic} \rangle (\mathbf{x}) - n(\mathbf{x}) \nabla^i \phi (\mathbf{x}) \right)
$$

EFT approach $\sum_{\text{Bias et al, }1507.06665}$ \blacksquare and \blacksquare and Schematically we have

Carrasco et al, 1206.2926

Blas et al, 1507.06665 Floerchinger et al, [1607.03453](http://arxiv.org/abs/arXiv:1607.03453) RG

ī

 $J^i_\sigma + J^i_1 \equiv -\frac{1}{\rho} \partial_j \tau^{ji}$ Effective stress-tensor for the long modes 1 ρ $\partial_j \tau^{ji}$ $J_{\sigma}^{\dagger}+J_{1}^{\dagger}\equiv --O_{j}\tau^{J}$ Effective stress-tensor for the long modes long wavelength fluctuations, though nothing stops us from going to higher order. By the stops us from going to

@h[⌧ *ij*]

…

expectation value on a long wavelength background. The resulting function depends only on

$$
\langle \left[\tau^{ij}\right]_{\Lambda}\rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v_l^k - \frac{3}{4} \frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3} \delta^{ij} \partial_k v_l^k\right)\right] + \Delta \tau^{ij} + \dots
$$

Expanion in the long modes

Issues: $Issues:$

scale and time dependence of the expansion parameters matching procedure scale and time dependence of the expansion parameters the diversity of the diversion of the actual value of $\frac{1}{2}$ in a given realization and $\frac{1}{2}$ matering procedure

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP, Villaescusa-Navarro, Viel, 1407.1342

$$
L_{UV} = 1, 2, 4 \text{ Mpc/h}
$$

$$
L_{UV}: \delta, v^i, J_1^i, J_\sigma^i
$$

 $L_{box} = 512$ Mpc/h $N_{particles} = (512)^3$

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP, Villaescusa-Navarro, Viel, 1407.1342

$$
L_{UV} = 1, 2, 4 \text{ Mpc/h}
$$

\n
$$
L_{UV}: \delta, v^i, J_1^i, J_\sigma^i
$$

\n
$$
L: \overline{\delta}, \overline{v}^i, \overline{J}_1^i, \overline{J}_\sigma^i
$$

\n
$$
W(R/L) = \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{L^3} e^{-\frac{R^2}{2L^2}}
$$

 $L_{box} = 512$ Mpc/h $N_{particles} = (512)^3$

$$
\langle \delta \delta \rangle_J = \langle \delta \delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta J J \delta \rangle_{J=0} + \cdots
$$

 $\bar{P}_{11}(k,\eta) \simeq \bar{P}_{11}^{lin}(k,\eta) + \bar{P}_{ss,11}^{1-\text{loop}}(k,\eta) - \Delta \bar{P}_{ss,11}^{h,1-\text{loop}}(k,\eta) + \Delta \bar{P}_{11}^{h, N-\text{body}}(k,\eta)$

 $(-PT + Nbody)_{UV}$

$$
\langle \delta \delta \rangle_J = \langle \delta \delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta J \delta \rangle_{J=0} + \cdots
$$

$$
\bar{P}_{11}(k,\eta) \simeq \bar{P}_{11}^{\rm lin}(k,\eta) + \bar{P}_{ss,11}^{\rm 1-loop}(k,\eta) - \Delta \bar{P}_{ss,11}^{h,1-loop}(k,\eta) + \Delta \bar{P}_{11}^{h, N-body}(k,\eta)
$$

$$
\langle \delta \delta \rangle_J = \langle \delta \delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta \rangle J \delta \rangle_{J=0} + \cdots
$$

$$
\bar{P}_{11}(k,\eta) \simeq \bar{P}_{11}^{lin}(k,\eta) + \bar{P}_{ss,11}^{1-\text{loop}}(k,\eta) - \Delta \bar{P}_{ss,11}^{h,1-\text{loop}}(k,\eta) + \Delta \bar{P}_{11}^{h,N-\text{body}}(k,\eta)
$$

$$
\langle \delta \delta \rangle_J = \langle \delta \delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2} \langle \delta \rangle_{J=0} + \cdots
$$

$$
\bar{P}_{11}(k,\eta) \simeq \bar{P}_{11}^{\text{lin}}(k,\eta) + \bar{P}_{ss,11}^{\text{1-loop}}(k,\eta) - \Delta \bar{P}_{ss,11}^{h,1-loop}(k,\eta) + \Delta \bar{P}_{11}^{h,N-body}(k,\eta)
$$

 $\langle \delta J \delta \rangle_{J=0}$

 $(-PT + Nbody)_{UV}$

 $\overline{}$

Time-dependence

$$
\bar{\varphi}_1(\eta, \mathbf{k}) \equiv e^{-\eta} \bar{\delta}(\eta, \mathbf{k}) \quad , \quad \bar{\varphi}_2(\eta, \mathbf{k}) \equiv e^{-\eta} \frac{-\theta(\eta, \mathbf{k})}{\mathcal{H}f} \qquad \eta \equiv \ln \frac{D_+(\tau)}{D_+(\tau_{\mathrm{in}})}
$$

 $h_a(\mathbf{k}, \eta) \equiv h_a^1(\mathbf{k}, \eta) + h_a^{\sigma}(\mathbf{k}, \eta)$,

$$
h_a^1(\mathbf{k}, \eta) = -i \frac{k^i J_1^i(\mathbf{k}, \eta)}{\mathcal{H}^2 f^2} e^{-\eta} \delta_{a2}, \qquad h_a^{\sigma}(\mathbf{k}, \eta) = -i \frac{k^i J_{\sigma}^i(\mathbf{k}, \eta)}{\mathcal{H}^2 f^2} e^{-\eta} \delta_{a2} \qquad g_{12}(\eta) = \frac{2}{5} \left(1 - e^{-5/2 \eta} \right)
$$

$$
\Delta \bar{P}_{11}^{h,\text{N-body}}(k,\eta) \equiv -2 \int_{\eta_{in}}^{\eta} ds \ g_{12}(\eta - s) \left\langle h_2(\mathbf{k},s) \,\bar{\varphi}_1(\mathbf{k}',\eta) \right\rangle'
$$

Ansatz for the time-dependence

$$
\langle h_2(\mathbf{k},s) \,\overline{\varphi}_1(\mathbf{k}',\eta) \rangle' \cong \left[\frac{D\left(s\right)}{D\left(\eta\right)} \right]^{\alpha(\eta)} \times \langle h_2(\mathbf{k},\eta) \,\overline{\varphi}_1(\mathbf{k}',\eta) \rangle' \quad , \quad s < \eta \; .
$$

PT limit: $\alpha(\eta) \rightarrow 2$

Checked independently

Results of the reconstruction

Results of the reconstruction

Good, but... why not simply run a N-body simulation?!

COSMOLOGY DEPENDENCE

Simulation Suite

 $L_{box} = 512 \text{ Mpc/h}$ $N_{particles} = (512)^3$

Ratios of UV source correlators

$$
\frac{\langle J\delta \rangle_i}{\langle J\delta \rangle_{REF}} \quad \text{From N-body}
$$

Scale-independent!!

Rescale by using PT information

$$
\widehat{\mathcal{R}}_{i}\left(k,\eta\right) \equiv \frac{\mathcal{R}_{i}\left(k,\eta\right)}{\mathcal{R}_{\text{REF}}\left(k,\eta\right)} \frac{\left(\frac{D_{\text{REF}}\left(\eta\right)}{D_{\text{REF}}\left(\eta\right)\right)}^{\alpha_{\text{REF}}\left(\eta\right)-2}}{\left(\frac{D_{i}\left(\eta\right)}{D_{i}\left(\eta\right)\right)}^{\alpha_{i}\left(\eta\right)-2}} \qquad \mathcal{R}_{i}\left(k,\eta\right) \equiv \frac{\left\langle h_{2}\right\rangle}{\frac{\left(\eta\right)}{D_{i}\left(\eta\right)}^{\alpha_{i}\left(\eta\right)-2}} \qquad \mathcal{R}_{i}\left(k,\eta\right) \equiv \frac{\left(\eta\right)}{D_{i}\left(\eta\right)} \qquad \mathcal{R}_{i}\left(k,\eta\right) \equiv \frac{\left(\
$$

Amplitude captured by PT!!

 $O(10)$

 $(\mathbf{k},\eta) \, \bar{\varphi}_1(\mathbf{k}',\eta) \rangle_i'$

 $\bar{\mathfrak{z}}^{h,1-\mathrm{loop}}_{ss,11,i}(k,\eta)$

The PS in 1-loop EFToLSS $\langle \left[\tau^{ij}\right]_\Lambda\rangle_{\delta_l}=p_b\delta^{ij}+\rho_b\left[c_s^2\delta_l\delta^{ij}-\frac{c_{bv}}{Ha}\delta^{ij}\partial_kv_l^k-\frac{c_{sv}}{4}\frac{c_{sv}}{Ha}\left(\partial^jv_l^i+\partial^iv_l^j-\frac{c}{3}\delta^{ij}\partial_kv_l^k\right)\right]+\Delta\tau^{ij}+\dots\ .$ $P_{11}(k, \eta) \simeq P_{11}^{lin}(k, \eta) + P_{ss,11}^{1-loop}(k, \eta) - 2(2\pi) c_{s(1)}^2$ $k²$ k_{NL}^2 $p(x) \simeq P_{11}^{lin}(k,\eta) + P_{ss,11}^{1-loop}(k,\eta) - 2(2\pi) c_{s(1)}^2 \frac{k^2}{12} P^{lin}(k,\eta)$, long wavelength fluctuations, though nothing stops us from τ and τ S/N in 1-100p er 101.55 \langle $\left[\tau^{ij}\right]_{\Lambda}\rangle_{\delta_l} = p_b\delta^{ij} + \rho_b$ $\sqrt{ }$ $c_s^2 \delta_l \delta^{ij} - \frac{c_b^2}{H}$ $\frac{c_{bv}}{Ha}\delta^{ij}\partial_k v^k_l$ $\frac{k}{l} - \frac{3}{4}$ 4 c_{sv}^2 $\frac{c_{sv}^2}{Ha}$ $\partial^j v^i_l + \partial^i v^j_l - \frac{2}{3}$ 3 $\left[\delta^{ij}\partial_k v^k_l\right)\right] + \Delta \tau^{ij} + \ldots \;.$ (25) $P_{11}(k, n) \simeq P_{11}^{lin}(k, n) + P_{11}^{1-loop}(k, n) - 2(2\pi) c^2$ $\begin{array}{ccc} \text{max} & \text{min} & \text{$

to reduce the scale dependence higher orders+resummations needed $\frac{1}{2}$ *p* and $\frac{1}{2}$ the coefficient dependence co requee are beare dependence

where we recall that *Plin* is the linear PS, and where the *k*-independent parameter either *(see Senatore Zaldarriaga, 1404.5954)* $($ see Senatore Zaldarriaga, 1404.5954)

Matching

0.050

"-² h< ^hHh^L ^f1HhL> ^ê ^H*k*² *^P*¹¹

The correction to the SPT result is given by $\Delta P^{h, Nbody}(k, \eta) - \Delta P^{h, SPT}(k, \eta)$ 0.010

0.050

"-² h< ^hHh^L ^f1HhL> ^ê ^H*k*² *^P*¹¹

 $\frac{1}{2}$

P^l (*p*) *P^l* (*q*) *W*˜ (*kL*)

$$
\Delta \bar{P}_{11}^{h,\mathrm{N-body}}(k,\eta) \equiv -2\int_{\eta_{in}}^{\eta} ds \; g_{12}(\eta - s) \left\langle h_2\left(\mathbf{k},s\right) \bar{\varphi}_1(\mathbf{k}',\eta) \right\rangle^2
$$

lin^L ; ^z=¹

lepends on two l e P ϵ $R_{\rm c}$ It depends on two scales: L_f and k

Ultimate reach of effective methods depends on PT!

Summary

- ✤ Semi-analytical methods and N-body are complementary: (flexibility, physical insight, speed)
- ✤ IR effects important and under control
- ✤ Intermediate scales treatable by (improved) SPT
- ✤ The UV is out of SPT reach but mildly cosmology dependent: effective approaches!