Testing General Relativity on cosmological scales

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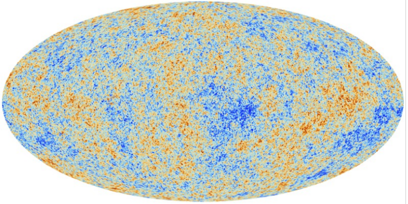
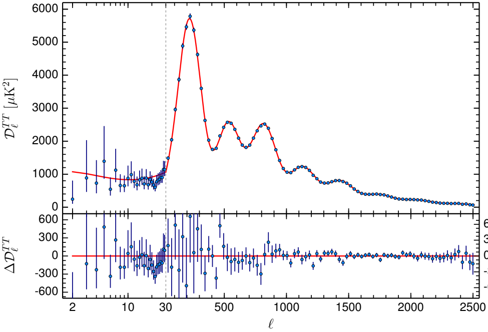
Outline

1. Introduction
2. What are very large scale galaxy catalogs really measuring?
3. The angular power spectrum and the correlation function of galaxy density fluctuations
   - The transversal power spectrum
   - The radial power spectrum
4. Real experiments: DES, Euclid, $\cdots$
5. 2nd order and bispectrum
6. Conclusions

The CMB

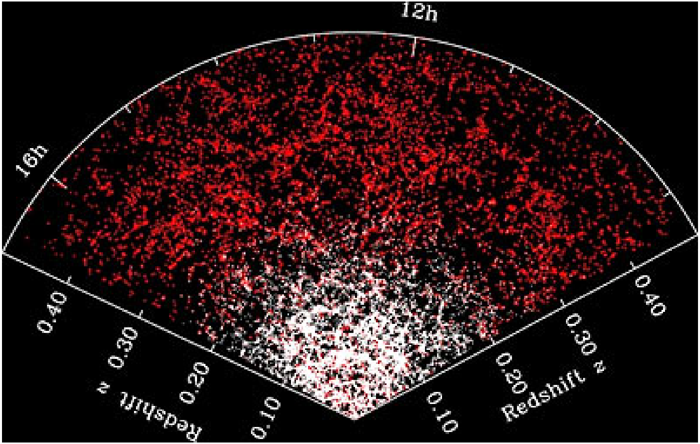


CMB sky as seen by Planck

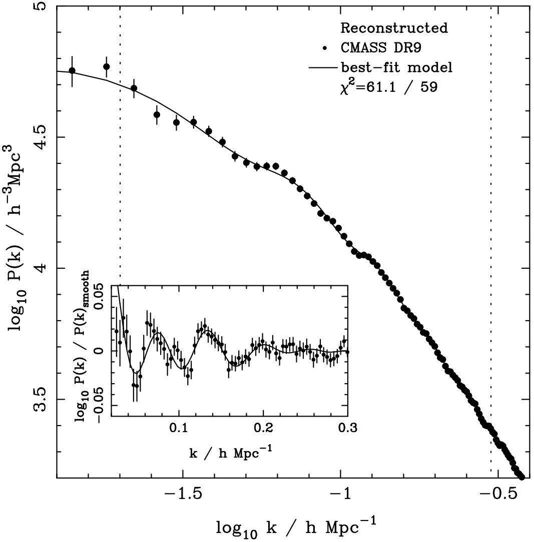


$D_\ell = \ell(\ell+1)C_\ell/(2\pi)$

The Planck Collaboration: Planck results 2015 XIII



M. Blanton and the Sloan Digital Sky Survey Team.



from Anderson et al. '12

SDSS-III (BOSS) power spectrum.

Galaxy surveys $\simeq$ matter density fluctuations, biasing and redshift space distortions.

But...

- We have to take fully into account that all observations are made on our past lightcone which is itself perturbed.
  We see density fluctuations which are further away from us, further in the past.
  We cannot observe 3 spatial dimensions but 2 spatial and 1 lightlike, more precisely we measure 2 angles and a redshift.

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- But of course much more for future surveys like DES, Euclid, WFIRST and SKA.

Cosmological distances

In a Friedmann Universe the (comoving) radial distance is

$$r(z) = \int_0^z \frac{dz'}{H(z')} = \frac{1}{H_0} \int_0^z \frac{dz'}{\sqrt{\Omega_m (1+z')^3 + \Omega_K (1+z')^2 + \Omega_\Lambda}}$$

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Depending on the observational situation we measure directly $r(z)$ or

$$d_A(z) = \frac{1}{(1+z)}\chi_K(r(z)) \quad \text{the angular diameter distance}$$

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- Whenever we convert a measured redshift and angle into a length scale, we make assumptions about the underlying cosmology.

What are very large scale galaxy catalogs really measuring?

We now consider fluctuations in the matter distribution and in the geometry first to linear order. (See C. Bonvin & RD [arXiv:1105.5080]; Challinor & Lewis, [arXiv:1105:5092], J. Yoo et al. 2009; J. Yoo 2010)

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$$\Delta(\mathbf{n}, z) = \frac{N(\mathbf{n}, z) - \bar{N}(z)}{\bar{N}(z)}.$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z') \rangle\,, \qquad \mathbf{n} \cdot \mathbf{n}' = \cos\theta\,.$$

This quantity is directly measurable $\Rightarrow$ gauge invariant.

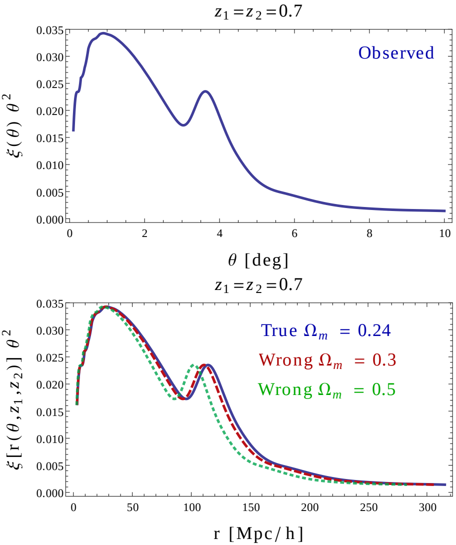
What are very large scale galaxy catalogs really measuring?

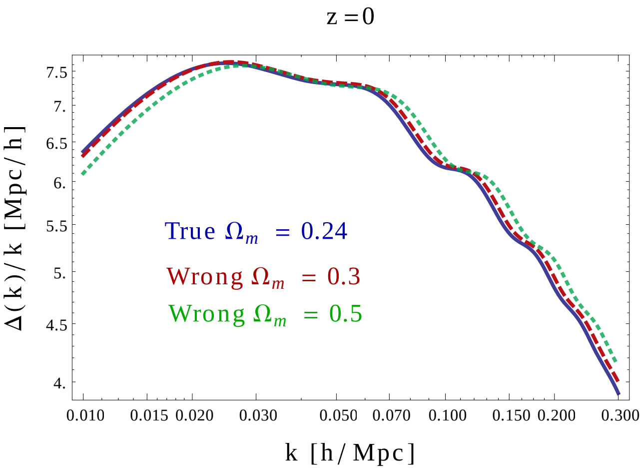
If we convert the measured $\xi(\theta, z_1, z_2)$ to a power spectrum, we have to introduce a cosmology, to convert angles and redshifts into length scales.

$$r(z_1, z_2, \theta) \overset{(K=0)}{=} \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta}.$$

$$r_i = r(z_i) = \int_0^{z_i} \frac{dz}{H(z)}$$

(Figure by F. Montanari)





(Figure by F. Montanari)

$$\Delta(k)/k = k^2 P(k)$$

Putting the density and volume fluctuations together one obtains the galaxy number density fluctuations to 1st order

$$\begin{aligned}\Delta(\mathbf{n},z) &= D_s - 2\Phi + \Psi + \frac{1}{\mathcal{H}}\left[\dot\Phi + \partial_r(\mathbf{V}\cdot\mathbf{n})\right] \\ &+ \left(\frac{\dot{\mathcal{H}}}{\mathcal{H}^2} + \frac{2}{r(z)\mathcal{H}}\right)\left(\Psi + \mathbf{V}\cdot\mathbf{n} + \int_0^{r(z)} dr(\dot\Phi + \dot\Psi)\right) \\ &+ \frac{1}{r(z)}\int_0^{r(z)} dr\left[2 - \frac{r(z)-r}{r}\Delta_\Omega\right](\Phi + \Psi).\end{aligned}$$

(C. Bonvin & RD '11)

The total galaxy density fluctuation per redshift bin, per sold angle

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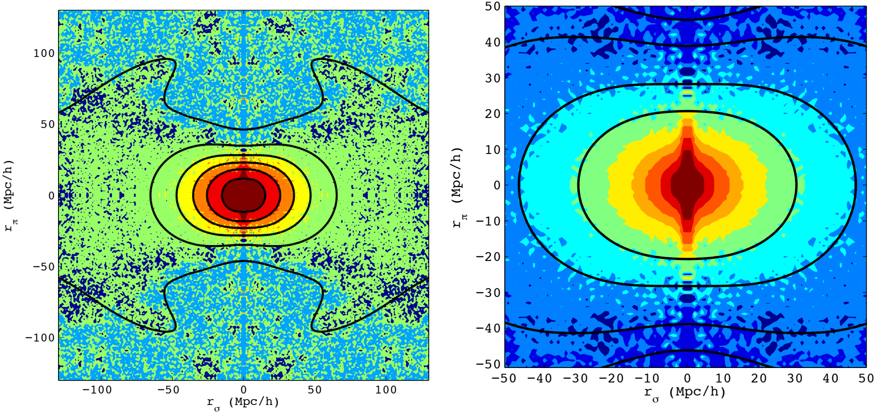
(C. Bonvin & RD '11)

Redshift space distortions in the BOSS survey

(from Reid et al. '12)



The angular power spectrum of galaxy density fluctuations

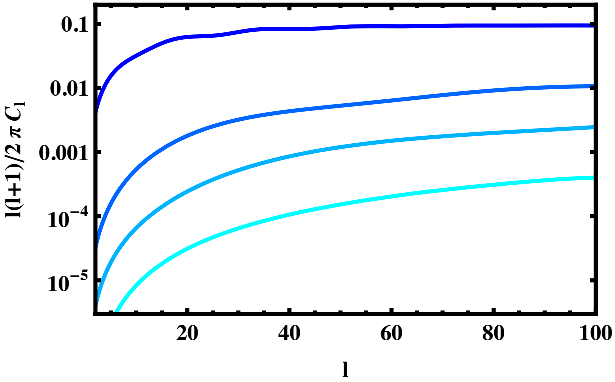
For fixed z, we can expand $\Delta(\mathbf{n}, z)$ in spherical harmonics,

$$\Delta(\mathbf{n}, z) = \sum_{\ell m} a_{\ell m}(z) Y_{\ell m}(\mathbf{n}), \qquad C_\ell(z, z') = \langle a_{\ell m}(z) a^*_{\ell m}(z') \rangle .$$

$$\xi(\theta, z, z') = \langle \Delta(\mathbf{n}, z) \Delta(\mathbf{n}', z') \rangle = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell(z, z') P_\ell(\cos\theta)$$

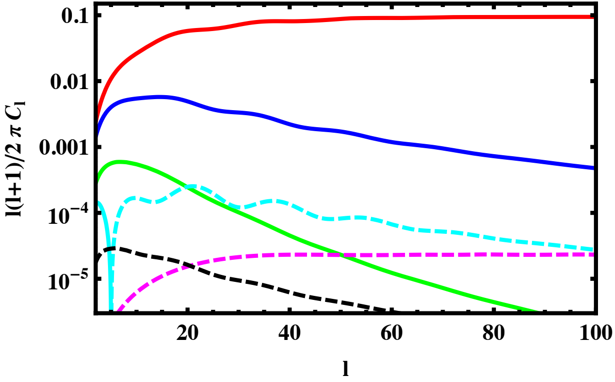
$$\cos\theta = \mathbf{n} \cdot \mathbf{n}'$$

The transverse power spectrum, $z' = z$ (from Bonvin & RD '11)



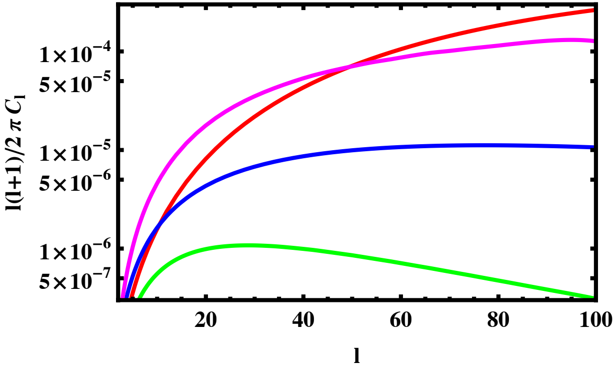
$z = 0.1$, $z = 0.5$, $z = 1$ and $z = 3$.

Contributions to the transverse power spectrum at redshift $z = 0.1$, $\Delta z = 0.01$ (from Bonvin & RD '11)



C_ℓ^{DD} (red), C_ℓ^{zz} (green), $2C_\ell^{Dz}$ (blue), $C_\ell^{Doppler}$ (cyan), $C_\ell^{lensing}$ (magenta) C_ℓ^{grav} (black).

Contributions to the transverse power spectrum at redshift $z = 3$, $\Delta z = 0.3$ (from Bonvin & RD '11)



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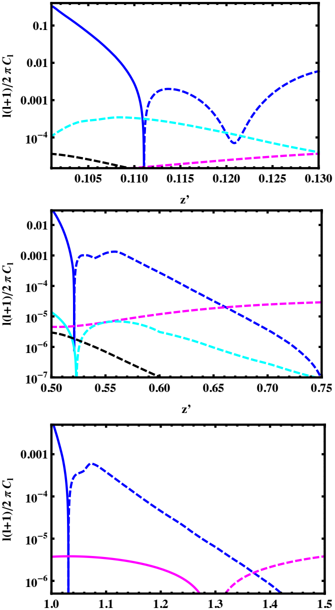
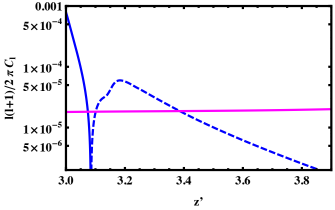
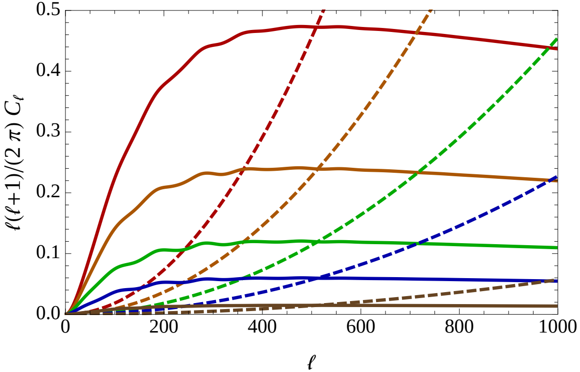




The radial power spectrum $C_\ell(z, z')$
for $\ell = 20$
Left, top to bottom: $z = 0.1,\ 0.5,\ 1,$
top right: $z = 3$

Standard terms (blue), $C_\ell^{lensing}$ (magenta),
$C_\ell^{Doppler}$ (cyan), C_ℓ^{grav} (black),
(from Bonvin & RD '11)



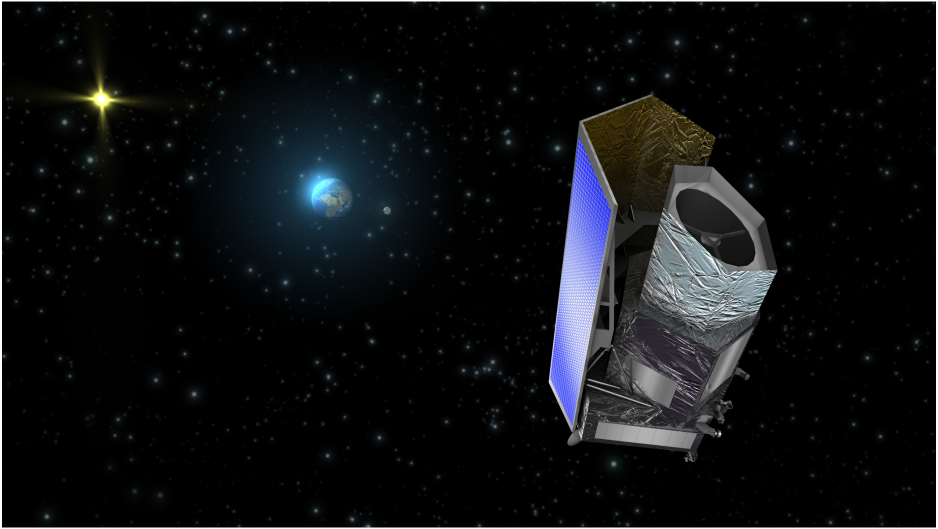
$\bar{z} = 0.55$
spectroscopic survey like DES
for shot-noise contribution.

(From Di Dio, Montanari, Lesgourgues, RD, 1307.1459 http://cosmology.unige.ch/tools)

The angular power spectrum C_ℓ (solid lines) and the shot-noise contribution (dashed lines) for different top-hat window functions of half-widths: $\Delta z = 0.1$, $\Delta z = 0.025$, $\Delta z = 0.0125$, $\Delta z = 0.00625$, $\Delta z = 0.003125$.

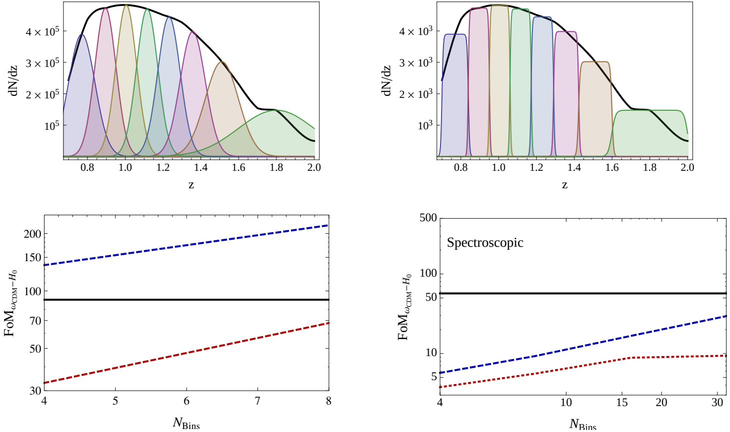
$$C_\ell^{obs}(z,z) = C_\ell(z,z) + \frac{1}{N(z)}$$

Euclid



(10^7 galaxy redshifts, 10^9 galaxies with photo-z)

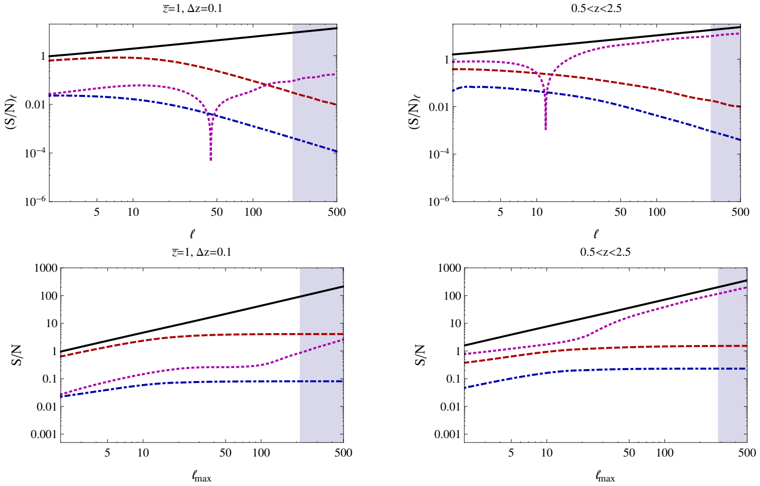
Real experiments (Euclid):



including cross-correlations, only auto-correlations

(From Di Dio, Montanari, RD, Lesgourgues, 1308.6186)

Real experiments (Euclid): Signal to Noise



Signal to noise for different contributions:
density, redshift-space distortions, lensing, potential
(From Di Dio, Montanari, RD, Lesgourgues, 1308.6186)

Measuring the lensing potential

At $z = z'$ density and rsd dominate the signal. Only at very low ℓ potential terms are relevant.

At $z < z'$ we truly measure $\langle D(z)\kappa(z')\rangle$.

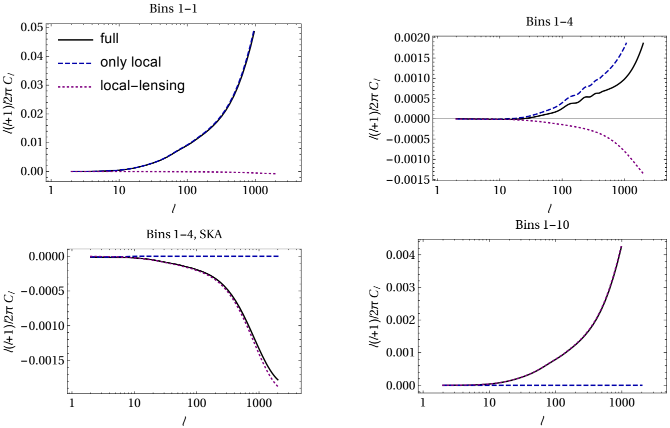
$$\kappa(\mathbf{n}, z) = \int_0^{r(z)} \frac{dr(r(z) - r)}{r(z)r} \Delta_2 \Psi(r\mathbf{n}, z)$$

Measuring the lensing potential

Well separated redshift bins measure mainly the lensing-density correlation:

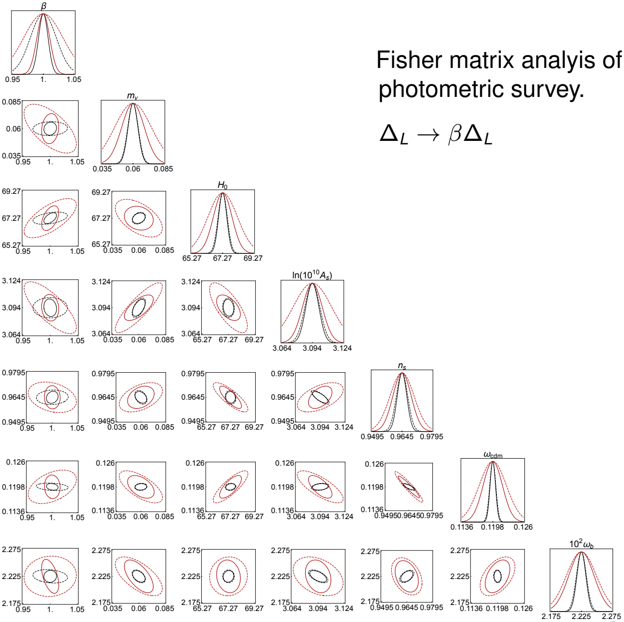
$$\langle \Delta(\mathbf{n}, z)\Delta(\mathbf{n}', z')\rangle \simeq \langle \Delta^L(\mathbf{n}, z)\delta(\mathbf{n}', z')\rangle \quad z > z'$$

$$\Delta^L(\mathbf{n}, z) = (2 - 5s(z))\kappa(\mathbf{n}, z)$$



(Montanari & RD) [1506.01369]

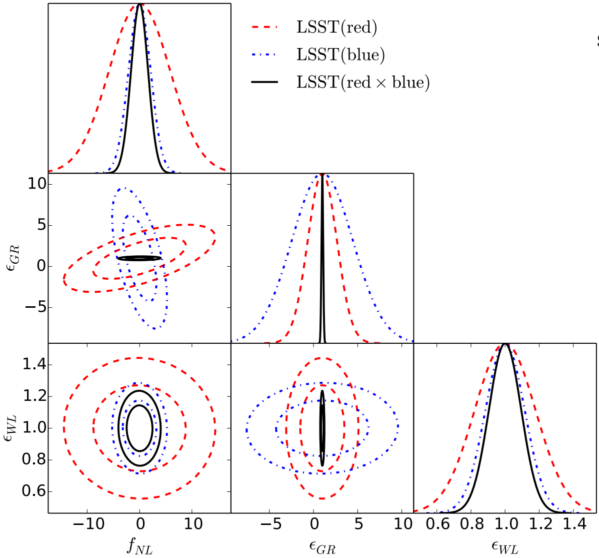
Testing GR with the lensing potential



Fisher matrix analyis of an Euclid-like photometric survey.

$\Delta_L \to \beta\Delta_L$

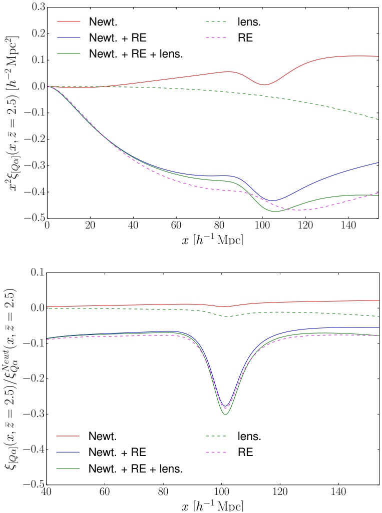
(Montanari & RD) [1506.01369]



standard parameters fixed

Alonso & Ferreira [1507.03550]

Measuring the relativistic terms with Quasar-Ly-α cross correlations



The antisymmetric part of the quasar–Ly-α cross correlation function.
Contrary to the quasars, the Ly-α signal has no lensing term.
The relativistic term is dominated by the Doppler contribution.

V. Iršič, E. Di Dio & M. Viel [1510.03436]

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D. Bertacca, R. Maartens, and C. Clarkson,[1405.4403,1406.0319]

J. Yoo and M. Zaldarriaga [1406.4140]

E. Di Dio, G. Marozzi, F. Montanari & RD [1407.0376]

2nd order number counts

The dominant terms are ($\propto (k/\mathcal{H})^4\Psi^2$)
(Di Dio, Marozzi, Montanari & RD, [1510.04202], Nielsen & RD [1606.02113])

$$\begin{aligned}\Delta^{(2)Leading}(\mathbf{n},z) \quad \simeq \quad & \delta^{(2)} + \mathcal{H}^{-1}\partial_r^2 v^{(2)} - 2\kappa^{(2)} + \mathcal{H}^{-2}\left(\partial_r^2 v\right)^2 + \mathcal{H}^{-2}\partial_r v \partial_r^3 v \\ & + \mathcal{H}^{-1}\left(\partial_r v\partial_r\delta + \partial_r^2 v\,\delta\right) - 2\delta\kappa + \nabla_a\delta\nabla^a\psi \\ & + \mathcal{H}^{-1}\left(-2\partial_r^2 v\,\kappa + \nabla_a\partial_r^2 v\nabla^a\psi\right) + 2\left(\kappa\right)^2 - 2\nabla_b\kappa\nabla^b\psi \\ & - \frac{2}{r(z)}\int_0^{r(z)} dr\frac{r(z)-r}{r}\Delta_2\left(\nabla^b\Psi_1\nabla_b\Psi_1\right) - 4\int_0^{r(z)}\frac{dr}{r}\nabla^a\Psi_1\nabla_a\kappa\,.\end{aligned}$$

$$\Delta^{(1)Leading} = \delta_\rho^{(1)} + \frac{1}{\mathcal{H}_s}\partial_r^2 v^{(1)} - 2\kappa^{(1)}$$

$$\psi = -2\int_0^{r(z)} dr\frac{r-r(z)}{r(z)r}\Psi\,, \qquad \kappa = -\Delta_2\psi$$

$$\Psi_1 = \frac{1}{r(z)}\int_0^{r(z)} dr\Psi$$

The bispectrum

$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \langle \Delta(\mathbf{n}_1, z_1)\, \Delta(\mathbf{n}_2, z_2)\, \Delta(\mathbf{n}_3, z_3) \rangle$$

Expanding in spherical harmonics gives

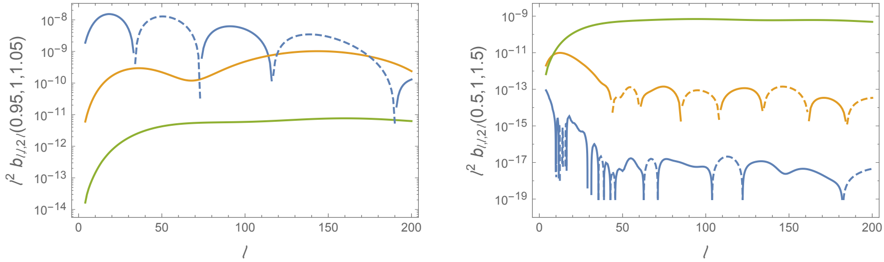
$$B(\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, z_1, z_2, z_3) = \sum B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}(z_1, z_2, z_3)\, Y_{\ell_1 m_1}(\mathbf{n}_1)\, Y_{\ell_2 m_2}(\mathbf{n}_2)\, Y_{\ell_3 m_3}(\mathbf{n}_3)\,,$$

statistical isotropy fully determines the *m*-dependence of these coefficients,

$$B_{\ell_1 \ell_2 \ell_3}^{m_1 m_2 m_3}(z_1, z_2, z_3) = \mathcal{G}_{\ell_1, \ell_2, \ell_3}^{m_1, m_2, m_3}\, b_{\ell_1, \ell_2, \ell_3}(z_1, z_2, z_3)\,,$$

where $\mathcal{G}_{\ell_1, \ell_2, \ell_3}^{m_1, m_2, m_3}$ is the Gaunt integral.

The bispectrum



(Di Dio, RD, Marozzi & Montanari, [1510.04202])

(density-density , density-lensing , lensing-lensing)

Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$ or $B(k_1, k_2, k_3)$. These are easier to measure (less noisy) but:
  • they require an fiducial input cosmology converting redshift and angles to length scales,
  $$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z')\cos\theta}\ .$$
  This complicates especially the determination of error bars in parameter estimation
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  • it is not evident how to correctly include lensing in the bispectrum.
- Future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation functions and spectra, $\xi(\theta, z, z')$ and $C_\ell(z, z')$ and $b_{\ell_1,\ell_2,\ell_2}(z_1, z_2, z_3)$ from the data.

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  • it is not evident how to correctly include lensing in the bispectrum.
- Future large & precise 3d galaxy catalogs like **Euclid** will be able to determine directly the measured 3d correlation functions and spectra, $\xi(\theta, z, z')$ and $C_\ell(z, z')$ and $b_{\ell_1,\ell_2,\ell_2}(z_1, z_2, z_3)$ from the data.
- These 3d quantities will of course be more noisy, but they also contain more information.

Conclusions

- So far cosmological LSS data mainly determined $\xi(r)$, or equivalently $P(k)$ or $B(k_1, k_2, k_3)$. These are easier to measure (less noisy) but:
  • they require an fiducial input cosmology converting redshift and angles to length scales,
  $$r = \sqrt{r(z)^2 + r(z')^2 - 2r(z)r(z')\cos\theta}\ .$$
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- These 3d quantities will of course be more noisy, but they also contain more information.
- These spectra are not only sensitive to the matter distribution (density) but also to the velocity via (redshift space distortions) and to the perturbations of spacetime geometry (lensing) .

Conclusions

- Using the antisymmetric part of the correlation function for different tracers is a promising tool to detect the relativistic terms.

Conclusions

- Using the antisymmetric part of the correlation function for different tracers is a promising tool to detect the relativistic terms.
- The spectra $C_\ell(z, z')$ and $b_{\ell_1,\ell_2,\ell_2}(z_1, z_2, z_3)$ depend sensitively and in several different ways on dark energy (growth factor, distance redshift relation), on the matter and baryon densities, bias, etc. Their measurements provide a new route to estimate cosmological parameters and to test general relativity.