IR and UV effects in the evolution of the Large Scale Structure

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The Big Bang And The Little Bangs - Non-Equilibrium Phenomena In Cosmology And In Heavy-Ion Collisions - Cern, 19/8/2016

Outline

- The coarse-grained Vlasov equation
- Why beyond standard PT (SPT)
- IR effects and safe resummations
- The UV failure of SPT and effective/mixed approaches

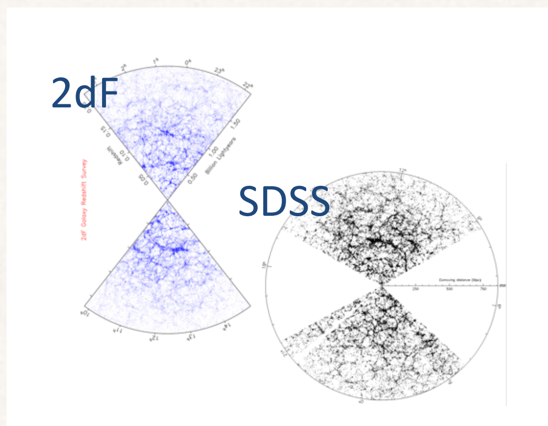
The LSS challenge(s)

Learn fundamental physics from Large Scale Structure measurements

Initial metric perturbations: spectra, non-gaussianity
Properties of the DM (cold, mixed, warm, fuzzy, …)
Neutrino masses
GR constraints
Properties of DE

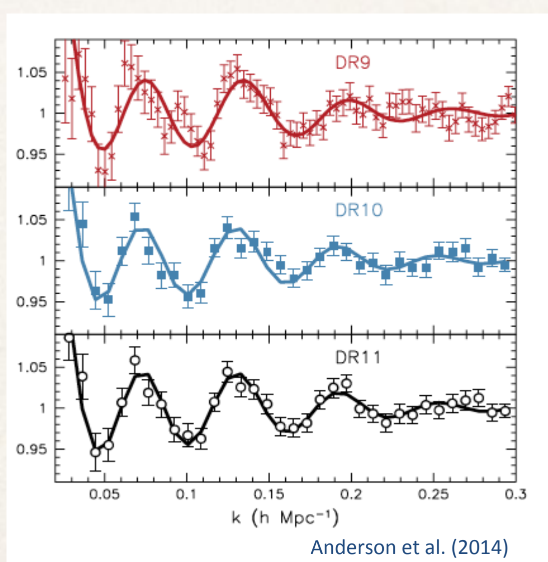
Data!

PAST:



O(10^5) spectroscopic redshifts of galaxies,
O(10^3)deg^2
at z< 0.7

PRESENT:



SDSS / BOSS
DES

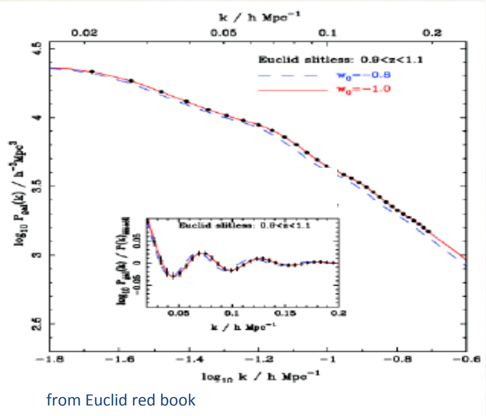
O(10^6) spectroscopic redshifts of galaxies,
O(10^4)deg^2
at z< 0.7

Data!

PRESENT/NEAR FUTURE:

O(10^9) photometric redshifts of galaxies, full sky (LSST)
at z< 1.5

NOT TOO FAR FUTURE:



EUCLID

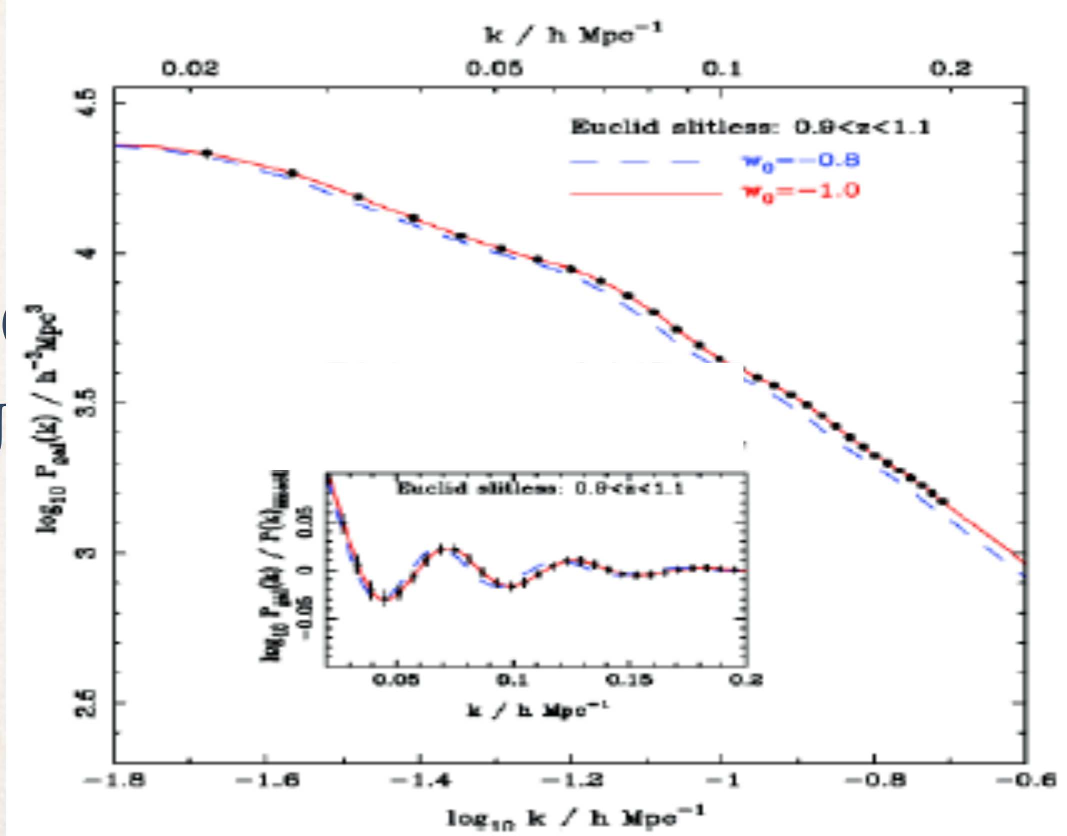
O(10^7) spectroscopic redshifts of galaxies,
O(10^9) photometric redshifts of galaxies,
full sky
at z< 1.5

Data!

PRESENT/NEAR FUTURE:

O(10^9) photometric redshifts of galaxies,
full sky (LSST)
at z< 1.5

NOT TO
FUTU

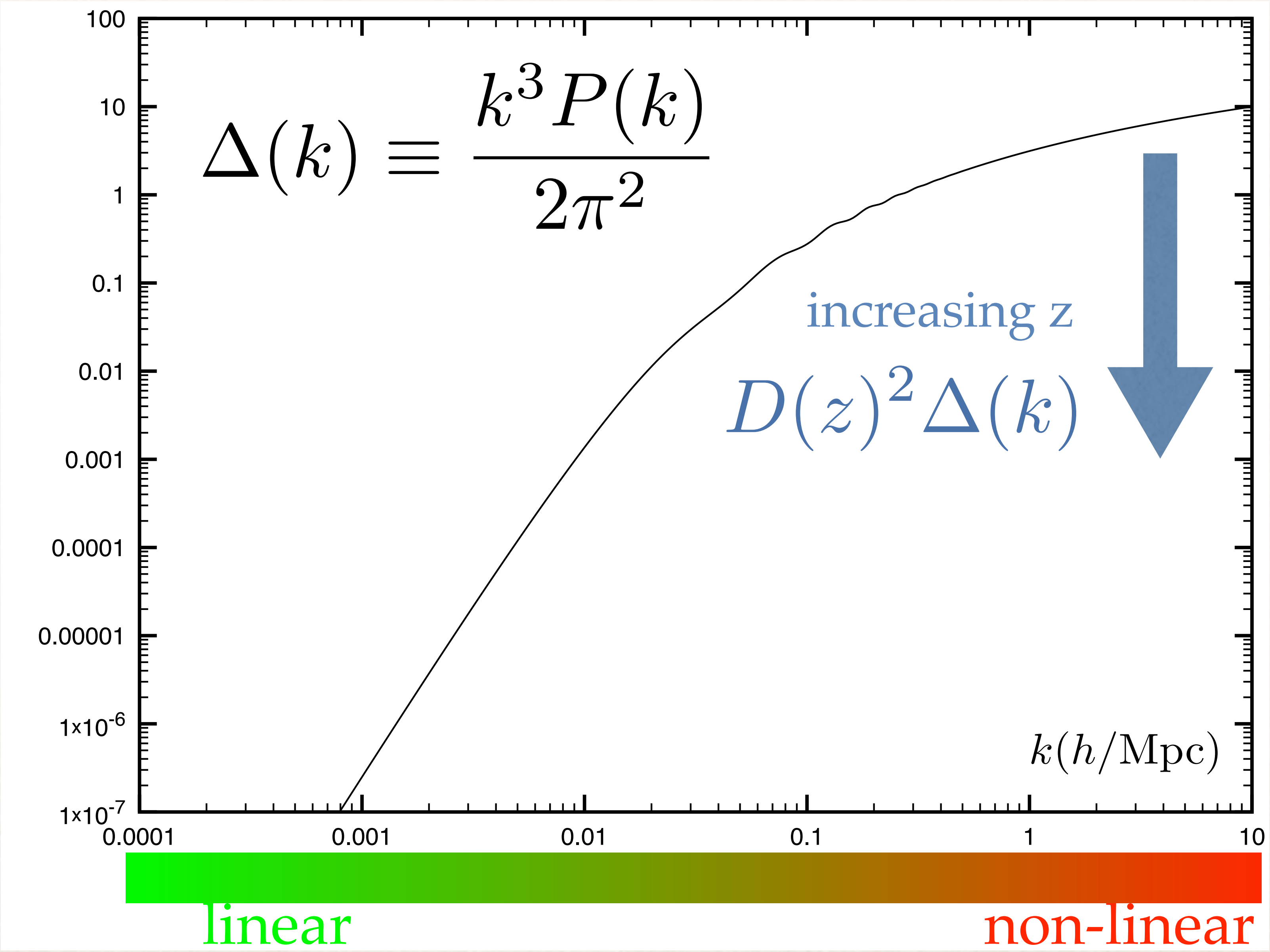


from Euclid red book

7) spectroscopic redshifts of galaxies,
9) photometric redshifts of galaxies,
y
.5

Linear and non-linear scales

linear Power Spectrum @z=0, ΛCDM



Vlasov Equation

Liouville theorem+ neglect non-gravitational interactions:

$$\frac{d}{d\tau} f_{mic} = \left[\frac{\partial}{\partial \tau} + \frac{p^i}{am}\frac{\partial}{\partial x^i} - am\nabla^i_x \phi(\mathbf{x}, \tau)\right] f_{mic}(\mathbf{x}, \mathbf{p}, \tau) = 0$$

moments:

$$n_{mic}(\mathbf{x}, \tau) = \int d^3p f_{mic}(\mathbf{x}, \mathbf{p}, \tau)$$ density

$$\mathbf{v}_{mic}(\mathbf{x}, \tau) = \frac{1}{n_{mic}(\mathbf{x}, \tau)} \int d^3p \, \frac{\mathbf{p}}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau)$$ velocity

$$\sigma^{ij}_{mic}(\mathbf{x}, \tau) = \frac{1}{n_{mic}(\mathbf{x}, \tau)} \int d^3p \, \frac{p^i}{am}\frac{p^j}{am} f_{mic}(\mathbf{x}, \mathbf{p}, \tau) - v^i_{mic}(\mathbf{x}, \tau) v^j_{mic}(\mathbf{x}, \tau)$$ velocity dispersion

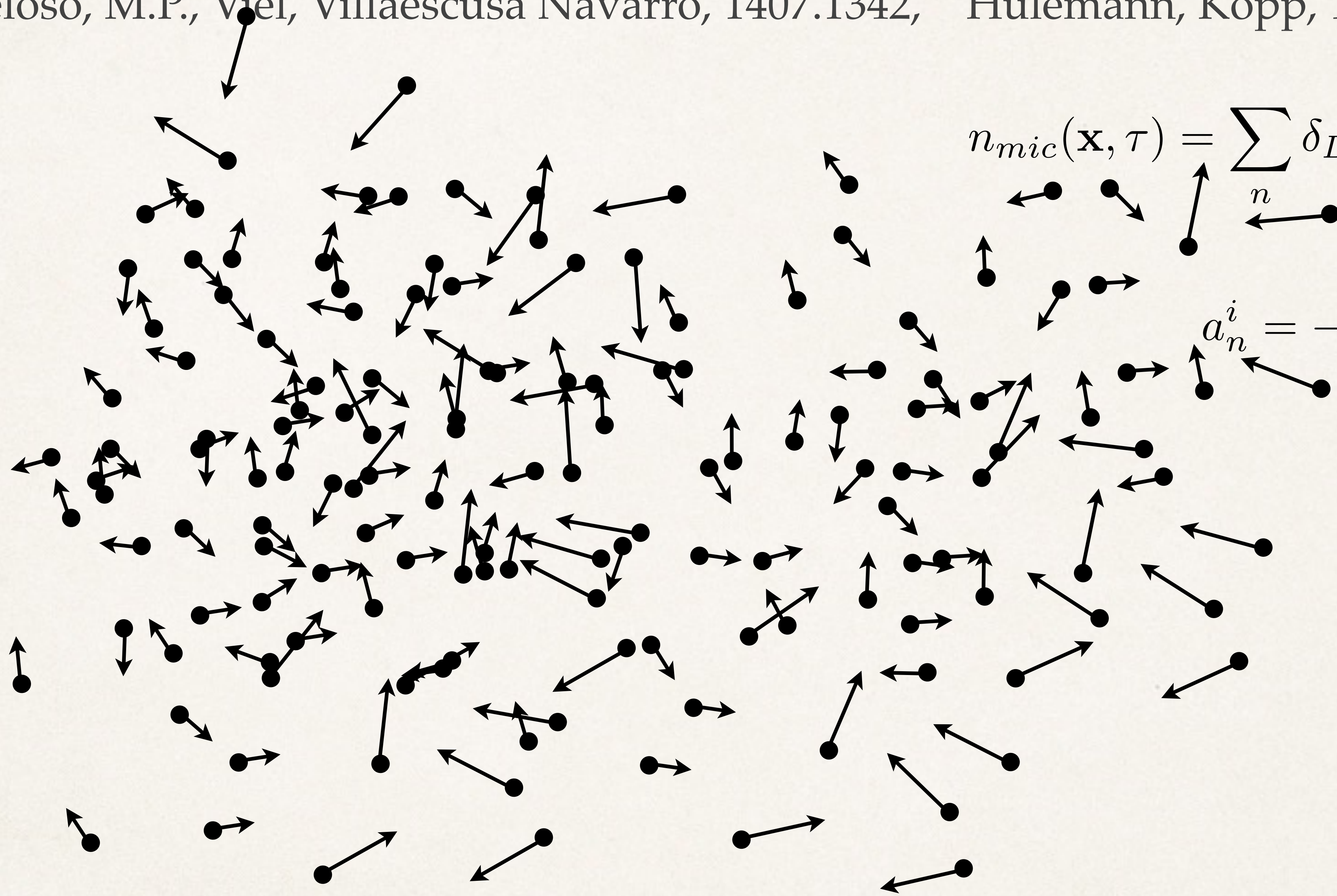
…

From particles to fluids

Buchert, Dominguez, '05, Pueblas Scoccimarro, '09, Baumann et al. '10

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203, Carrasco, Hertzberg, Senatore,1206.2976 .

Manzotti, Peloso, M.P., Viel, Villaescusa Navarro, 1407.1342, Hulemann, Kopp, 1407.4810 ...



$$n_{mic}(\mathbf{x},\tau) = \sum_n \delta_D(\mathbf{x} - \mathbf{x}_n(\tau))\,,$$

$$v_n^i = \dot{x}_n(\tau)\,,$$

$$a_n^i = -\nabla_x^i \phi_{mic}(\mathbf{x},\tau)$$

$$f_{mic}(x,p,\tau) = \sum_n \delta_D(x - x_n(\tau))\delta_D(p - p_n(\tau))$$

Satisfies the "Vlasov eq."

From particles to fluids

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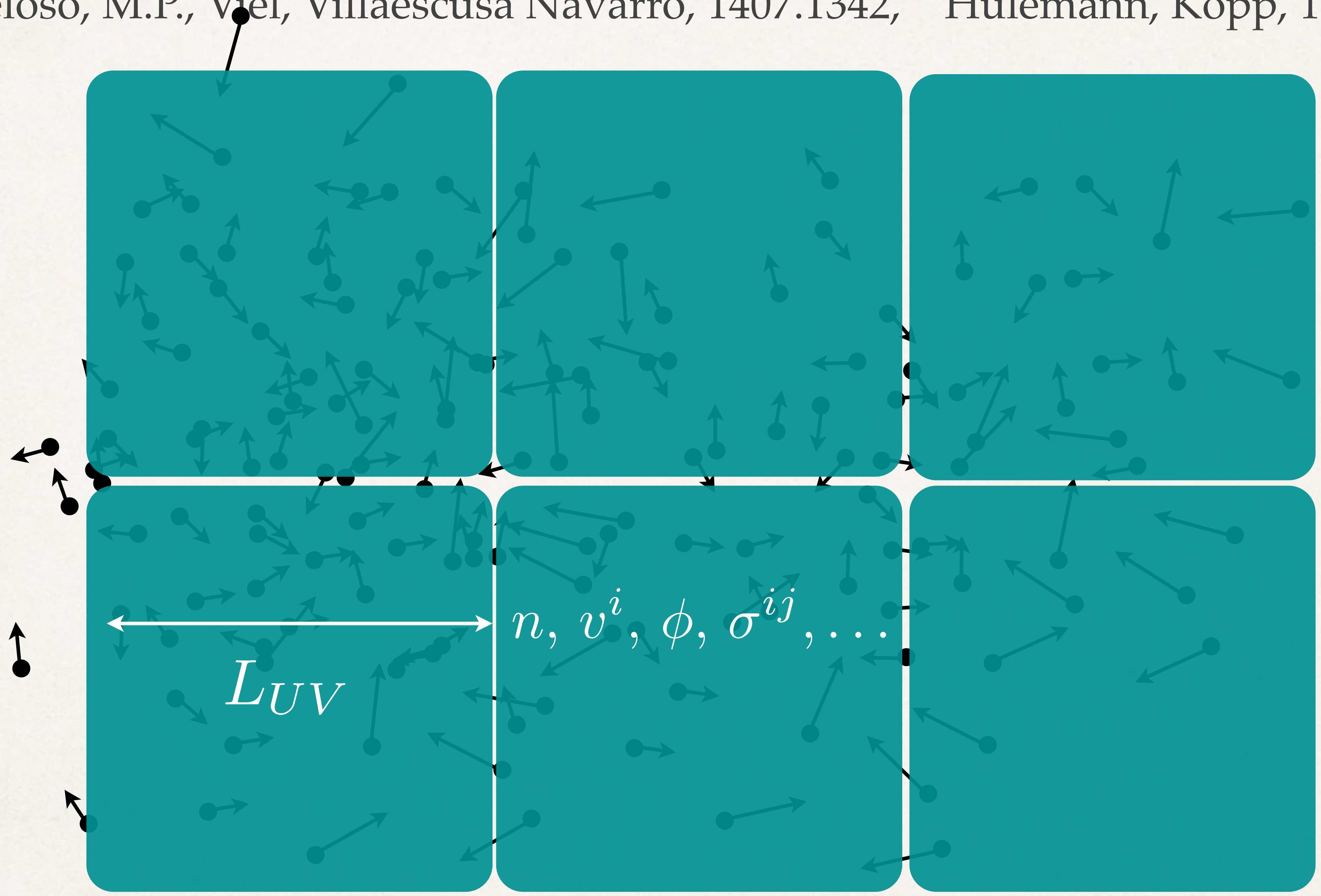

$$f(x,p,\tau) \equiv \frac{1}{V}\int d^3y\,\mathcal{W}(y/L_{UV})\,f_{mic}(x+y,p,\tau)$$

Coarse-grained Vlasov equation

large scales

$$\left[\frac{\partial}{\partial \tau} + \frac{p^i}{am}\frac{\partial}{\partial x^i} - am\nabla_x^i \phi(\mathbf{x},\tau)\frac{\partial}{\partial p^i}\right] f(\mathbf{x},\mathbf{p},\tau) =$$
$$am\left[\langle \frac{\partial}{\partial p^i} f_{mic}\nabla^i \phi_{mic}\rangle_{L_{UV}}(\mathbf{x},\mathbf{p},\tau) - \frac{\partial}{\partial p^i} f(\mathbf{x},\mathbf{p},\tau)\nabla_x^i \phi(\mathbf{x},\tau)\right]$$

short scales

$$\langle g\rangle_{L_{UV}}(\mathbf{x}) \equiv \frac{1}{V_{UV}}\int d^3y\, \mathcal{W}(y/L_{UV})g(\mathbf{x}+\mathbf{y})$$

$$\phi = \langle \phi_{mic}\rangle_{L_{UV}}$$

$$f = \langle f_{mic}\rangle_{L_{UV}}$$

Vlasov equation in the L_uv → 0 limit!

Taking moments…

Exact large scale dynamics for density and velocity fields

$$\frac{\partial}{\partial \tau}\delta(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta(\mathbf{x}))v^i(\mathbf{x})\right] = 0$$

$$\frac{\partial}{\partial \tau}v^i(\mathbf{x}) + \mathcal{H}v^i(\mathbf{x}) + v^k(\mathbf{x})\frac{\partial}{\partial x^k}v^i(\mathbf{x}) = -\nabla^i_x\phi(\mathbf{x}) - \underline{J^i_\sigma(\mathbf{x}) - J^i_1(\mathbf{x})}$$

$$\nabla^2\phi(\mathbf{x}) = \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{x})$$

$$n(\mathbf{x}) = n_0(1+\delta(\mathbf{x})) = n_0(1+\langle\delta_{mic}\rangle(\mathbf{x}))$$

$$v^i(\mathbf{x}) = \frac{\langle(1+\delta_{mic})v^i_{mic}\rangle(\mathbf{x})}{1+\delta(\mathbf{x})}$$

external input on UV-physics needed to close the system

$$J^i_\sigma(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\frac{\partial}{\partial x^k}(n(\mathbf{x})\sigma^{ki}(\mathbf{x}))$$

$$J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\left(\langle n_{mic}\nabla^i\phi_{mic}\rangle(\mathbf{x}) - n(\mathbf{x})\nabla^i\phi(\mathbf{x})\right)$$

Single stream regime

Set $\sigma^{ij} = \omega^{ijk} = \cdots = \nabla \times \mathbf{v} = 0$

↕

$\left(J^i_\sigma = J^i_1 = 0\right)$

$$f(\mathbf{x}, \mathbf{p}, \tau) = n(\mathbf{x}, \tau)\, \delta_D(\mathbf{p} - am\mathbf{v}(\mathbf{x}, \tau))$$

System described by $\delta(\mathbf{x}, \tau),\ \theta(\mathbf{x}, \tau) \equiv \nabla \cdot \mathbf{v}(\mathbf{x}, \tau)$

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot ((1 + \delta)\mathbf{v})$$ continuity

$$\frac{\partial\, \mathbf{v}}{\partial\, \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi$$ Euler

$$\nabla^2\phi = \frac{3}{2}\,\Omega_M\,\mathcal{H}^2\,\delta$$ Poisson

Single stream regime

Set $\sigma^{ij} = \omega^{ijk} = \cdots = \nabla \times \mathbf{v} = 0$

$\updownarrow$

$\left(J^i_\sigma = J^i_1 = 0\right)$

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System described by $\delta(\mathbf{x}, \tau),\ \theta(\mathbf{x}, \tau) \equiv \nabla \cdot \mathbf{v}(\mathbf{x}, \tau)$

$$\frac{\partial \delta}{\partial \tau} + \nabla \cdot ((1 + \delta)\mathbf{v}) \qquad \text{continuity}$$

$$\frac{\partial \mathbf{v}}{\partial \tau} + \mathcal{H}\mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \phi \qquad \text{Euler}$$

$$\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta \qquad \text{Poisson}$$

warning: self-consistent … but ultimately wrong!

Linear Perturbation Theory

$$\frac{\partial \delta}{\partial \tau} + \theta = 0$$

$$\frac{\partial \theta}{\partial \tau} + \mathcal{H}\theta = -\nabla^2 \phi$$

$$\nabla^2 \phi = \frac{3}{2} \Omega_M \mathcal{H}^2 \delta$$

$\Rightarrow$

$$\ddot{\delta} + \mathcal{H}\dot{\delta} = \frac{3}{2}\Omega_M \mathcal{H}^2 \delta$$

linear GR equation for $k \gg \mathcal{H}$

Solution : $\delta^{(1)}(\mathbf{k},\tau) = -\dfrac{\theta^{(1)}(\mathbf{k},\tau)}{\mathcal{H} f_\pm} = \delta(\mathbf{k},\tau_{in}) D_\pm(\tau)$

growing/decaying mode

$(D_\pm(\tau_{in}) = 1)$

$$\left(f_\pm \equiv \frac{d \ln D_\pm}{d \ln a} \right)$$

For EdS (Ω_M=1): $D_\pm = \left(\dfrac{a(\tau)}{a(\tau_{in})} \right)^{f_\pm}$ $\quad f_+ = 1,\ f_- = -3/2$

Standard Perturbation Theory

It is an expansion of the density and velocity fields in terms of the initial conditions

Compact notation: $\eta = \log(a/a_{in})$

The continuity+Euler+Poisson system reads:

$$\left(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta)\right)\varphi_b(\mathbf{k},\eta) = e^\eta \int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{q}-\mathbf{p})\gamma_{abc}(\mathbf{k},\mathbf{q},\mathbf{p})\varphi_b(\mathbf{q},\eta)\varphi_c(\mathbf{p},\eta)$$

linear

nonlinear

$$\Omega_{ab}(\eta) = \begin{pmatrix} 1 & -1 \\ -\frac{3}{2}\Omega_M(\eta) & 2 + \frac{d\log\mathcal{H}}{d\eta} \end{pmatrix}$$

$$\gamma_{112}(\mathbf{k},\mathbf{q},\mathbf{p}) = \gamma_{121}(\mathbf{k},\mathbf{p},\mathbf{q}) = \frac{\mathbf{k}\cdot\mathbf{p}}{2p^2}$$

$$\gamma_{222}(\mathbf{k},\mathbf{q},\mathbf{p}) = \frac{k^2\,\mathbf{q}\cdot\mathbf{p}}{2\,q^2p^2}$$

Standard Perturbation Theory

It is an expansion of the density and velocity fields in terms of the initial conditions

Compact notation: $\begin{pmatrix} \varphi_1(\eta, \mathbf{k}) \\ \varphi_2(\eta, \mathbf{k}) \end{pmatrix} \equiv e^{-\eta} \begin{pmatrix} \delta(\eta\,, \mathbf{k}) \\ -\theta(\eta\,, \mathbf{k})/\mathcal{H} \end{pmatrix}$ $\qquad \eta = \log(a/a_{in})$

The continuity+Euler+Poisson system reads:

$$(\delta_{ab}\partial_\eta + \Omega_{ab}(\eta))\,\varphi_b(\mathbf{k},\eta) = e^\eta \int \frac{d^3q}{(2\pi)^3}\frac{d^3p}{(2\pi)^3}\delta_D(\mathbf{k}-\mathbf{q}-\mathbf{p})\gamma_{abc}(\mathbf{k},\mathbf{q},\mathbf{p})\varphi_b(\mathbf{q},\eta)\varphi_c(\mathbf{p},\eta)$$

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Iterative solution (EdS)

$$\varphi_a^{(1)}(\mathbf{k},\eta) = g_{ab}(\eta)\varphi_b^{in}(\mathbf{k})$$

linear solution

$$g_{ab}(\eta) = \left[\begin{pmatrix} 3/5 & 2/5 \\ 3/5 & 2/5 \end{pmatrix} + e^{-5/2\,\eta}\begin{pmatrix} 2/5 & -2/5 \\ -3/5 & 3/5 \end{pmatrix}\right]\Theta(\eta)$$

linear propagator

$$\varphi_a^{(2)}(\mathbf{k},\eta) = \int_0^\eta ds\, g_{ab}(\eta - s)\, e^s\, I_{\mathbf{k},\mathbf{q},\mathbf{p}}\gamma_{bcd}(\mathbf{k},\mathbf{q},\mathbf{p})\varphi_c^{(1)}(\mathbf{q},s)\varphi_d^{(1)}(\mathbf{p},s)$$

2nd order solution

$$\varphi_a^{(n)}(\mathbf{k},\eta) = I_{\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n}} F^{(n)}_{a,b_1,\cdots,b_n}(\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n};\eta)\varphi_{b_1}^{in}(\mathbf{q_1})\cdots\varphi_{b_n}^{in}(\mathbf{q_n})$$

nth order solution

$$\left(I_{\mathbf{k},\mathbf{q_1},\cdots,\mathbf{q_n}} \equiv \int \frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3q_n}{(2\pi)^3}\delta_D(\mathbf{k} - \sum_{i=1}^{n}\mathbf{q_i})\right)$$

MODE MODE COUPLING

Correlators

If the initial conditions are gaussian, then only correlators involving an even number of fields are non-vanishing

tree-level

Power spectrum: $\langle \varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{k}',\eta)\rangle = \langle \varphi_a^{(1)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\rangle$

$$+\langle \varphi_a^{(1)}(\mathbf{k},\eta)\varphi_b^{(3)}(\mathbf{k}',\eta)\rangle + \langle \varphi_a^{(3)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\rangle$$

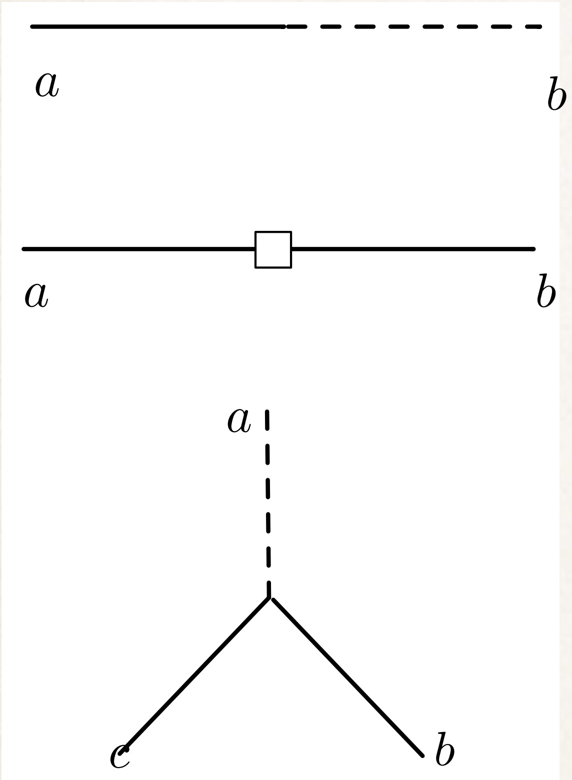
one-loop

$$+\langle \varphi_a^{(2)}(\mathbf{k},\eta)\varphi_b^{(2)}(\mathbf{k}',\eta)\rangle + O((\varphi^{in})^6)$$

Bispectrum: $\langle \varphi_a(\mathbf{k},\eta)\varphi_b(\mathbf{k}',\eta)\varphi_c(\mathbf{k}'',\eta)\rangle = \langle \varphi_a^{(2)}(\mathbf{k},\eta)\varphi_b^{(1)}(\mathbf{k}',\eta)\varphi_c^{(1)}(\mathbf{k}'',\eta)\rangle$

tree-level $+2\ \text{permutations} + O((\varphi^{in})^6)$

Diagrammar

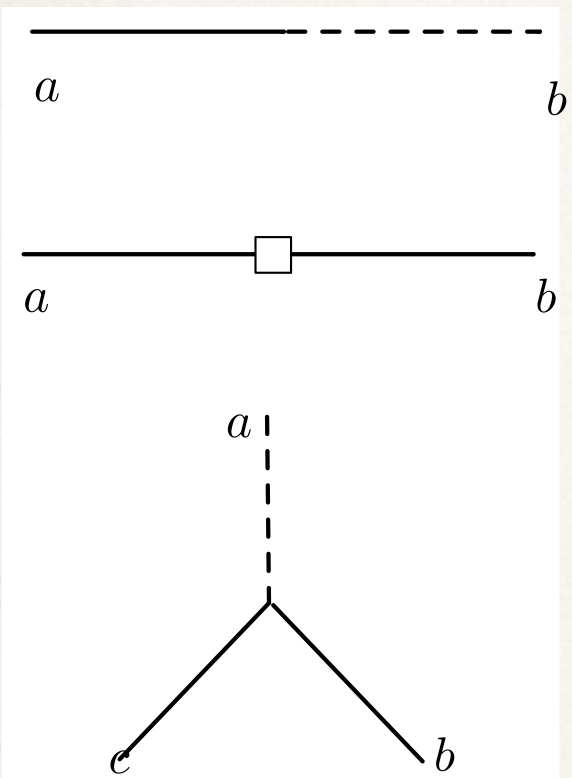


propagator (linear growth factor): $-i\, g_{ab}(\eta_a,\, \eta_b)$

linear power spectrum: $P^L_{ab}(\eta_a,\, \eta_b;\, \mathbf{k})$

interaction vertex: $-i\, e^{\eta}\, \gamma_{abc}(\mathbf{k_a},\, \mathbf{k_b},\, \mathbf{k_c})$

Diagrammar



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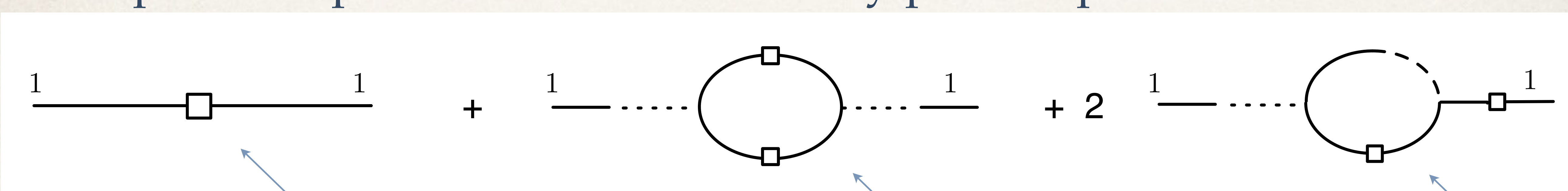
Example: 1-loop correction to the density power spectrum:

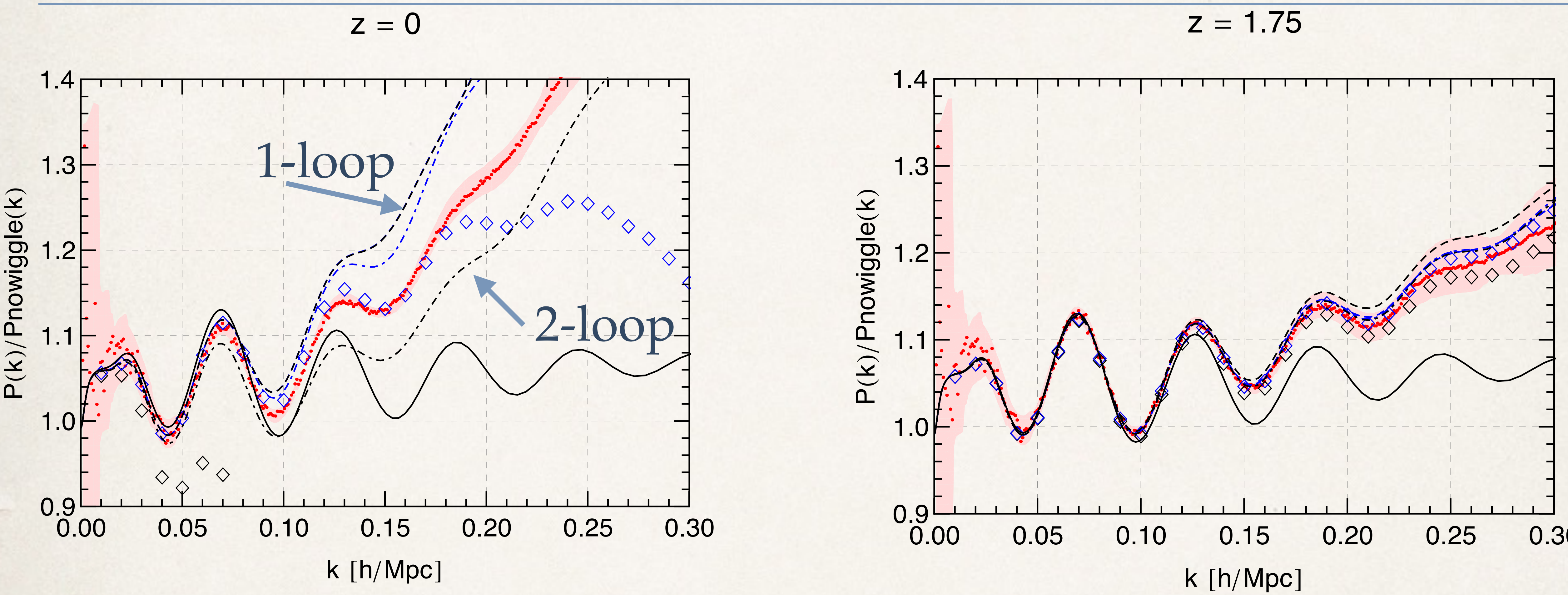


Linear Power spectrum

"P22"

"P13"

Performance of standard PT

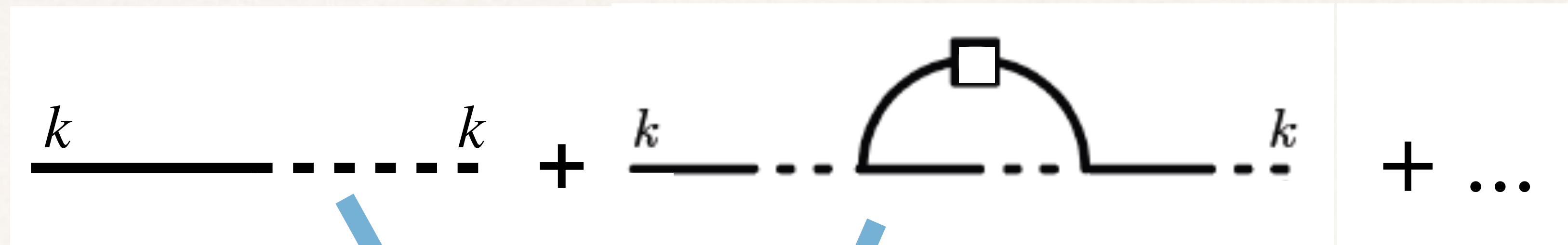


Blas et al 1309.3308

The expansion fails to converge at low redshifts/small scales

PT in the BAO range

1-loop propagator @ large k:



$$G_{ab}(k;\eta_a,\,\eta_b) = g_{ab}(\eta_a,\,\eta_b)\left[1 - k^2\sigma^2\,\frac{(e^{\eta_a} - e^{\eta_b})^2}{2}\right] + O(k^4\sigma^4)$$

$$\left(\sigma^2 \equiv \frac{1}{3}\int d^3q\,\frac{P^0(q)}{q^2}\right)$$

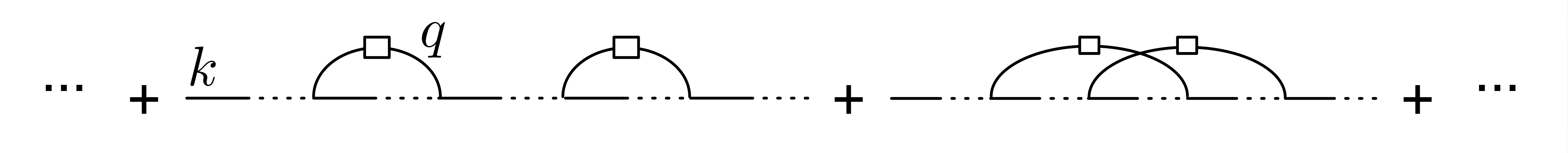
$(\sigma\,e^{\eta_a})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$ in the BAO range!

2-loop

the PT series blows up in the BAO range

... but it can be resummed

$k \gg q$ (Crocce-Scoccimarro '06)



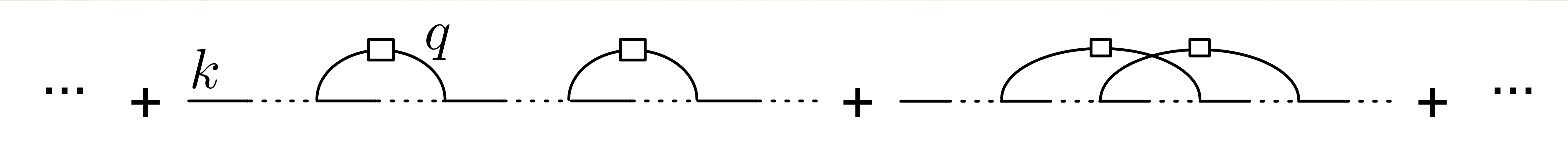
$$G(k;\eta,\eta_{in}) = \frac{\langle \delta(k,\eta)\delta(k,\eta_{in})\rangle}{\langle \delta(k,\eta_{in})\delta(k,\eta_{in})\rangle} \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

... but it can be resummed

$k \gg q$ (Crocce-Scoccimarro '06)



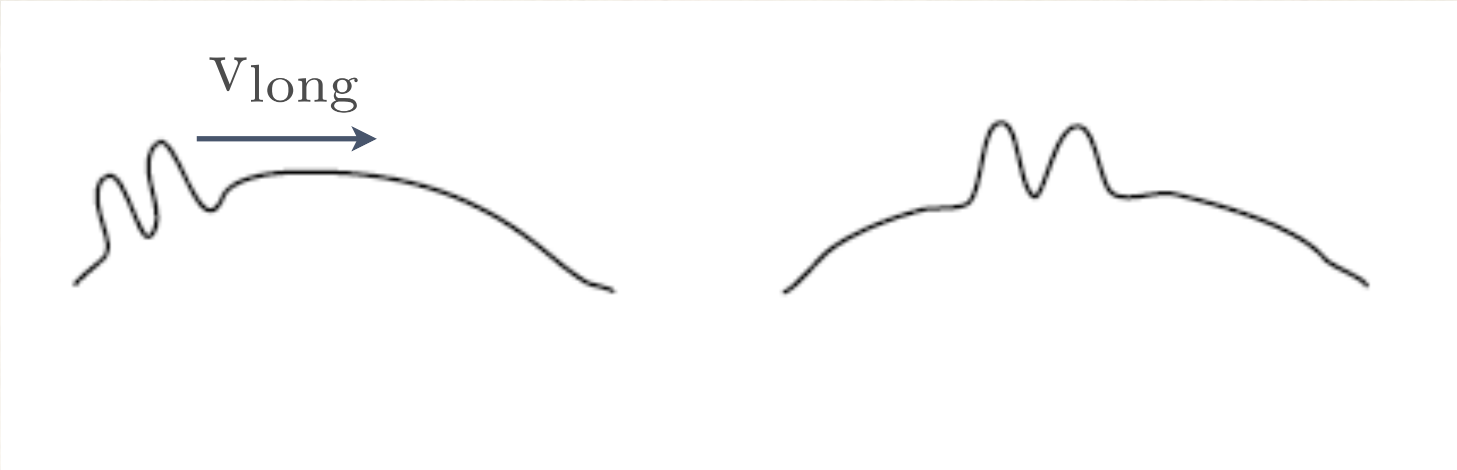
$$G(k;\eta,\eta_{in}) = \frac{\langle \delta(k,\eta)\delta(k,\eta_{in})\rangle}{\langle \delta(k,\eta_{in})\delta(k,\eta_{in})\rangle} \sim e^{-\frac{k^2\sigma^2}{2}e^{2\eta}}$$

\`coherence momentum' $k_{ch} = (\sigma\, e^{\eta})^{-1} \simeq 0.15\,\mathrm{h\,Mpc}^{-1}$

damping in the BAO range!

RPT: use G, and not g, as the linear propagator

Physical meaning of the IR resummation



$$\bar{\delta}_\alpha(\mathbf{x},\tau) = \delta_\alpha(\mathbf{x} - \mathbf{D}_\alpha(\mathbf{x},\tau),\tau)$$

$$\mathbf{D}_\alpha(\mathbf{x},\tau) \equiv \int_{\tau_{in}}^{\tau} d\tau' \mathbf{v}_{\alpha,\mathrm{long}}(\mathbf{x},\tau') \simeq \mathbf{D}_\alpha(\tau)$$

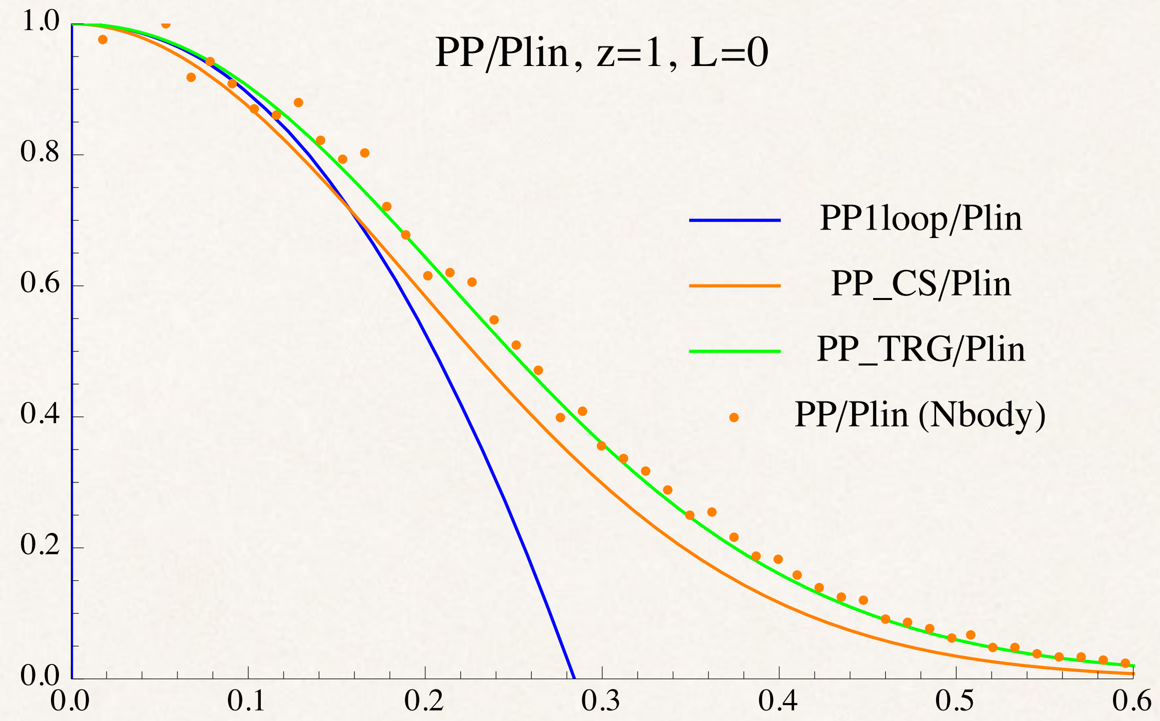
$$\langle \delta_\alpha(\mathbf{k},\tau)\delta_\alpha(\mathbf{k}',\tau')\rangle = \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle \langle e^{-i\mathbf{k}\cdot(\mathbf{D}_\alpha(\tau)-\mathbf{D}_\alpha(\tau'))}\rangle$$

$$= \langle \bar{\delta}_\alpha(\mathbf{k},\tau)\bar{\delta}_\alpha(\mathbf{k}',\tau')\rangle e^{\frac{-k^2\sigma_v^2(D(\tau)-D(\tau'))^2}{2}}$$

$$\sigma_v^2 = -\frac{1}{3\mathcal{H}^2 f^2}\int^{\Lambda} d^3q \langle v^i_{long}(q) v^i_{long}(q)\rangle' = \frac{1}{3}\int^{\Lambda} d^3q \frac{P^0(q)}{q^2}$$

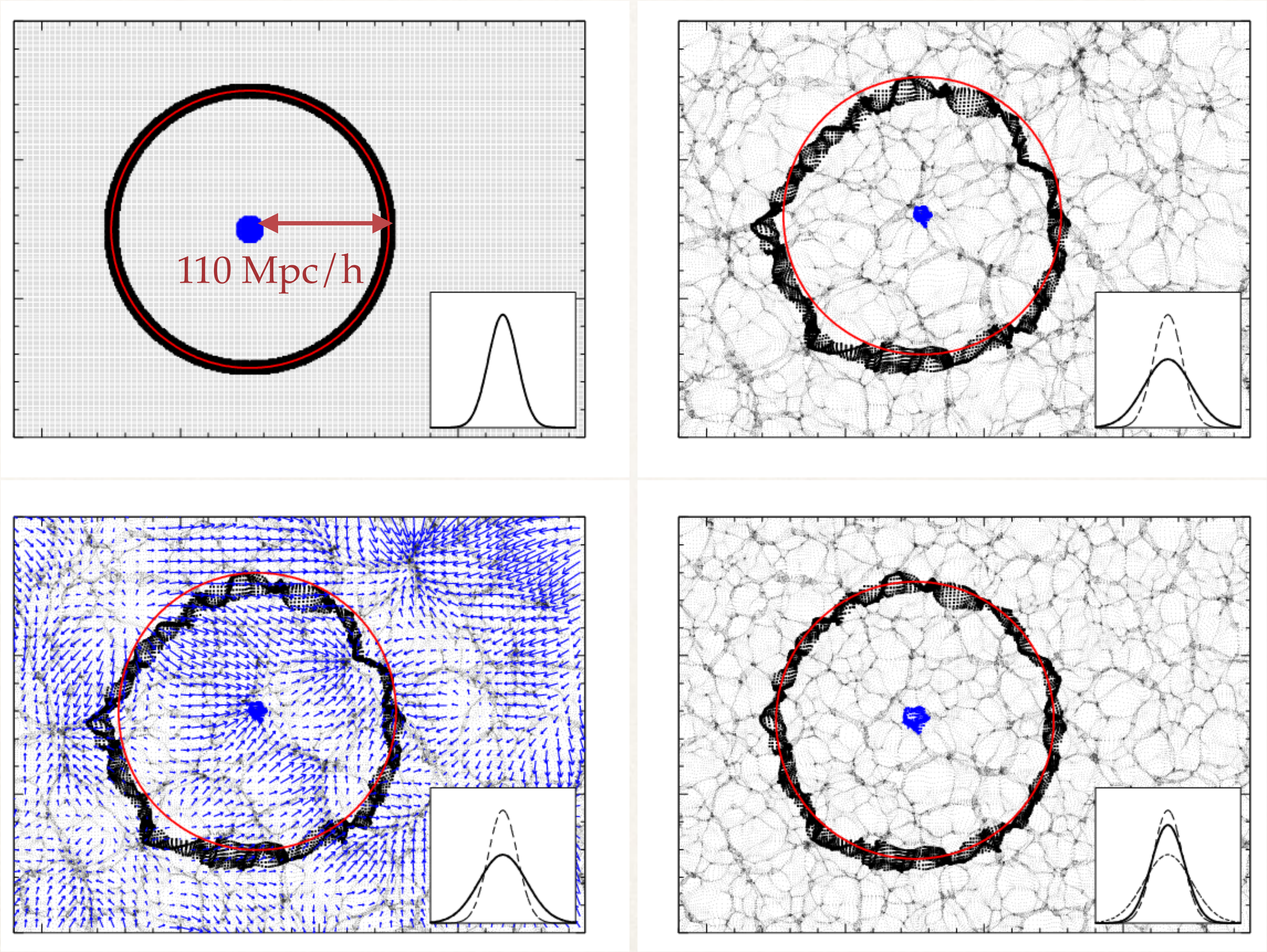
PT at any finite order truncates the full exponential behavior (P_{13}, P_{15},…)
(IR) Resummations take into account the large scale bulk motions at all PT orders

Zel'dovich and beyond



Large k limit: Zel'dovich
Small k limit: 1-loop
Interpolation built in the equation!

Large scale flows and BAO's

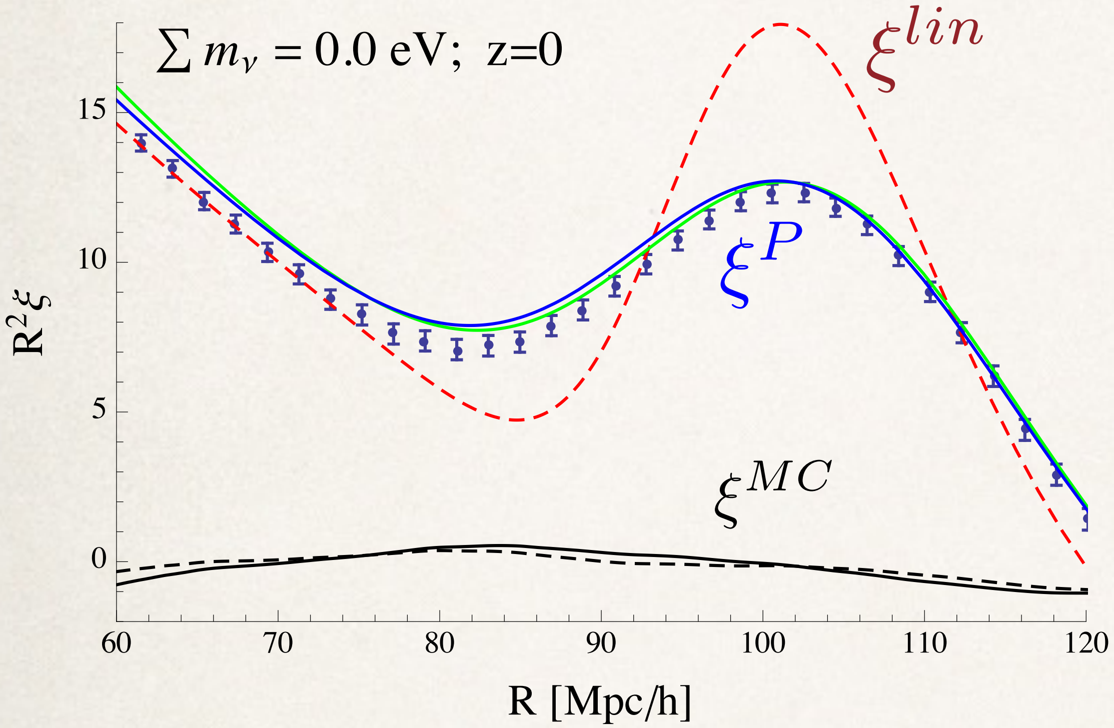


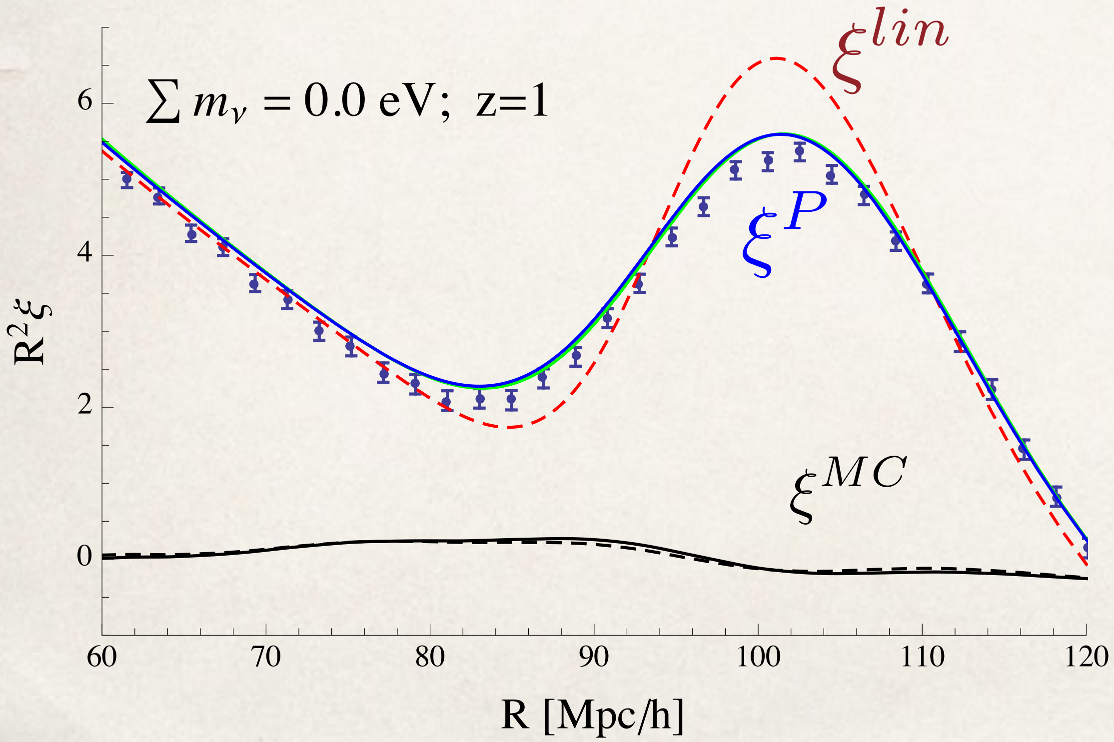
O(6 Mpc/h)
displacements

reconstruction

Seo et al, 0910.5005, Padmanabhan et al 1202.0090, Tassev, Zaldarriaga 1203.6066, …

Effect on the Correlation Function





Most of the information on the BAO peak is contained in the propagator part

The widening of the peak can be reproduced by Zel'dovich approximation (and improvements of it)

The widening of the peak contains robust physical information (not a parameter to marginalize!)

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

(simplified) Zel'dovich approximation

$$G^{Zeld}(k,z) = \mathrm{e}^{-\frac{k^2 \sigma_v^2(z)}{2}}$$

$$\sigma_v^2(z) = \frac{1}{3} \int \frac{d^3q}{(2\pi)^3} \frac{P^{lin}(q,z)}{q^2}$$

$$P_{11}^{P}(k,z) = e^{-\frac{k^2 \sigma_v^2(z)}{2}} P^{lin}(k;z)$$

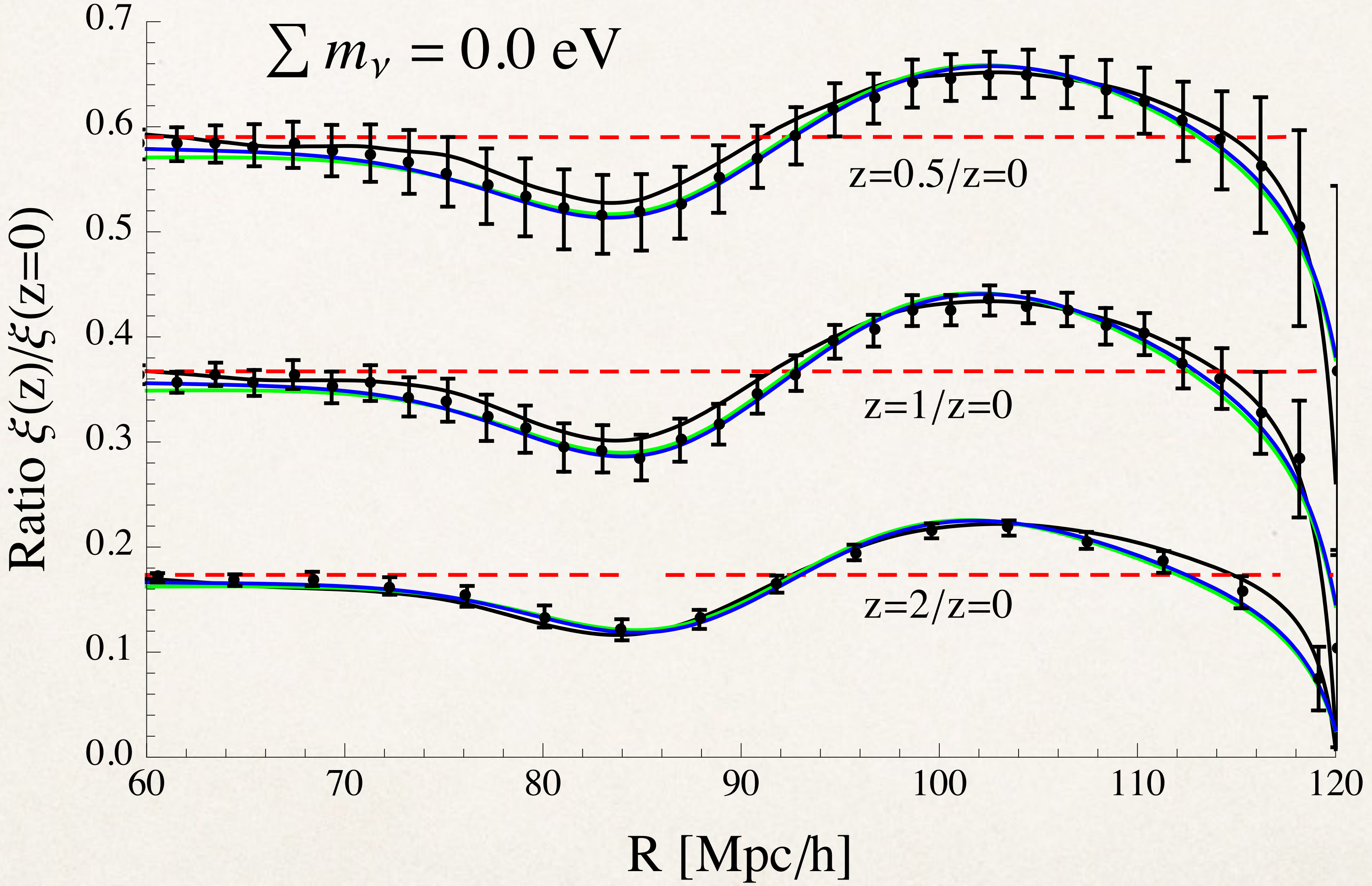
linear velocity dispersion:
contains information on linear PS, growth factor,…

$$\delta\xi(R) = \frac{1}{2\pi^2} \int dq\, q^2\, \delta P^{lin}(q) \left(\frac{\sin(qR)}{qR}\, e^{-q^2\sigma_v^2} - \frac{1}{3} \frac{\xi_2(R)}{q^2 R^2} \right)$$

$$\xi_n(R) \equiv \frac{1}{2\pi^2 R} \int_0^\infty dq\, q\, (qR)^n \sin(qR)\, P(q)$$

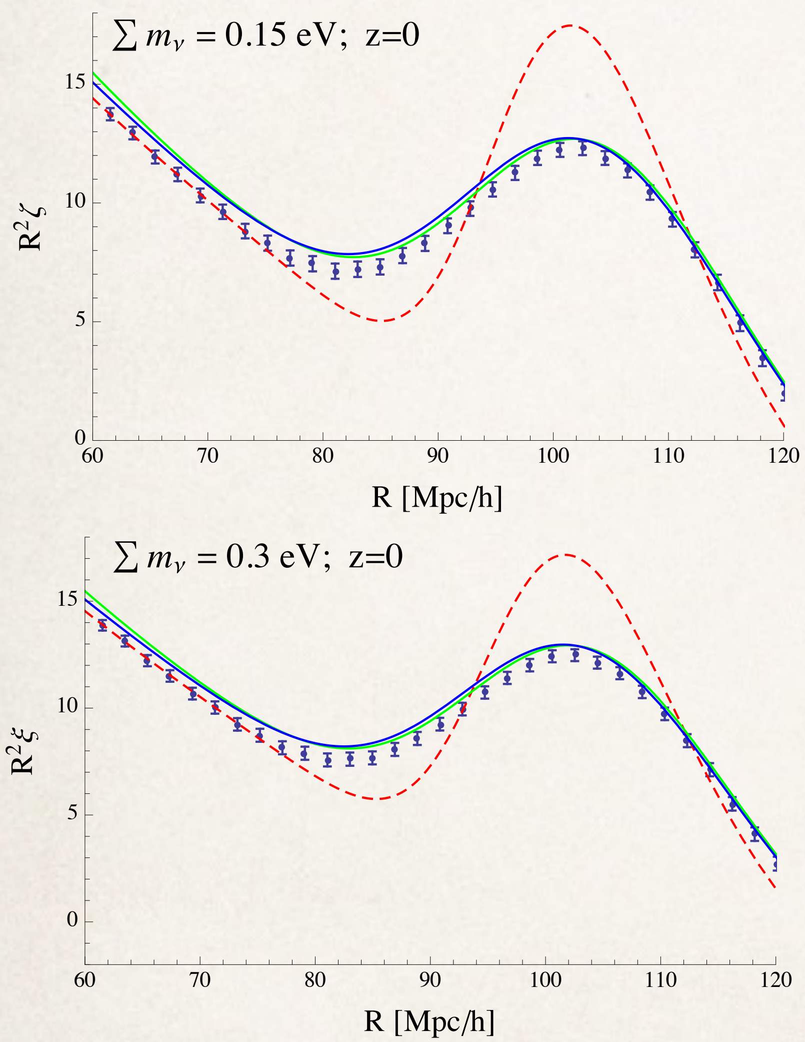
Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

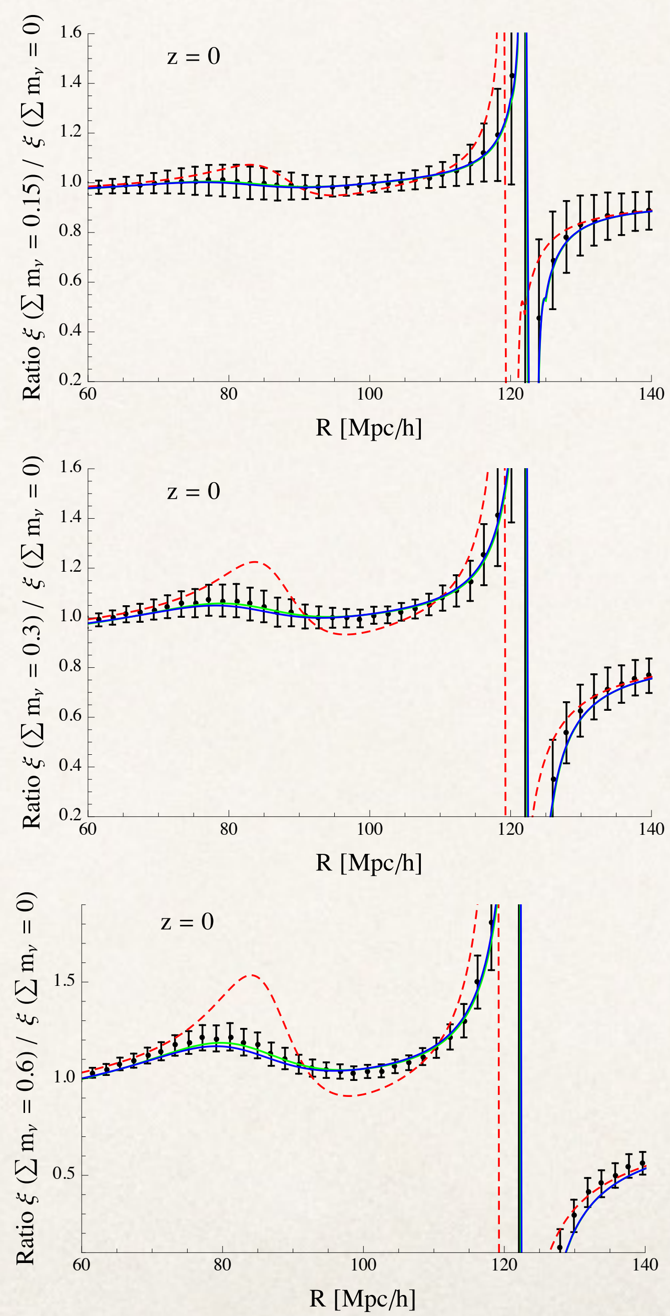
Redshift ratios



Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Effect of Massive neutrinos on BAO peak





$$P_{11}^{P}(k,z) = e^{-\frac{k^2\sigma_v^2(z)}{2}} P^{lin}(k;z)$$

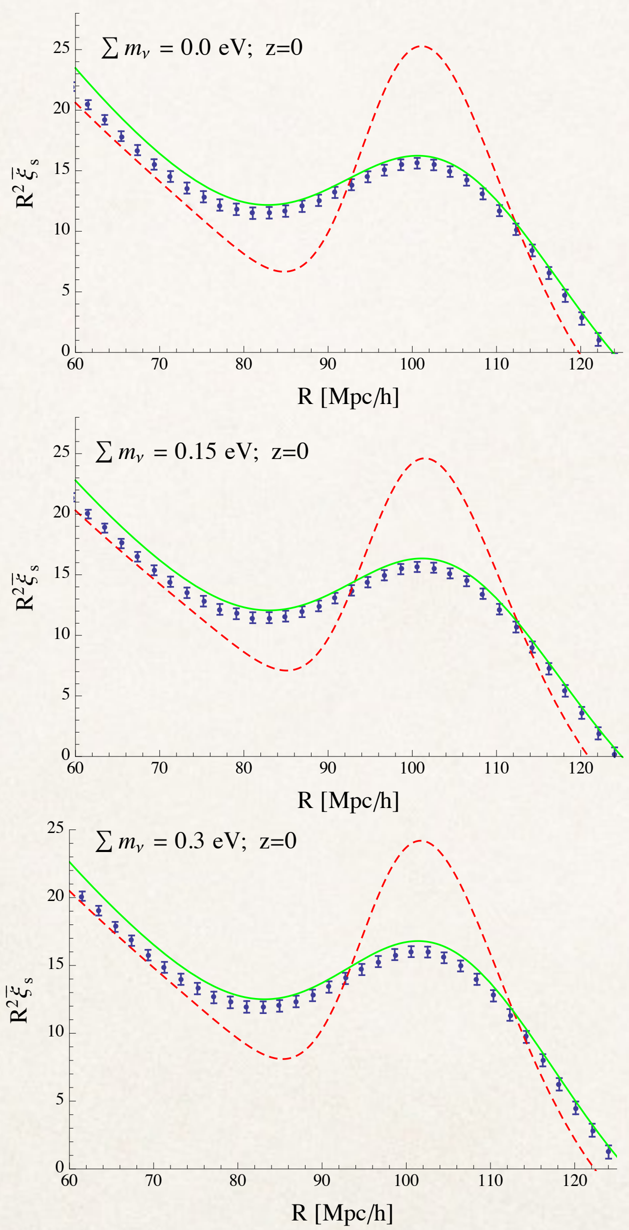
increasing neutrino masses,
Plin decreases, but also velocity dispersion decreases.

$$\sum m_\nu = 0.15 \quad \text{eV} \qquad \downarrow 0.6\%$$

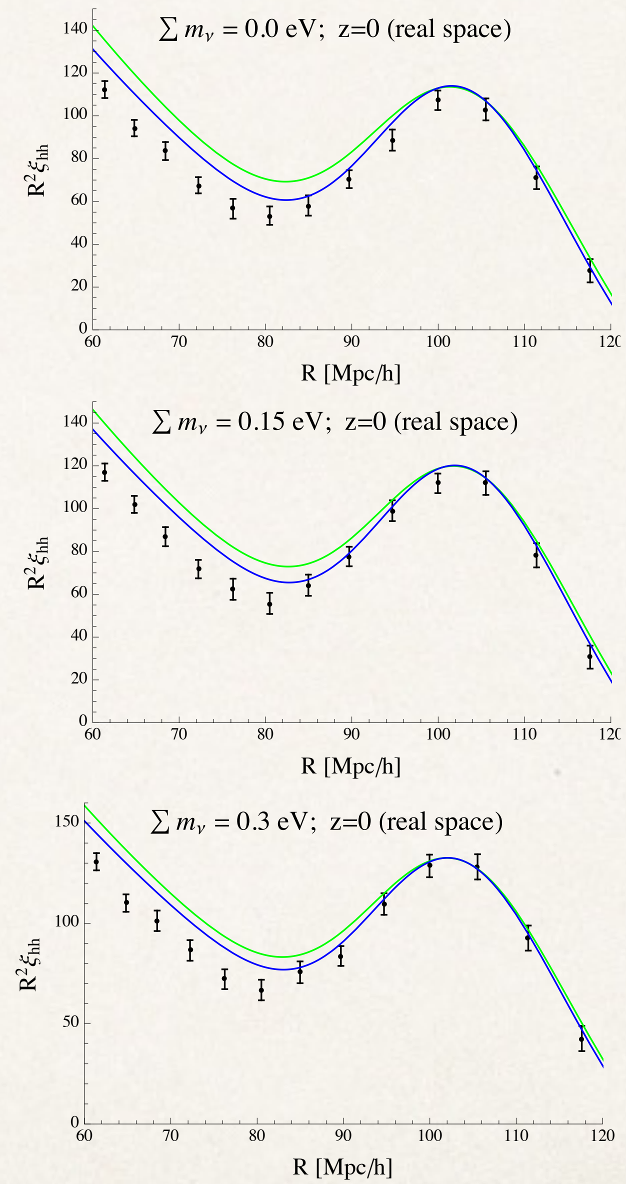
$$\sum m_\nu = 0.3 \quad \text{eV} \qquad \uparrow 1.2\%$$

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Massive neutrinos



Redshift space



Halos

Peloso, MP, Viel, Villaescusa-Navarro, 1505.07477

Mode coupling-Response functions

The nonlinear PS is a functional of the initial one (in a given cosmology and assuming no PNG): $P_{ab}[P^0](\mathbf{k},\eta)$

SPT is an expansion around $P^0(q) = 0$

$$P_{ab}[P^0](\mathbf{k};\eta) = \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \left. \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)}\right|_{P^0=0} P^0(\mathbf{q}_1)\cdots P^0(\mathbf{q}_n)$$

n=1 linear order (= "0-loop")
n=2 "1-loop"
...

$a, \cdots, d = 1$ density
$a, \cdots, d = 2$ velocity div.

Mode coupling-Response functions

We can instead expand around a reference PS: $P^0(q) = \bar{P}^0(q)$

$$P_{ab}[P^0](\mathbf{k};\eta) = P_{ab}[\bar{P}^0](\mathbf{k};\eta) + \sum_{n=1}^{\infty} \frac{1}{n!} \int d^3q_1 \cdots d^3q_n \; \frac{\delta^n P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)}\Big|_{P^0=\bar{P}^0} \delta P^0(\mathbf{q}_1)\cdots\delta P^0(\mathbf{q}_n)\,,$$

$$= P_{ab}[\bar{P}^0](\mathbf{k};\eta) + \int \frac{dq}{q}\, K_{ab}(k,q;\eta)\,\delta P^0(q) + \cdots\,,$$

$$\delta P^0(\mathbf{q}) \equiv P^0(\mathbf{q}) - \bar{P}^0(\mathbf{q})$$

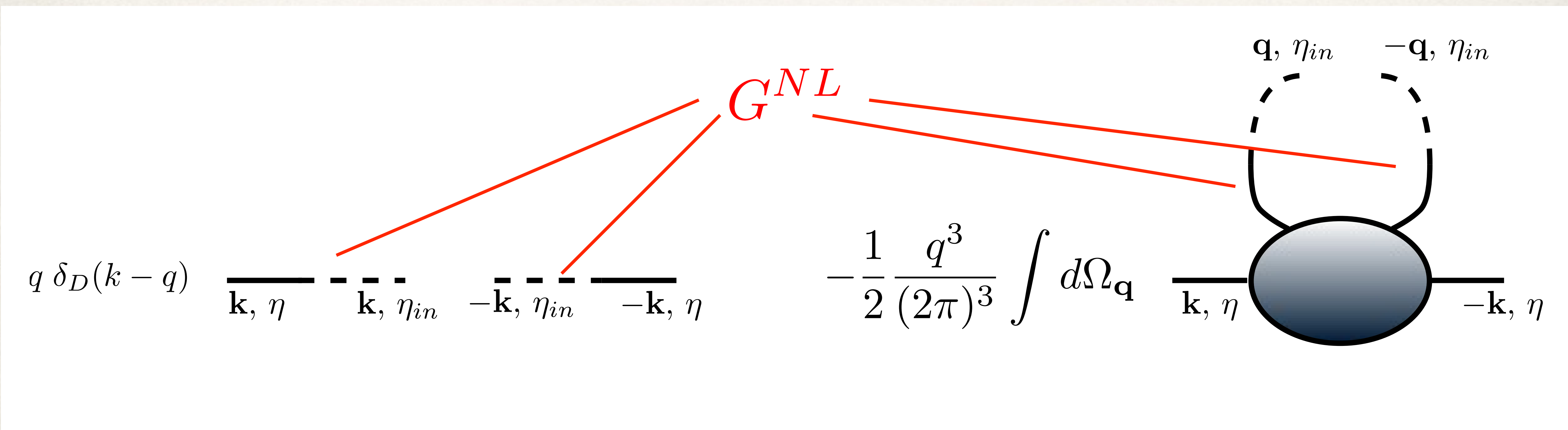
Linear response function: $K_{ab}(k,q;\eta) \equiv q^3 \int d\Omega_{\mathbf{q}} \; \frac{\delta P_{ab}[P^0](\mathbf{k};\eta)}{\delta P^0(\mathbf{q})}\Big|_{P^0=\bar{P}^0}$

Non-perturbative (gets contributions from all SPT orders)

Key object for more efficient interpolators ?

The non-perturbative LRF

$$K_{ab}(k,q;\eta) = q\,\delta_D(k-q)\,G_{ac}(k;\eta,\eta_{in})u_c\,G_{bd}(k;\eta,\eta_{in})u_d$$
$$-\frac{1}{2}\frac{q^3}{(2\pi)^3}\int d\Omega_{\mathbf{q}}\,\langle\varphi_a(\mathbf{k};\eta)\chi_c(-\mathbf{q};\eta_{in})\chi_d(\mathbf{q};\eta_{in})\varphi_b(-\mathbf{k};\eta)\rangle'_c\,u_c u_d\,,$$



$$G_{ab}(k;\eta,\eta_{in}) = \langle\frac{\delta\varphi_a(\mathbf{k},\eta)}{\delta\varphi_b(\mathbf{k},\eta_{in})}\rangle' = -i\langle\varphi_a(\mathbf{k},\eta)\chi_b(-\mathbf{k},\eta_{in})\rangle'$$

methods from Matarrese, MP, ‘07

IR consistency relations

M. Peloso, M.P.
1302.0223/1310.7915
A. Kehagias, A. Riotto et al
1302.0130
Creminelli et al. 1309.3557
P. Valageas 1311.1236

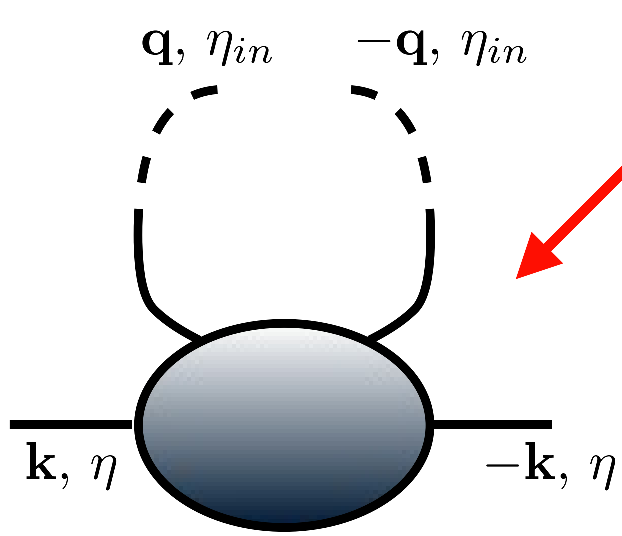
the effect of a long wavelength (time dependent) velocity mode can be exactly reabsorbed by a change of coordinates in the uniform limit

<u>exact</u> relations between N and N+1 point functions

$$v^j(\mathbf{q}) = i\frac{q^j}{q^2}\theta(\mathbf{q}) \propto i\frac{q^j}{q^2}\delta(\mathbf{q})$$

No 1/q dependence for soft q

$$-\frac{1}{2}\frac{q^3}{(2\pi)^3}\int d\Omega_{\mathbf{q}}$$

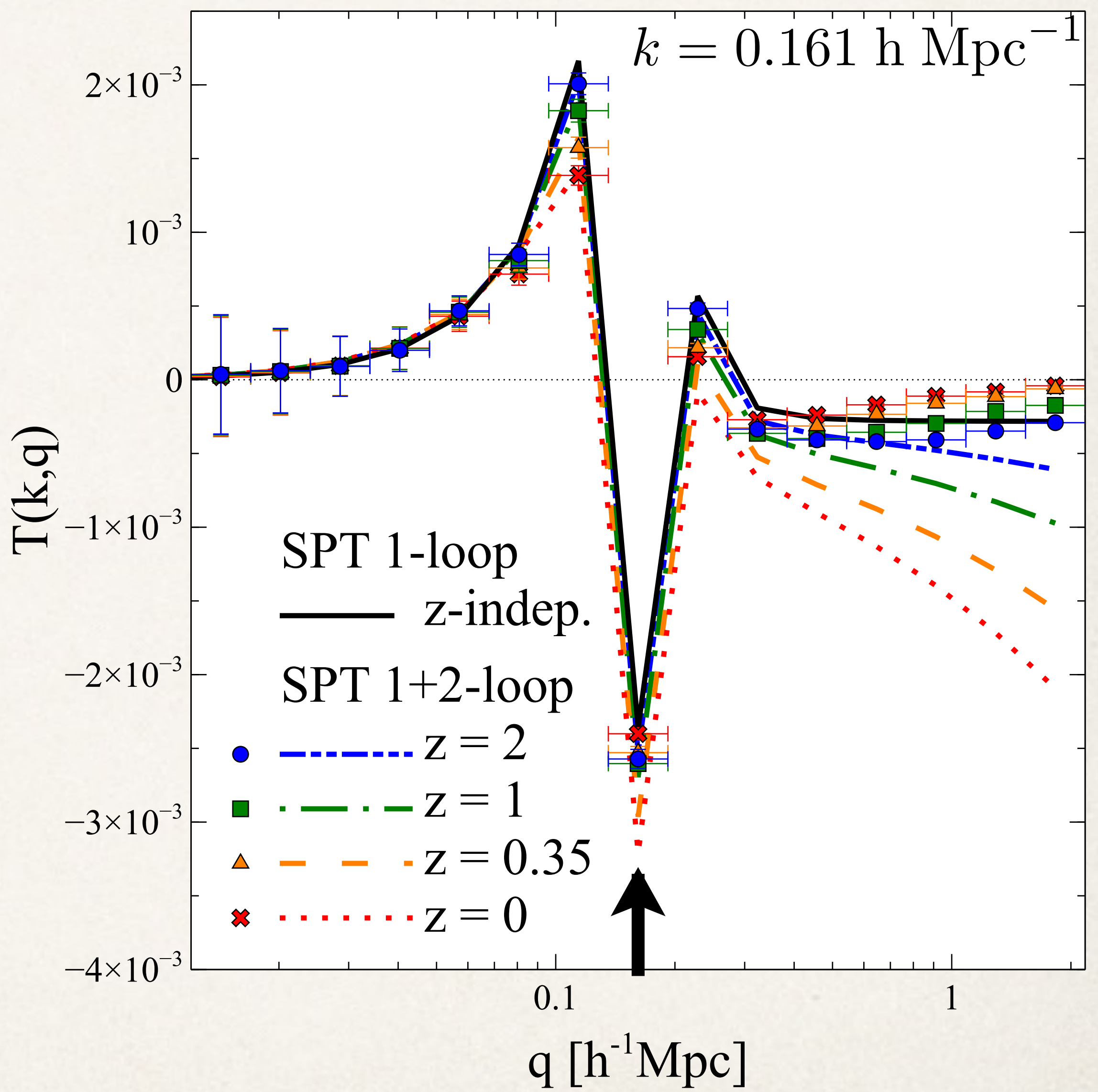


$$K_{ab}(k,q) \propto q^3 \quad \text{for} \quad q \ll k$$

IR screening

Sensitivity of the nonlinear PS at scale k on a change of the initial PS at scale q:

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



no O(q) terms

Nishimichi, Bernardeau, Taruya 1411.2970

… Little, Weinberg, Park, 1991

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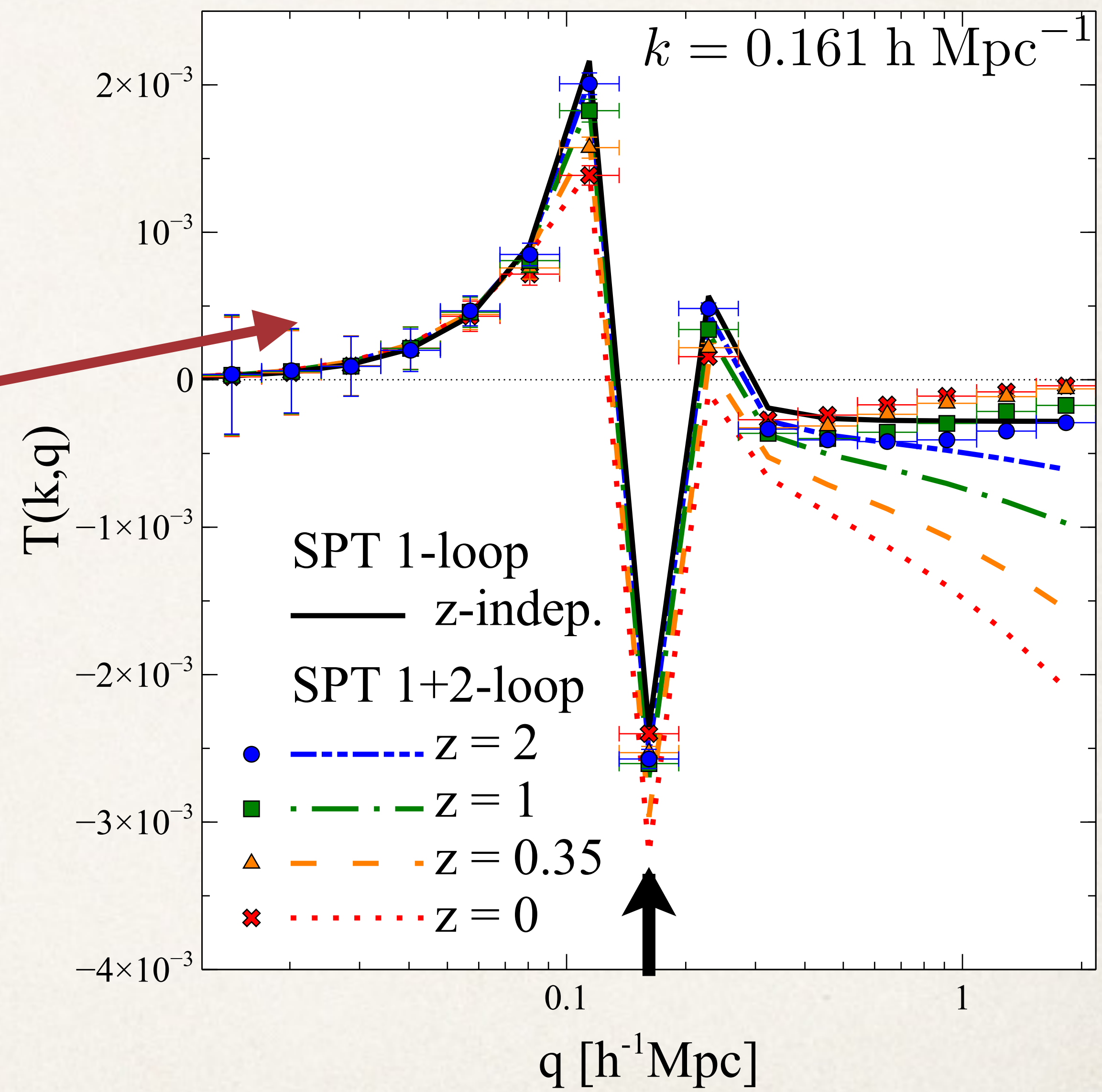
Sensitivity of the nonlinear PS at scale k on a change of the initial PS at scale q:

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$

IR: "Galilean invariance"

$K(k,q;z) \sim q^3$

no O(q) terms



Nishimichi, Bernardeau, Taruya 1411.2970

… Little, Weinberg, Park, 1991

Resummations and IR sensitivity

PS in Lagrangian framework

$\mathbf{\Psi}(\mathbf{q}) = \mathbf{x} - \mathbf{q}$ displacement field

$(1+\delta(\mathbf{x}))\, d^3x = d^3q$ mass conservation

$$\delta(\mathbf{k}) = \int d^3x\, \delta(\mathbf{x}) e^{-i\mathbf{k}\cdot\mathbf{x}} = \int d^3q\, e^{-i\mathbf{k}\cdot\mathbf{q}} \left(e^{-i\mathbf{k}\cdot\mathbf{\Psi}(\mathbf{q})} - 1\right)$$

$$P(k;\tau,\tau') = \int d^3r\ \mathrm{e}^{-i\,\mathbf{k}\cdot\mathbf{r}} \left(\langle \mathrm{e}^{-i\mathbf{k}\cdot\Delta\mathbf{\Psi}}\rangle - 1\right)$$

$$\Delta\mathbf{\Psi} = \mathbf{\Psi}(\mathbf{q},\tau) - \mathbf{\Psi}(\mathbf{q}',\tau'),$$

$$\mathbf{r} = \mathbf{q} - \mathbf{q}'$$

Zel′dovich approximation: displacement field from linear PT

$$\mathbf{\Psi}_Z(\mathbf{q},\tau) = \int_0^\tau d\tau'' \mathbf{v}(\mathbf{q},\tau'') = i\, D_+(\tau) \int \frac{d^3k}{(2\pi)^3}\, \mathrm{e}^{i\,\mathbf{q}\cdot\mathbf{k}}\, \frac{\mathbf{k}}{k^2}\, \delta_l(\mathbf{k},0),$$

(gaussian field)

$$\langle \mathrm{e}^{-i\mathbf{k}\cdot\Delta\mathbf{\Psi}_Z} \rangle = \mathrm{e}^{-\frac{1}{2}k^i k^j \langle \Delta\Psi^i_Z \Delta\Psi^j_Z \rangle}$$

$$P_Z(k;\tau,\tau') = \int d^3r\ \cos(\mathbf{k}\cdot\mathbf{r}) \left[\mathrm{e}^{-\frac{k^2\sigma^2}{2}(D-D')^2}\, \mathrm{e}^{-DD'\left(k^2\sigma^2 - I(\mathbf{k},\mathbf{r})\right)} - 1 \right]$$

$D \equiv D_+(\tau)$, $D' \equiv D_+(\tau')$ linear growth factor

$$I(\mathbf{k},\mathbf{r}) = \int \frac{d^3p}{(2\pi)^3}\, \mathrm{e}^{i\,\mathbf{p}\cdot\mathbf{r}}\, \frac{(\mathbf{p}\cdot\mathbf{k})^2}{p^4}\, P^0(p)\,, \qquad I(\mathbf{k},0) = k^2\sigma^2$$

All orders in SPT, good testing ground for resummations

$e^{-DD'(k^2\sigma^2 - I(\mathbf{k},\mathbf{r}))}$ insensitive to IR velocity modes with momentum p $p < O(1/r) \simeq O(\mathrm{Min}[k, 1/l_{BAO}])$

SPT: $$e^{-DD'(k^2\sigma^2 - I(\mathbf{k},\mathbf{r}))} = \sum_{n=0}^{N} \frac{(-1)^n}{n!} \left(DD'(k^2\sigma^2 - I(\mathbf{k},\, \mathbf{r}))\right)^n$$

No spurious IR dependence

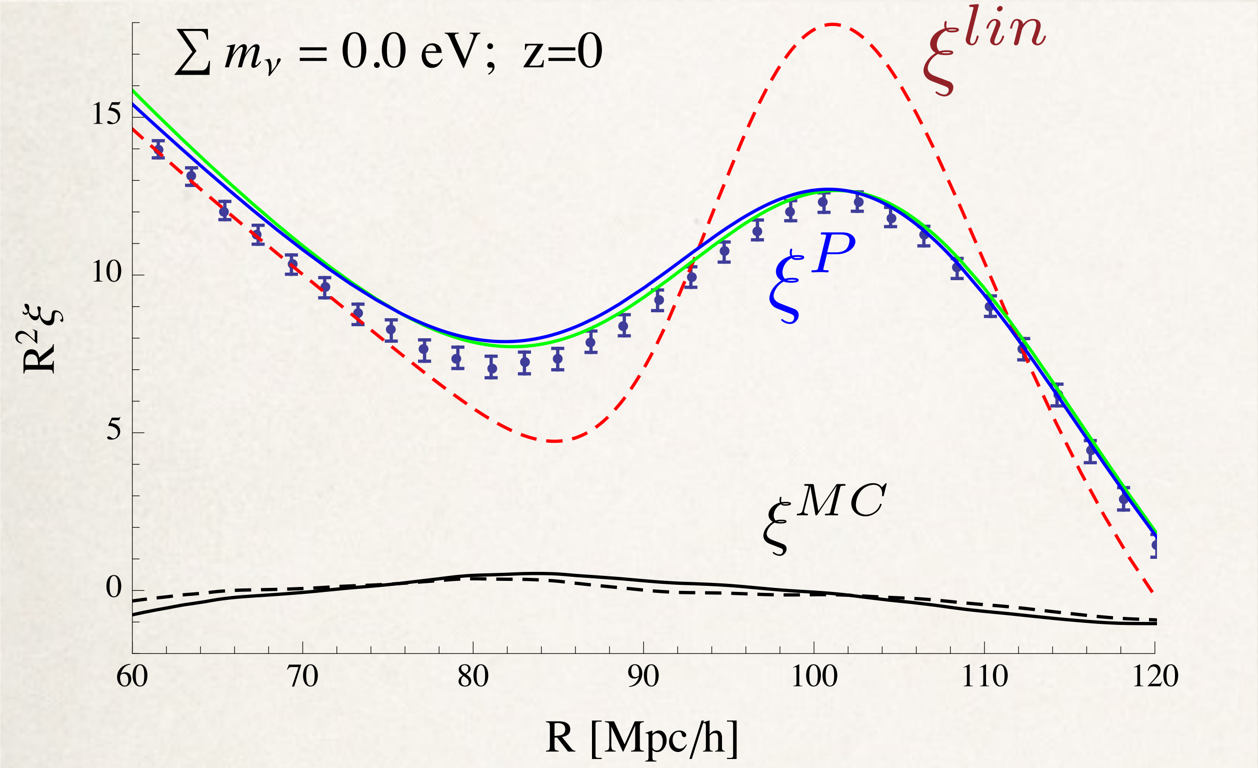
but truncated "propagator" $e^{-\frac{k^2\sigma^2}{2}(D-D')^2} = \sum_{n=0}^{N} \frac{(-1)^n}{n!} \left(\frac{k^2\sigma^2}{2}(D-D')^2\right)^n$

RPT & Co.: $$e^{-DD'(k^2\sigma^2 - I(\mathbf{k},\mathbf{r}))} \simeq e^{-DD'k^2\sigma^2} \sum_{n=0}^{N} \frac{1}{n!} \left(DD'I(\mathbf{k},\, \mathbf{r}))^n\right)$$

Spurious IR dependence at order N+1

intact "propagator" $e^{-\frac{k^2\sigma^2}{2}(D-D')^2}$

How much does it matter?



Not so much on the BAO feature

Broadband effect $\sim (k^2\sigma^2)^{N+1}$

IR-UV connection

IR safe resummations

General idea: define $\sigma^2(\bar{p}) = \frac{1}{3}\int \frac{d^3p}{(2\pi)^3} f\left(\frac{p}{\bar{p}}\right) \frac{P^0(p)}{p^2}$ ex: $f(x) = e^{-x^2}$

$$e^{-DD'(k^2\sigma^2 - I(\mathbf{k},\mathbf{r}))} \simeq e^{-DD'k^2(\sigma^2 - \sigma^2(\bar{p}))} \sum_{n=0}^{N} \frac{(DD')^n}{n!} \left(I(\mathbf{k},\, \mathbf{r}) - k^2\sigma^2(\bar{p})\right)^n$$

Properties:

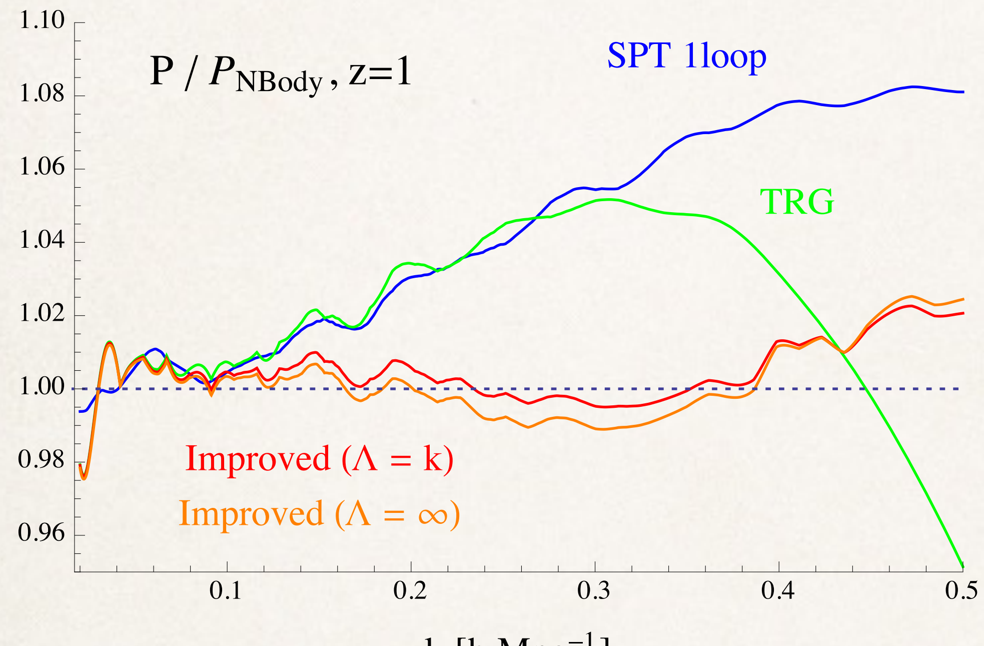
1) approaches SPT as $\bar{p} \to \infty$;

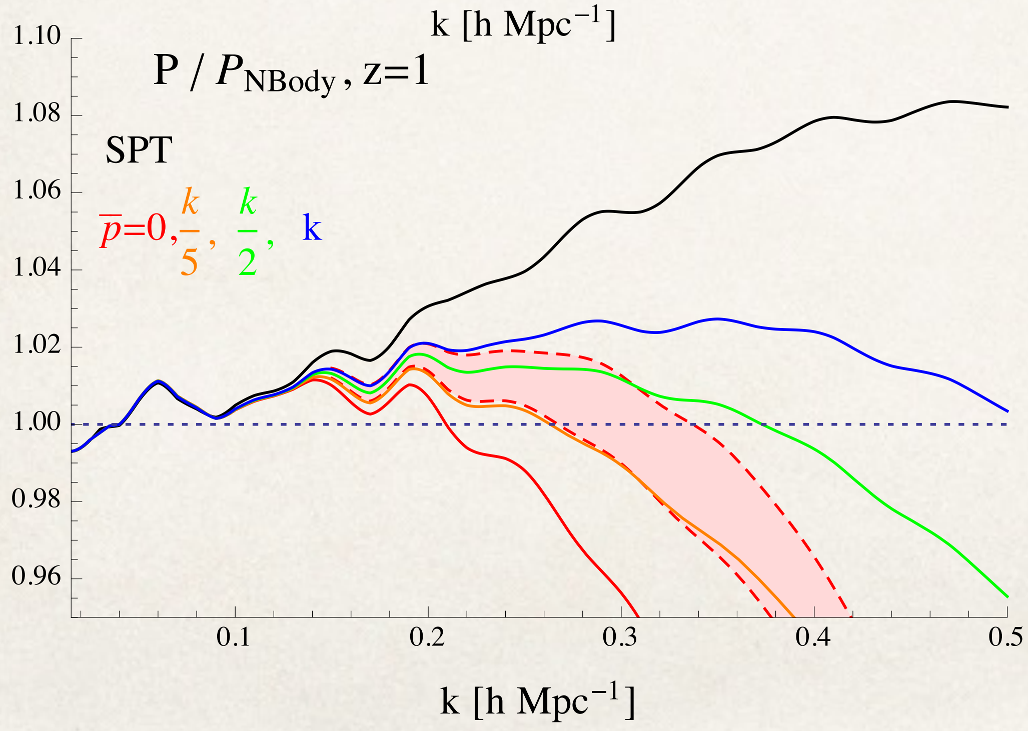
2) for any finite $\bar{p}$ is IR safe;

3) RPT corresponds to $\bar{p} = 0$ strictly. “Singular” non IR safe point.

Restoring the IR behaviour

M. Peloso, MP (in preparation)





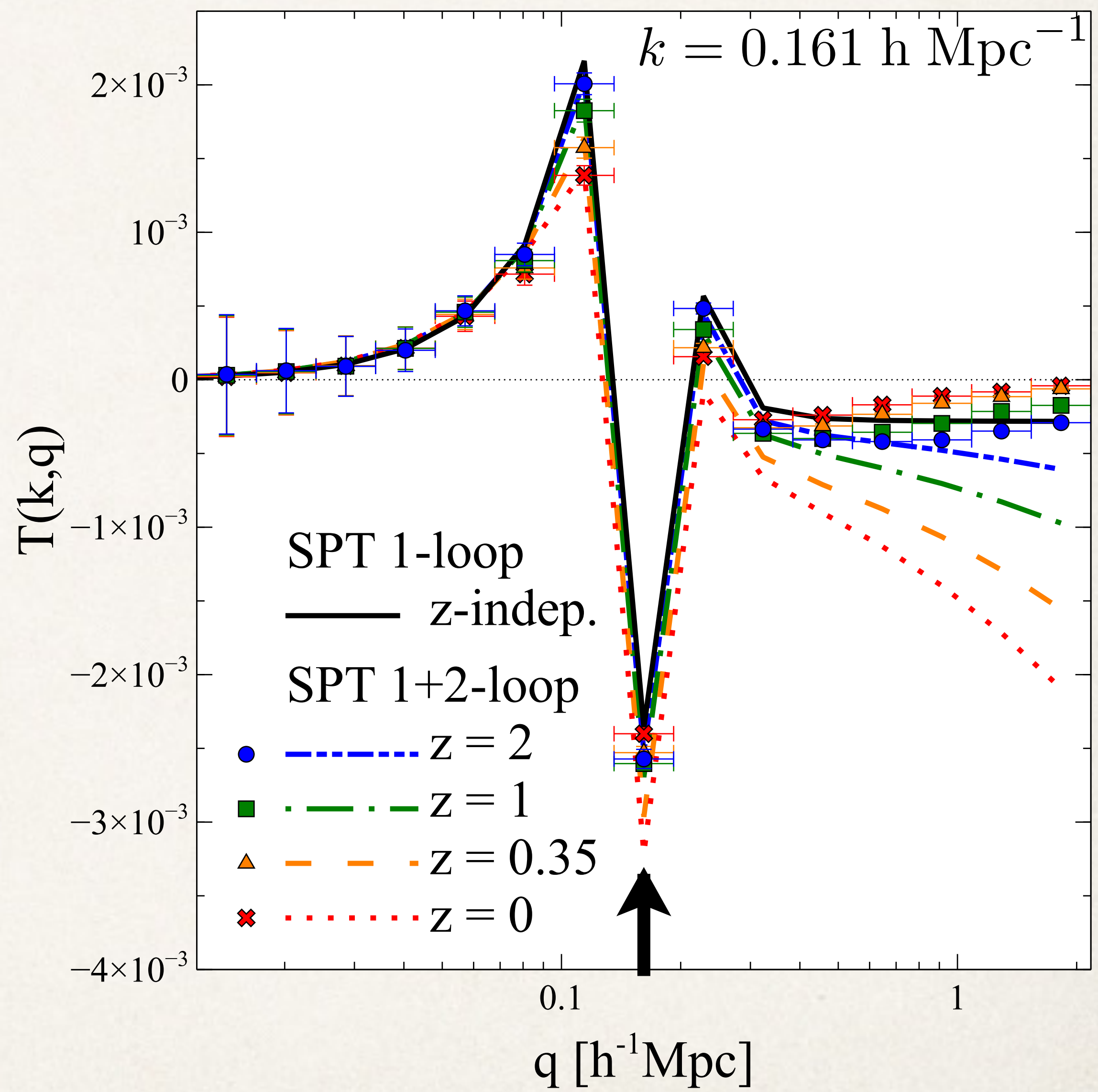
direct subtraction of
O(q) contributions to K(k,q)

IR safe evolution equations

UV screening

Sensitivity of the nonlinear PS at scale k on a change of the initial PS at scale q:

$$K(k,q;z) = q\frac{\delta P^{\rm nl}(k;z)}{\delta P^{\rm lin}(q;z)}$$



Nishimichi, Bernardeau, Taruya 1411.2970

… Little, Weinberg, Park, 1991

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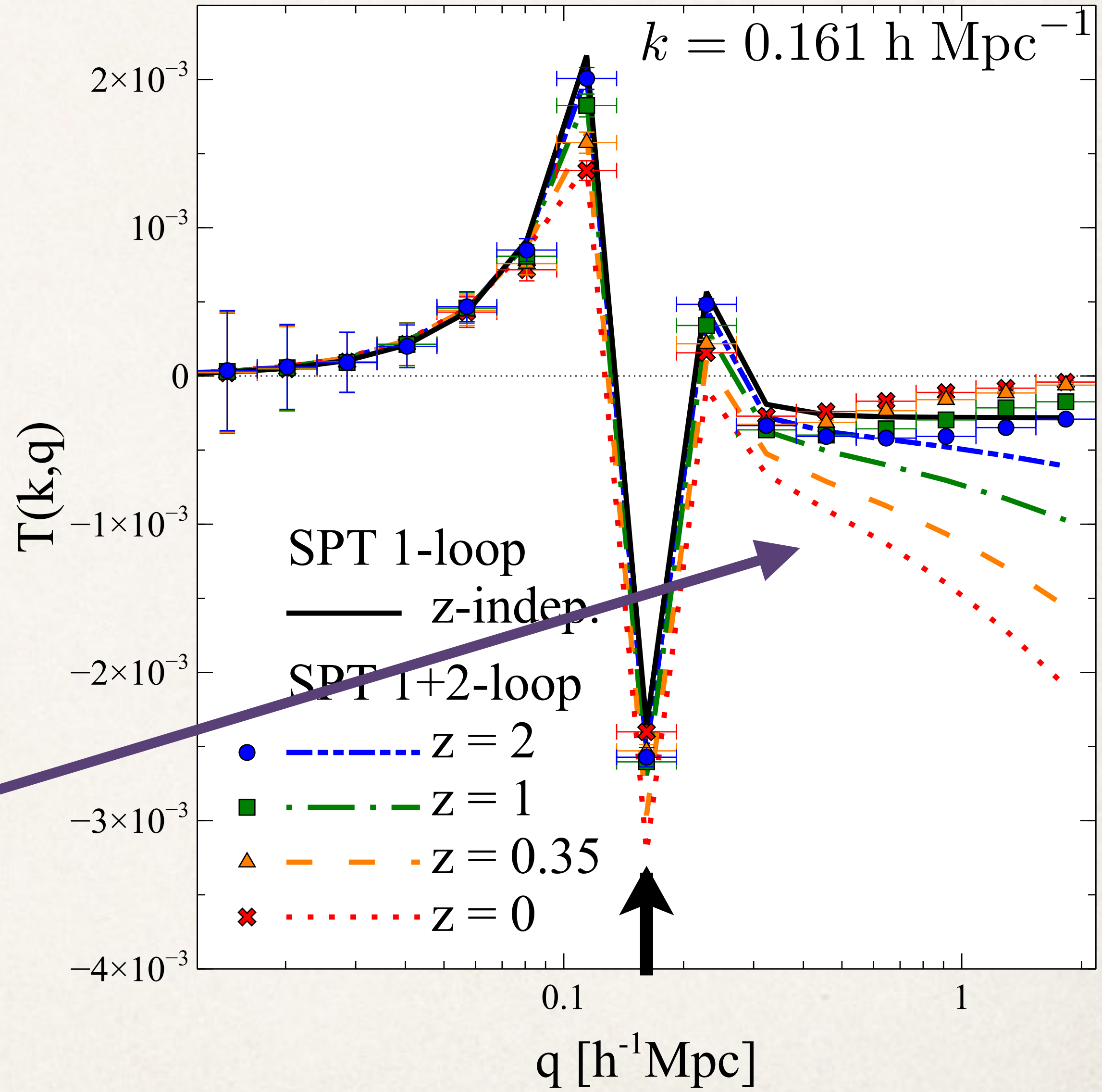


<u>PT overpredicts the sensitivity of UV scales on intermediate ones</u>

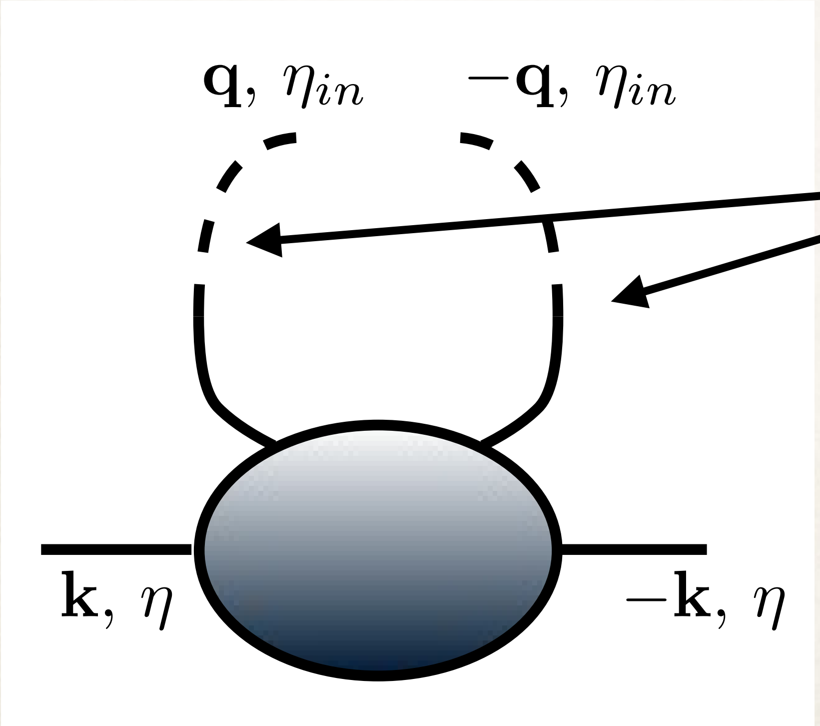
Nishimichi, Bernardeau, Taruya 1411.2970

… Little, Weinberg, Park, 1991

UV screening

The effect of virialized structures on larger scales is screened (Peebles '80, Baumann et al 1004.2488, Blas et al 1408.2995).

However, the departure from the PT predictions starts at small q's: is it only a virialization effect?

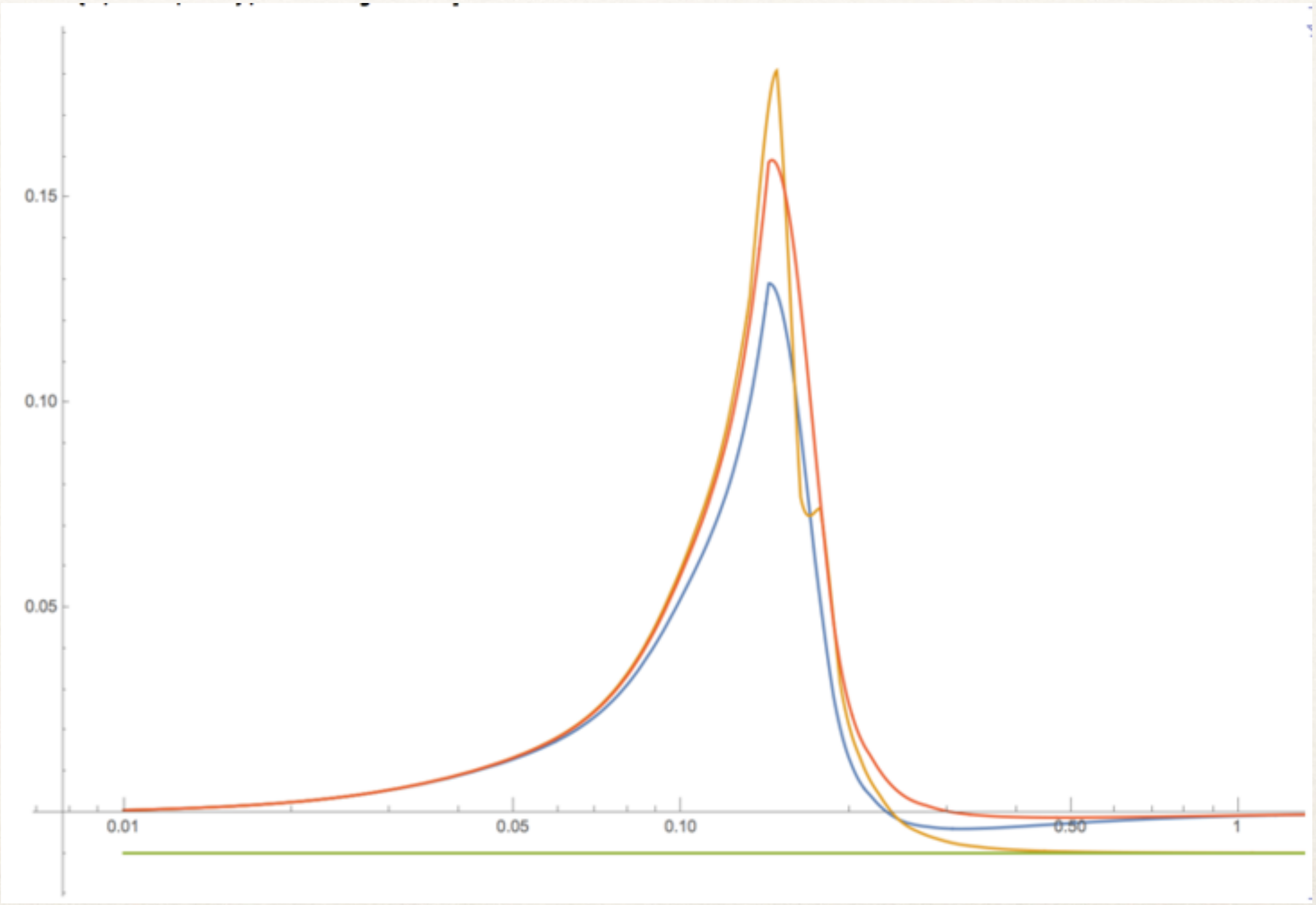


$e^{-\frac{q^2\sigma_v^2}{2}}$ damped propagators!

(compare SPT: linear propagator g=O(1))

memory of initial substructures is largely lost

NL propagator and LRF

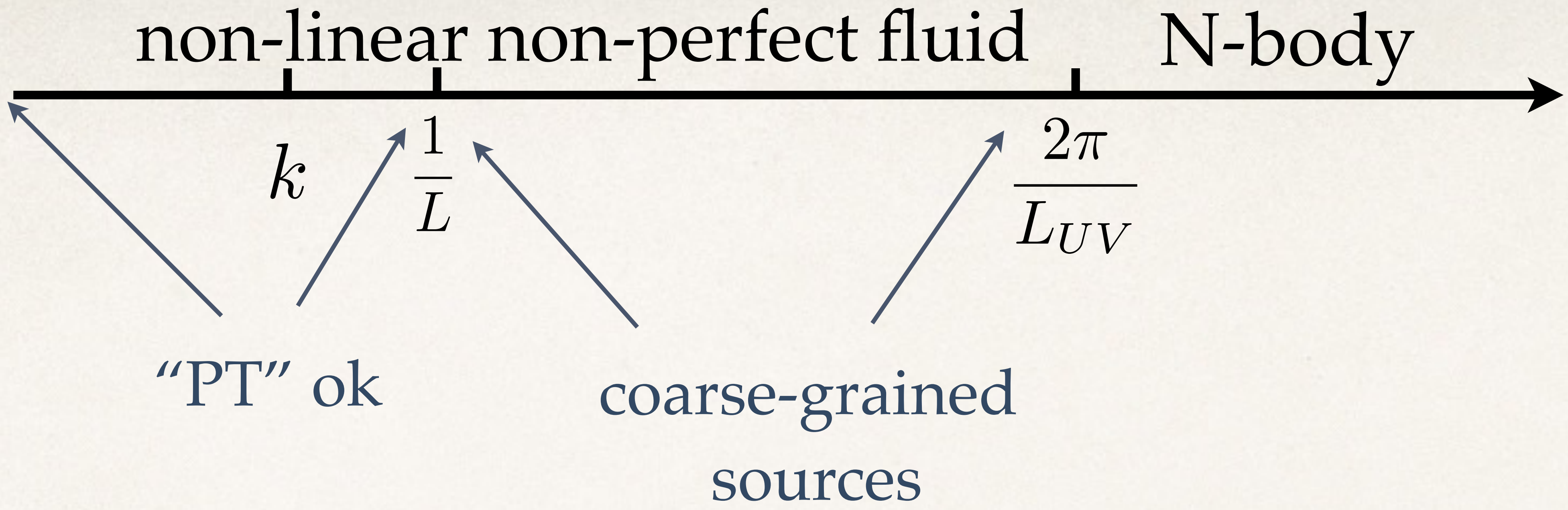


UV lessons

- SPT fails when loop momenta become higher than the nonlinear scale ($q \gtrsim 0.4$ h/Mpc)
- The real response to modifications in the UV regime is mild
- Most of the cosmology dependence is on intermediate scales

Dealing with the UV

- General idea: take the UV physics from N-body simulations and use (IR resummed) PT only for the large and intermediate scales



Physics at k must be independent on L, L_uv
("Wilsonian approach")

Expansion in sources:

$$\langle \delta\delta \rangle_J = \langle \delta\delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2}\langle \delta J J \delta \rangle_{J=0} + \cdots$$

computed in PT with cutoff at 1/L

measured from simulations

M.P., G. Mangano, N. Saviano, M. Viel, 1108.5203

Manzotti, Peloso, MP, Villaescusa-Navarro, Viel, 1407.1342

Exact large scale dynamics for density and velocity fields

$$\frac{\partial}{\partial \tau}\delta(\mathbf{x}) + \frac{\partial}{\partial x^i}\left[(1+\delta(\mathbf{x}))v^i(\mathbf{x})\right] = 0$$

$$\frac{\partial}{\partial \tau}v^i(\mathbf{x}) + \mathcal{H}v^i(\mathbf{x}) + v^k(\mathbf{x})\frac{\partial}{\partial x^k}v^i(\mathbf{x}) = -\nabla^i_x\phi(\mathbf{x}) - \underline{J^i_\sigma(\mathbf{x}) - J^i_1(\mathbf{x})}$$

$$\nabla^2\phi(\mathbf{x}) = \frac{3}{2}\Omega_M\mathcal{H}^2\delta(\mathbf{x})$$

$$n(\mathbf{x}) = n_0(1+\delta(\mathbf{x})) = n_0(1+\langle\delta_{mic}\rangle(\mathbf{x}))$$

$$v^i(\mathbf{x}) = \frac{\langle(1+\delta_{mic})v^i_{mic}\rangle(\mathbf{x})}{1+\delta(\mathbf{x})}$$

external input on UV-physics needed

$$J^i_\sigma(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\frac{\partial}{\partial x^k}(n(\mathbf{x})\sigma^{ki}(\mathbf{x}))$$

$$J^i_1(\mathbf{x}) \equiv \frac{1}{n(\mathbf{x})}\left(\langle n_{mic}\nabla^i\phi_{mic}\rangle(\mathbf{x}) - n(\mathbf{x})\nabla^i\phi(\mathbf{x})\right)$$

EFT approach

Carrasco et al, 1206.2926

…

Blas et al, 1507.06665

Floerchinger et al, 1607.03453 ← RG

$$J^i_\sigma + J^i_1 \equiv -\frac{1}{\rho}\partial_j \tau^{ji}$$

Effective stress-tensor for the long modes

$$\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v_l^k - \frac{3}{4}\frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3}\delta^{ij}\partial_k v_l^k \right) \right] + \Delta\tau^{ij} + \ldots \ .$$

Expansion in the long modes

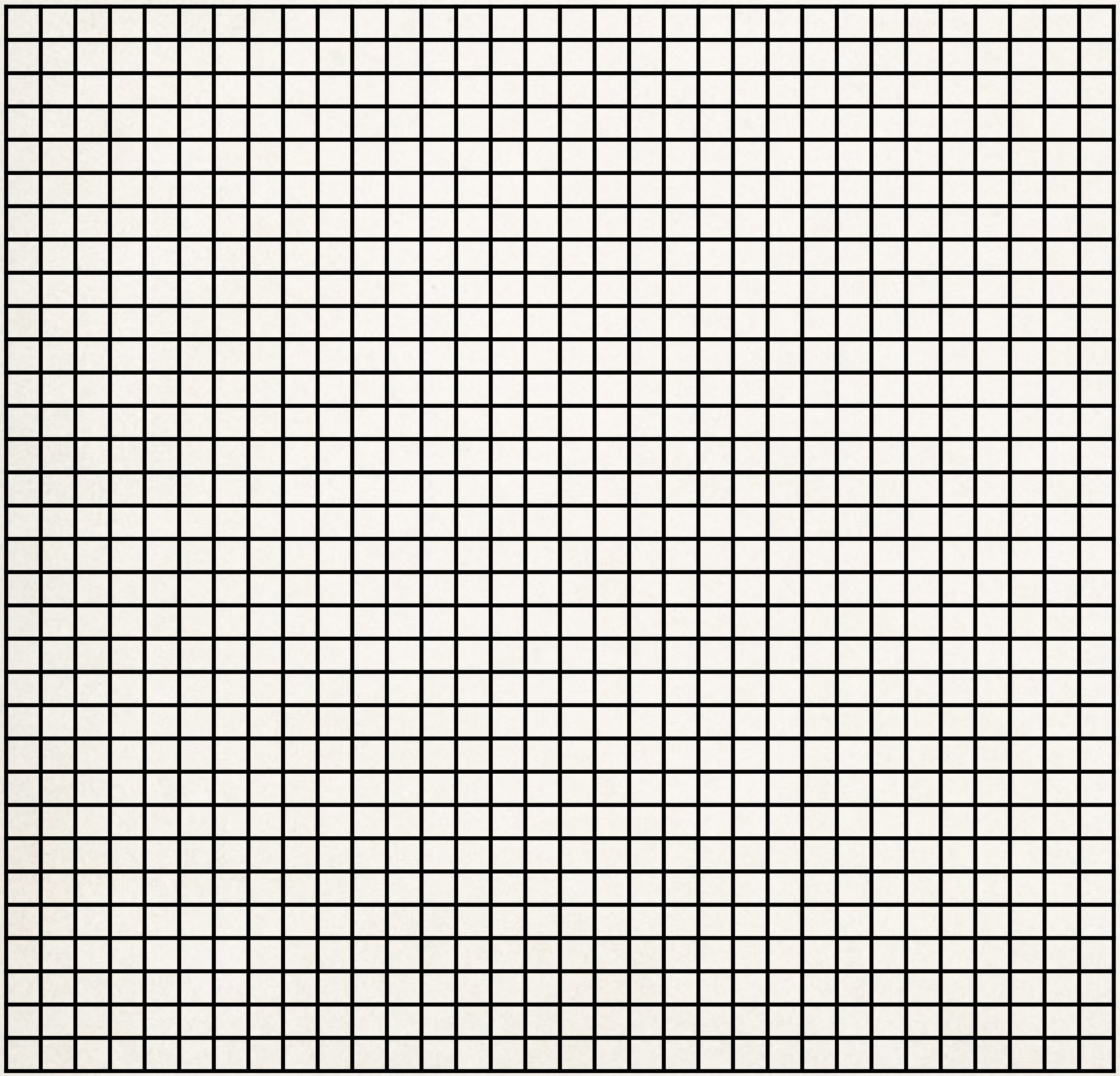
Issues:

scale and time dependence of the expansion parameters

matching procedure

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP,
Villaescusa-Navarro, Viel, 1407.1342



$$L_{UV} = 1,\ 2,\ 4\ \mathrm{Mpc/h}$$

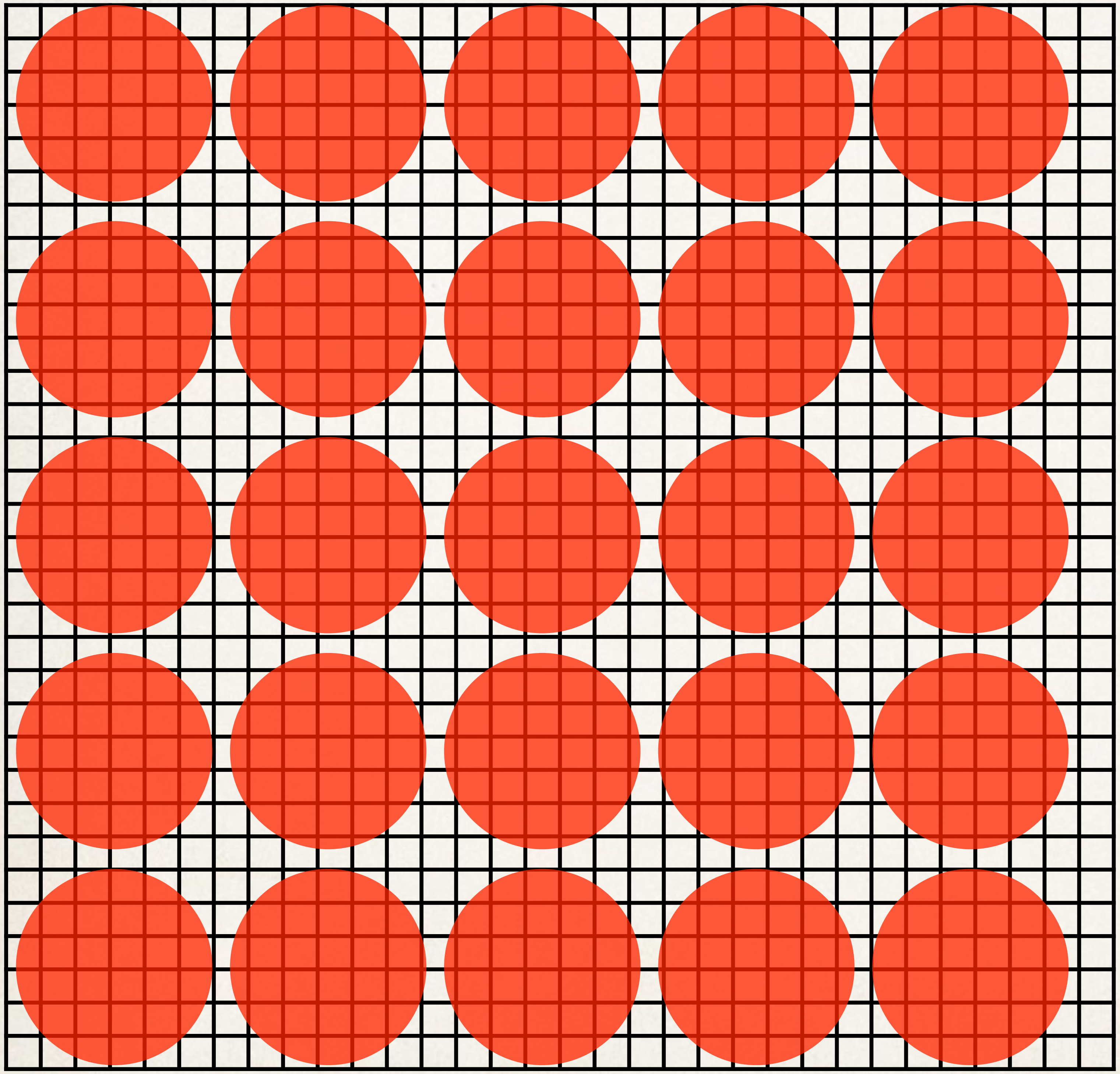
$$L_{UV}:\ \ \delta,\ v^i,\ J_1^i\,,\ J_\sigma^i$$

$L_{box} = 512\,\mathrm{Mpc/h}$

$N_{particles} = (512)^3$

Measuring the sources in Nbody simulation

Manzotti, Peloso, MP,
Villaescusa-Navarro, Viel, 1407.1342



$$L_{UV} = 1,\, 2,\, 4\ \mathrm{Mpc/h}$$

$$L_{UV}:\ \delta,\, v^i,\, J_1^i\,,\, J_\sigma^i$$

$$L:\ \bar{\delta},\, \bar{v}^i,\, \bar{J}_1^i,\, \bar{J}_\sigma^i$$

$$\mathcal{W}(R/L) = \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{L^3}\, e^{-\frac{R^2}{2L^2}}$$

$$L_{box} = 512\,\mathrm{Mpc/h} \qquad N_{particles} = (512)^3$$

Expansion in UV sources

$$\langle \delta\delta \rangle_J = \langle \delta\delta \rangle_{J=0} + \langle \delta J \delta \rangle_{J=0} + \frac{1}{2}\langle \delta J J \delta \rangle_{J=0} + \cdots$$

$$\bar{P}_{11}(k,\eta) \simeq \bar{P}_{11}^{lin}(k,\eta) + \bar{P}_{ss,11}^{1-\text{loop}}(k,\eta) - \Delta\bar{P}_{ss,11}^{h,1-\text{loop}}(k,\eta) + \Delta\bar{P}_{11}^{h,\text{N}-\text{body}}(k,\eta)$$

$$\langle \delta J \delta \rangle_{J=0}$$

(- PT + Nbody)_UV

Expansion in UV sources

$$\langle\delta\delta\rangle_J = \langle\delta\delta\rangle_{J=0} + \langle\delta J\delta\rangle_{J=0} + \frac{1}{2}\langle\delta JJ\delta\rangle_{J=0} + \cdots$$

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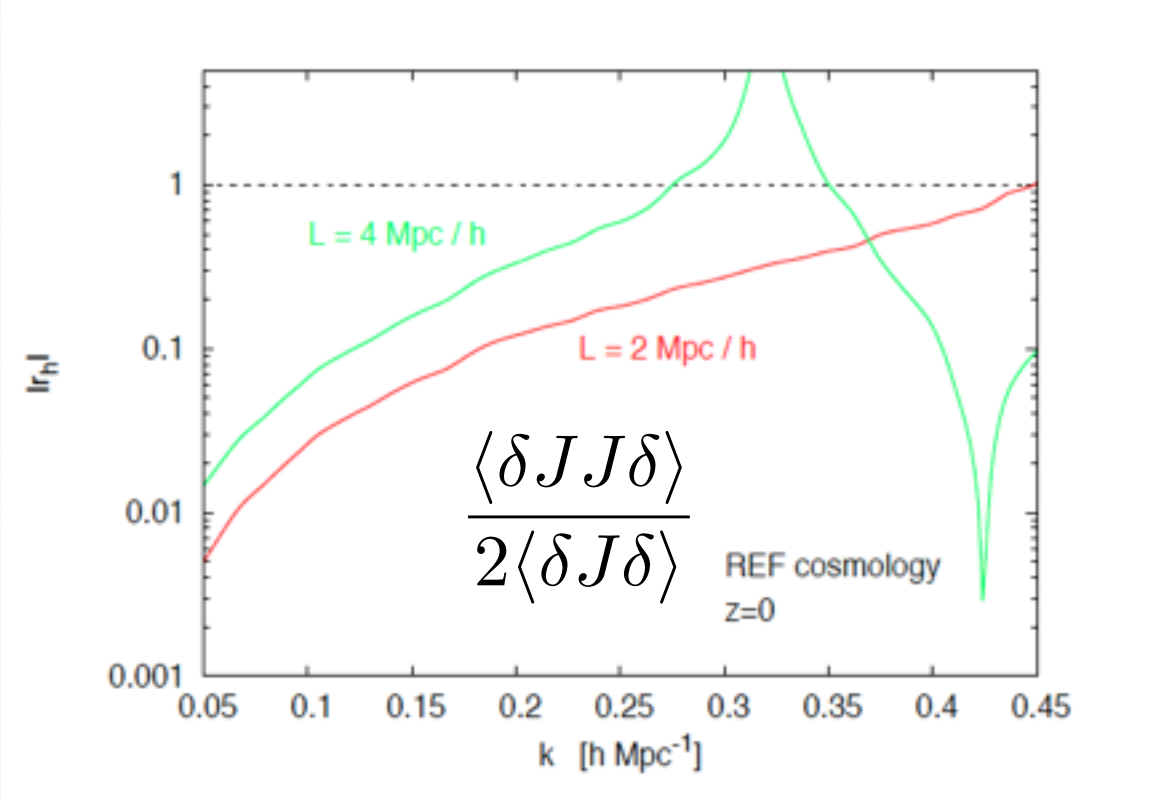
$$\langle\delta J\delta\rangle_{J=0}$$

(- PT + Nbody)_UV



$$\frac{\langle\delta JJ\delta\rangle}{2\langle\delta J\delta\rangle}$$

Expansion in UV sources

$$\langle\delta\delta\rangle_J = \langle\delta\delta\rangle_{J=0} + \langle\delta J\delta\rangle_{J=0} + \frac{1}{2}\langle\delta JJ\delta\rangle_{J=0} + \cdots$$

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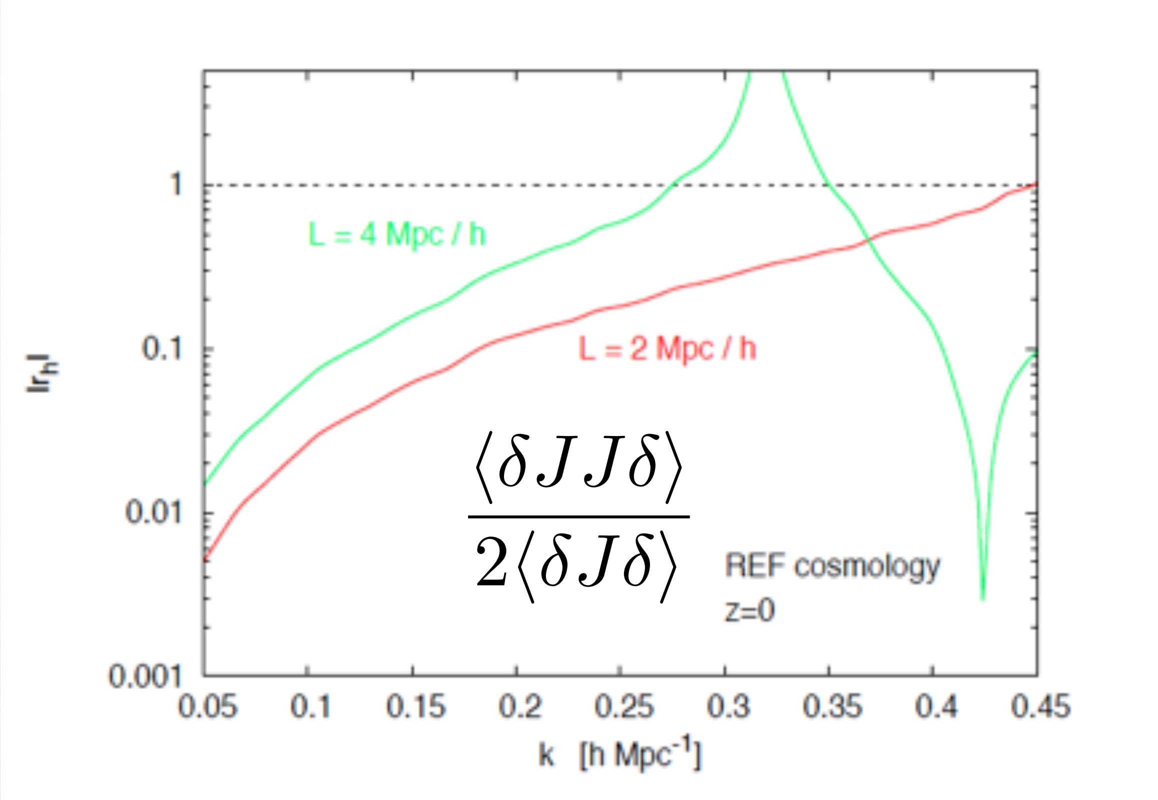

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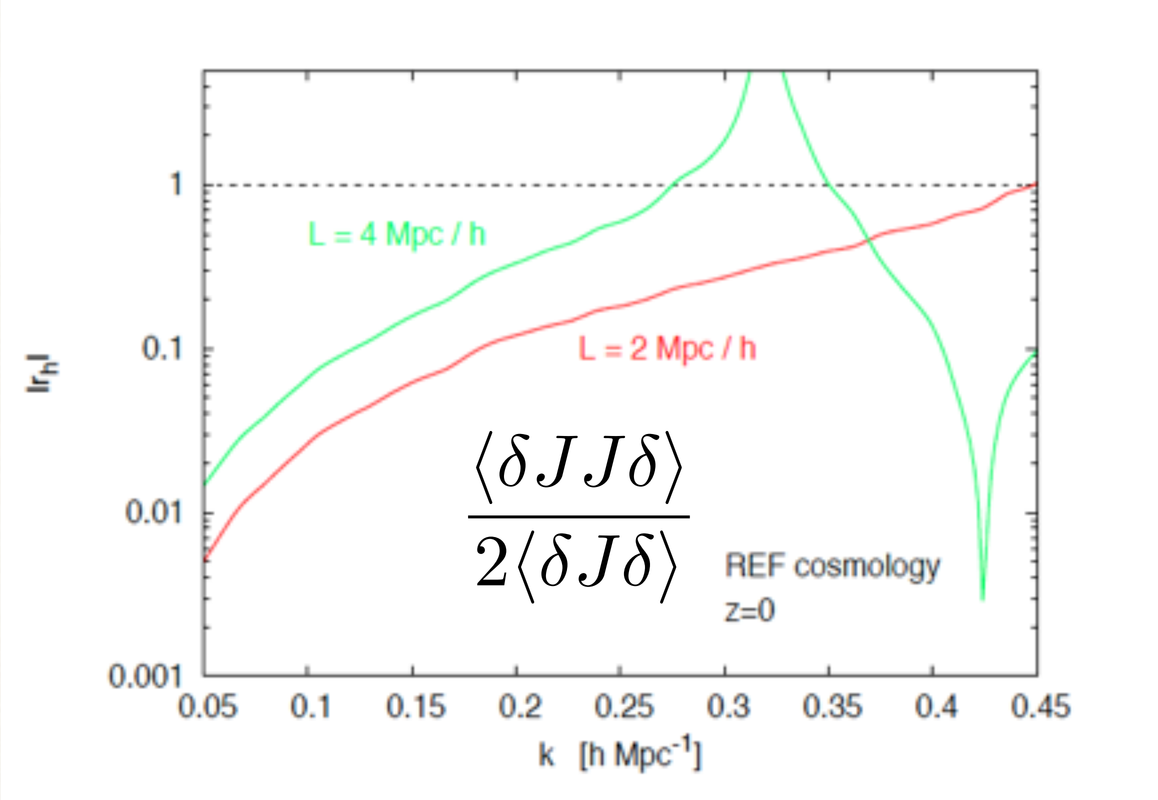
$$\langle\delta J\delta\rangle_{J=0}$$

(- PT + Nbody)_UV



Time-dependence

$$\bar{\varphi}_1(\eta, \mathbf{k}) \equiv e^{-\eta}\bar{\delta}(\eta, \mathbf{k}) \quad , \quad \bar{\varphi}_2(\eta, \mathbf{k}) \equiv e^{-\eta}\frac{-\bar{\theta}(\eta, \mathbf{k})}{\mathcal{H} f}$$

$$\eta \equiv \ln \frac{D_+(\tau)}{D_+(\tau_{\rm in})}$$

$$h_a(\mathbf{k}, \eta) \equiv h_a^1(\mathbf{k}, \eta) + h_a^\sigma(\mathbf{k}, \eta) \,,$$

$$h_a^1(\mathbf{k}, \eta) = -i\,\frac{k^i J_1^i(\mathbf{k}, \eta)}{\mathcal{H}^2 f^2} e^{-\eta}\,\delta_{a2} \,, \qquad h_a^\sigma(\mathbf{k}, \eta) = -i\,\frac{k^i J_\sigma^i(\mathbf{k}, \eta)}{\mathcal{H}^2 f^2} e^{-\eta}\,\delta_{a2}$$

$$g_{12}(\eta) = \frac{2}{5}\left(1 - e^{-5/2\,\eta}\right)$$

$$\Delta \bar{P}_{11}^{h,\mathrm{N-body}}(k, \eta) \equiv -2 \int_{\eta_{in}}^{\eta} ds\; g_{12}(\eta - s)\,\langle h_2(\mathbf{k}, s)\,\bar{\varphi}_1(\mathbf{k}', \eta)\rangle'$$

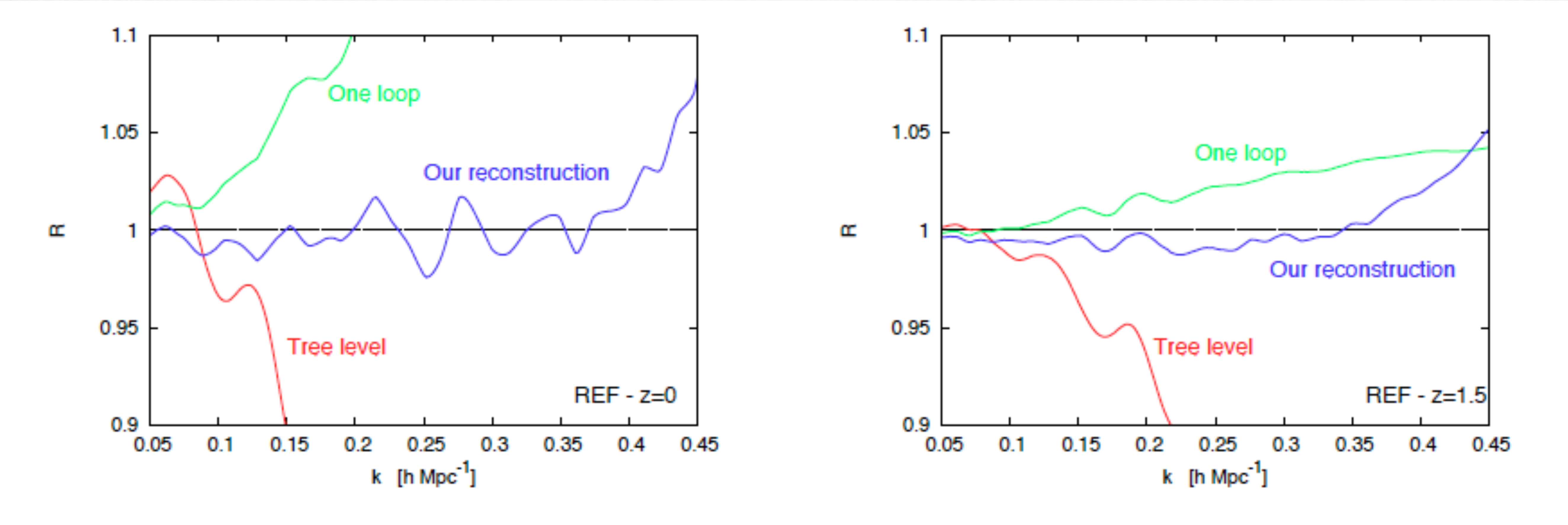
Ansatz for the time-dependence

$$\langle h_2(\mathbf{k}, s)\,\bar{\varphi}_1(\mathbf{k}', \eta)\rangle' \cong \left[\frac{D(s)}{D(\eta)}\right]^{\alpha(\eta)} \times \langle h_2(\mathbf{k}, \eta)\,\bar{\varphi}_1(\mathbf{k}', \eta)\rangle' \quad , \quad s < \eta \,.$$

PT limit: $\alpha(\eta) \to 2$

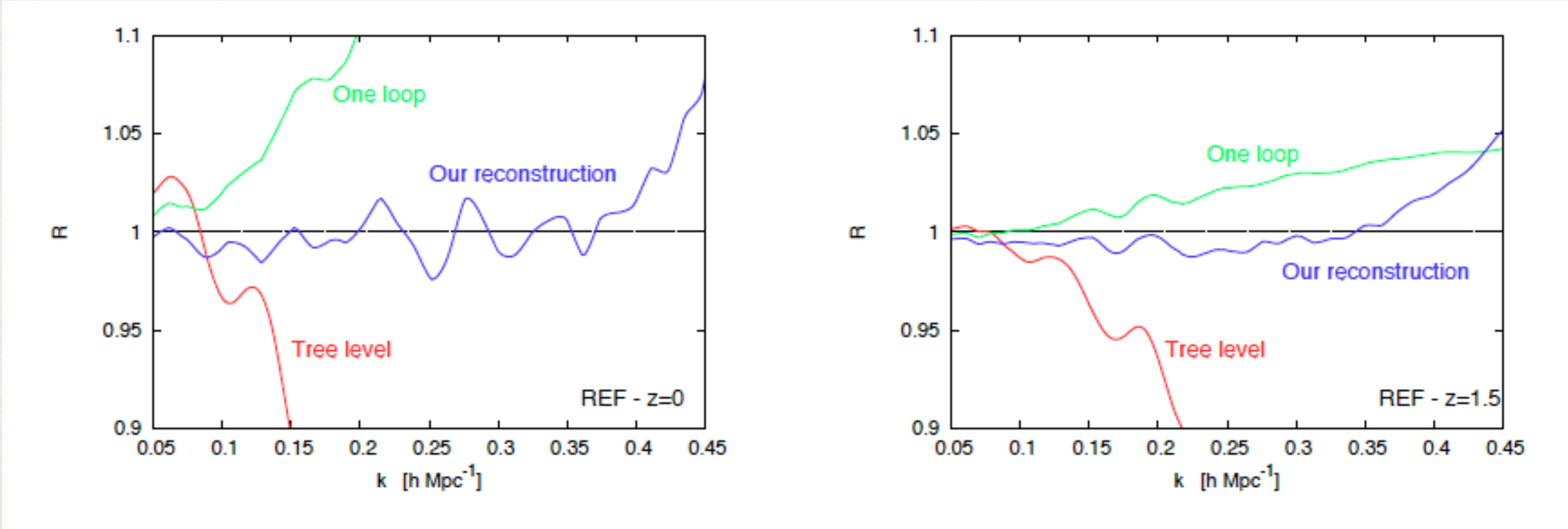
Checked independently

Results of the reconstruction



REF z	$\alpha_{\mathrm{rec.}}$	err.$_{\mathrm{rec.}}$	err.$_{1-\mathrm{loop,stand.}}$	$\alpha_{\mathrm{scaling}}$
0	$1.76^{+0.06}_{-0.05}$	1.0%	15%	1.81
0.25	$1.81^{+0.08}_{-0.08}$	1.2%	12%	1.82
0.5	$1.88^{+0.12}_{-0.11}$	1.3%	8.5%	1.85
1	$2.00^{+0.16}_{-0.14}$	1.0%	4.7%	1.92
1.5	$2.08^{+0.19}_{-0.16}$	0.8%	2.4%	

Results of the reconstruction



REF z	$\alpha_{\rm rec.}$	err.$_{\rm rec.}$	err.$_{\rm 1-loop,stand.}$	$\alpha_{\rm scaling}$
0	$1.76^{+0.06}_{-0.05}$	1.0%	15%	1.81
0.25	$1.81^{+0.08}_{-0.08}$	1.2%	12%	1.82
0.5	$1.88^{+0.12}_{-0.11}$	1.3%	8.5%	1.85
1	$2.00^{+0.16}_{-0.14}$	1.0%	4.7%	1.92
1.5	$2.08^{+0.19}_{-0.16}$	0.8%	2.4%	

Good, but... why not simply run a N-body simulation?!

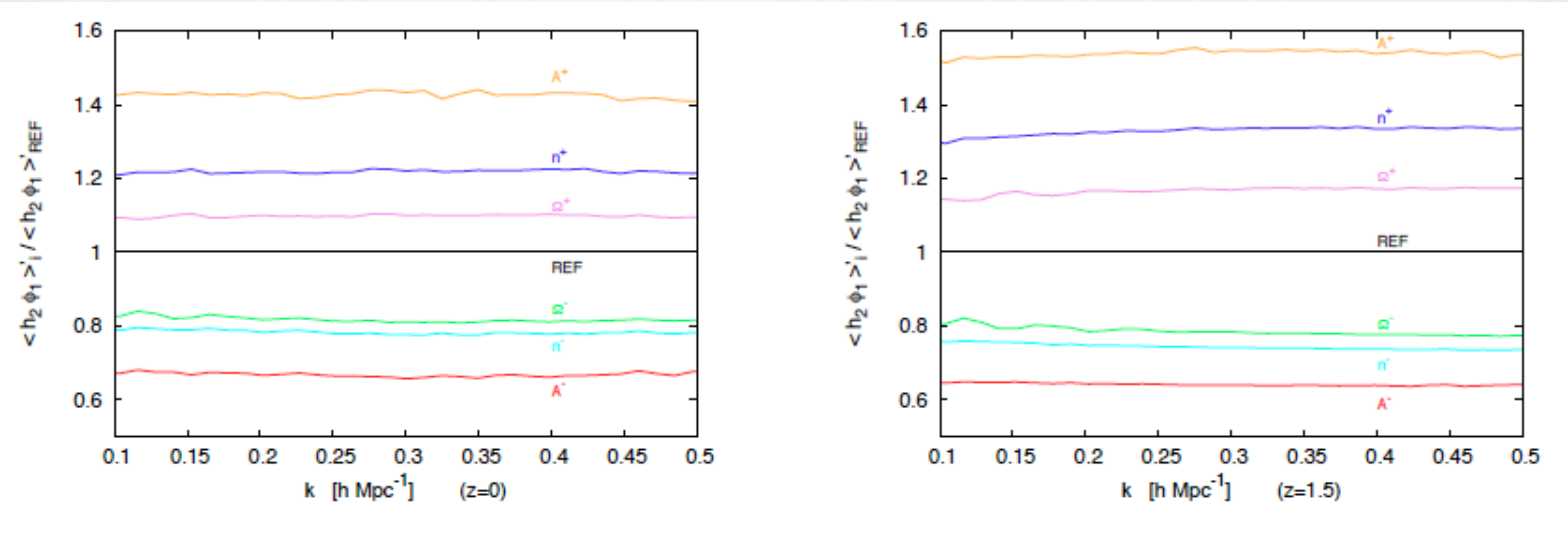
COSMOLOGY DEPENDENCE

Simulation Suite

Name	Ω_m	Ω_b	Ω_Λ	h	n_s	$A_s\,[10^{-9}]$
REF	0.271	0.045	0.729	0.703	0.966	2.42
A_s^-	0.271	0.045	0.729	0.703	0.966	1.95
A_s^+	0.271	0.045	0.729	0.703	0.966	3.0
n_s^-	0.271	0.045	0.729	0.703	0.932	2.42
n_s^+	0.271	0.045	0.729	0.703	1.000	2.42
Ω_m^-	0.247	0.045	0.753	0.703	0.966	2.42
Ω_m^+	0.289	0.045	0.711	0.703	0.966	2.42

$L_{box} = 512\,\mathrm{Mpc/h}$ $\qquad$ $N_{particles} = (512)^3$

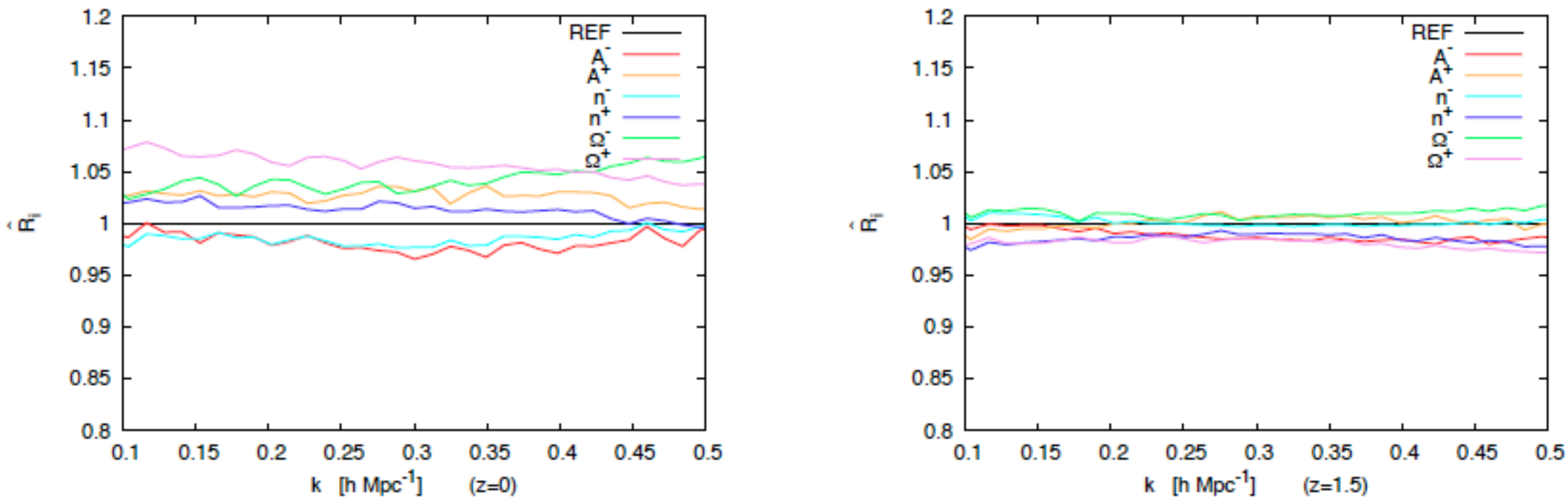
Ratios of UV source correlators



$$\frac{\langle J\delta \rangle_i}{\langle J\delta \rangle_{REF}}$$ From N-body

Scale-independent!!

Rescale by using PT information



$$\widehat{\mathcal{R}}_i\,(k,\eta) \equiv \frac{\mathcal{R}_i\,(k,\eta)}{\mathcal{R}_{\mathrm{REF}}\,(k,\eta)}\,\frac{\left(\frac{D_{\mathrm{REF}}(\eta)}{D_{\mathrm{REF}}(\eta_*)}\right)^{\alpha_{\mathrm{REF}}(\eta)-2}}{\left(\frac{D_i(\eta)}{D_i(\eta_*)}\right)^{\alpha_i(\eta)-2}}$$

$$\mathcal{R}_i(k,\eta) \equiv \frac{\langle h_2\,(\mathbf{k},\eta)\,\bar{\varphi}_1(\mathbf{k}',\eta)\rangle_i'}{\Delta\bar{P}^{h,1-\mathrm{loop}}_{ss,11,i}(k,\eta)}$$

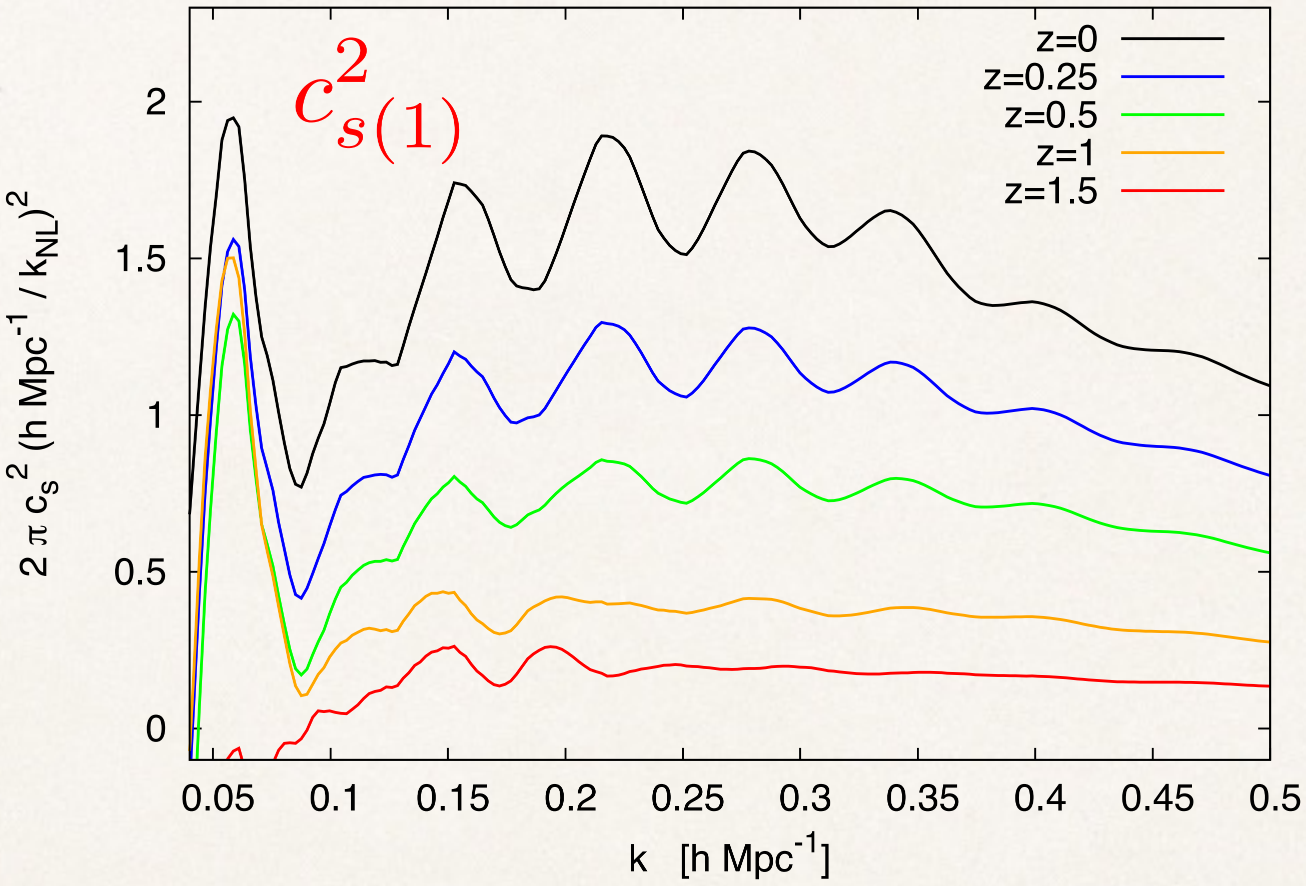
=O(10)

Amplitude captured by PT!!

The PS in 1-loop EFToLSS

$$\langle [\tau^{ij}]_\Lambda \rangle_{\delta_l} = p_b \delta^{ij} + \rho_b \left[c_s^2 \delta_l \delta^{ij} - \frac{c_{bv}^2}{Ha} \delta^{ij} \partial_k v_l^k - \frac{3}{4} \frac{c_{sv}^2}{Ha} \left(\partial^j v_l^i + \partial^i v_l^j - \frac{2}{3} \delta^{ij} \partial_k v_l^k \right) \right] + \Delta\tau^{ij} + \ldots$$

$$P_{11}(k,\eta) \simeq P_{11}^{lin}(k,\eta) + P_{ss,11}^{1-\mathrm{loop}}(k,\eta) - 2(2\pi)\, c_{s(1)}^2 \frac{k^2}{k_{NL}^2} P^{lin}(k,\eta)\,,$$



higher orders+resummations needed to reduce the scale dependence

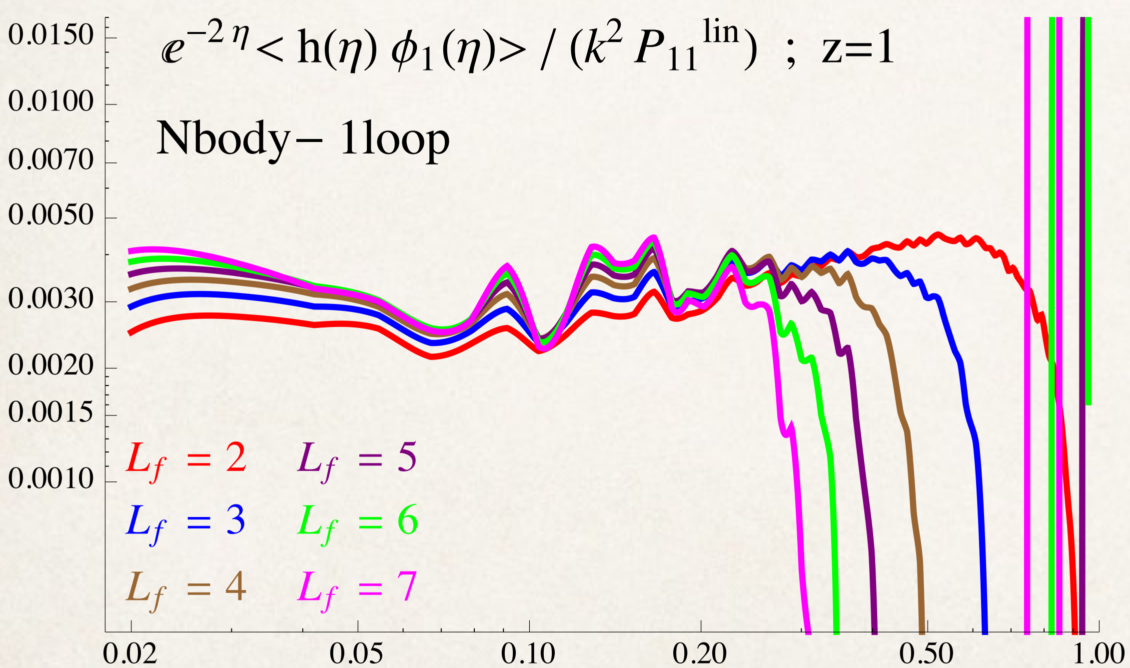
(see Senatore Zaldarriaga, 1404.5954)

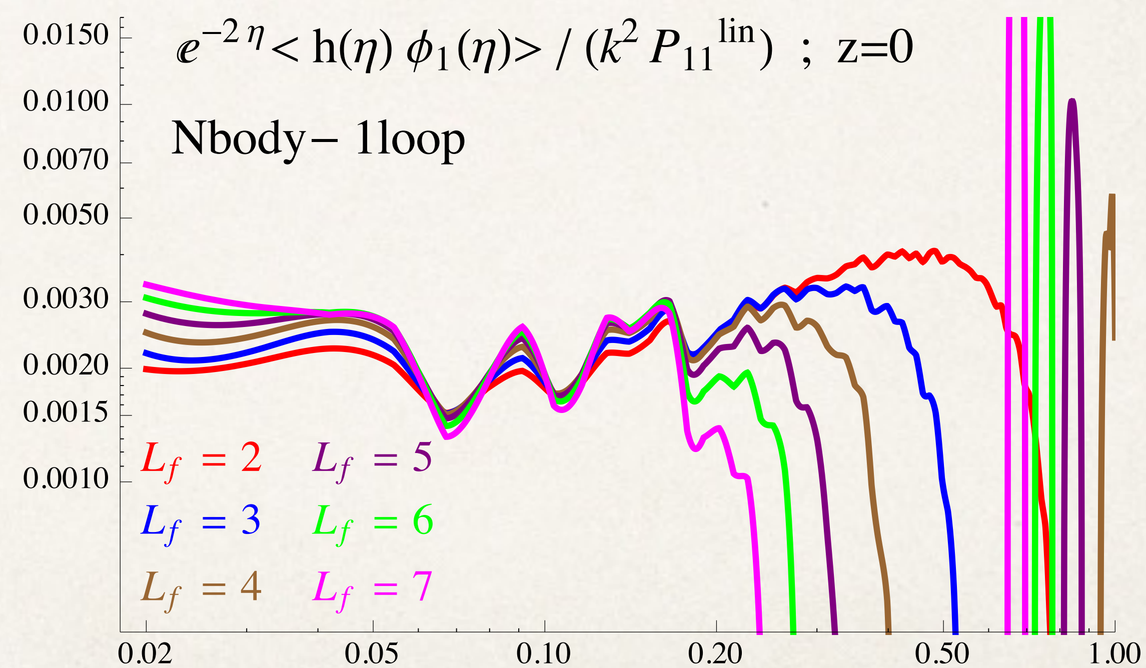
Matching

The correction to the SPT result is given by $\Delta P^{h,Nbody}(k,\eta) - \Delta P^{h,SPT}(k,\eta)$

$$\Delta \bar{P}_{11}^{h,\mathrm{N-body}}(k,\eta) \equiv -2\int_{\eta_{in}}^{\eta} ds\, g_{12}(\eta - s)\, \langle h_2(\mathbf{k}, s)\, \bar{\varphi}_1(\mathbf{k}',\eta)\rangle'$$

It depends on two scales: L_f and k





Ultimate reach of effective methods depends on PT!

Summary

- Semi-analytical methods and N-body are complementary: (flexibility, physical insight, speed)
- IR effects important and under control
- Intermediate scales treatable by (improved) SPT
- The UV is out of SPT reach but mildly cosmology dependent: effective approaches!