



This project is funded
by the European Union

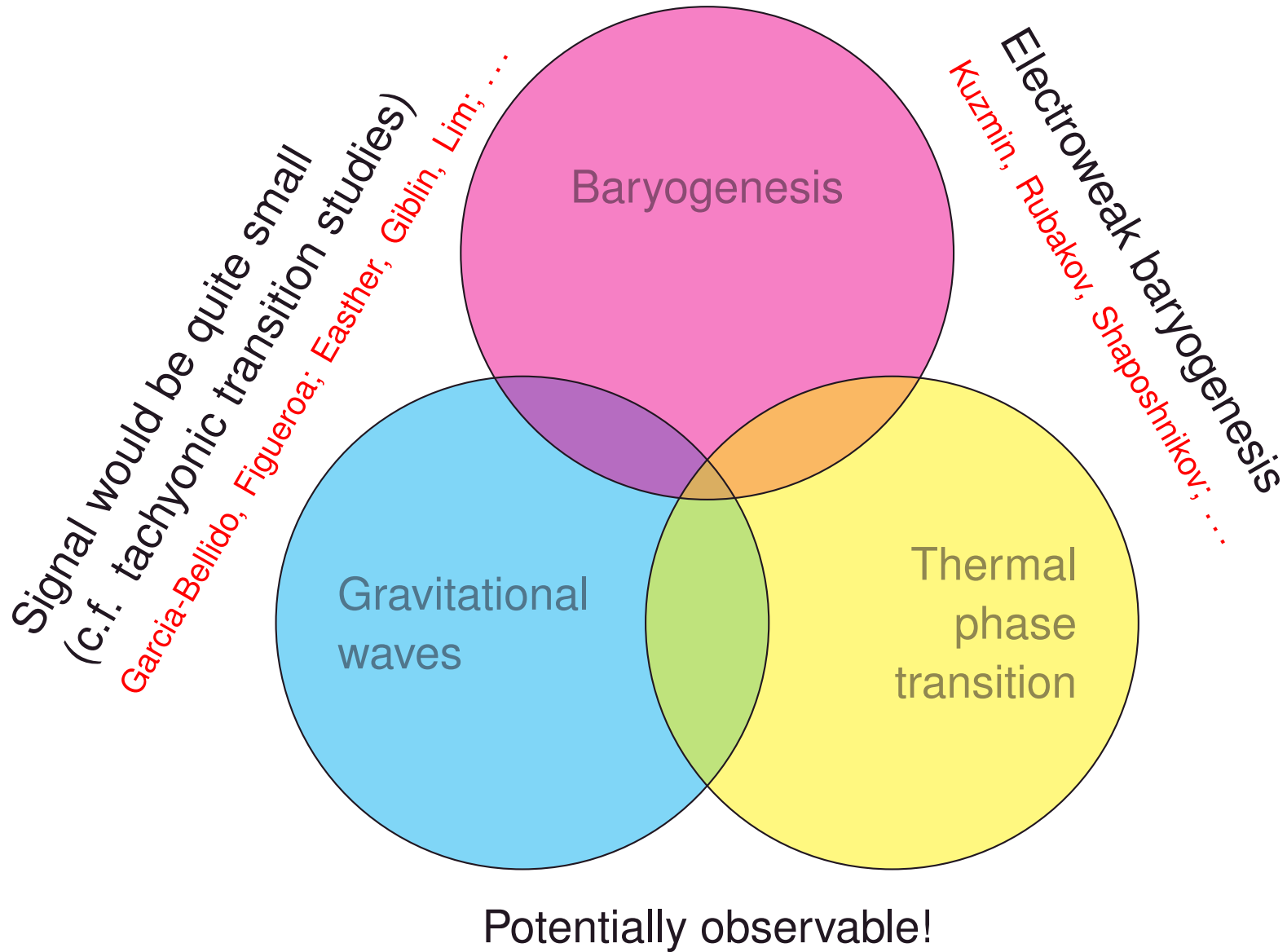
Simulating a first-order electroweak phase transition

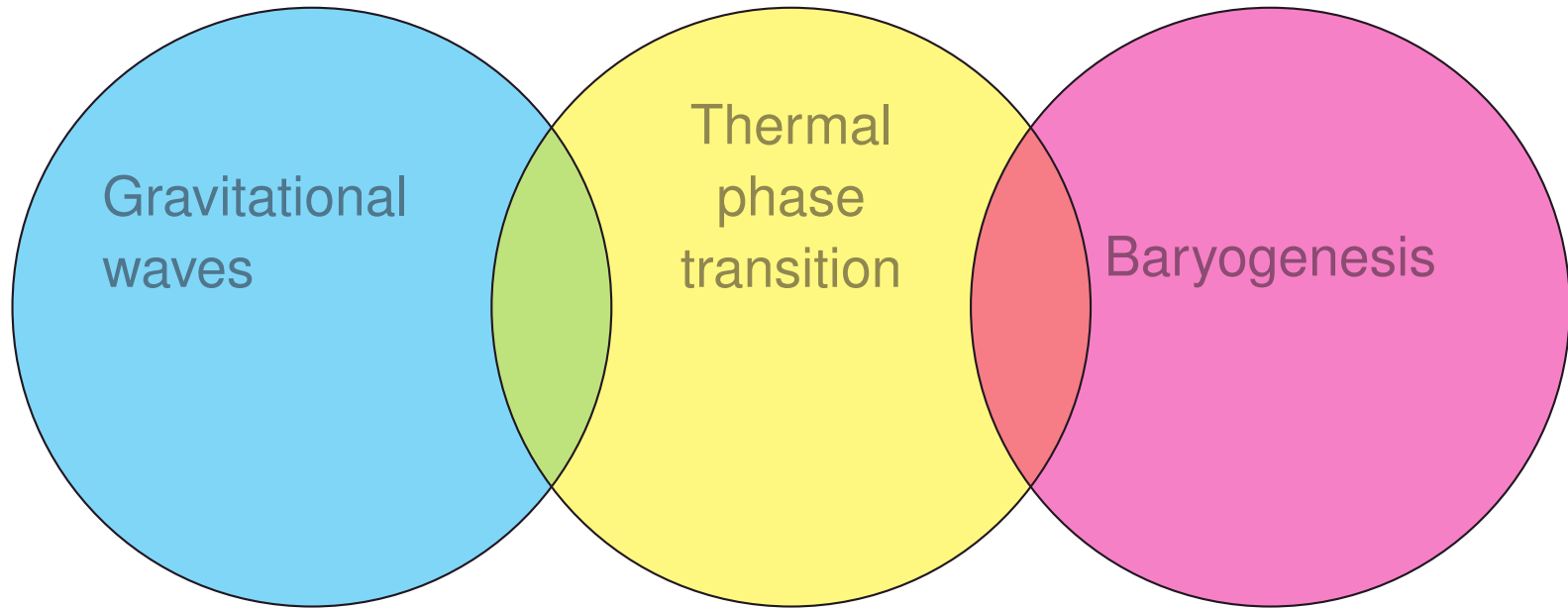
*PRL 112, 041301 (2014) [arXiv:1304.2433],
PRD 92, 123009 (2015) [arXiv:1504.03291],
JCAP 1604 (2016) 001 [arXiv:1512.06239],
and PRD 93, 124037 (2016) [arXiv:1604.08429].*

David J. Weir

CERN Theory Workshop, 25 August 2016

Pick any two?





- First order EWPT can produce observable gravitational wave signatures
- For future projects (including eLISA), the EWPT *is* a scientific objective
- It's *possible* to believe that a phase transition that produces observable GWs also could explain baryogenesis

Gravitational waves

Gravitational waves **Weinberg**

GR predicts that time-dependent stress energy sources gravitational waves

- Weak field approximation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}; \quad |h_{\mu\nu}| \ll 1$$

- After some algebra

$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\text{TT}}$$

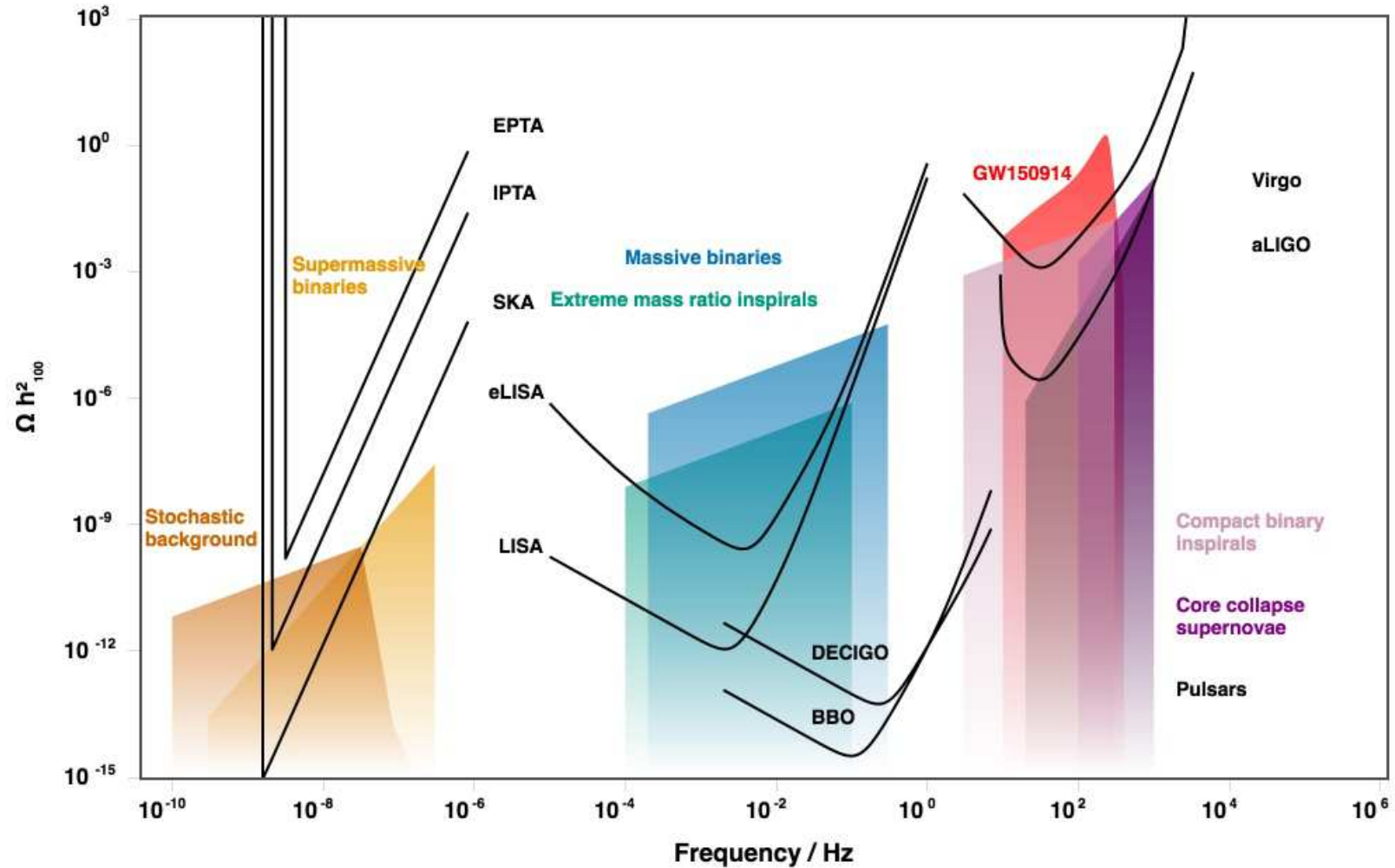
- Source (transverse traceless part of T_{ij}):
 - Astrophysics (neutron stars, black holes)
 - Cosmology (defects, phase transitions, reheating)
- After production, immediately ‘decouple’ – can directly probe (e.g.) EWPT



‘ripples in spacetime’

Gravitational wave sources

Lots of potential sources...




... lots of potential detectors ...

GWs are now a Thing

... it was therefore only a matter of time before we saw something.




 **Lawrence M. Krauss** ✓
@LKrauss1 Follow

Rumor of a gravitational wave detection at LIGO detector. Amazing if true. Will post details if it survives.

RETWEETS 635 LIKES 666

1:39 p.m. - 25 Sep 2015

 **Lawrence M. Krauss** ✓
@LKrauss1 Follow

The chirp heard round the world. Gravitational wave frequency in the audible range.

RETWEETS 238 LIKES 492

7:50 a.m. - 11 Feb 2016

PRL **116**, 061102 (2016) Selected for a **Viewpoint** in *Physics* week ending
PHYSICAL REVIEW LETTERS 12 FEBRUARY 2016

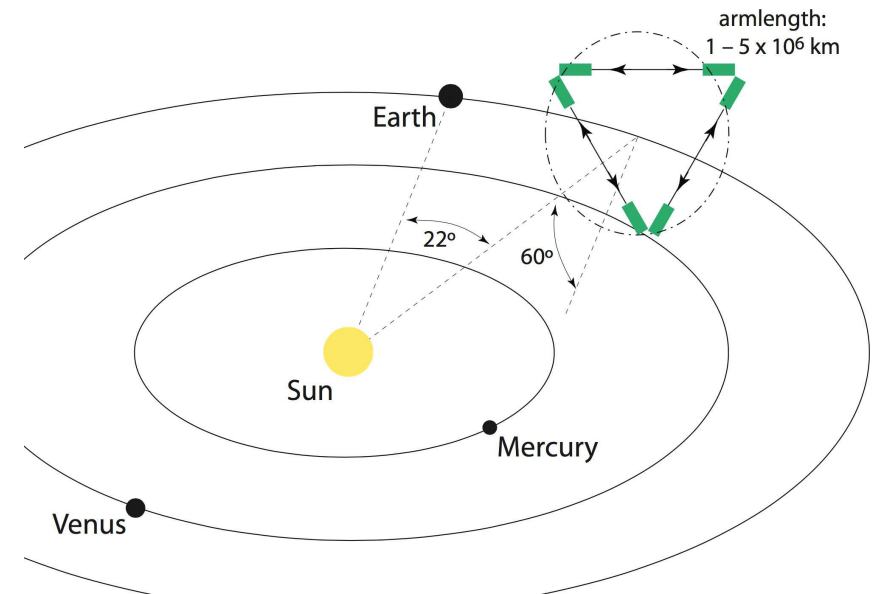
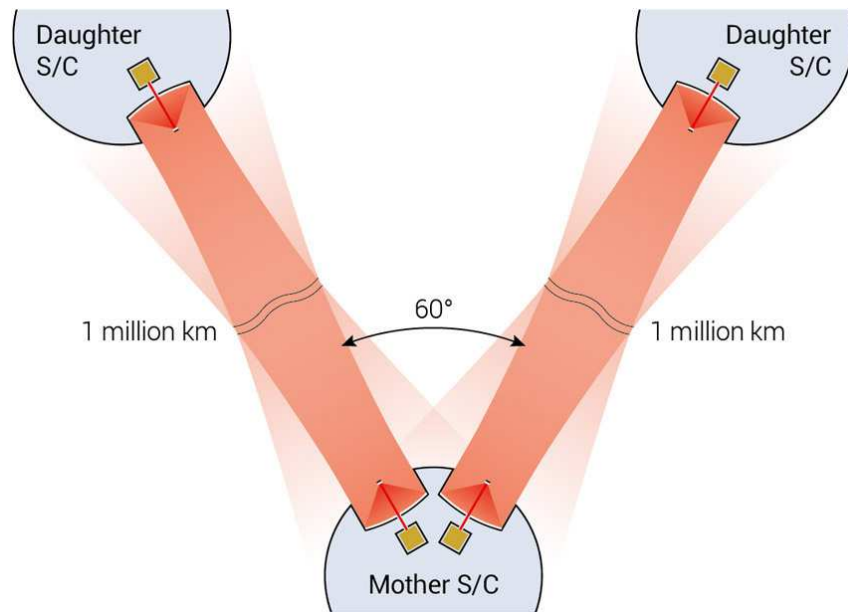


Observation of Gravitational Waves from a Binary Black Hole Merger

B. P. Abbott *et al.**

(LIGO Scientific Collaboration and Virgo Collaboration)
(Received 21 January 2016; published 11 February 2016)

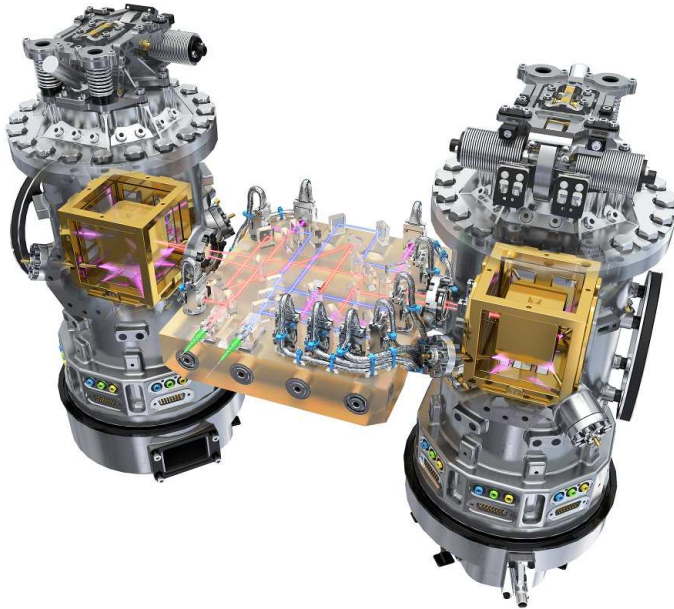
What's "next": [e]LISA



- eLISA would have two arms (four laser links), 1M km separation
- Launch as ESA's third large-scale mission (L3) in c.2034
- Cheaper version of LISA (one less arm, smaller separation, higher noise floor, shorter duration)
- In light of events (aLIGO; LISA Pathfinder; international collaboration):
 - Restore missing arm?
 - Increase separation?
 - Extend mission duration?
 - Drop the 'e'???

LISA Pathfinder is orbiting Earth-Sun L_1

Interferometer + test masses – technology demonstrator



[Science & Environment](#)

Lisa Pathfinder launches to test space 'ripples' technology

By Jonathan Amos
BBC Science Correspondent

🕒 3 December 2015 | [Science & Environment](#)



Lisa Pathfinder's Vega rocket clears the pad at the Kourou spaceport

Europe has launched the Lisa Pathfinder satellite, an exquisite space physics experiment.

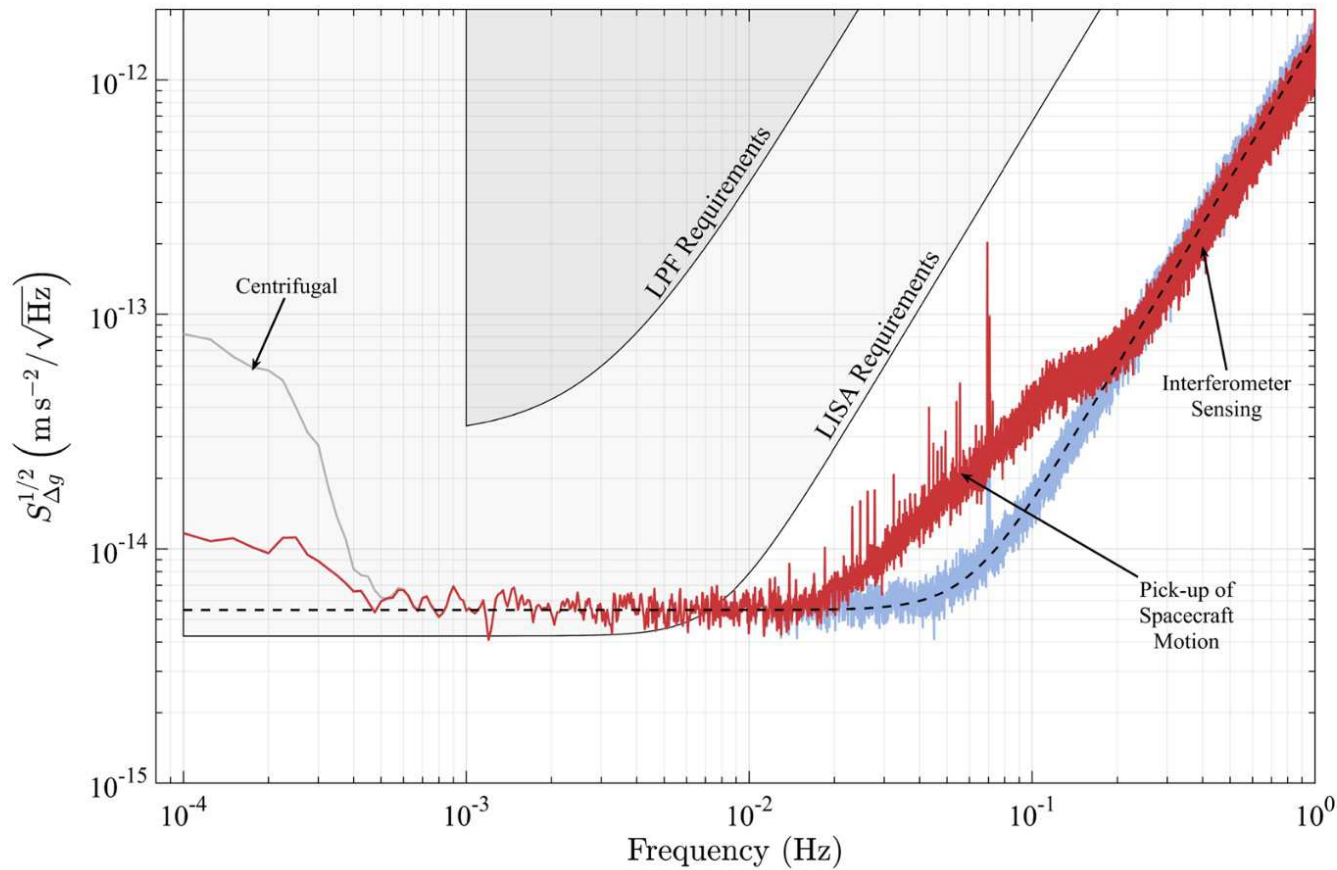
LISA Pathfinder exceeds expectations

Exceeded design expectations by a factor of five!

PRL **116**, 231101 (2016)

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week ending
10 JUNE 2016



Close to requirements for LISA.

Thermal phase transitions

Thermal phase transitions

Extended Standard Model with first-order PT.
Around temperature T_* ,

- Bubbles nucleate in false vacuum
 - with rate β
- Bubbles expand, liberate latent heat
 - characterised by α_{T_*}
- Bubbles interact with plasma
 - deposit kinetic energy with efficiency κ
- Friction from plasma acts on bubble walls
 - walls move with velocity v_{wall}
- Bubbles collide
 - producing gravitational waves

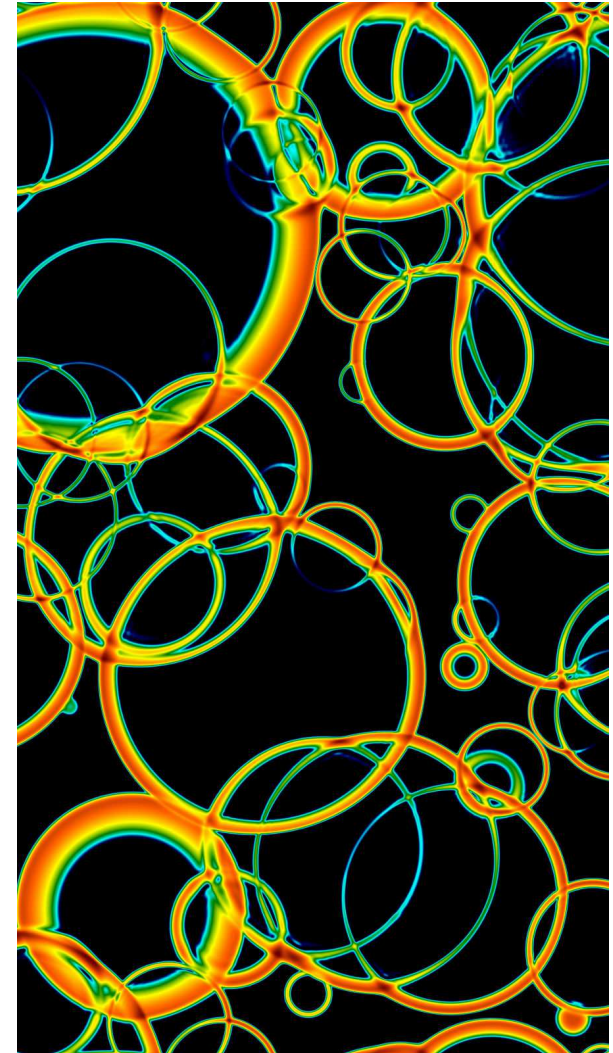
β , α_{T_*} , v_{wall} (and T_*):

3 (+1) parameters are all you need

Espinosa, Konstandin, No, Servant;

Kamionkowski, Kosowsky, Turner

(can get κ from α_{T_*} and v_{wall})



What the metric sees at a thermal phase transition

- Bubbles nucleate, most energy goes into plasma, then:
 1. $h^2\Omega_\phi$: Bubble walls and shocks collide – ‘envelope phase’
 2. $h^2\Omega_{sw}$: Sound waves set up after bubbles have collided, before expansion dilutes KE – ‘acoustic phase’
 3. $h^2\Omega_{turb}$: MHD turbulence – ‘turbulent phase’
- These sources then add together to give the observed GW power:

$$h^2\Omega_{GW} \approx h^2\Omega_\phi + h^2\Omega_{sw} + h^2\Omega_{turb}$$

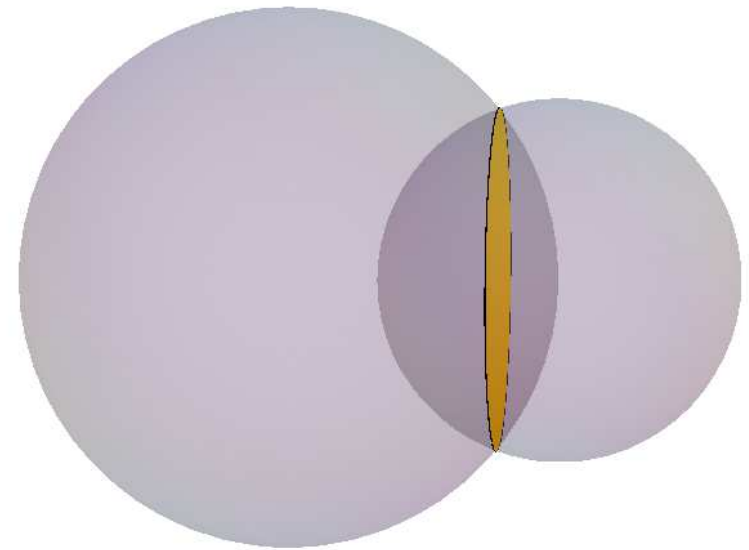
- Each phase’s contribution depends on the nature of the phase transition.
- Now: explore steps 1-2 through two types of simulations:
 1. The ‘envelope approximation’ $\rightarrow h^2\Omega_\phi$
 2. A field ϕ (‘Higgs’) coupled by friction to a fluid U^μ (‘plasma’) $\rightarrow h^2\Omega_{sw}$

1: Envelope [and thin wall] approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kamionowsky and Turner

- Thin, hollow bubbles, no fluid
- Bubbles expand with velocity v_w
- Stress-energy tensor $\propto R^3$ on wall
- Overlapping bubbles \rightarrow GWs
- Keep track of solid angle
- Collided portions of bubbles disappear, sourcing gravitational waves
- Resulting power spectrum is simple
 - One length scale (average bubble radius R_*)
 - Two power laws ($\omega^3, \sim \omega^{-1}$)
 - Amplitude

\Rightarrow 4 numbers define spectral form



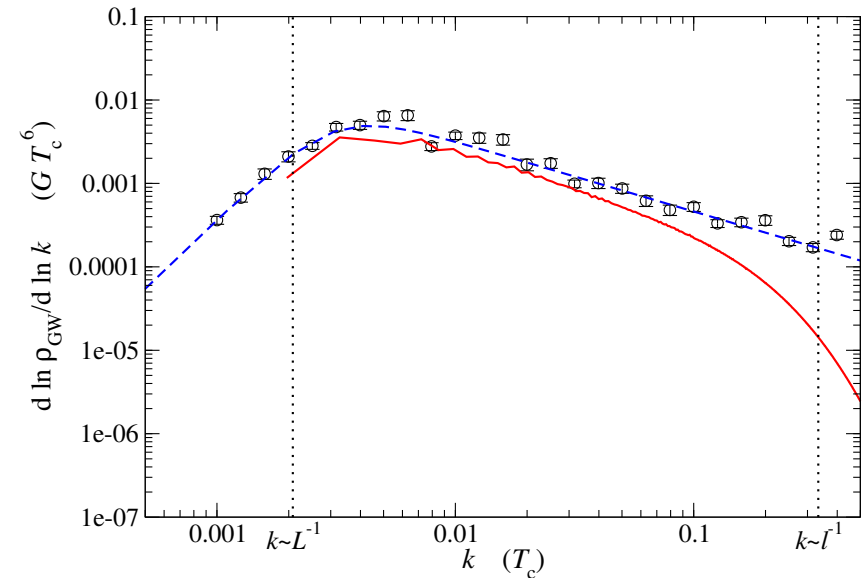
1: Making predictions with the envelope approximation

Espinosa, Konstandin, No and Servant; Huber and Konstandin

4-5 numbers parametrise the transition:

- α_{T_*} , vacuum energy fraction
- v_w , bubble wall speed
- κ_ϕ , conversion ‘efficiency’ into gradient energy $(\nabla\phi)^2$
- Transition rate:
 - H_* , Hubble rate at transition
 - β , bubble nucleation rate

→ ansatz for $h^2\Omega_\phi$



NB: applied to colliding shocks in a *thermal* transition ($\kappa = \kappa_f$), energy in GWs is

$$\rho_{\text{GW}} \propto \frac{0.11 v_w^3}{0.42 + v_w^2} \left(\frac{H_*}{\beta} \right)^2 \frac{\kappa_f^2 \alpha^2}{(\alpha + 1)^2}$$

assumes the shocks are **thin** and disappear after the bubbles collide: this is an underestimate; the dominant source from the fluid KE is sound waves... 12/28

2: Coupled field and fluid system

- Scalar ϕ + ideal fluid u^μ
 - Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits

Ignatius, Kajantie, Kurki-Suonio and Laine

$$\partial_\mu T^{\mu\nu} = \partial_\mu (T_{\text{field}}^{\mu\nu} + T_{\text{fluid}}^{\mu\nu}) = 0$$

- Parameter η sets the scale of friction due to plasma

$$\partial_\mu T_{\text{field}}^{\mu\nu} = \eta u^\mu \partial_\mu \phi \partial^\nu \phi \quad \partial_\mu T_{\text{fluid}}^{\mu\nu} = -\eta u^\mu \partial_\mu \phi \partial^\nu \phi$$

- Effective potential $V(\phi, T)$ does not need to be realistic

$$V(\phi, T) = \frac{1}{2}\gamma(T^2 - T_0^2)\phi^2 - \frac{1}{3}AT\phi^3 + \frac{1}{4}\lambda\phi^4$$

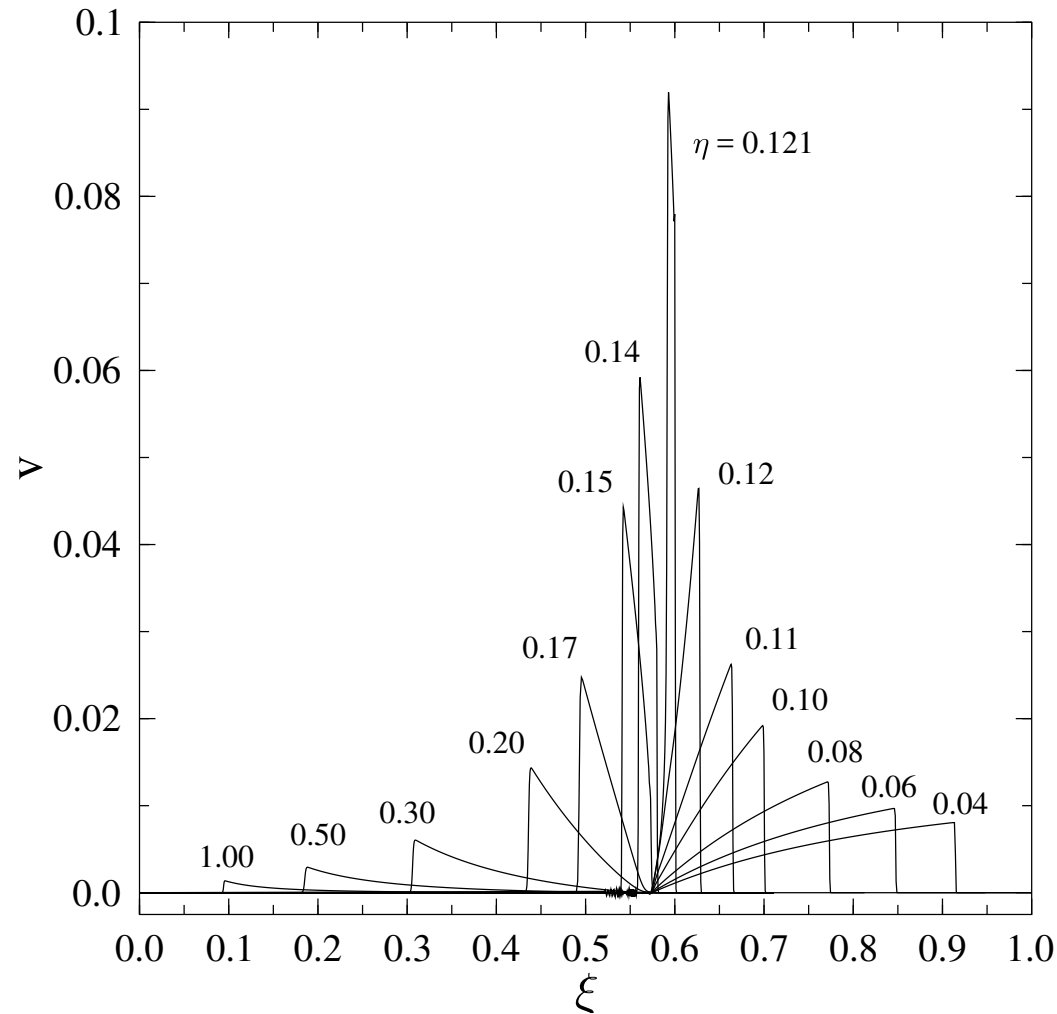
- γ, T_0, A, λ chosen to match scenario of interest
- Equations of motion (+ continuity equation)

$$\partial_\mu \partial^\mu \phi + \frac{\partial V(\phi, T)}{\partial \phi} = -\eta u^\mu \partial_\mu \phi$$

$$\partial_\mu \{[\epsilon + p] u^\mu u^\nu - g^{\mu\nu} [p - V(\phi, T)]\} = \left(\eta u^\mu \partial_\mu \phi + \frac{\partial V(\phi, T)}{\partial \phi} \right) \partial^\nu \phi$$

2: Wall velocities and shock profiles: the η parameter **Kurki-Suonio and Laine**

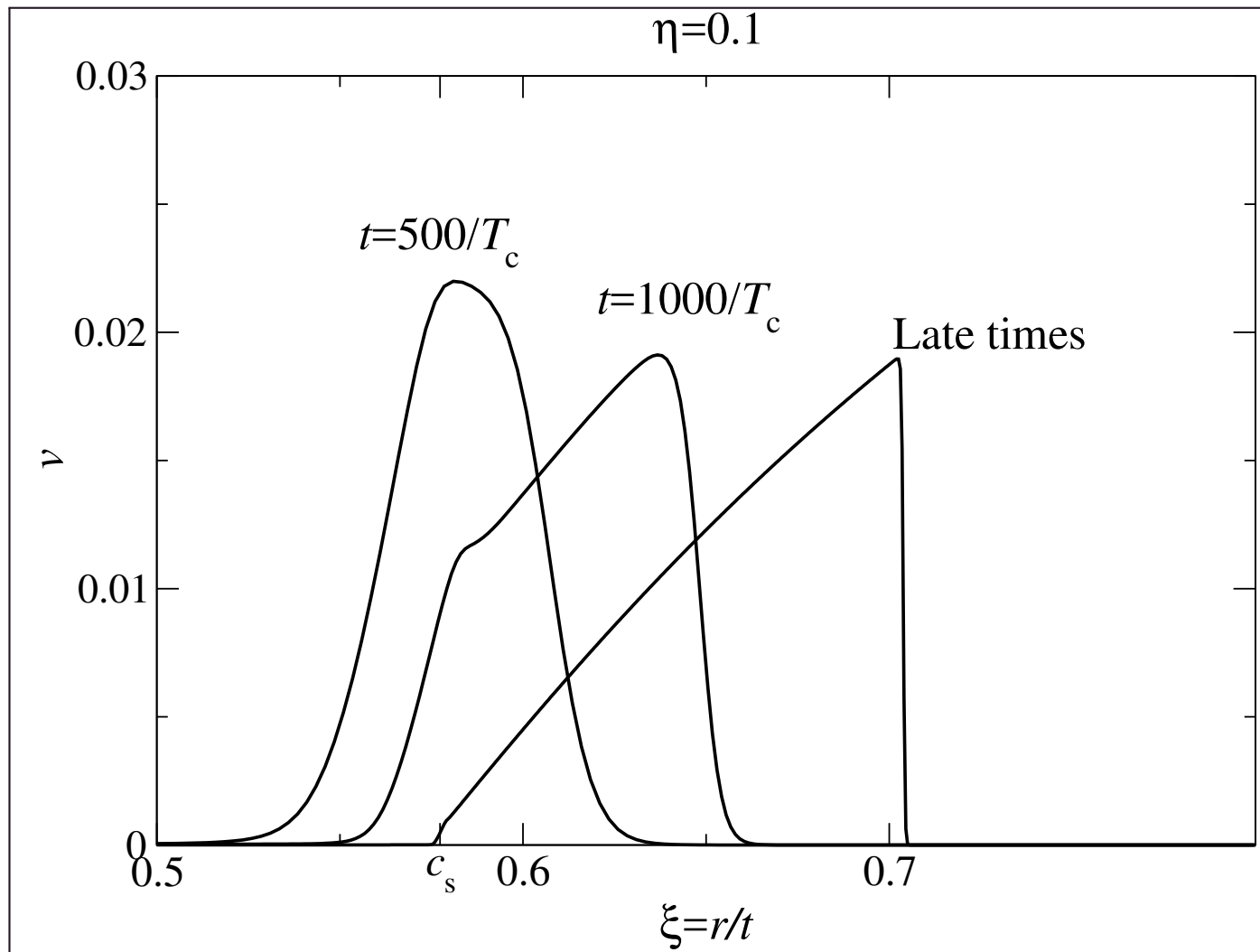
- The value of η sets the velocity of bubble wall v_w



Deflagrations ($v_w < c_s$, shock leads); detonations ($v_w > c_s$, shock trails)

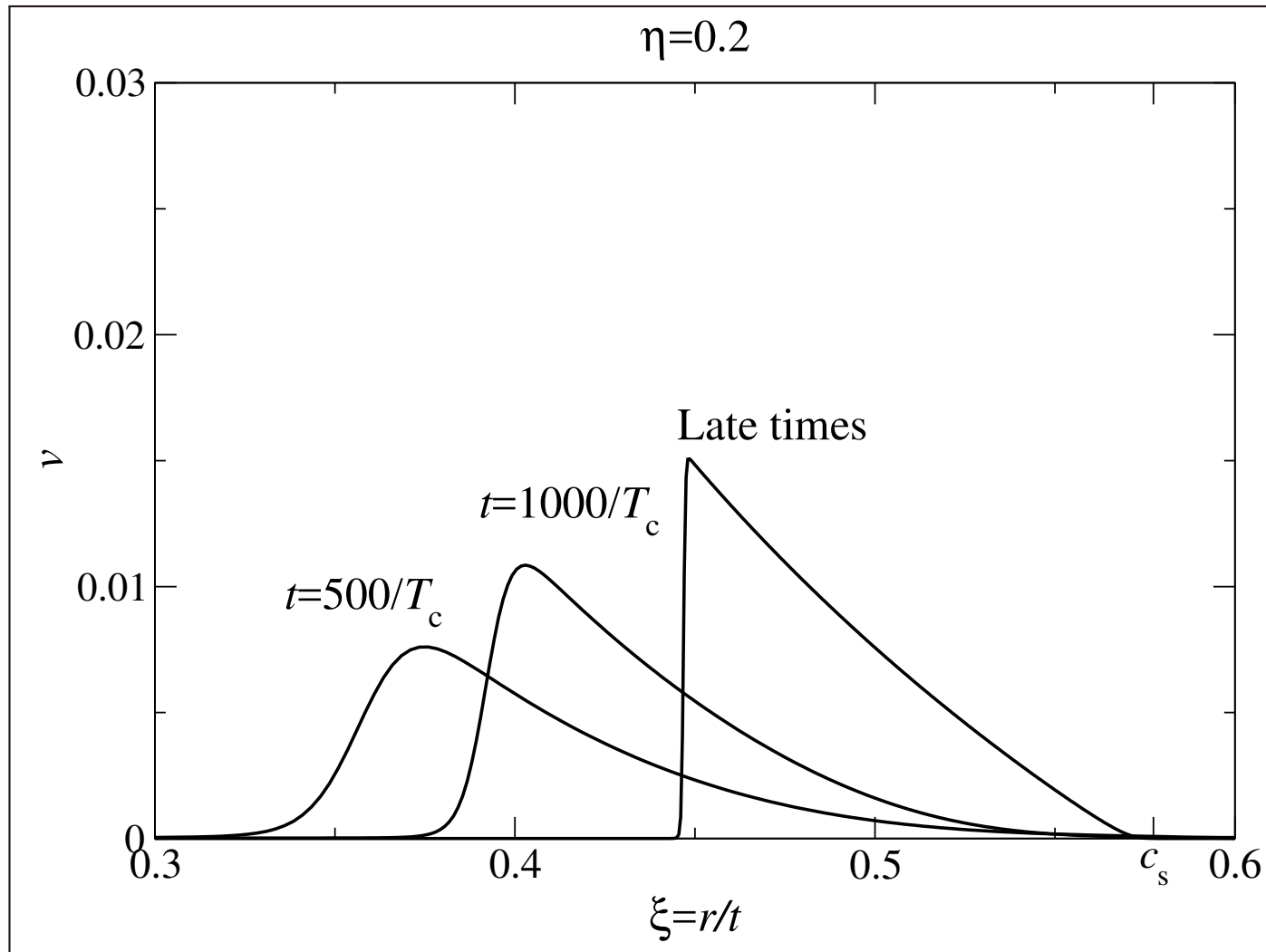
2: Velocity profile development - detonation

Here, $\eta = 0.1$ (detonation)



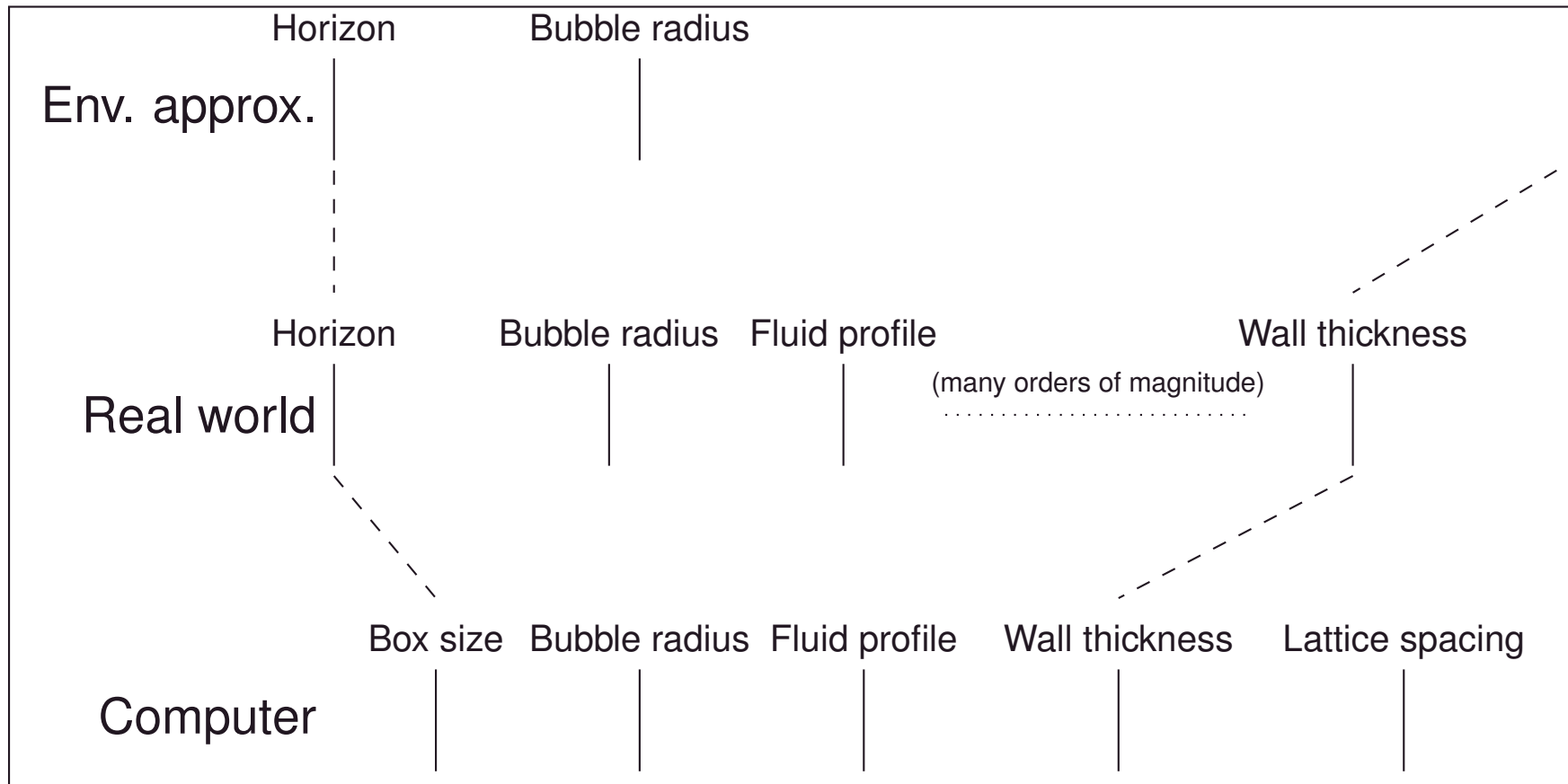
2: Velocity profile development - deflagration

Here, $\eta = 0.2$ (deflagration)



2: Dynamic range issues

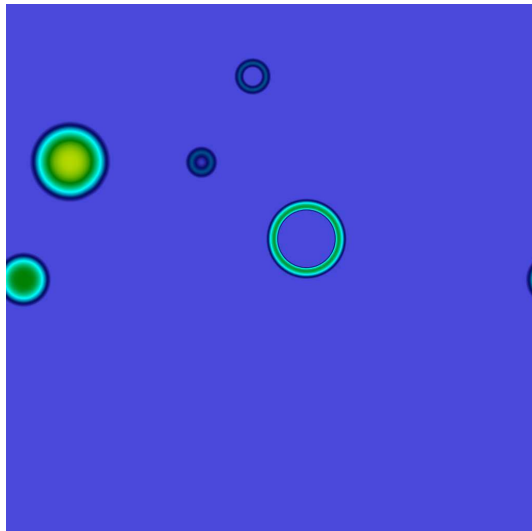
- Most realtime lattice simulations in the early universe have a single [nontrivial] length scale
- Here, many length scales important



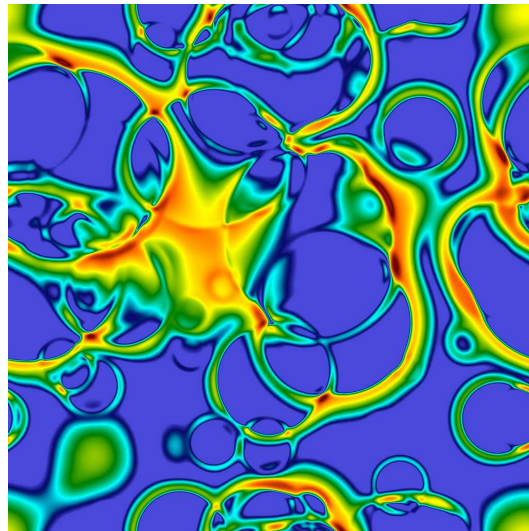
- Recently completed simulations with 4200^3 lattices, $\delta x = 2/T_c$
→ approx 1M CPU hours each (~ 17.6 M total)

2: Simulation slice example [optional movie]

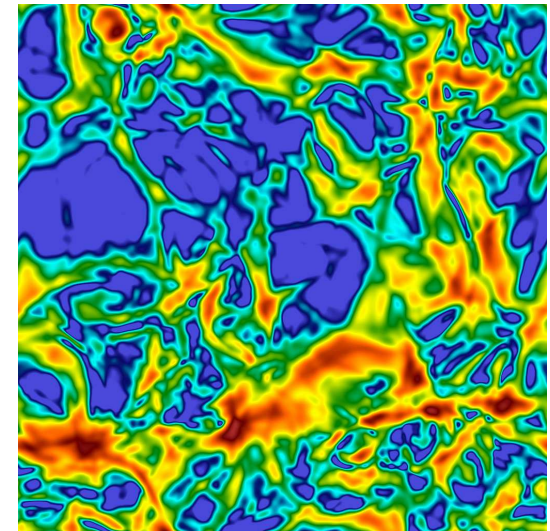
Simulations at 1024^3 , deflagration, fluid kinetic energy density, ~ 250 bubbles



$$t = 500 T_c^{-1}$$



$$t = 750 T_c^{-1}$$



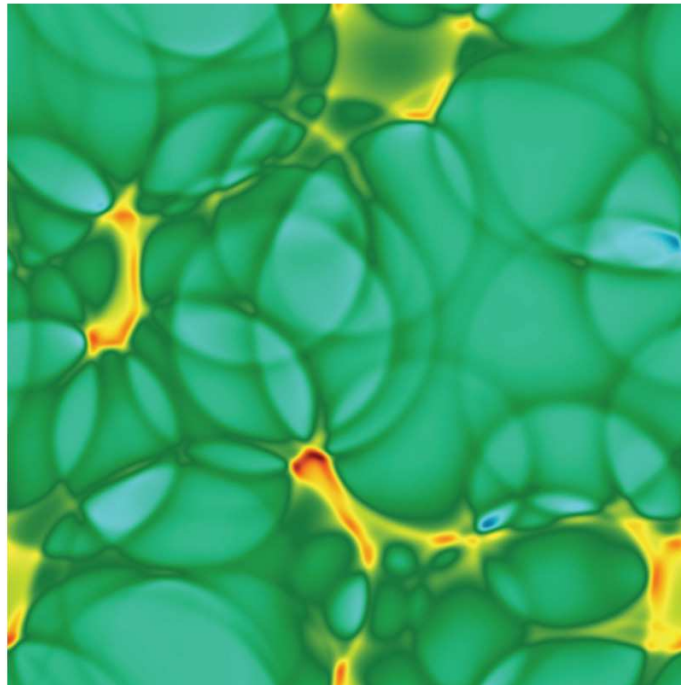
$$t = 1000 T_c^{-1}$$

2: Simulation slice example

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Cover Image: Phys. Rev. Lett. Vol. 112, Iss. 4



Simulated energy density of a fluid-fluid system at the end of initial coalescence: A model for gravitational wave generation in the early Universe.

From the article:

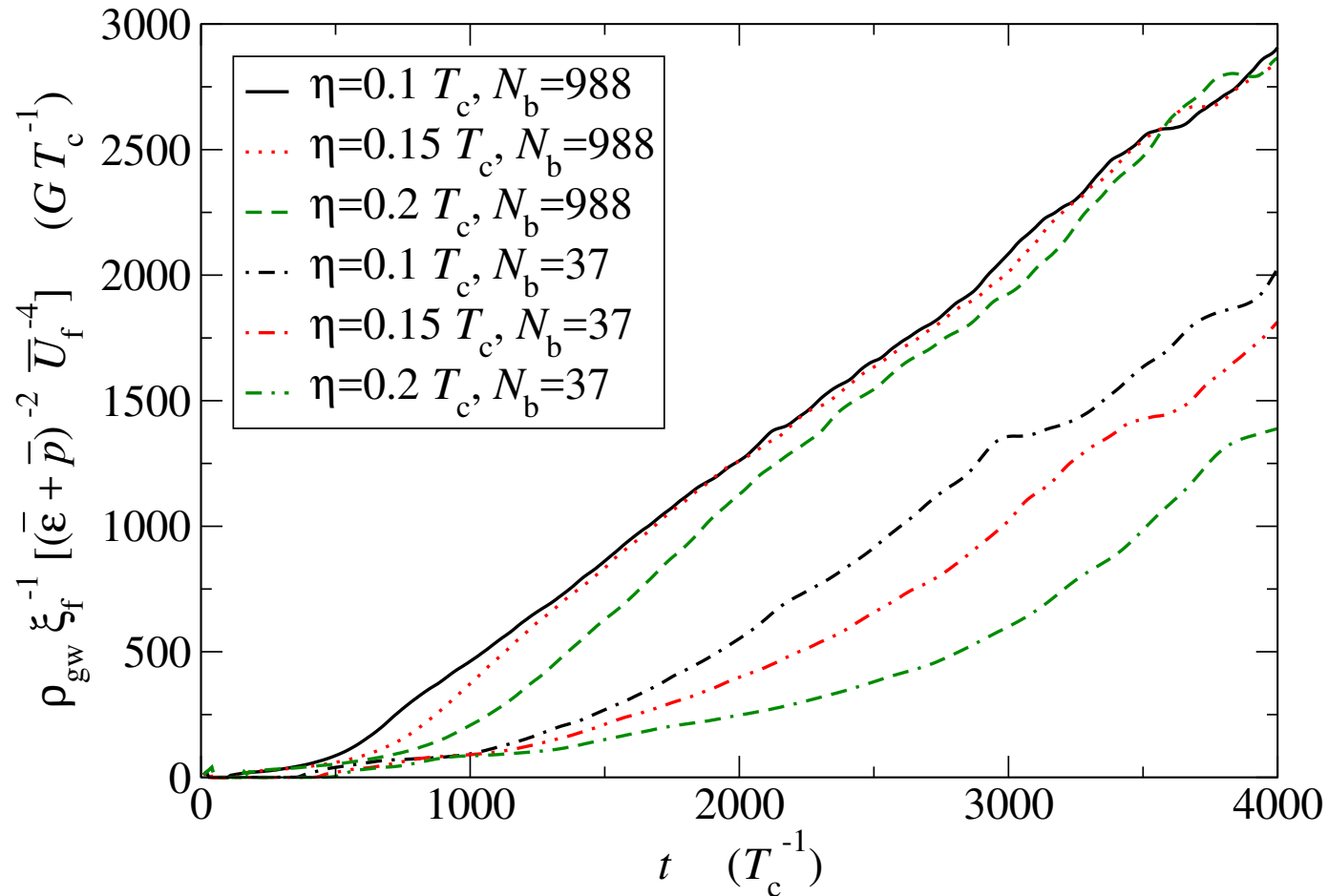
[Gravitational Waves from the Sound of a First Order Phase Transition](#)

Mark Hindmarsh, Stephan J. Huber, Kari Rummukainen, and David J. Weir

Phys. Rev. Lett. **112**, 041301 (2014)

2: Acoustic waves source linear growth of gravitational waves

- Energy density in gravitational waves ρ_{gw} :



- Stationary source
- Total energy generically scales as $\rho_{\text{GW}} \propto t [G \xi_f (\bar{\epsilon} + \bar{p})^2 \bar{U}_f^4]$

2: Lifetime of sound waves and increase in GW power

- Does the acoustic source matter?
 - Sound is damped by (bulk and) shear viscosity

Arnold, Dogan and Moore; Arnold, Moore and Yaffe

$$\left(\frac{4}{3}\eta_s + \zeta\right) \nabla^2 V_{\parallel}^i + \dots \Rightarrow \tau_{\eta}(R) \sim \frac{R^2 \epsilon}{\eta_s}$$

- Compared to $\tau_{H_*} \sim H_*^{-1}$, on length scales

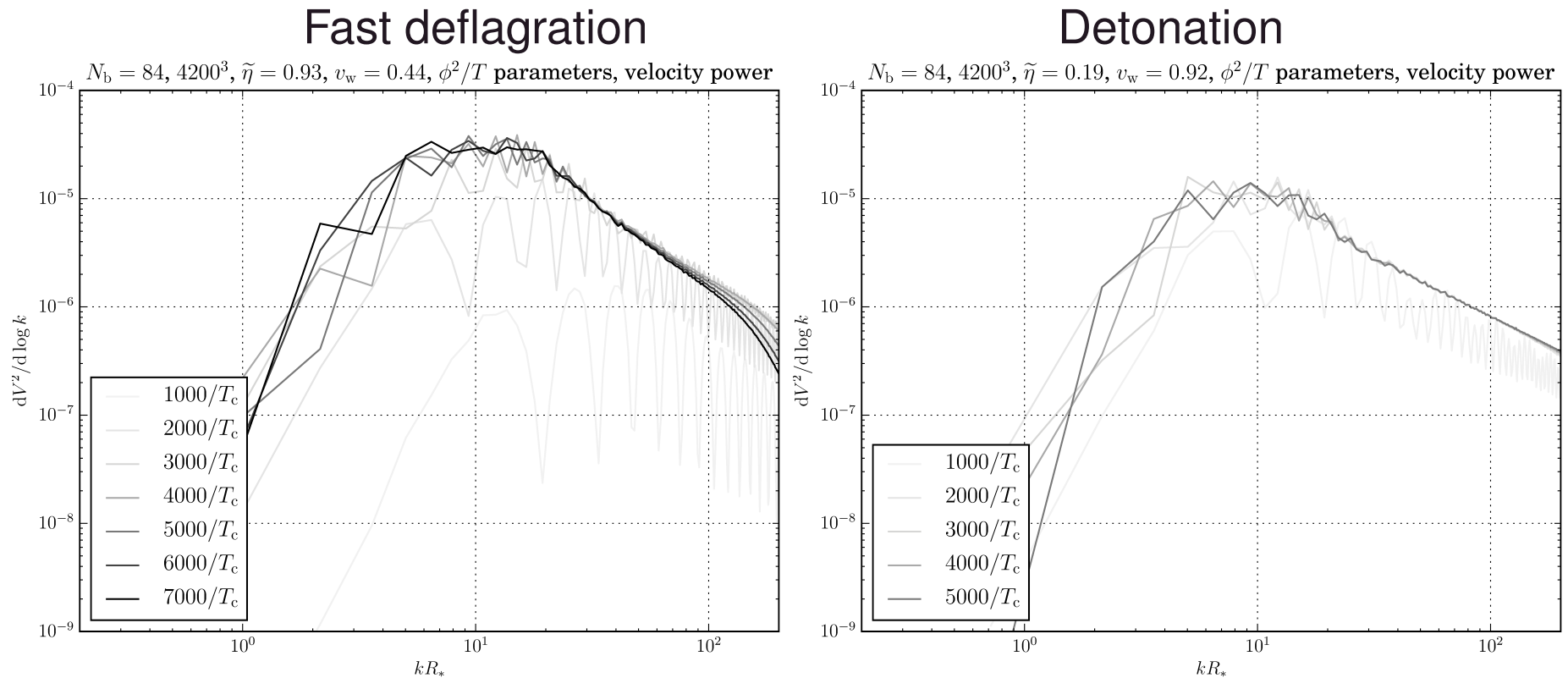
$$R^2 \gg \frac{1}{H_*} \frac{\eta_s}{\epsilon} \sim 10^{-11} \frac{v_w}{H_*} \left(\frac{T_c}{100 \text{ GeV}}\right)$$

the Hubble damping is faster than shear viscosity damping.

- Does the acoustic source enhance GWs?
 - Yes, we have

$$\Omega_{\text{GW}} \approx \left(\frac{\kappa\alpha}{\alpha+1}\right)^2 (H_*\tau_{H_*})(H_*\xi_f) \Rightarrow \frac{\Omega_{\text{GW}}}{\Omega_{\text{GW}}^{\text{envelope}}} \gtrsim 60 \frac{\beta}{H_*}.$$

2: Velocity power spectra and power laws

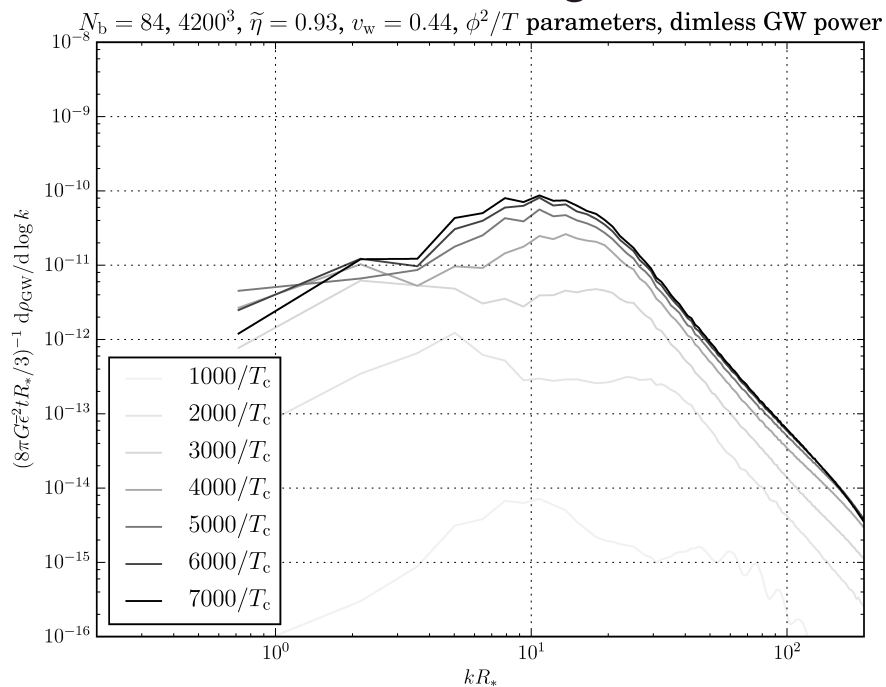


- Weak transition: $\alpha_{T_N} = 0.01$
- Power law behaviour above peak is between k^{-2} and k^{-1}
- “Ringing” due to simultaneous bubble nucleation, not physically important

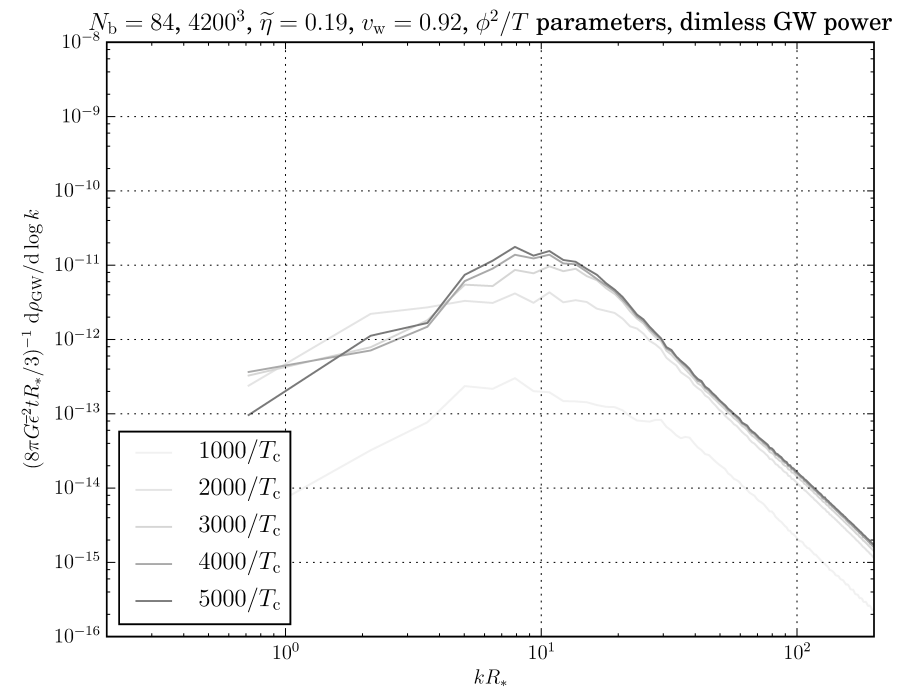
2: GW power spectra and power laws

- Sourced by T_{ij}^f only

Fast deflagration



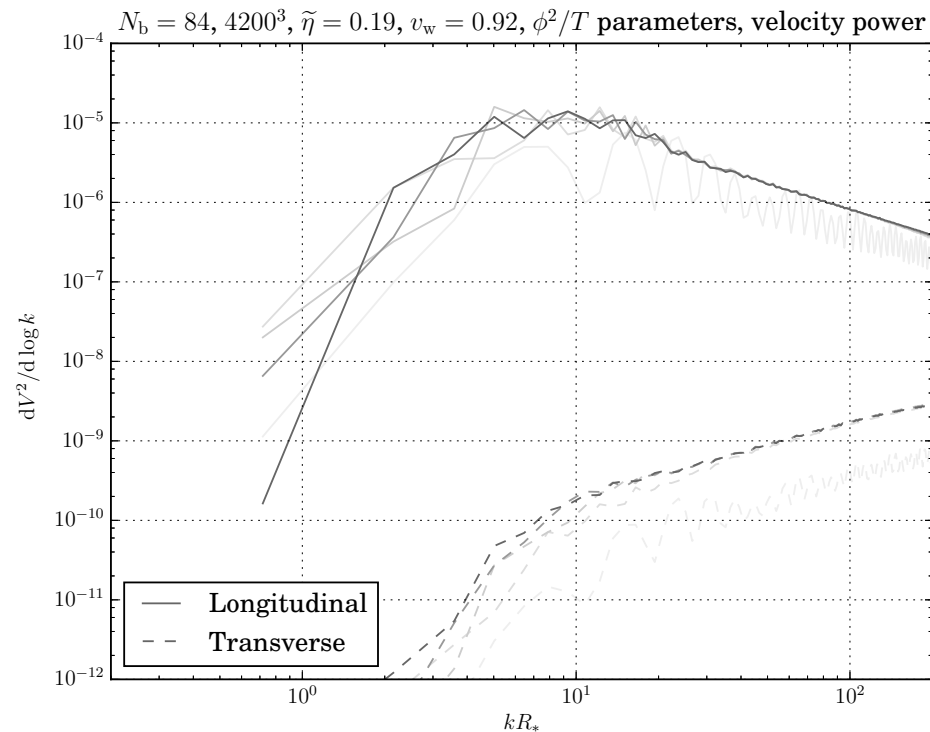
Detonation



- Approximate k^{-3} to k^{-4} power spectrum at high k
- Expect causal k^3 at low k
- Curves scaled by t : source 'on' continuously until turbulence/expansion

→ power law ansatz for $h^2 \Omega_{\text{sw}}$

3: Transverse versus longitudinal modes – turbulence?



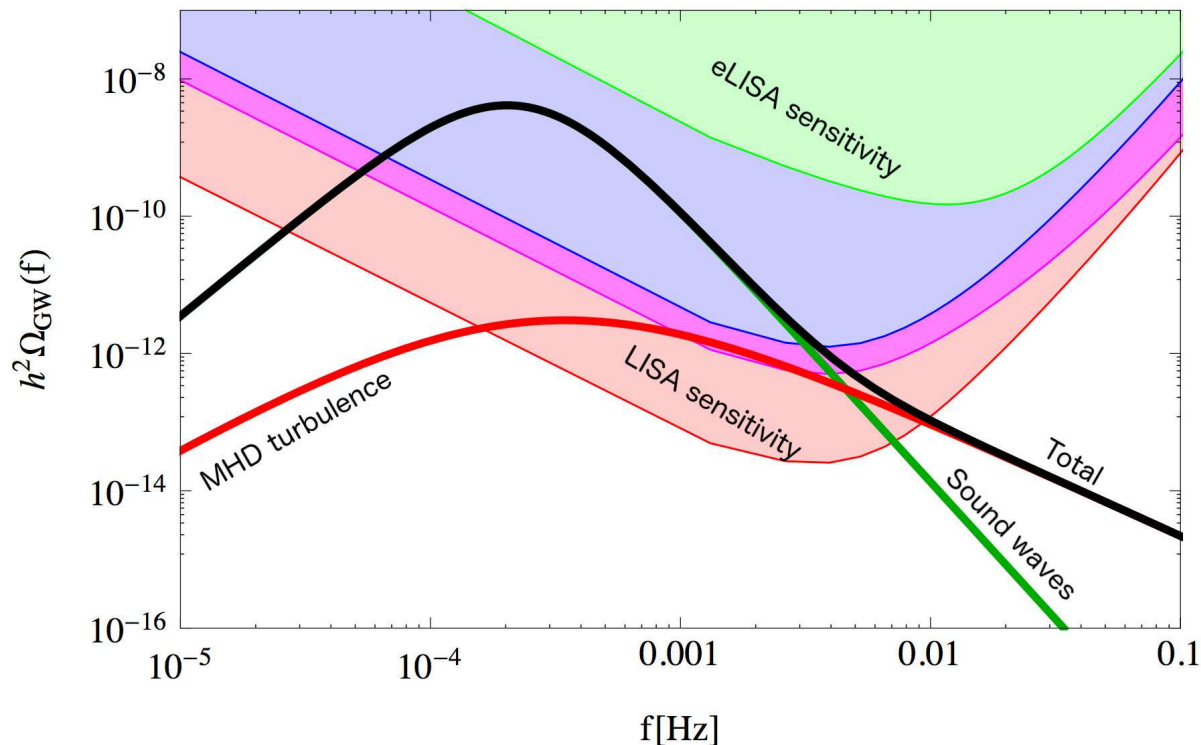
- Weak transition (small α): physics is linear; most power is in the longitudinal modes – acoustic waves, not turbulence
- Is turbulence is something that would happen later? [Pen and Turok](#)
- Power spectrum would have causal k^3 then $k^{-5/3}$ from Kolmogorov velocity power spectrum [Caprini, Durrer and Servant](#)

→ power law ansatz for $h^2 \Omega_{\text{turb}}$

Putting it all together - $h^2\Omega_{\text{gw}}$

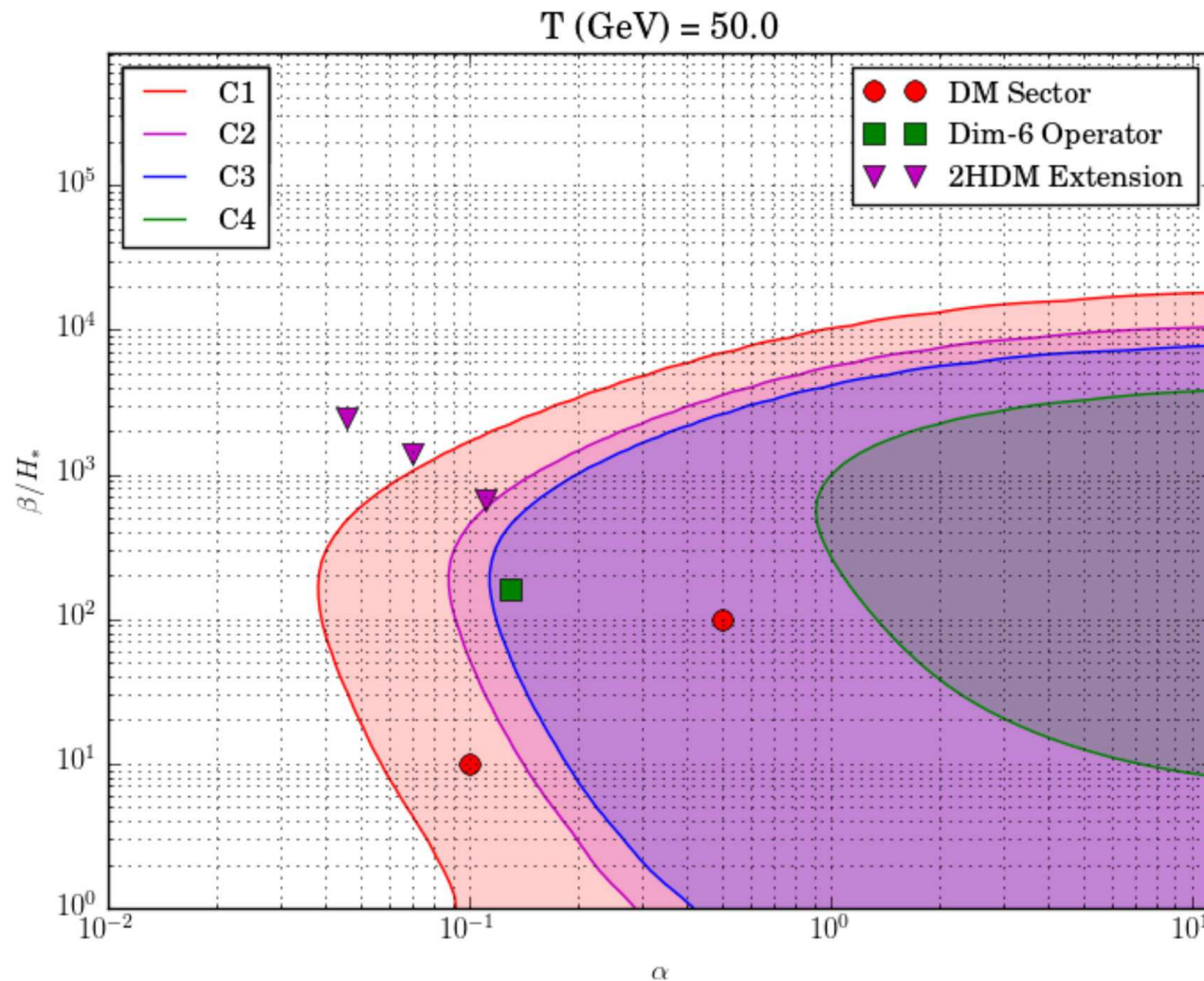
- We have three sources, $\approx h^2\Omega_{\phi}, h^2\Omega_{\text{sw}}, h^2\Omega_{\text{turb}}$
- We know how they vary as a function of $T_*, \alpha_T, v_w, \beta$
- So we can (tentatively) say whether eLISA can detect the phase transition associated with a given model...

(example with $T_* = 100\text{GeV}, \alpha_{T_*} = 0.5, v_w = 0.95, \beta/H_* = 10$)



Putting it all together - physical models to GW power spectra

Map your favourite theory to $(T_*, \alpha_{T_*}, v_w, \beta)$; we can put it on a plot like this

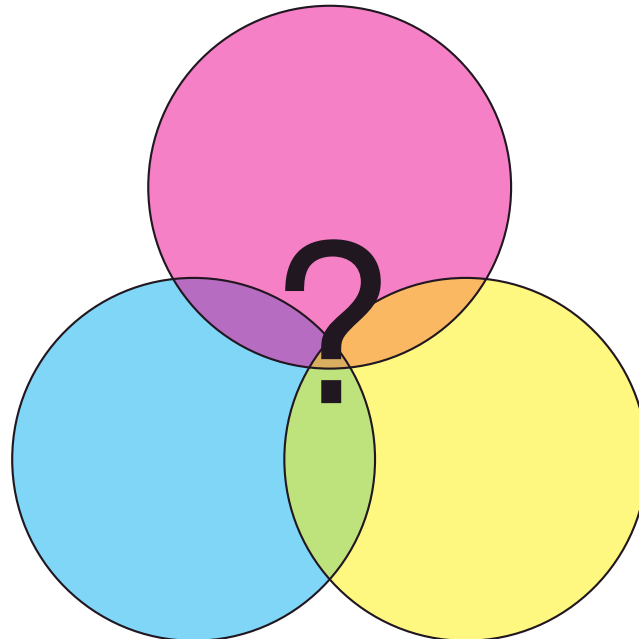


... and tell you if it is detectable by the different [e]LISA cases.

Baryogenesis... ?

Can a thermal phase transition yield baryogenesis and GWs?

- The folklore is that lower wall velocities are better, because they allow the \mathcal{CP} -violating processes to take place [Megevand](#); [Joyce](#), [Prokopec](#), [Turok](#)
- In particular subsonic wall velocities are required
- But energy in GWs goes as v_w^3 , so faster walls are preferred for an observable gravitational wave power spectrum
- In some cases it seems that v_w just shy of c_s works well [Fromme](#), [Huber](#), [Seniuch](#)
- Can we get baryogenesis and GWs from a viable model?



Summary and outlook

- Now:
 - Have a good [cosmological] understanding of what happened during a first order PT
 - Recent work shows source may be stronger than previously thought
 - Many models of first order EWPTs can produce observable gravitational waves – forms part of eLISA science case
- Next:
 - Gravitational wave detectors now firmly on the agenda, and eLISA has support; mission could be improved; launch date could come forward from 2034
 - Strong transitions, turbulence, instabilities still poorly understood
 - Wall velocities; connections with baryogenesis – need [more] model-specific computations?!