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Cosmological constraints on the Higgs-gravity coupling

Arttu Rajantie

CERN TH Institute

24 August 2016

Herranen, Markkanen, Nurmi & AR, PRL113(2014)211102
Herranen, Markkanen, Nurmi & AR, PRL115(2015)241301
AR & Stopyra, arXiv:1606.00849
Figueroa, AR & Torrenti, in progress

Higgs cosmology

27 – 28 March 2017
The Royal Society at Chicheley Hall, Buckinghamshire

Organised by Professor Arttu Rajantie, Dr Malcolm Fairbairn, Dr Tommi Markkanen and Dr Astrid Eichhorn

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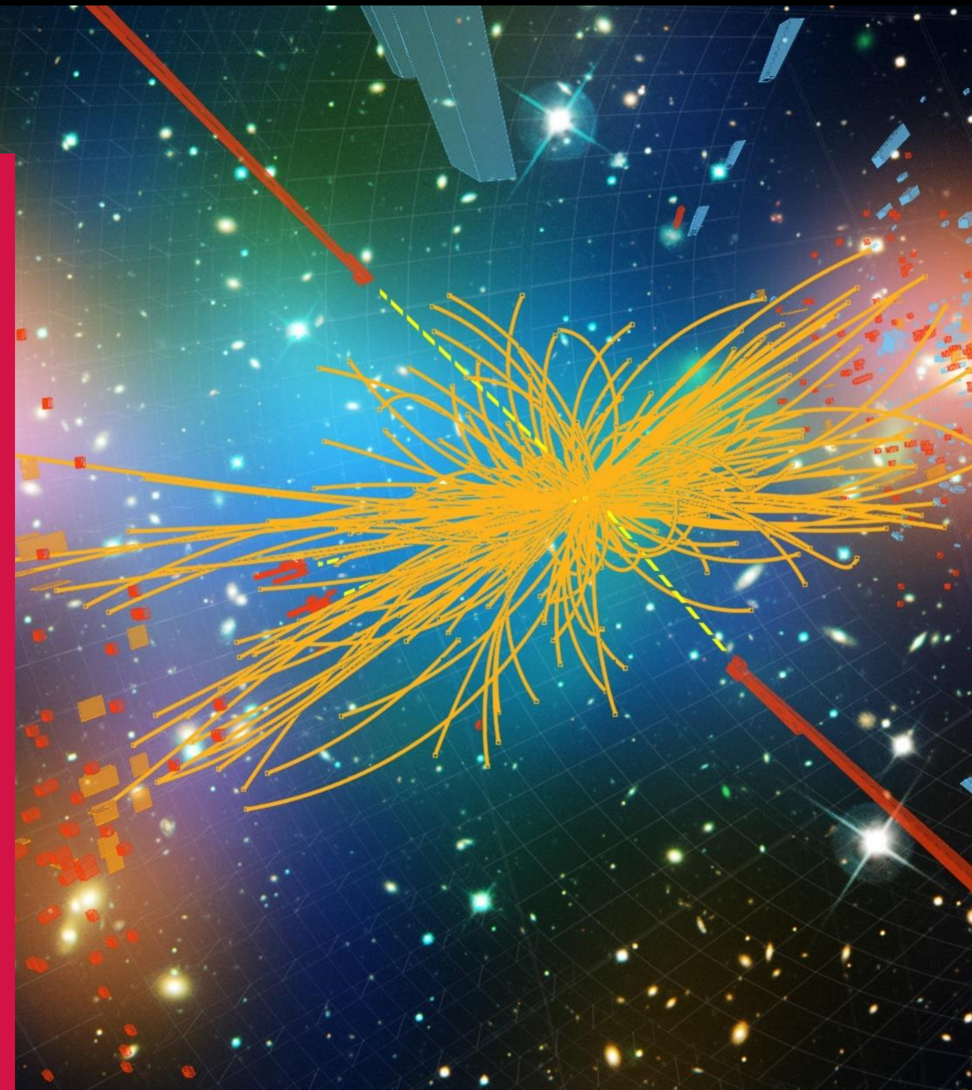
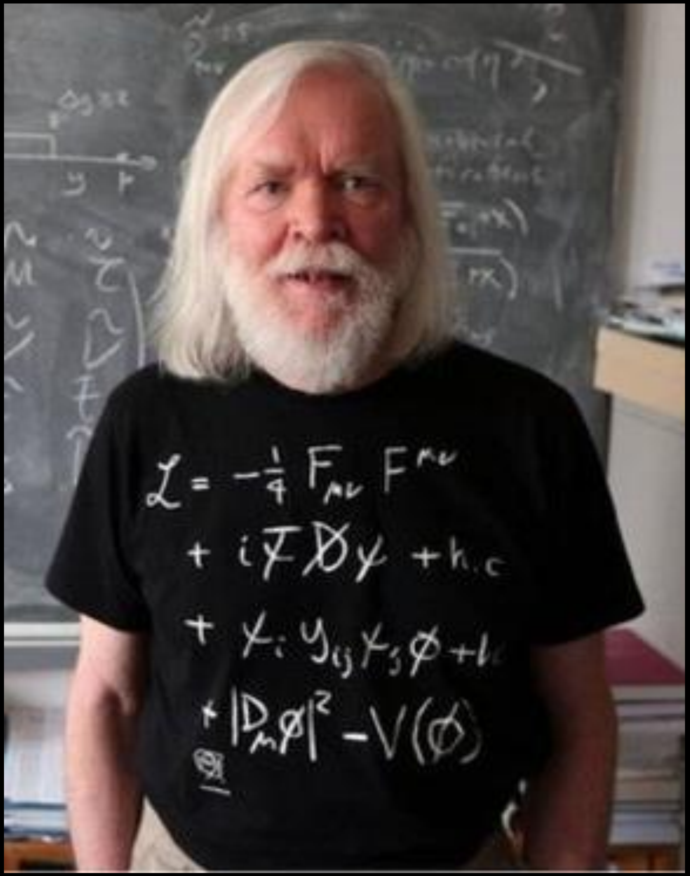


The Standard Model

- Six leptons, six quarks, three gauge fields + Higgs scalar ϕ
- 19 parameters – all have been measured
- All renormalisable terms allowed by symmetries in Minkowski space



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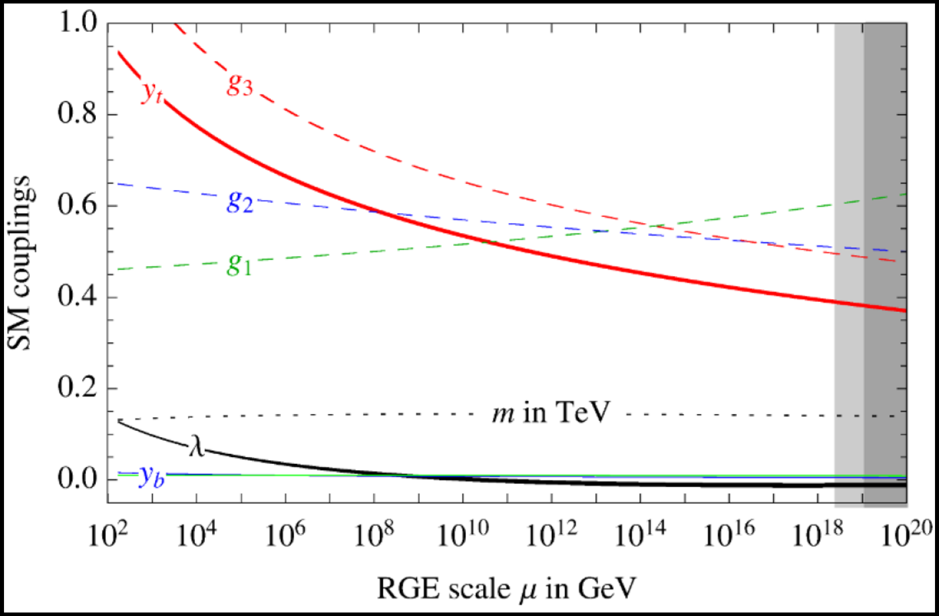
The Standard Model

- Six leptons, six quarks, three gauge fields + Higgs scalar ϕ
- 19 parameters – all have been measured
- All renormalisable terms allowed by symmetries in Minkowski space
- Can be extrapolated all the way to the Planck scale



(Buttazzo et al 2013)

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Cosmos may be 'inherently unstable'

COMMENTS (1000)

By Jonathan Amos
Science correspondent, BBC News, Boston

Scientists say they may be able to determine the eventual fate of the cosmos as they probe the properties of the Higgs boson.

A concept known as vacuum instability could result, billions of years from now, in a new universe opening up in the present one and replacing it.

It all depends on some precise numbers related to the Higgs that researchers are currently

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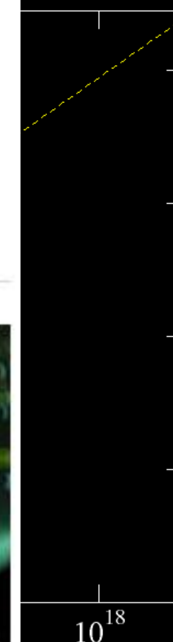
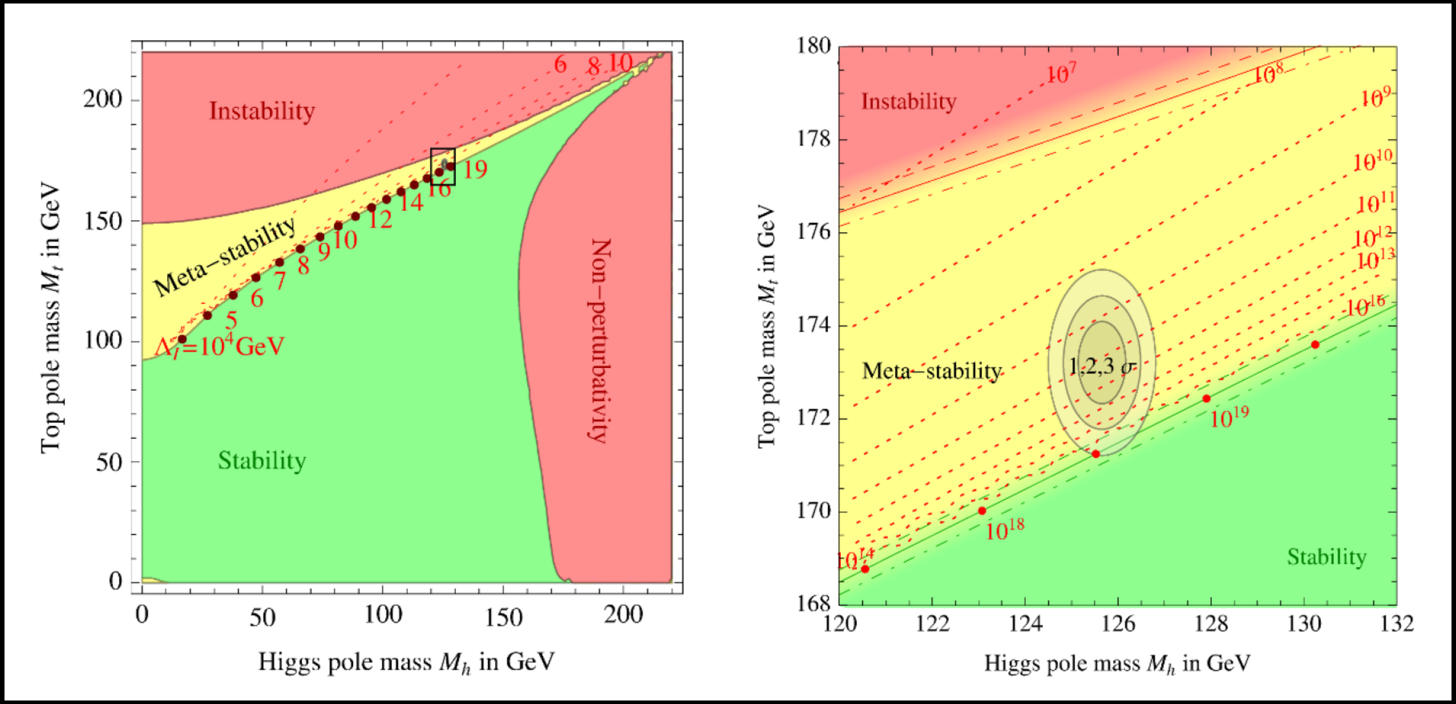
Collisions at the LHC in Geneva have refined a mass for the Higgs-like particle



Instability Bounds

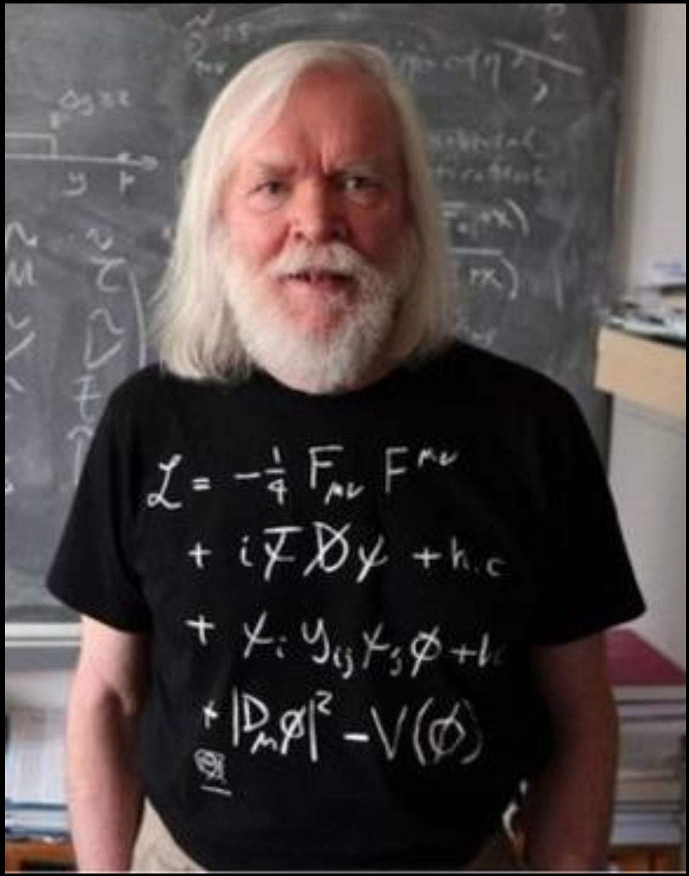


(Buttazzo et al. 2013)

- Vacuum lifetime $\sim \exp(S_{\rm bounce}) \sim \exp\left(\frac{8\pi^2}{3|\lambda|}\right)$
- Longer than the age of the Universe – We are safe!

The Standard Model

- Six leptons, six quarks, three gauge fields + Higgs scalar ϕ
- 19 parameters – all have been measured
- All renormalisable terms allowed by symmetries in Minkowski space



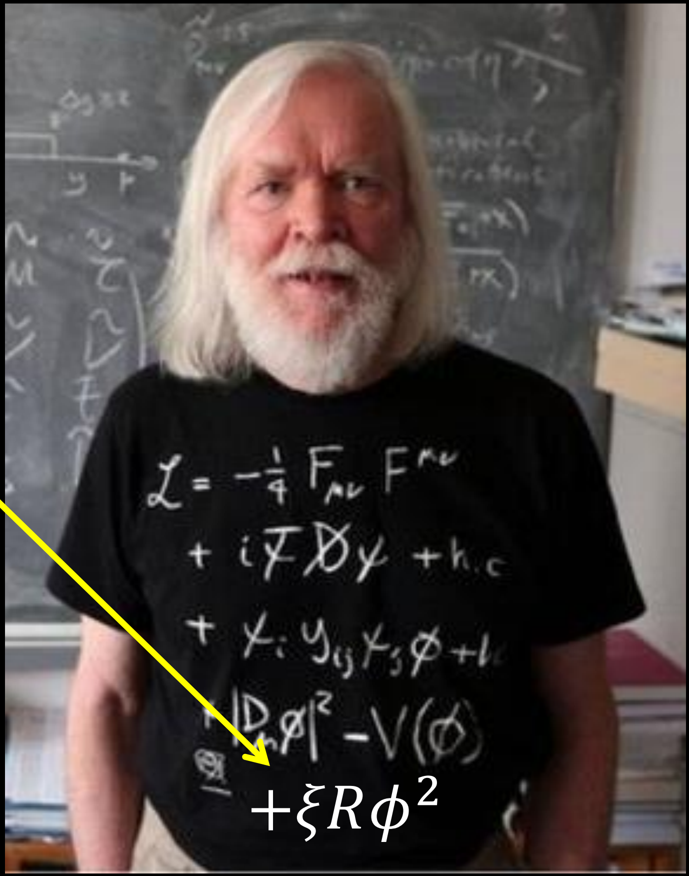
Higgs-Gravity Coupling

- Curved spacetime:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

(Chernikov&Tagirov 1968)

- One more renormalisable term: Higgs-curvature coupling ξ
- Required for renormalisability, runs with energy – Cannot be set to zero!
- Last unknown parameter in the Standard Model



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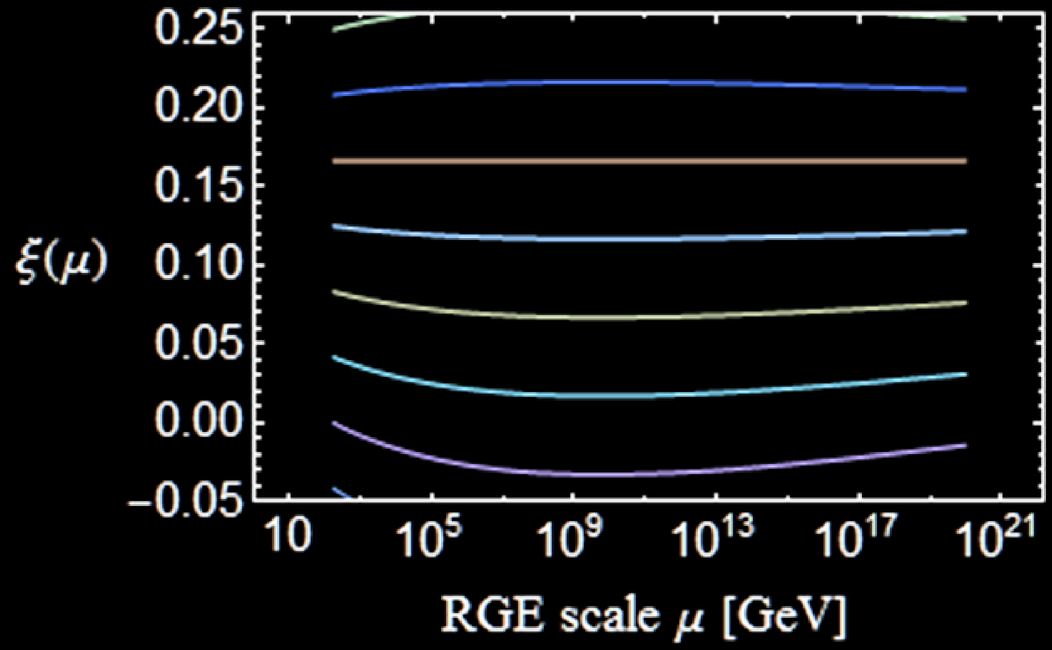
Running ξ

$$\mu\frac{d\xi}{d\mu} = \left(\xi - \frac{1}{6}\right)\frac{12\lambda + 6y_t^2 - \frac{3}{2}g'^2 - \frac{9}{2}g^2}{16\pi^2}$$

- Becomes negative if $\xi_{\mathrm{EW}} = 0$
- Conformal value $\xi = 1/6$ RG invariant at 1 loop



Vacuum Instability and ξ

- Tunneling rate: $\Gamma \sim e^{-B}$, where
  $B =$ action of the Euclidean "bounce" solution (Coleman 1977)
- Coupled field and Einstein equations
- Perturbative gravitational correction (Isidori et al 2007)
  $\Delta B = \frac{32\pi^2}{45 M_{Pl}^2 \rho^2 |\lambda|^2}$, where $\rho =$ radius of the instanton
- AR & Stopyra, arXiv:1606.00849:
  Full numerical solution including ξ

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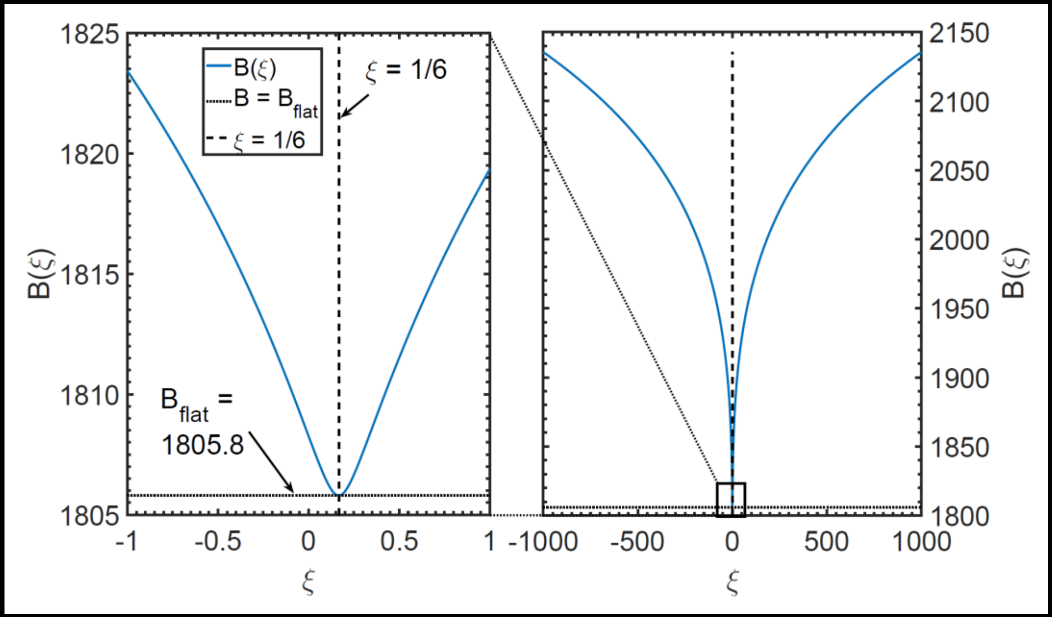
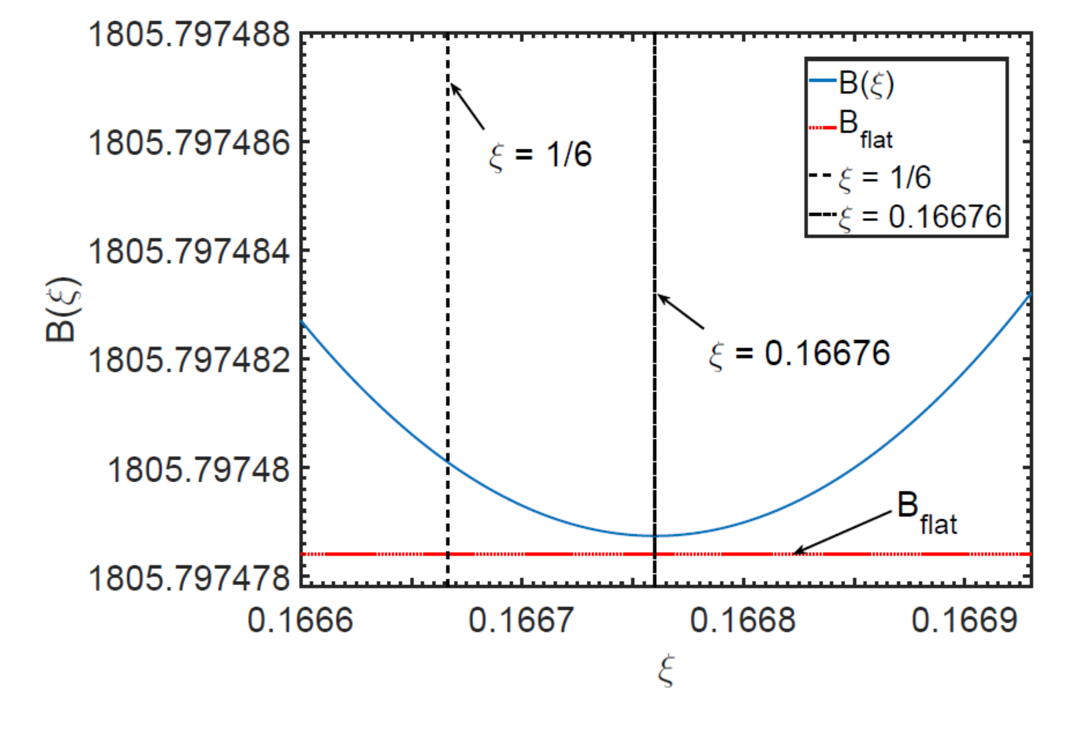
Vacuum Instability and ξ



AR & Stopyra, arXiv:1606.00849

- Tunneling rate $\Gamma \sim e^{-B}$: Larger $|\xi|$ ⇨ more stable vacuum
- Perturbative $\Delta B = \frac{32\pi^2(1-6\xi)^2}{45 M_{Pl}^2 \rho^2 \lambda^2}$ (Salvio et al, arXiv:1608:02555)

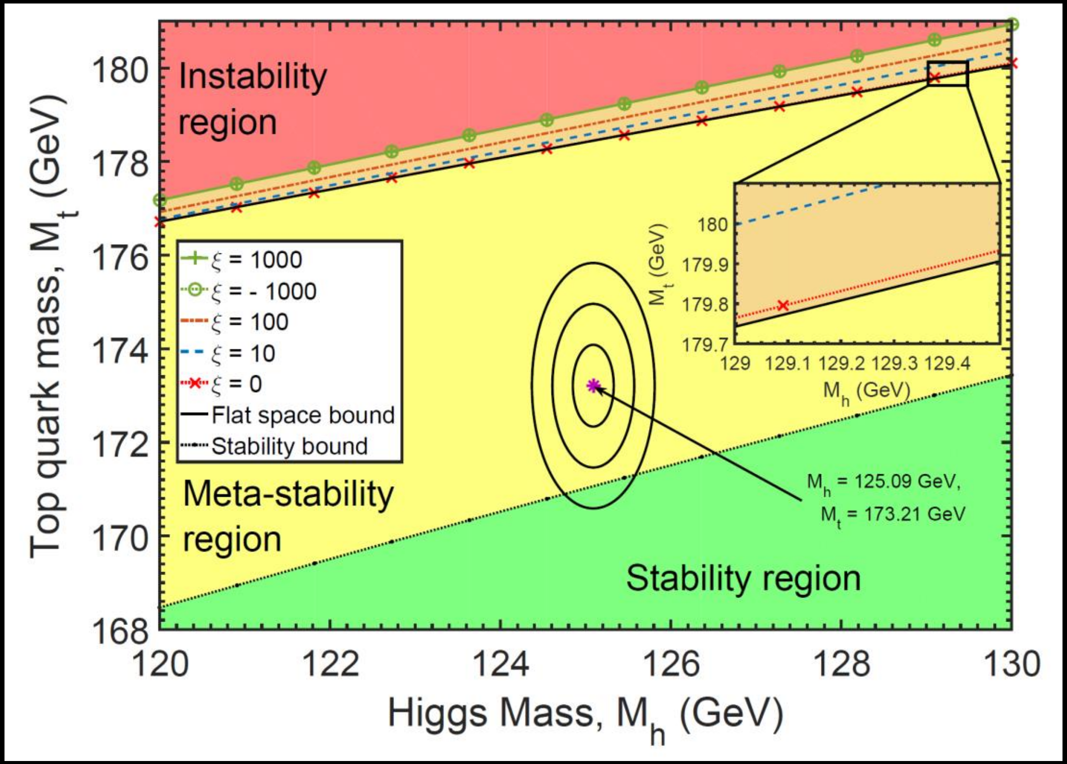
Vacuum Instability Today



AR & Stopyra, arXiv:1606.00849

- Fastest at $\xi \approx 1/6$, where almost the same as Minkowski

Effect on Stability



AR & Stopyra, arXiv:1606.00849

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Measuring ξ

- Curved spacetime:

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \xi R \phi^\dagger \phi$$

- Ricci scalar R very small today – Difficult to measure
- Colliders: Suppresses Higgs couplings (Atkins&Calmet 2012)
  - LHC Bound $|\xi| \lesssim 2.6 \times 10^{15}$
  - Future (?) ILC: $|\xi| \lesssim 4 \times 10^{14}$
- In contrast, R was high in the early Universe: Cosmological effects?

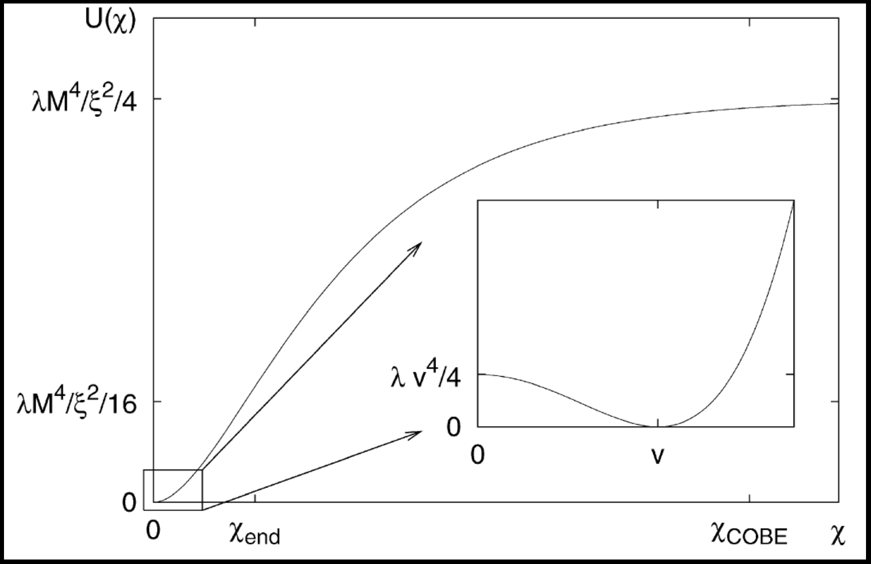
Example: Higgs Inflation (Bezrukov&Shaposhnikov 2008)

- Higgs as inflaton: Dominates energy density
- Jordan → Einstein frame: $\phi \sim \frac{M_{Pl}}{\sqrt{\xi}} \exp\left(\frac{\chi}{\sqrt{6}M_{Pl}}\right)$

$$V(\phi) = \frac{\lambda\phi^4}{4} \rightarrow U(\chi) \sim \frac{\lambda M_{Pl}^4}{4\xi^2}\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\chi}{M_{Pl}}\right)\right]$$

- During inflation

$$\phi \gtrsim M_{Pl}/\sqrt{\xi}$$



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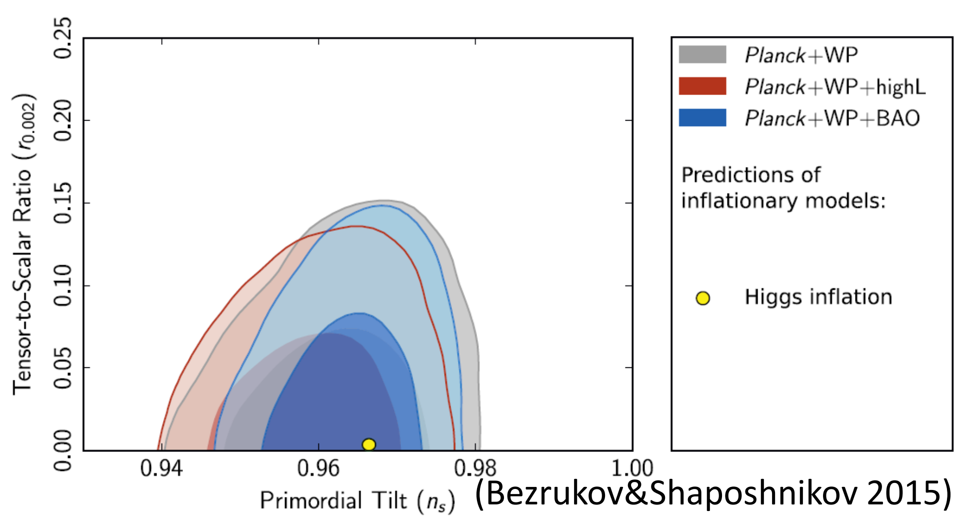
- CMB amplitude
  ⇨ $\xi \sim -49000\sqrt{\lambda}$
- Spectral index
  $n_s \approx 0.97$
- Tensor/scalar ratio:
  $r \approx 0.003$



(Bezrukov&Shaposhnikov 2015)

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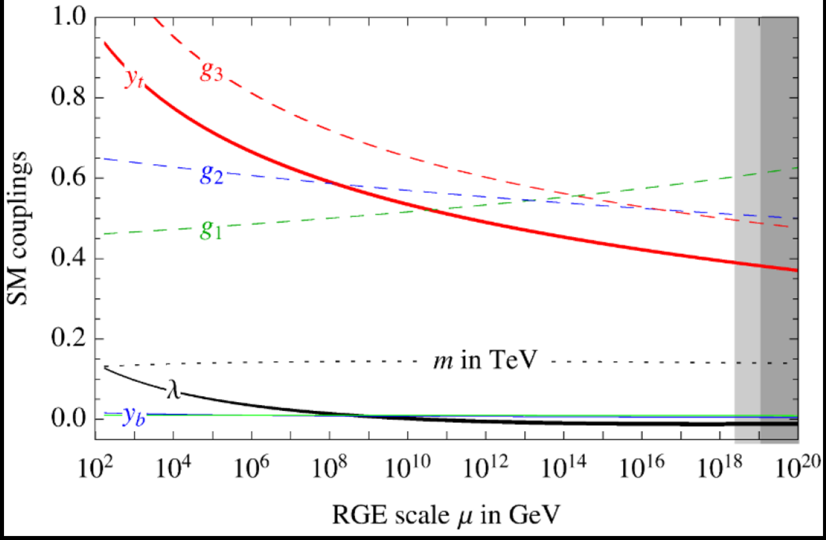
- Problem: Eff. potential

$$V_{\text{eff}}(\phi) \approx \frac{1}{4}\lambda(\phi)\phi^4 < 0$$

for $\phi \gtrsim 10^{10}\text{GeV}$



(Buttazzo et al 2013)

Higgs During Inflation

- Instead of Higgs inflation,
  assume inflaton sector decoupled from the Higgs:
  Higgs dynamics in expanding background
- Ricci scalar in FRW spacetime:

$$R = 6\left(\frac{\dot{a}^2}{a^2} + \frac{\ddot{a}}{a}\right) = 3(1 - 3w)H^2$$

  - Radiation dominated ($w = 1/3$): $R = 0$
  - Matter dominated ($w = 0$): $R = 3H^2$
  - Inflation / de Sitter ($w = -1$): $R = 12H^2$
- Effective Higgs mass term $m_{\mathrm{eff}}^2 = m_{\mathrm{H}}^2 + \xi R$:
  Time-dependent and very high in the early Universe

Higgs During Inflation

- Let us consider $\xi = 0$ first
- Inflation: $H \lesssim 9 \times 10^{13}\ \mathrm{GeV}$ (Planck+BICEP2 2015)
- Higgs mass $m_{\mathrm{H}} \approx 125\ \mathrm{GeV} \ll H$:
  Light scalar field – nearly scale invariant fluctuations
- Fokker-Planck equation (Starobinsky&Yokoyama 1994)

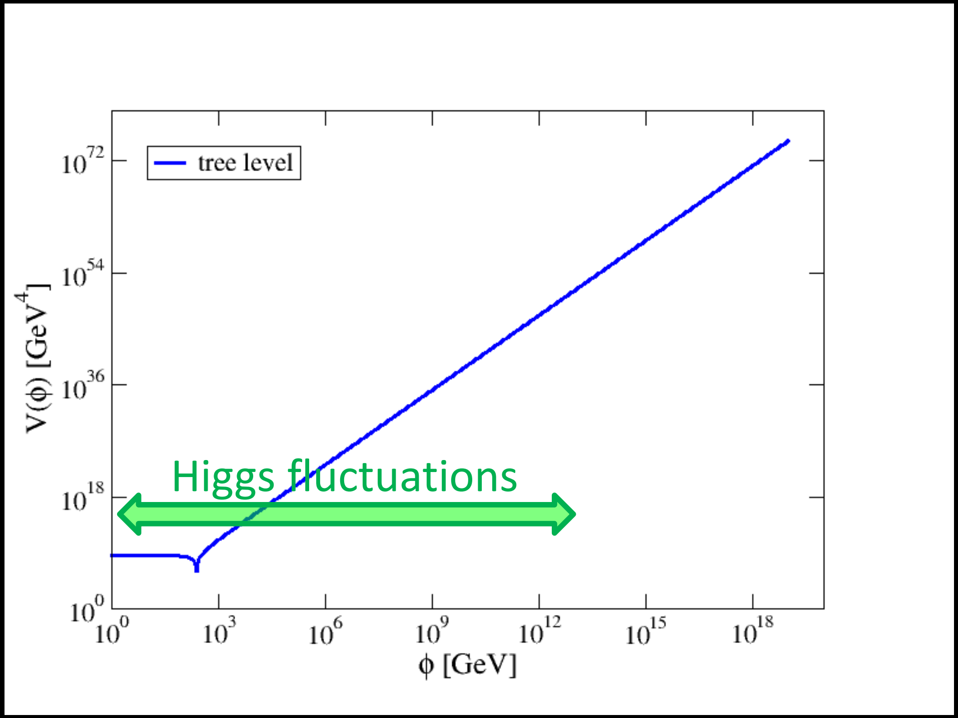
$$\frac{\partial P[\phi]}{\partial t} = \frac{\partial}{\partial \phi}\left[\frac{H^3}{8\pi^2}\frac{\partial P}{\partial \phi} + \frac{V'(\phi)}{3H}P\right]$$

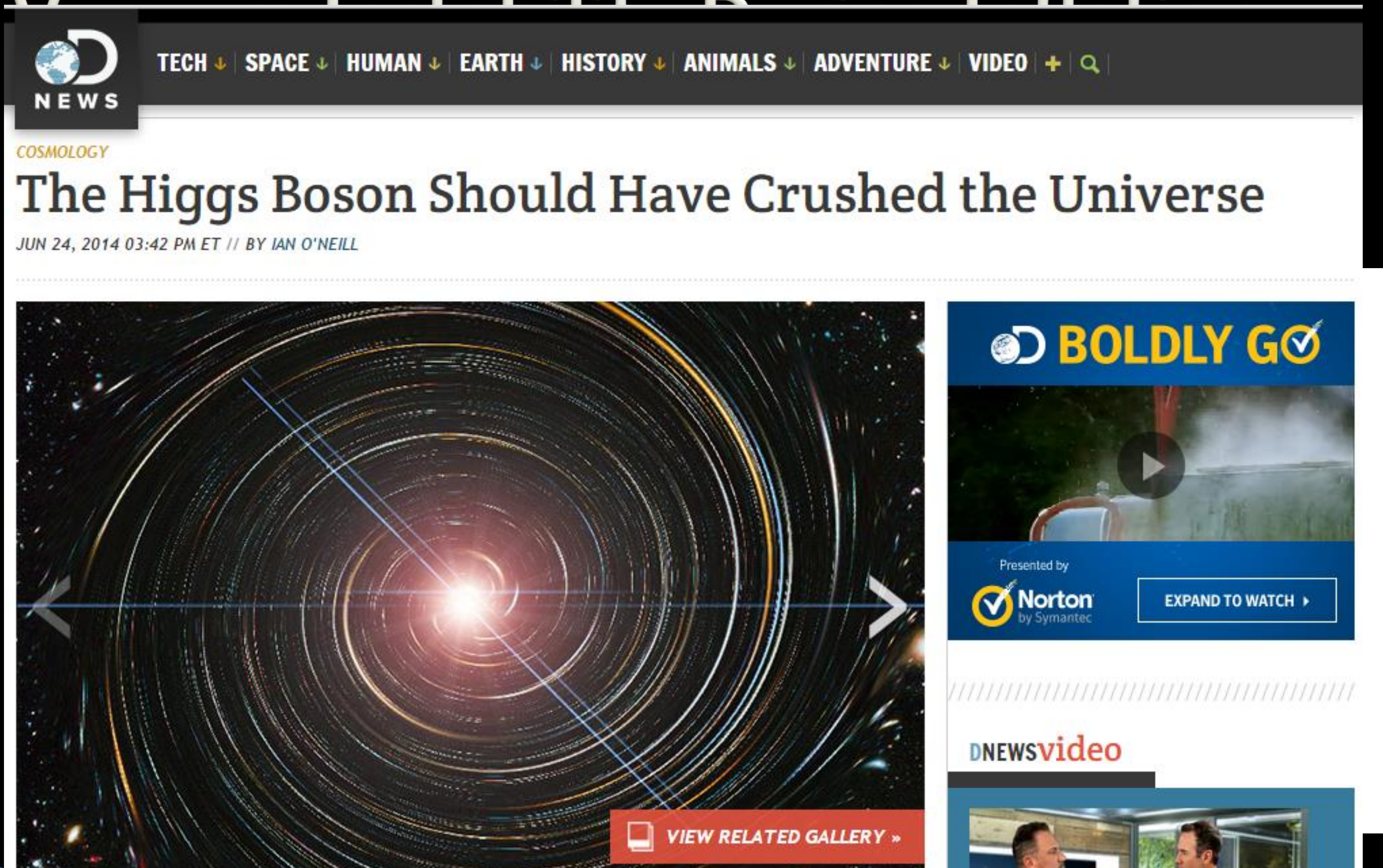
- Equilibrium distribution

$$P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$$

Higgs Fluctuations

- Equilibrium $P(\phi) \propto \exp\left[-\frac{8\pi^2}{3H^4}V(\phi)\right]$
- Tree-level potential $V(\phi) = \lambda(\phi^2 - v^2)^2$
- Nearly scale-invariant fluctuations with amplitude $\phi \sim \lambda^{1/4}H$

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Gravity to the Rescue

- Inflation: Constant $R = 12H^2$
- Effective mass term

$$m_{\text{eff}}^2 = m_{\text{H}}^2 + \xi R = m_{\text{H}}^2 + 12\xi H^2$$

- Tree level: (Espinosa et al 2008)
  - $\xi > 0$: Increases barrier height
    Makes the low-energy vacuum more stable
  - $\xi < 0$: Decreases barrier height
    Makes the low energy vacuum less stable
- Also contributes to loop corrections

One-Loop Potential

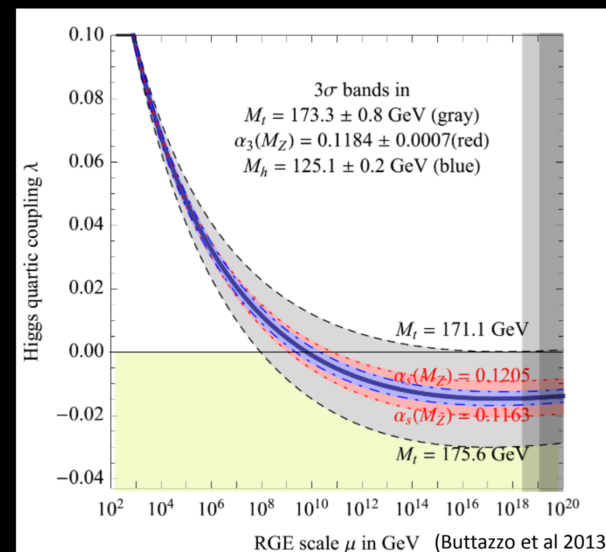
$$V_{\mathrm{eff}} = -\frac{1}{2}m^2(t)\phi^2(t) + \frac{1}{2}\xi(t)R\phi^2(t) + \frac{1}{4}\lambda(t)\phi^4(t) + \sum_{i=1}^{9}\frac{n_i}{64\pi^2}M_i^4(\phi)\left[\log\frac{|M_i^2(\phi)|}{\mu^2(t)} - c_i\right],$$

(HMNR 2014)

$$M_i^2(\phi) = \kappa_i\phi^2(t) - \kappa_i' + \theta_i R,$$

Φ	i	n_i	κ_i	κ_i'	θ_i	c_i
	1	2	$g^2/4$	0	$1/12$	$3/2$
$W^\pm$	2	6	$g^2/4$	0	$1/12$	$5/6$
	3	-2	$g^2/4$	0	$-1/6$	$3/2$
	4	1	$(g^2+g'^2)/4$	0	$1/12$	$3/2$
Z^0	5	3	$(g^2+g'^2)/4$	0	$1/12$	$5/6$
	6	-1	$(g^2+g'^2)/4$	0	$-1/6$	$3/2$
t	7	-12	$y_t^2/2$	0	$1/12$	$3/2$
ϕ	8	1	3λ	m^2	$\xi - 1/6$	$3/2$
χ_i	9	3	λ	m^2	$\xi - 1/6$	$3/2$

- Note: For $R \gg \phi$, there are $\log R/\mu^2$ terms
- RG improved potential becomes
$$V(\phi) \approx \lambda\left(R^{1/2}\right)\phi^4$$
- Negative for all ϕ, no barrier if $\xi = 0$



(Buttazzo et al 2013)

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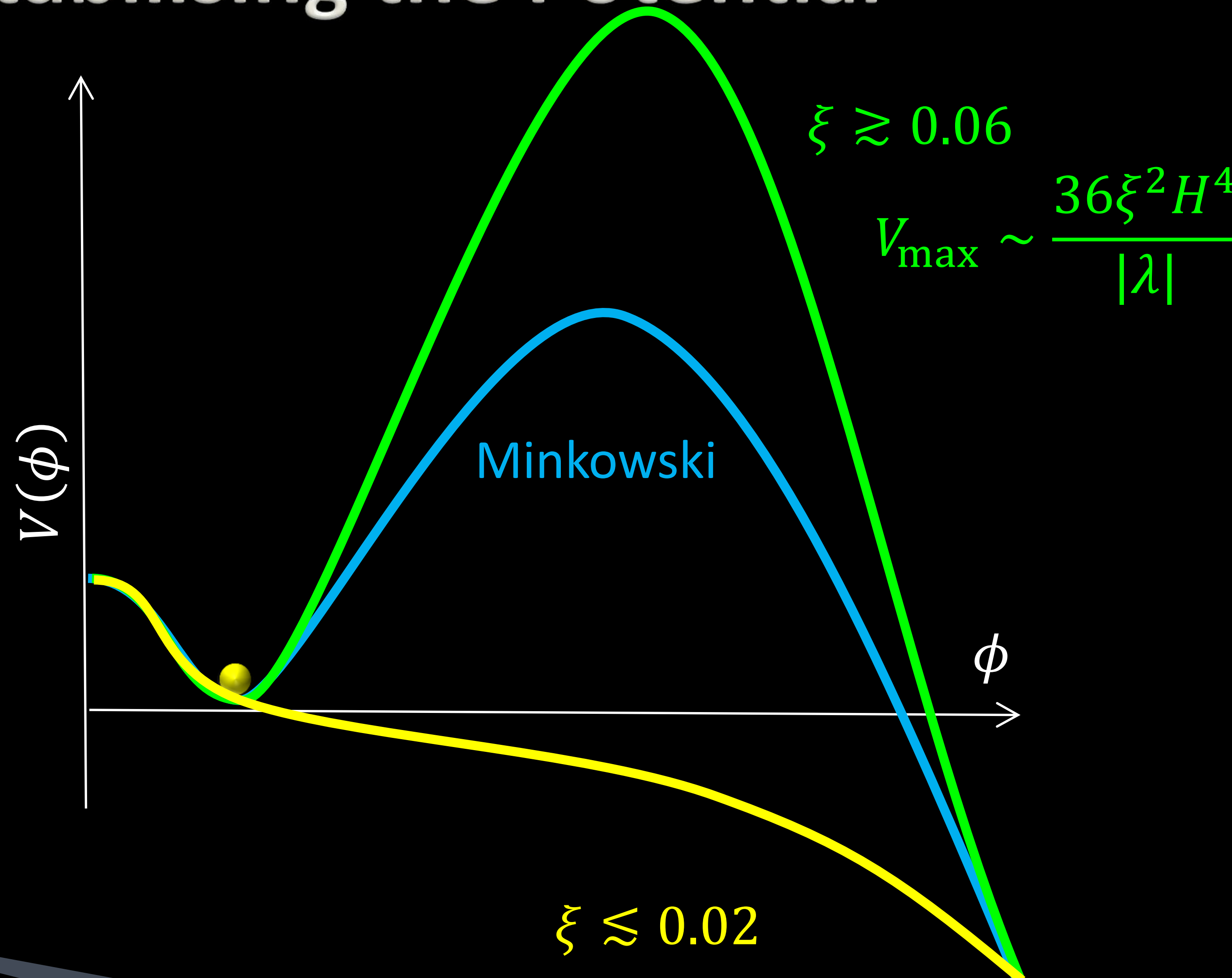
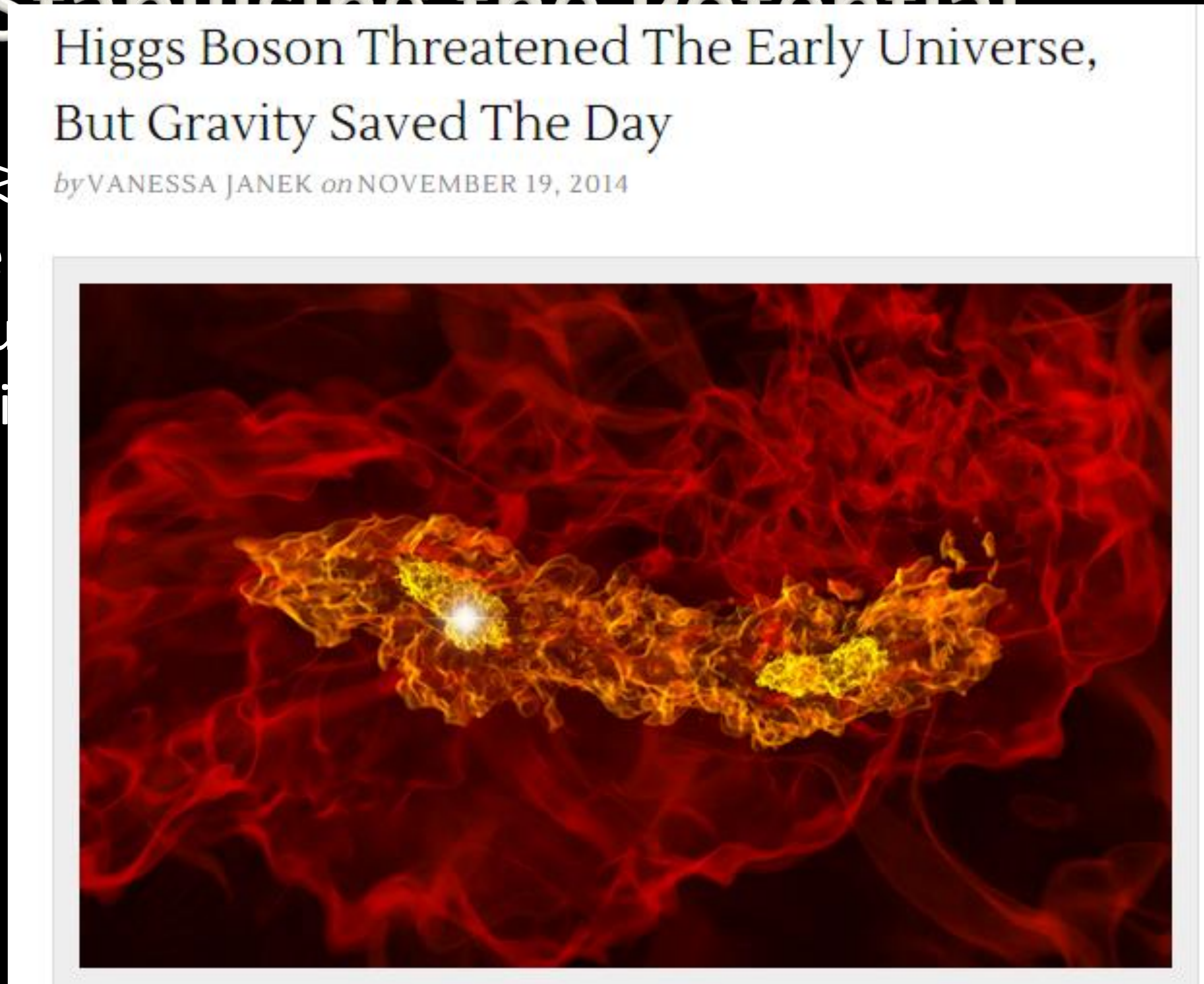
(De-)Stabilising the Potential



(De)Stabilising the Potential

- If $H \gtrsim$ ...
  there...
  vacuu...
  inflati...


Higgs Boson Threatened The Early Universe, But Gravity Saved The Day

by VANESSA JANEK on NOVEMBER 19, 2014

(HMNR 2014)

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End of Inflation

- Reheating:
  Inflation ($R = 12H^2$) → radiation domination ($R = 0$)
- Effective Higgs mass $m_{\rm eff}^2 = m_{\rm H}^2 + \xi R$ changes rapidly:
  - Excites the Higgs field
  - "Geometric preheating" (Bassett&Liberati 1998, Tsujikawa et al. 1999)
- Could this throw it over the barrier,
  triggering the instability?

Inflaton Oscillations

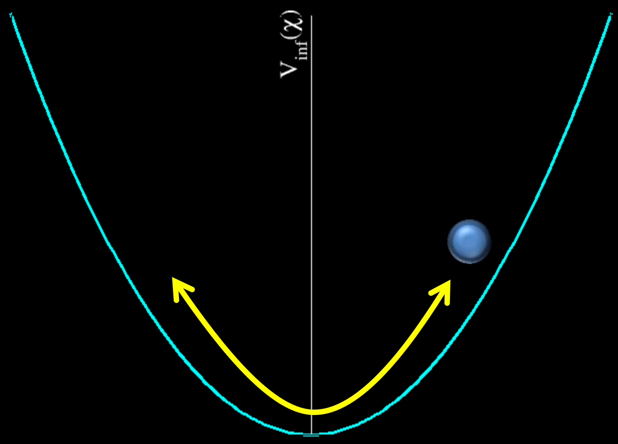
- After inflation, Universe is dominated by an almost uniform inflaton field χ oscillating in its potential $V_{\rm inf}(\chi)$
- Ricci scalar

$$R = \frac{1}{M_{Pl}^2}\left[4V_{\rm inf}(\chi) - \dot{\chi}^2\right]$$

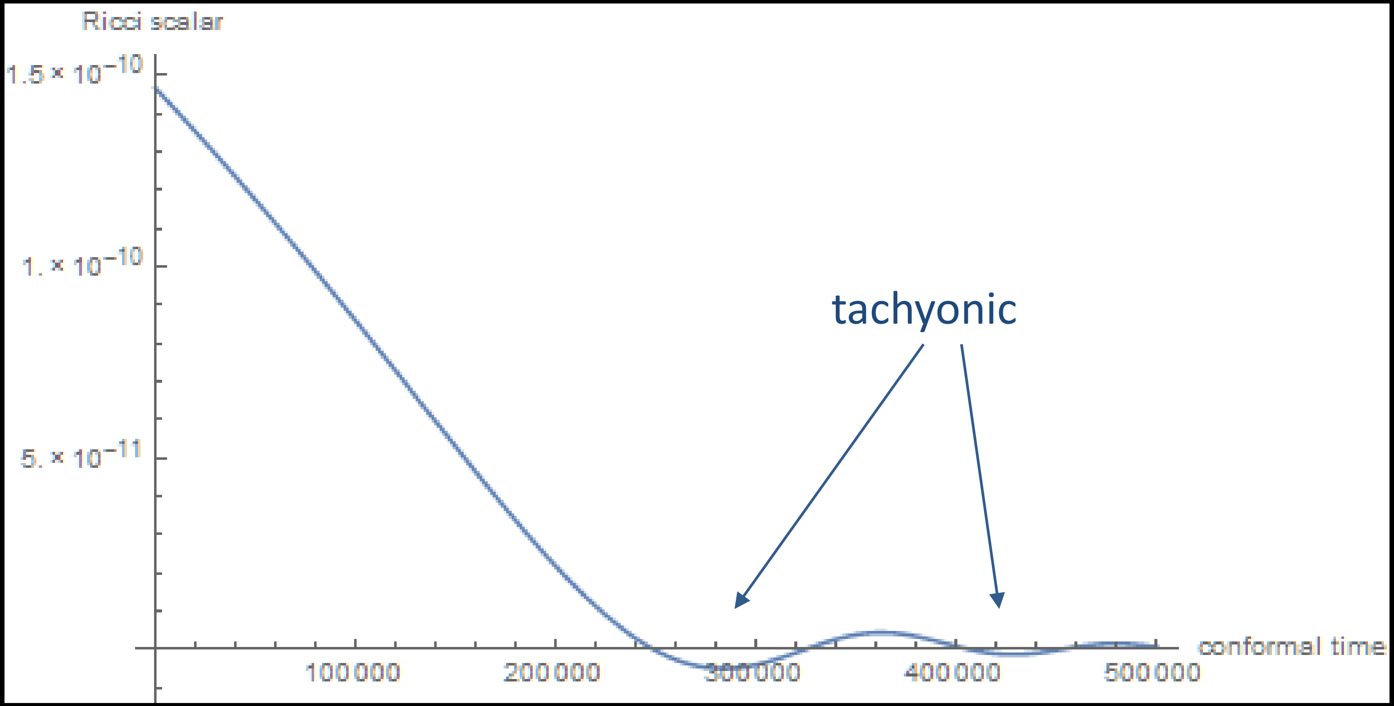
- At the minimum

$$R \sim -\frac{V_{\rm inf}(\chi_{\rm ini})}{M_{Pl}^2} < 0$$

- Tachyonic Higgs mass $m_{\rm eff}^2 < 0$ if $\xi > 0$ (HMNR 2015)



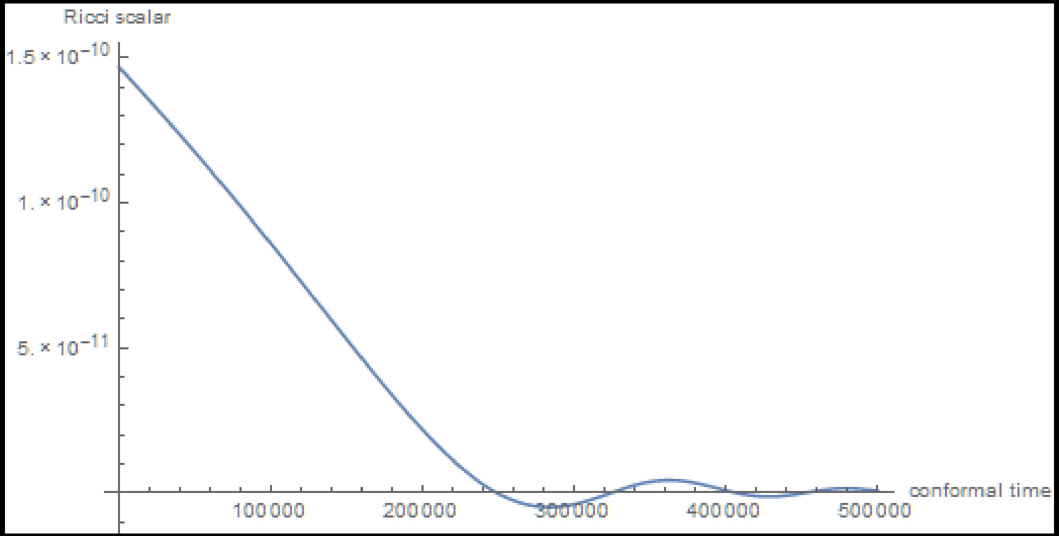
Inflaton Oscillations



- Example: $m^2\chi^2$ chaotic inflation

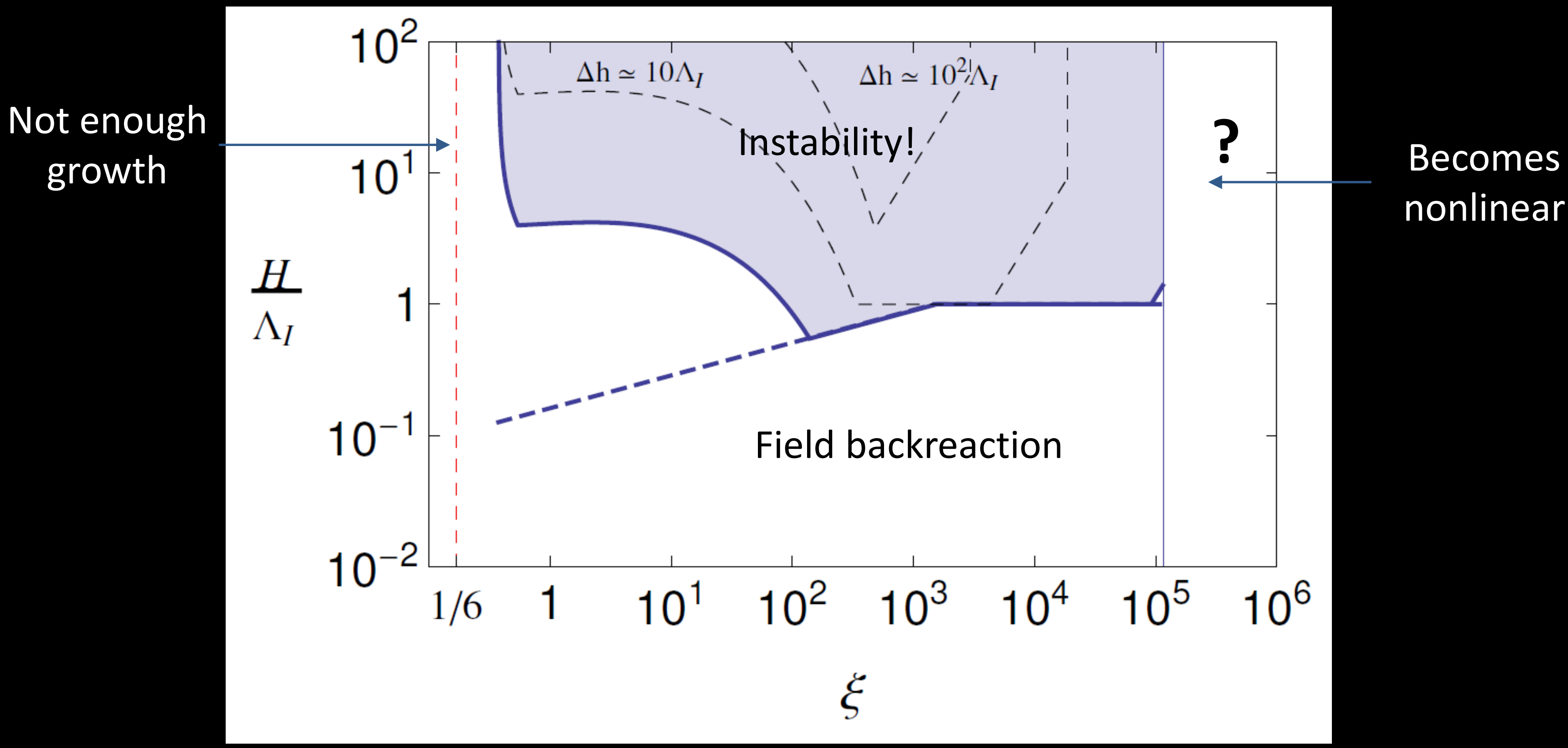
Tachyonic Growth

- Tachyonic growth rate $\mu \sim \sqrt{\xi|R|} \sim \xi^{1/2} m \chi_{\rm ini}/M_{\rm Pl}$
- Duration in time $\Delta t \sim 1/m$
- Growth during first oscillation $e^{\mu \Delta t} \sim e^{\xi^{1/2}\chi_{\rm ini}/M_{\rm Pl}}$
- Higgs variance at horizon scale:



$$\langle \phi^2 \rangle_H \sim \frac{2}{3\sqrt{3}\xi}\left(\frac{H}{2\pi}\right)^2 e^{\frac{2\sqrt{\xi}\chi_{\rm ini}}{M_{\rm Pl}}} \sim \frac{H^2}{\xi} e^{2\sqrt{\xi}}$$ (HMNR 2015)

Vacuum Instability



(HMNR 2015)

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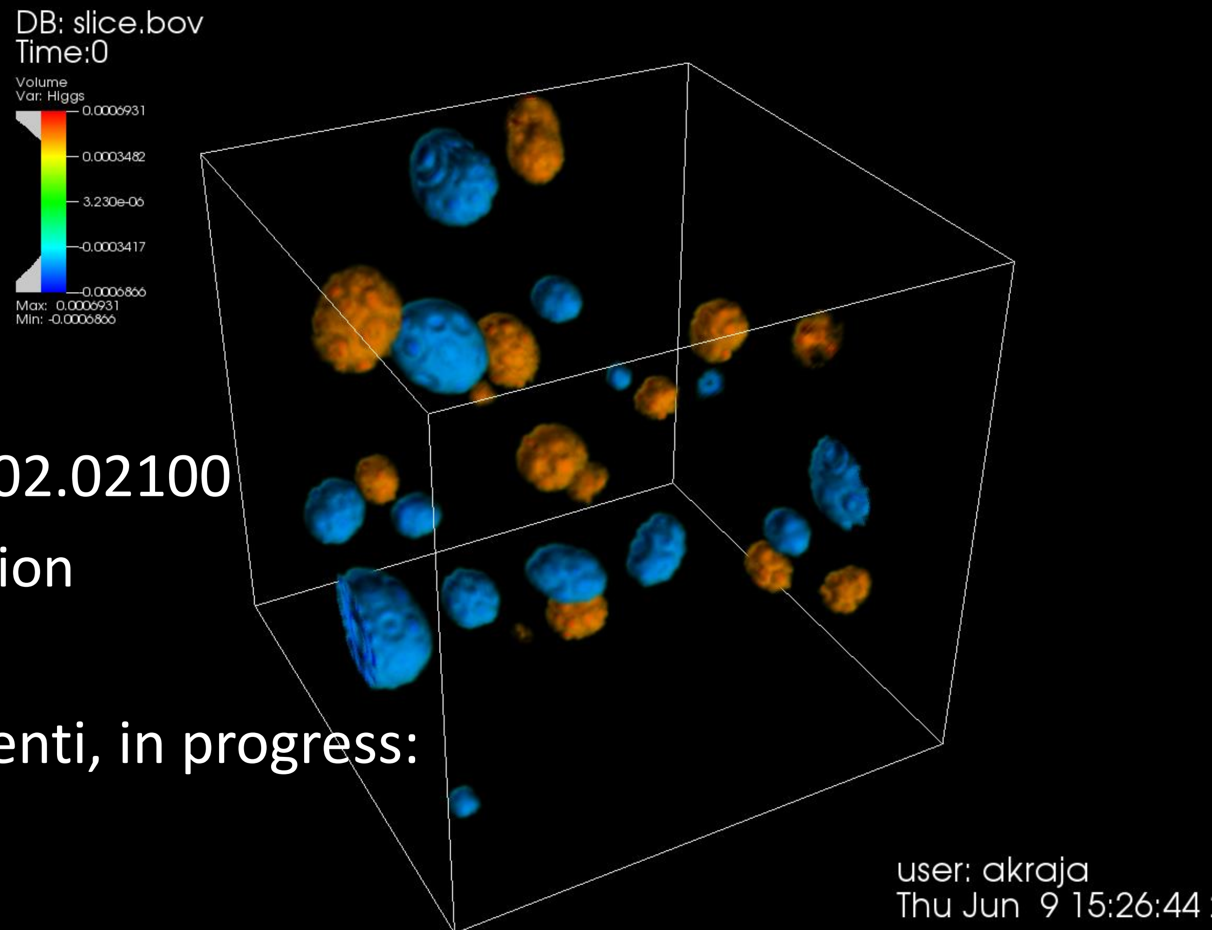
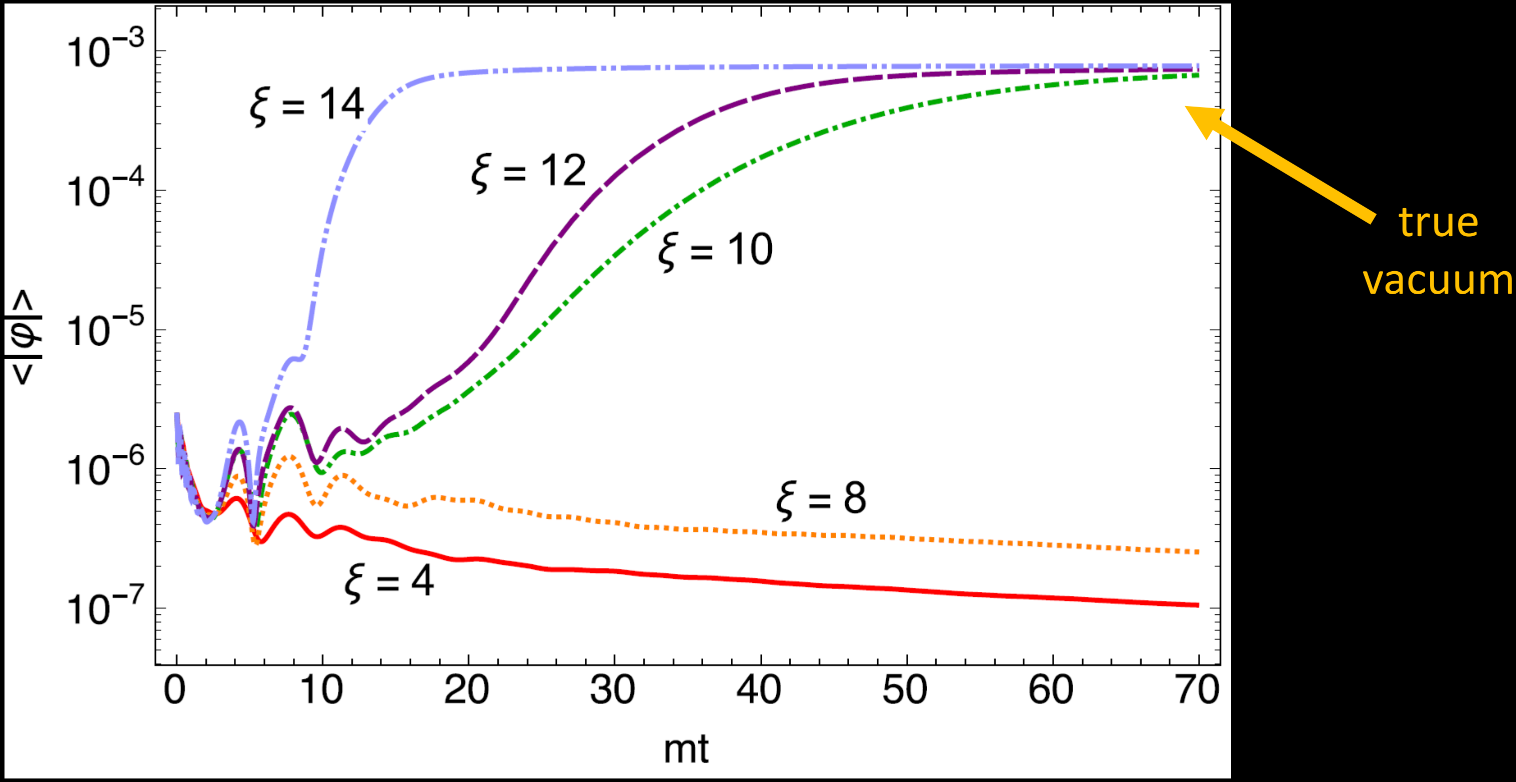
Detailed Calculations

- Ema et al, arXiv:1602:00483
  - Lattice simulation
  - $\xi \lesssim 20$
- Kohri et al, arXiv:1602.02100
  - Linearised calculation
  - $\xi \lesssim 40$
- Figueroa, AR & Torrenti, in progress:
  - Lattice simulations



Lattice Simulations: $m^2\chi^2$



Figueroa, AR & Torrenti, in progress

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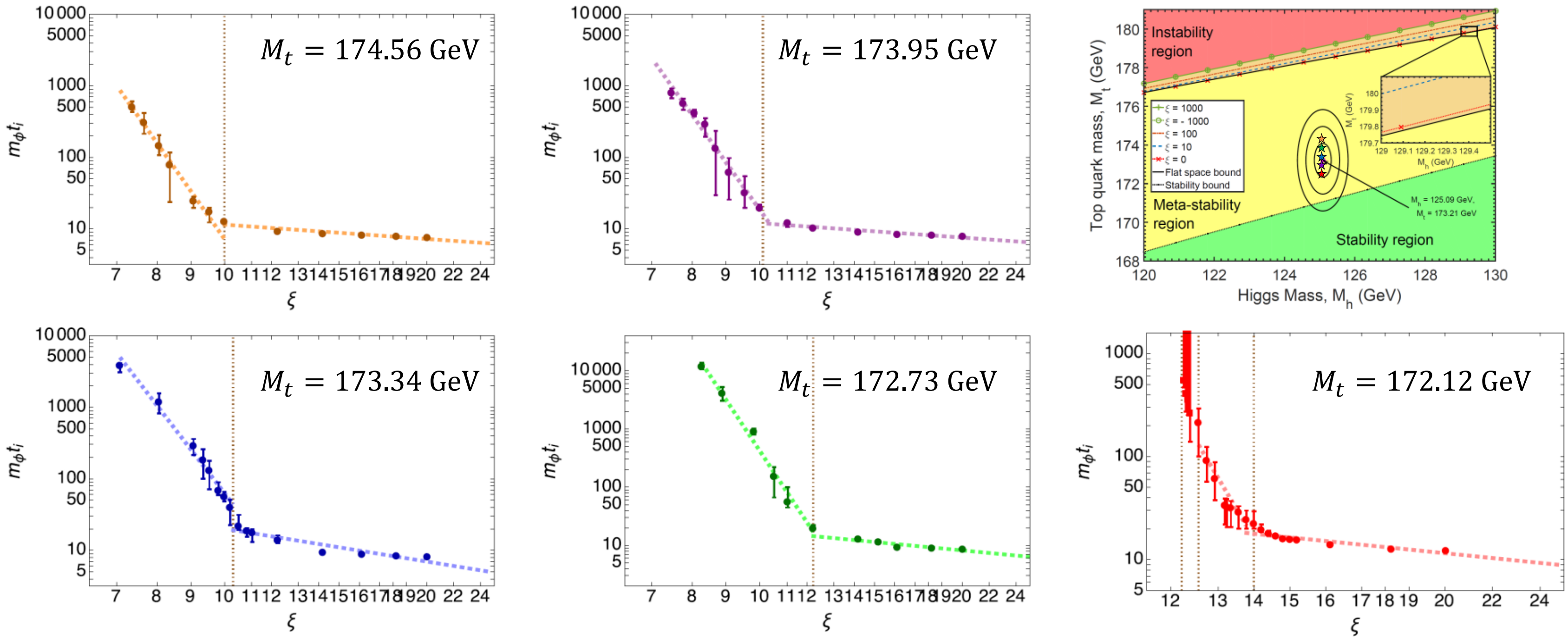
Instability Time



Figueroa, AR & Torrenti, in progress

- Stability depends on speed of reheating, top mass
- $M_t = 173.34$ GeV: vacuum decays before $mt = 100$ if $\xi \gtrsim 9$

Constraints on ξ

- Minimal scenario:
  Standard Model + $m^2\phi^2$ chaotic inflation,
  no direct coupling to inflaton

$$0.06 \lesssim \xi \lesssim 9$$

- 15 orders of magnitude stronger than the LHC bound

$$|\xi| \lesssim 2.6 \times 10^{15}$$

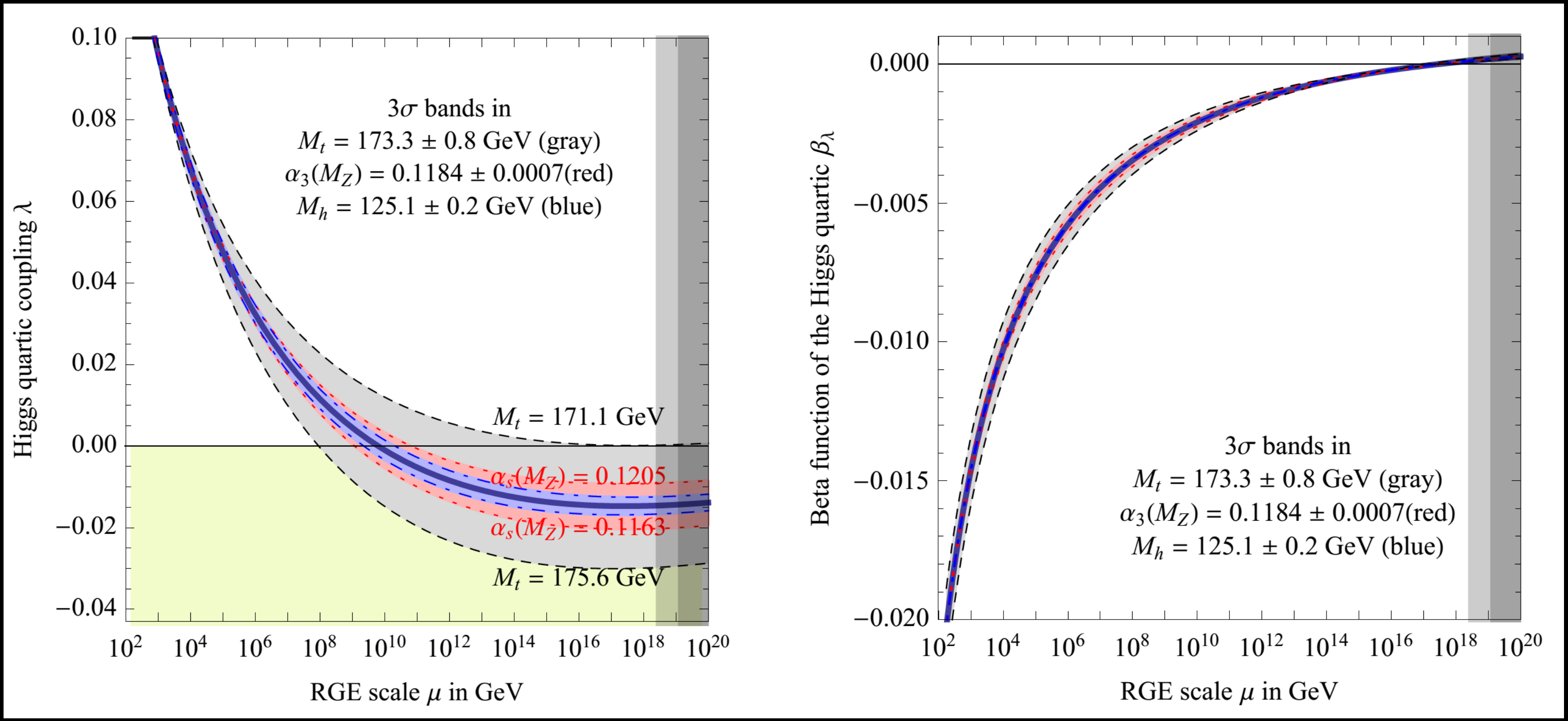
- Caveats:
  - Inflaton coupling – Would still need $|\xi| \lesssim O(1)$
  - New physics – Could stabilise potential altogether
- Future work: Standard Model couplings, de Sitter tunneling

Summary

- Last unknown parameter in the Standard Model: Higgs-gravity coupling ξ
  - Required for renormalisability in curved spacetime

- Assuming no new physics except inflation:
  - Triggers vacuum decay during inflation unless $\xi \gtrsim 0.06$
  - Triggers vacuum decay after inflation unless $\xi \lesssim 9$
  - Gravitational effects slow down vacuum decay today – Fastest rate for near-conformal coupling $\xi \approx 0.16676$

Running Coupling



- λ becomes negative at $\mu \gtrsim 10^{10}$ GeV

Buttazzo et al., JHEP 2013