Hydrodynamics without (local) Equilibrium

Jorge Casalderrey-Solana

Non-Conformal Holographic Hydrodynamization

Jorge Casalderrey-Solana

MARIE CURIE ACTIONS

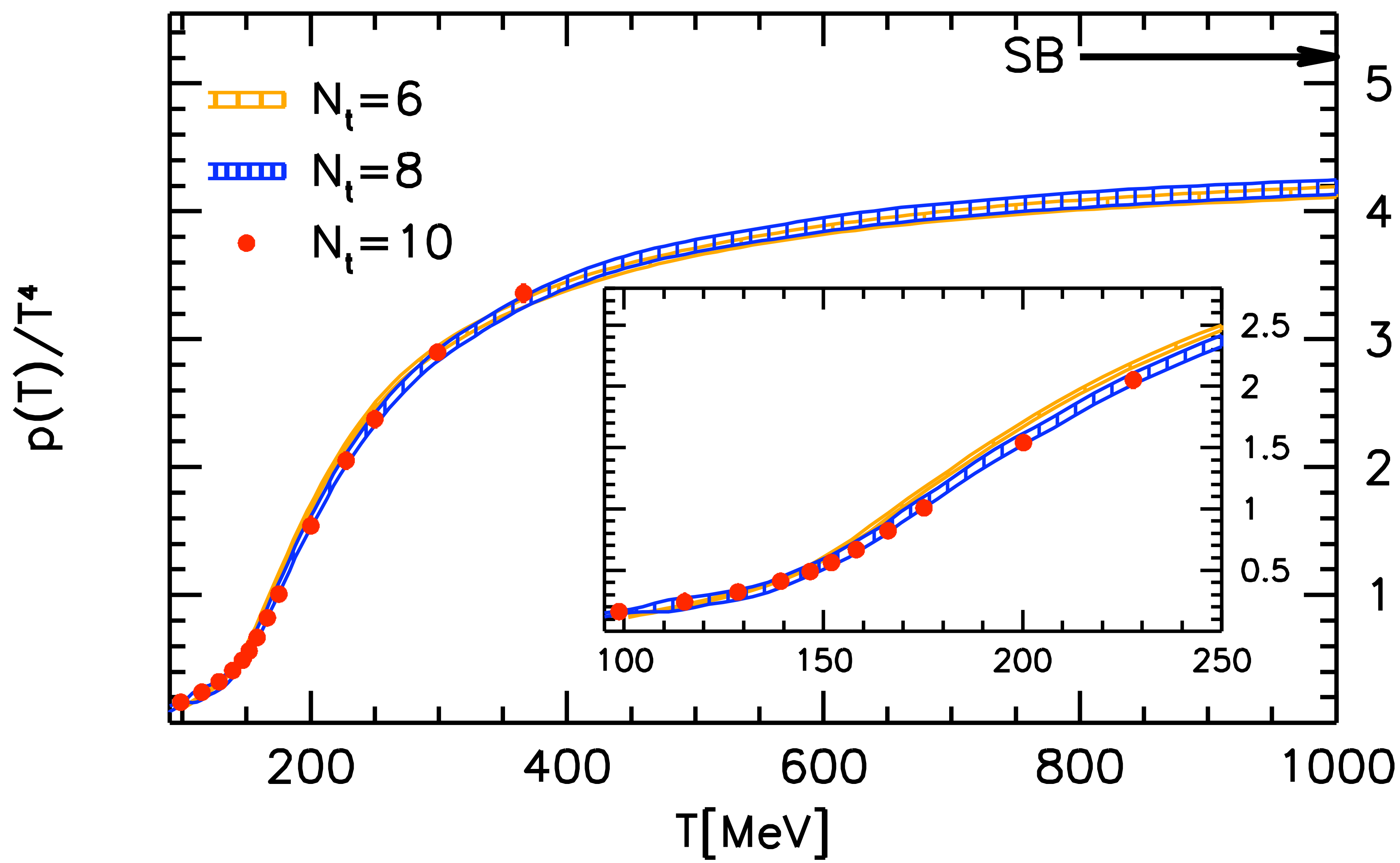
PEOPLE

Oxford Physics

THE ROYAL SOCIETY

Equation of State

Wuppertal-Budapest Col. arXiv: 1007.2580

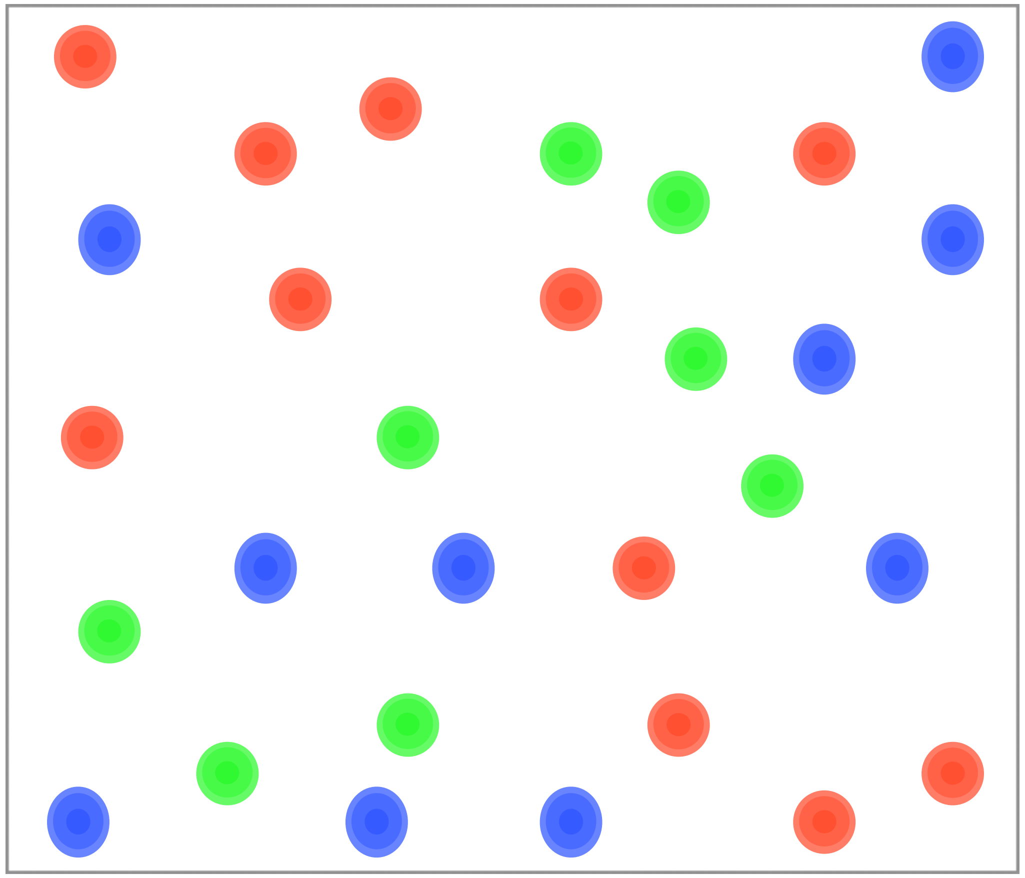


Rapid cross over transition:

- Deconfined matter: Quark Gluon Plasma

A Gas of Quarks and Gluons

At T>>Λ:

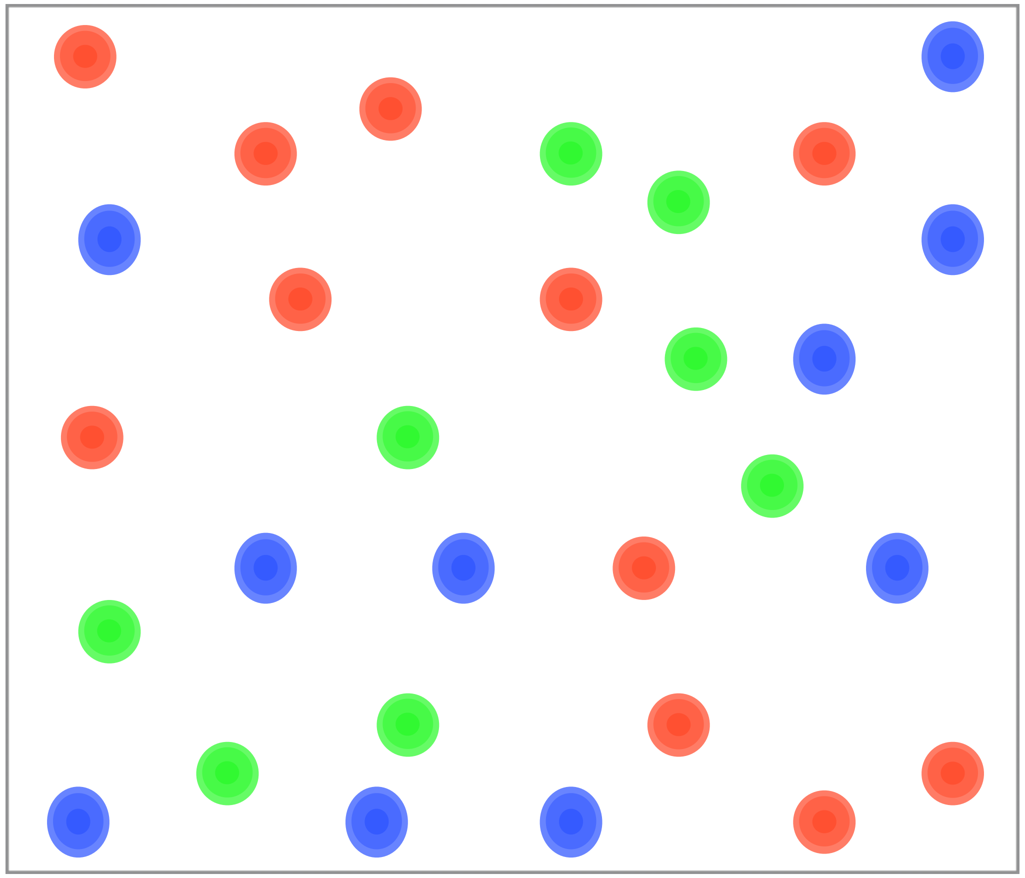


$$\frac{1}{T} \ll \frac{1}{\sqrt{\alpha_s}\,T} \ll \frac{1}{\alpha_s T}$$

inter-particle spacing | Interaction range | mean free path

A Gas of Quarks and Gluons

At $T \gg \Lambda$:



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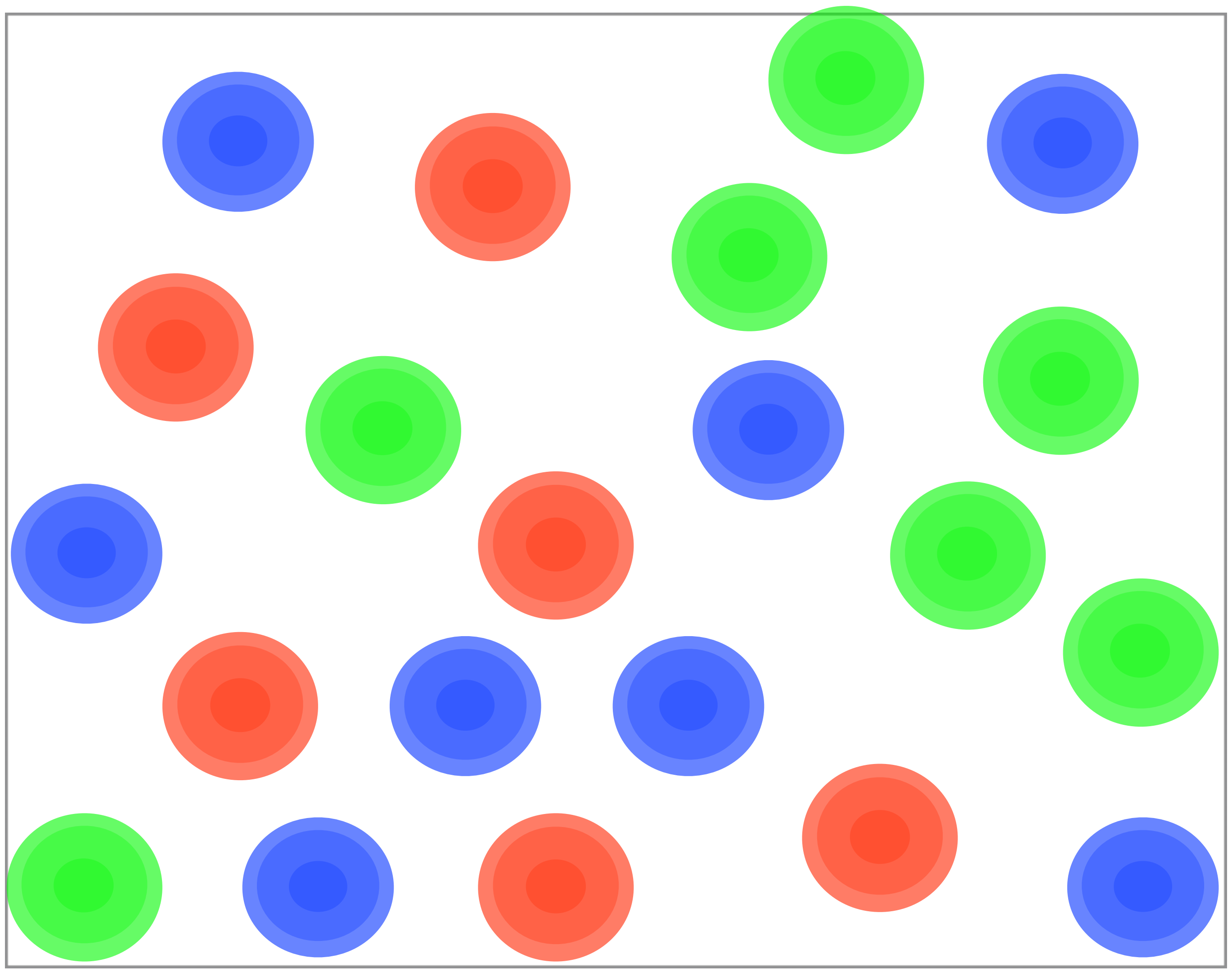
inter-particle spacing | Interaction range | mean free path

Resummations can extend the validity of perturbative methods to much lower temperatures!

What is the correct picture of the plasma?

At T~0.2 GeV



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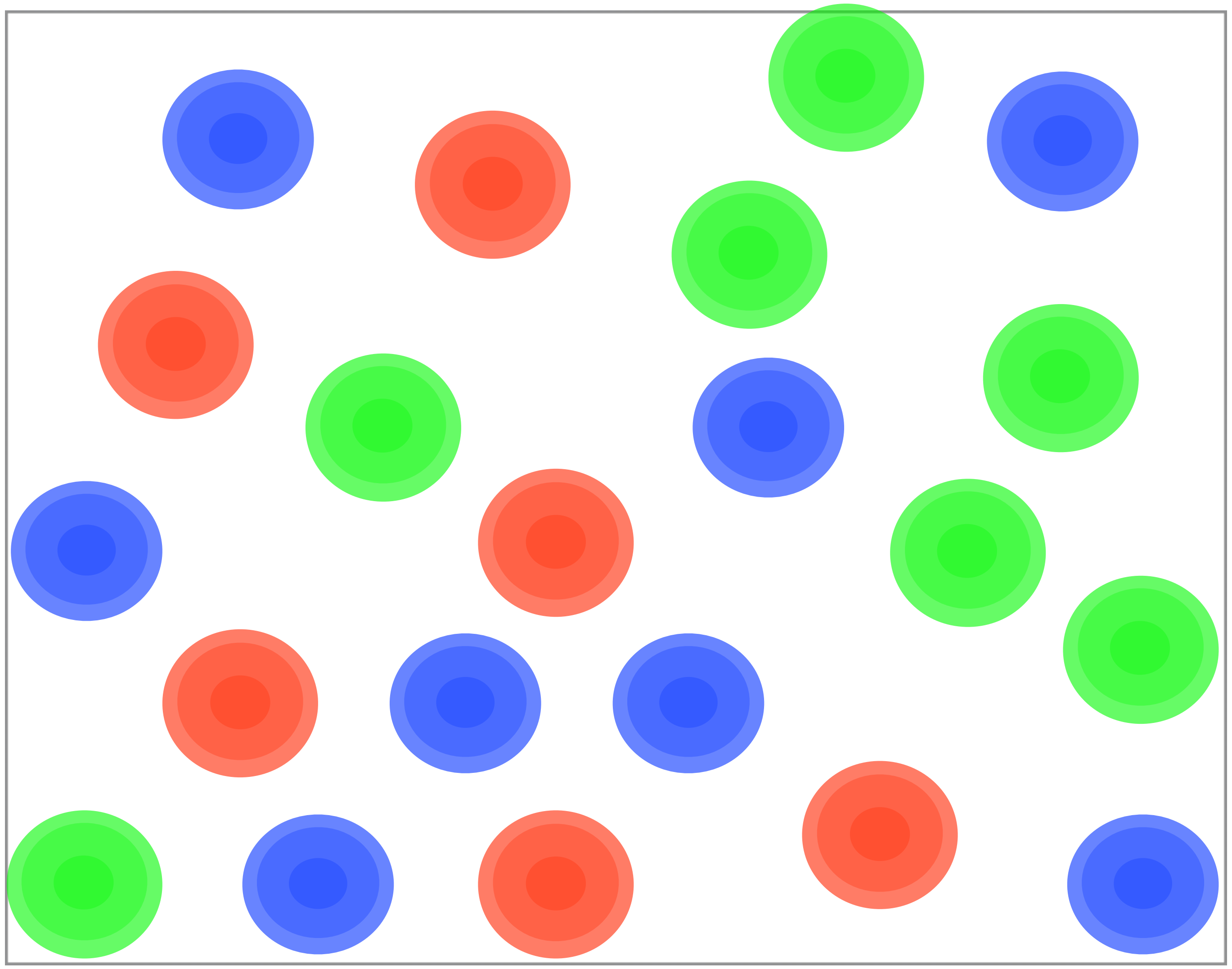

α_s=0.5

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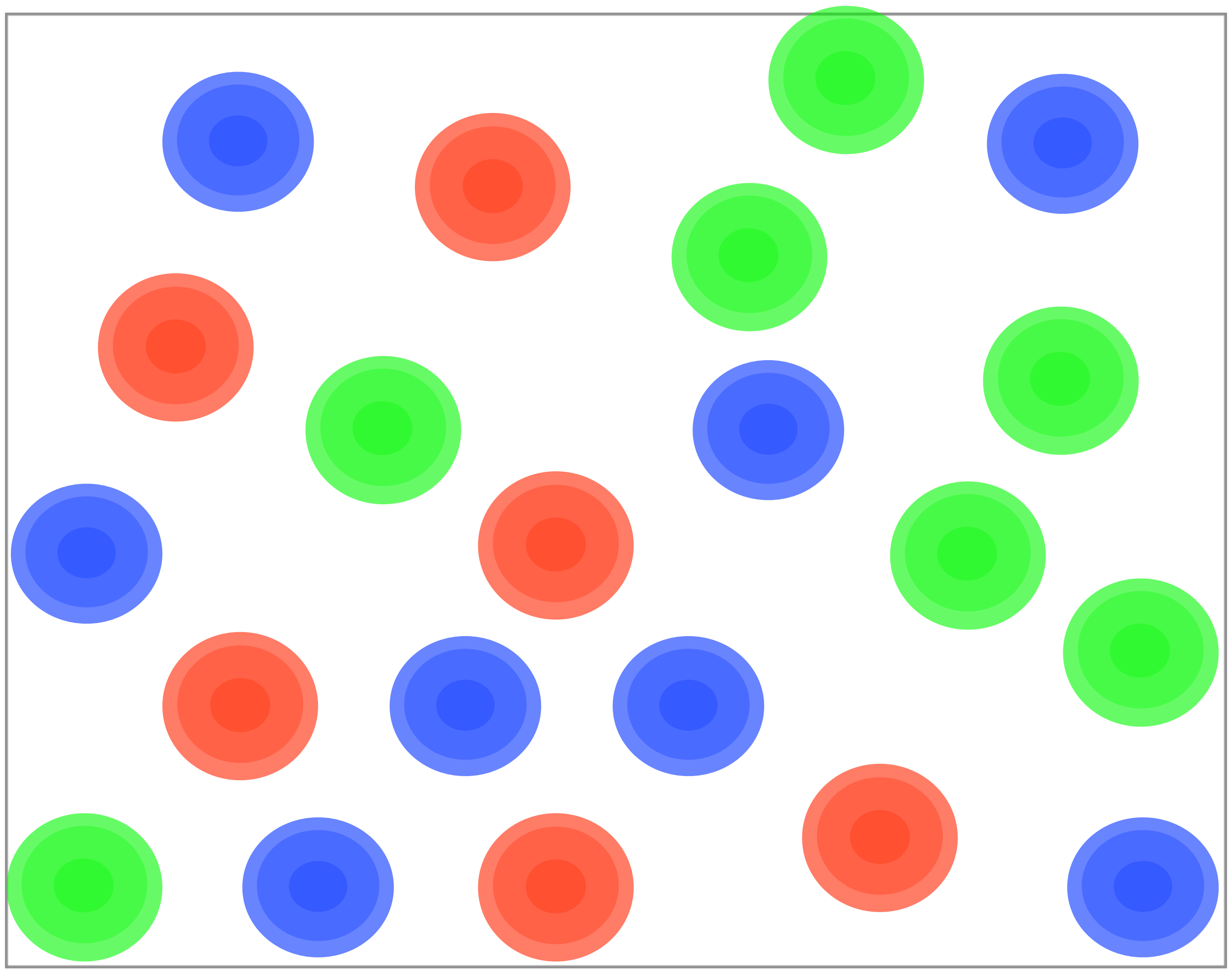

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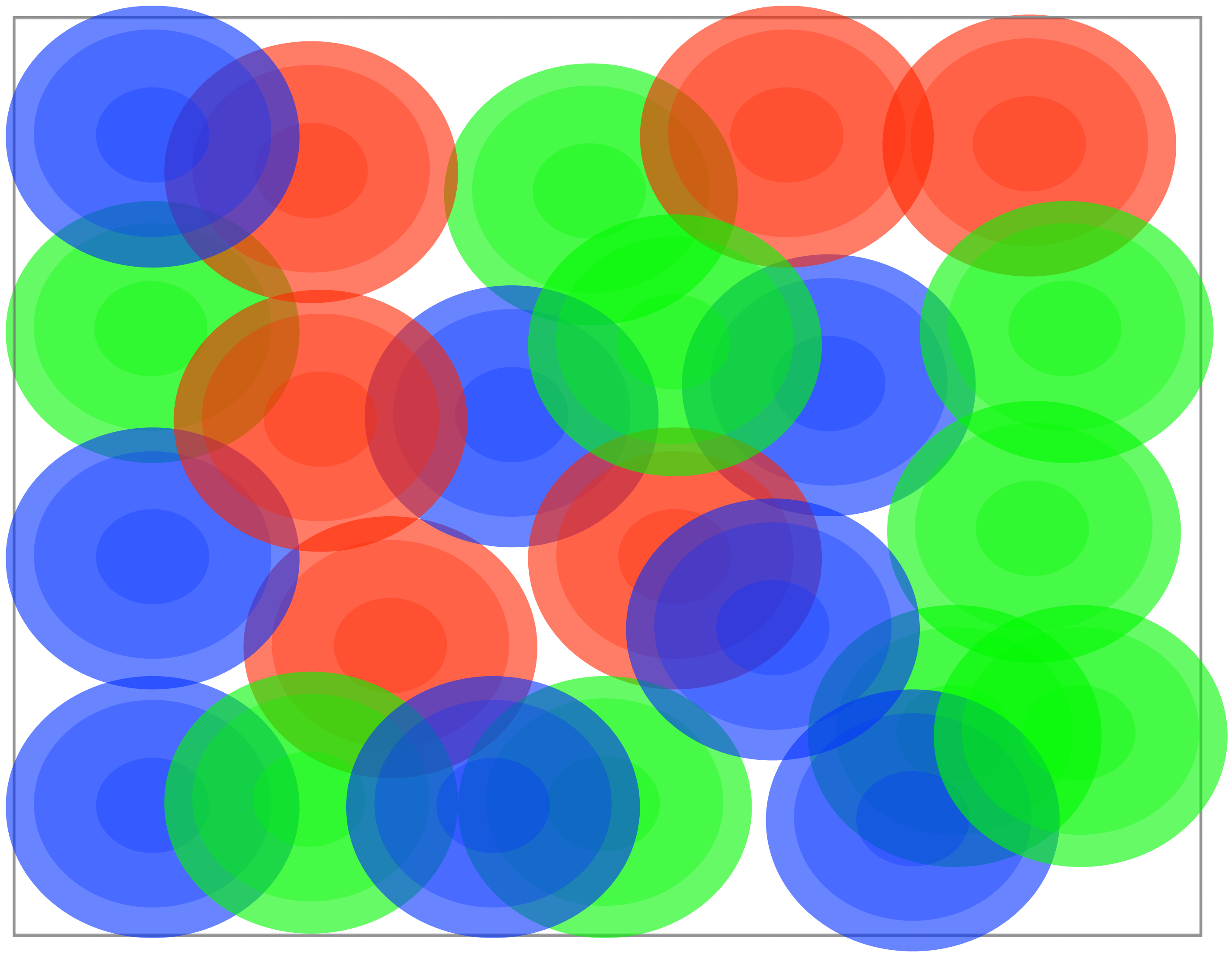


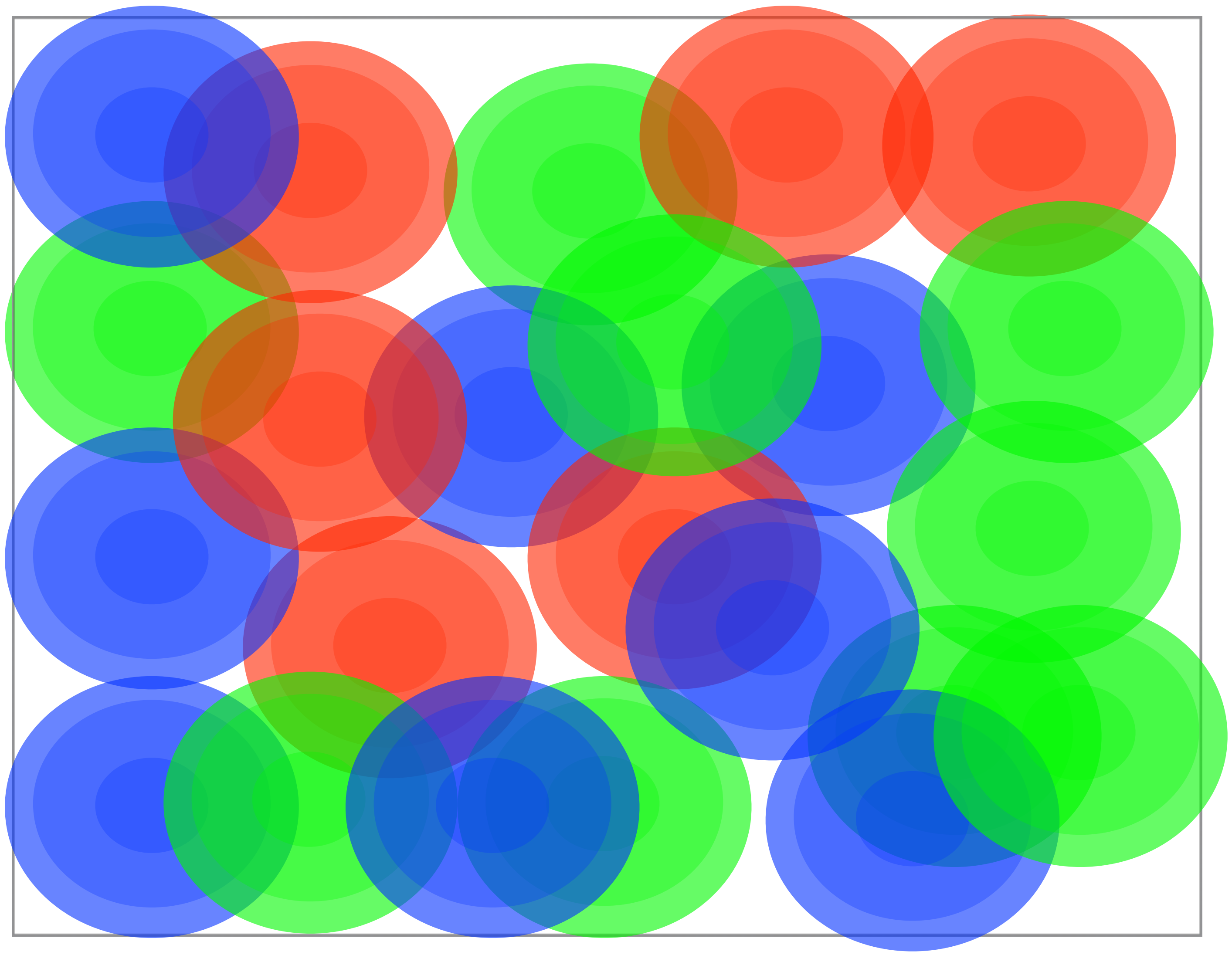
α_s=0.5

T~ $\sqrt{\alpha_s}$ T~ α_s T

Is it a system without long lived excitations?

What is the correct picture of the plasma?

At T~0.2 GeV



α_s=0.5

T~ $\sqrt{\alpha_s}$ T~ α_s T

Is it a system without quasiparticles?

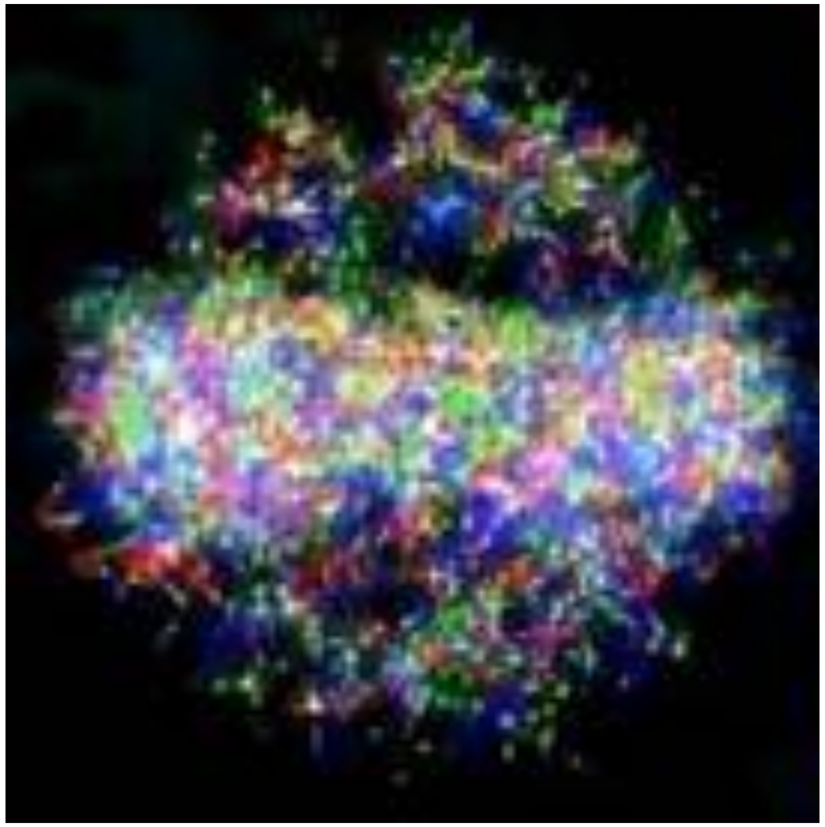
The Little Bang

Very strong collective effects

- Emission of 17000 particles correlated with the impact parameter

- Correlation measured in terms of Fourier coefficients

- Hydrodynamic explosion



The quark gluon plasma is a very good fluid

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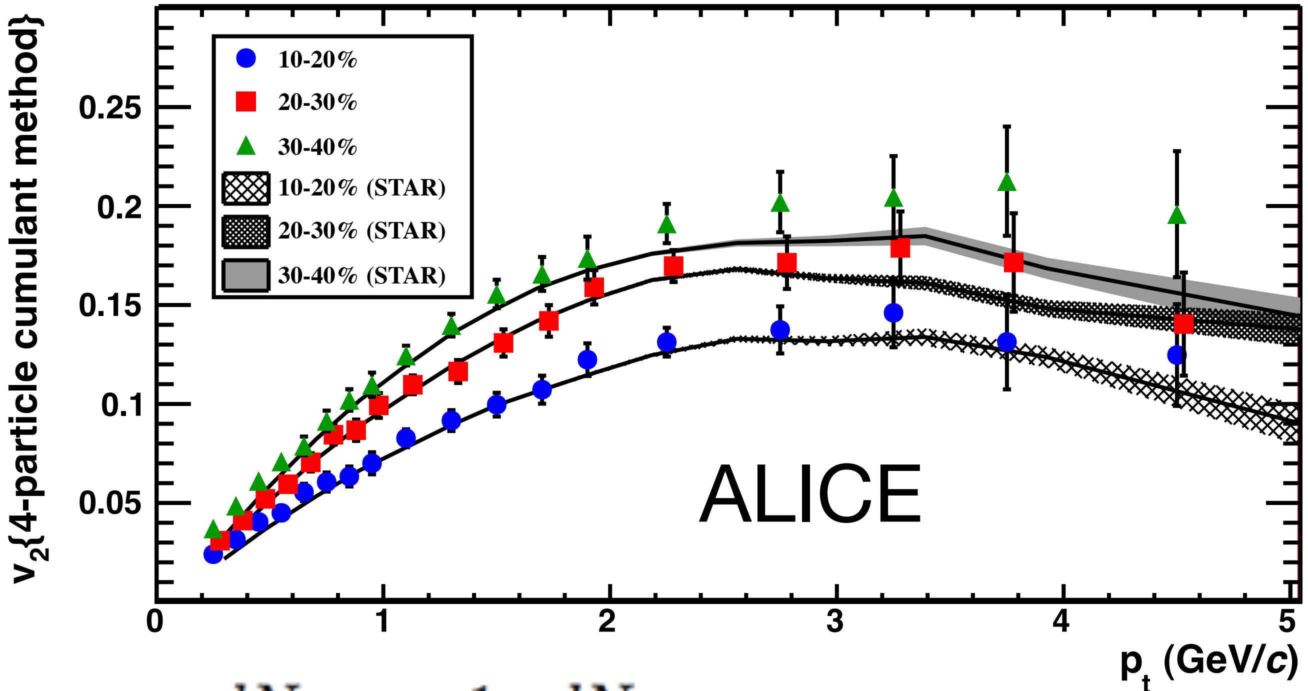
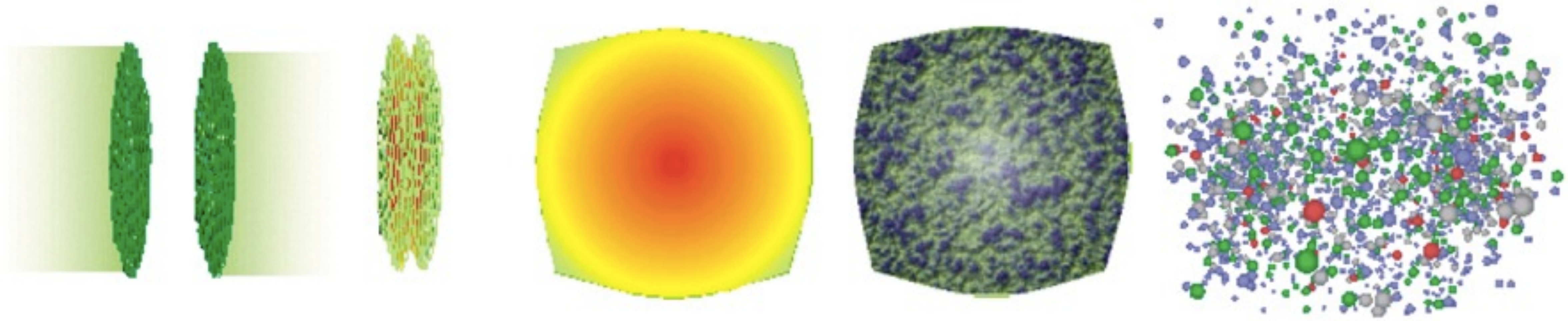


$$\frac{dN}{d^2p_T d\phi} = \frac{1}{2\pi}\frac{dN}{d^2p_T}\left(1 + 2v_2(p_t)\cos 2\phi + \ldots\right)$$

The quark gluon plasma is a very good fluid

From Initial to Final State in HIC



Initial State | Collision | Out-of-equilibrium | Collective expansion | Final stage

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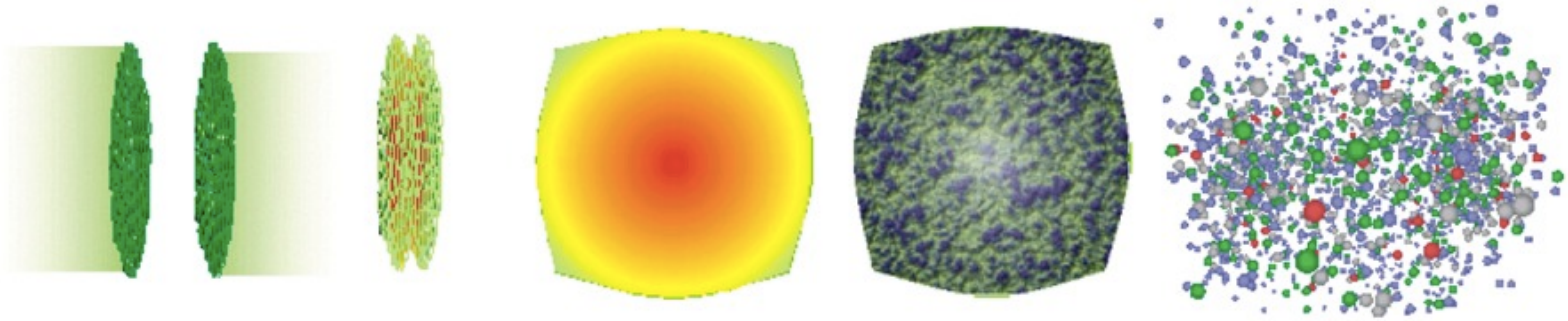
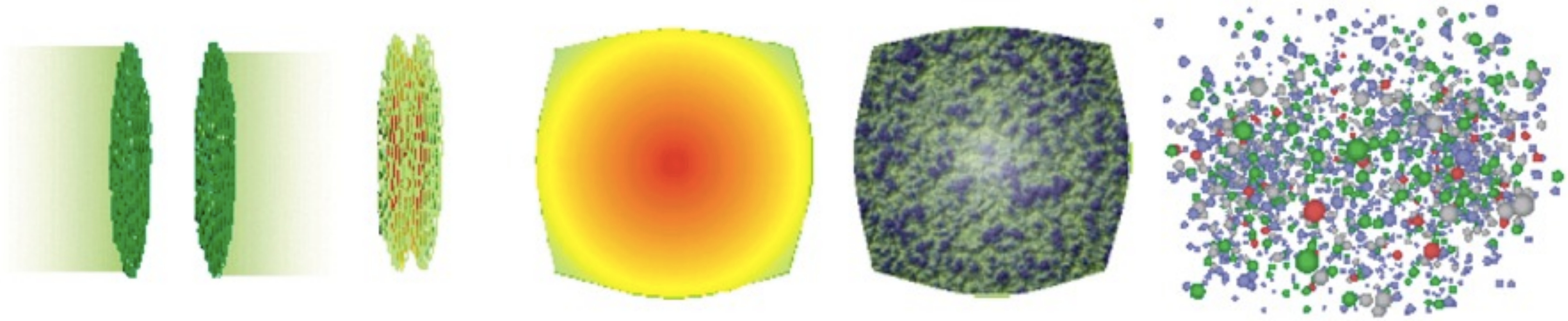


Initial State | Collision | Out-of-equilibrium | Collective expansion | Final stage

Properties of nuclei at high energy

From Initial to Final State in HIC



Initial State

Collision

Out-of-equilibrium

Collective expansion

Final stage

Hydrodynamics
Strong collectivity

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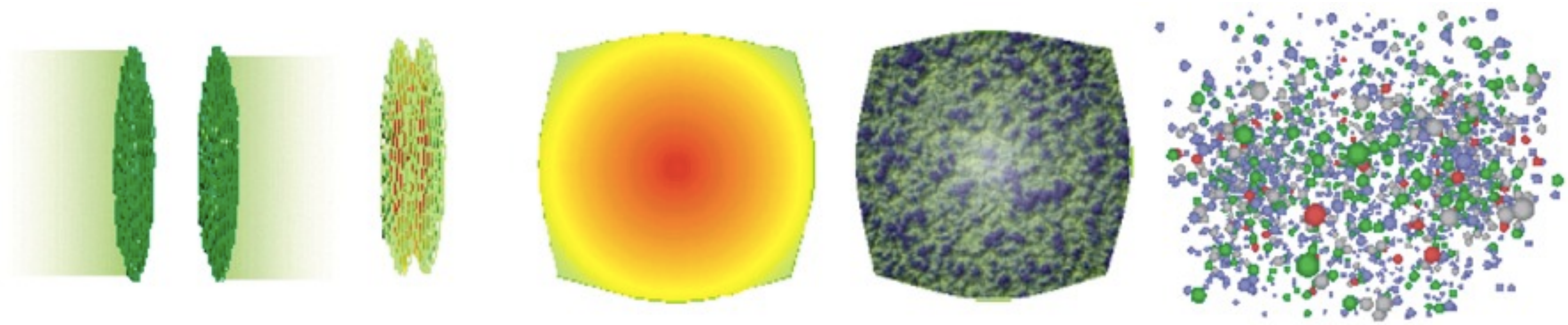

Initial State — Collision — Out-of-equilibrium — Collective expansion — Final stage

Liberation of partons
Perturbation theory

Hydrodynamics
Strong collectivity

Properties of nuclei
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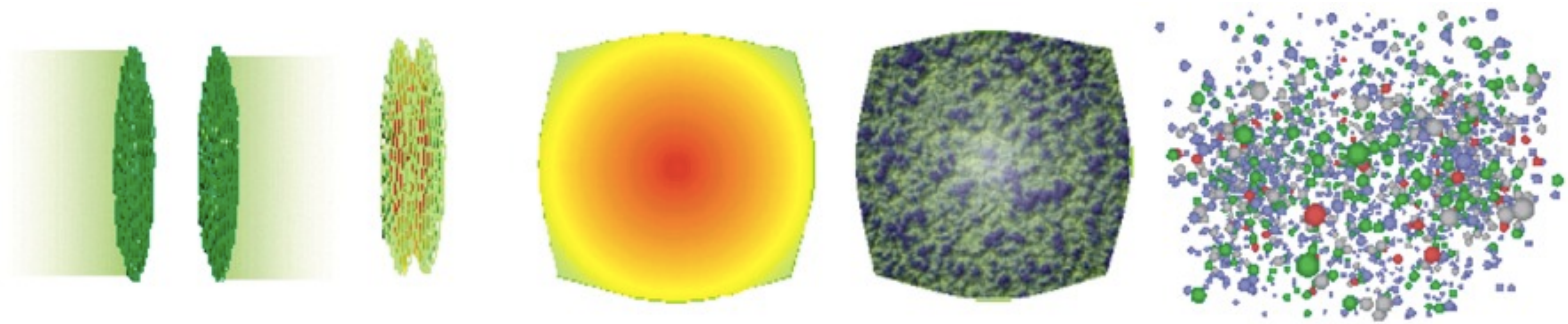
From Initial to Final State in HIC



Initial State → Properties of nuclei at high energy

Collision → Liberation of partons; Perturbation theory

Out-of-equilibrium → Equilibration, Isotropization, Hydrodynamization

Collective expansion → Hydrodynamics; Strong collectivity

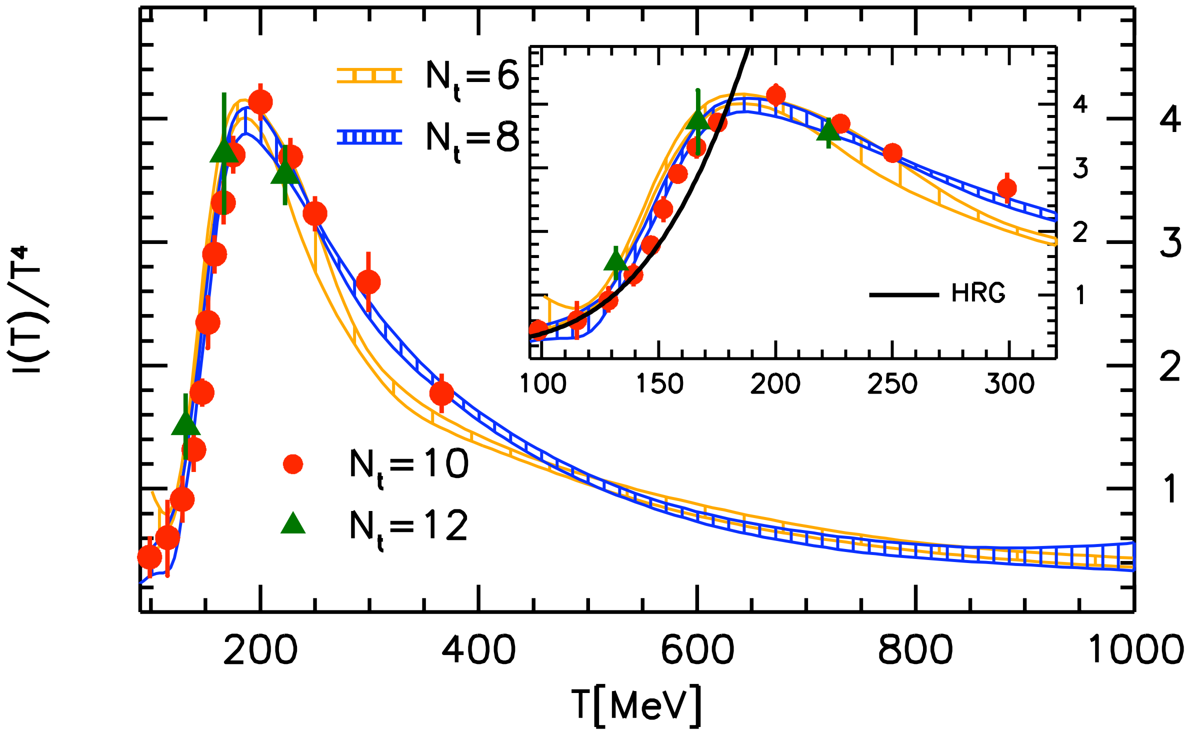
Final stage

Non-Conformal QCD

- Most out-off-equilibrium analyses focus on conformal theories

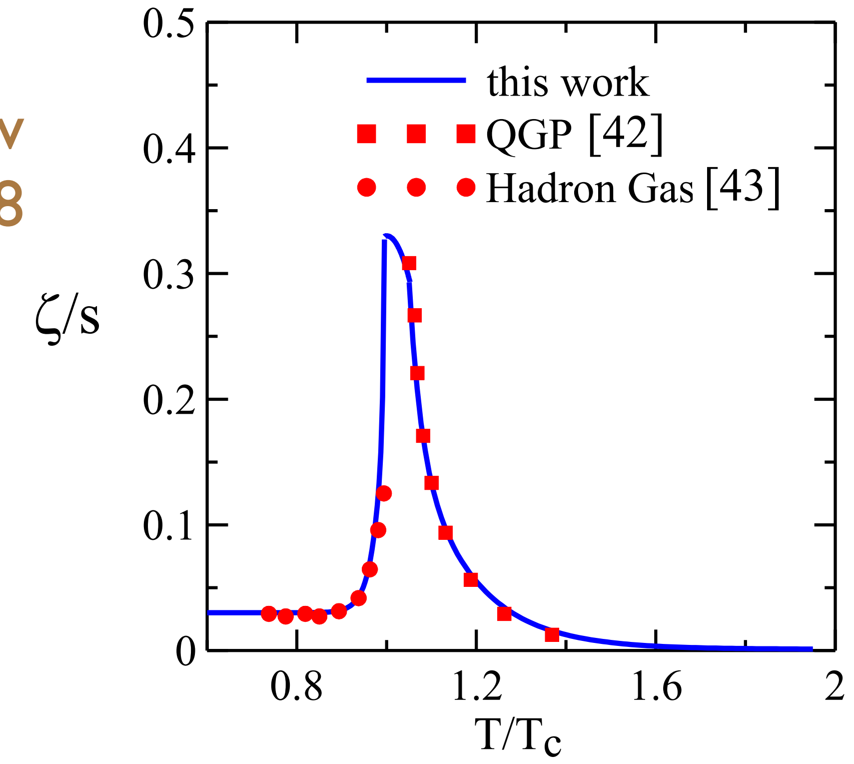
- However, QCD is non-conformal, specially close to the transition

Borsanyi et al. 2010



Karsch, Kharzeev and Tuchin 2008

⇒



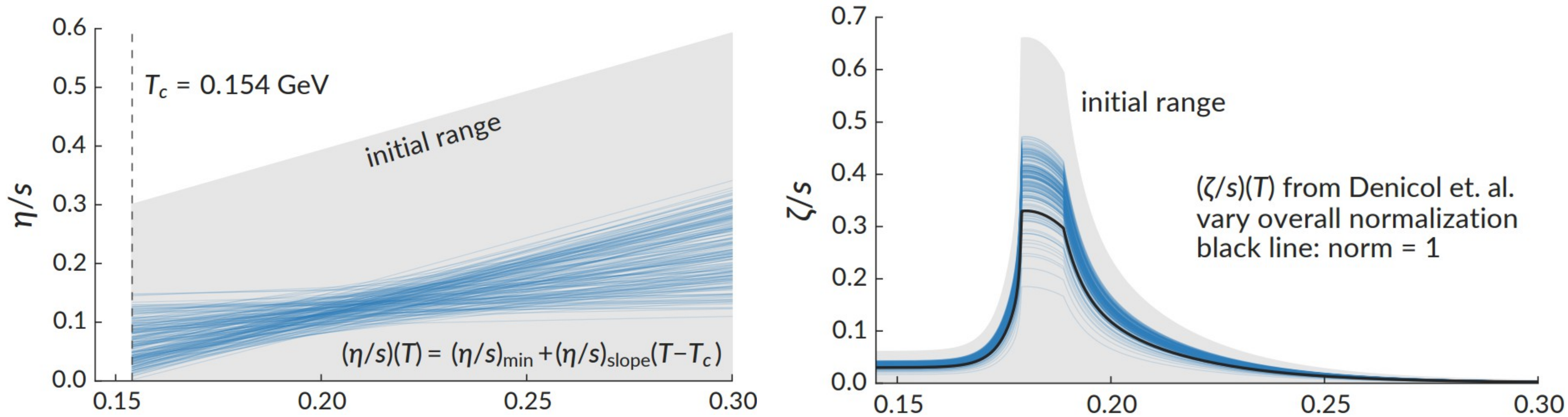
- Bulk viscosity effects become important to accurately describe heavy ion data

Ryu et al. 2015

Extracting Transport Coefficients

Global fit to several sets of data

J. Bernhard, J.S. Moreland, S. Bass, J. Liu, U. Heinz arXiv:1605.03954



$$\left(\frac{\eta}{s}\right)_{\mathrm{T_c}} = 0.08 \pm 0.05$$

Implication of η/s Value

Implication of η/s Value

- It is the smallest value ever measured in any substance.

The Quark Gluon Plasma is the most perfect fluid!

$$\left.\frac{\eta}{s}\right|_{water} \approx 380 \left.\frac{\eta}{s}\right|_{QGP}$$

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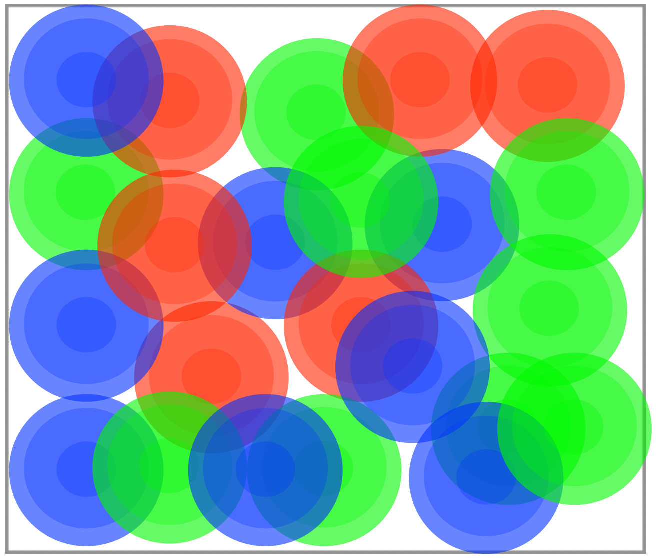

Boltzmann equation $\Rightarrow$ $\tau_{qp} \sim 5\,\frac{\eta}{s}\,\frac{1}{T} \sim \frac{1}{T}$

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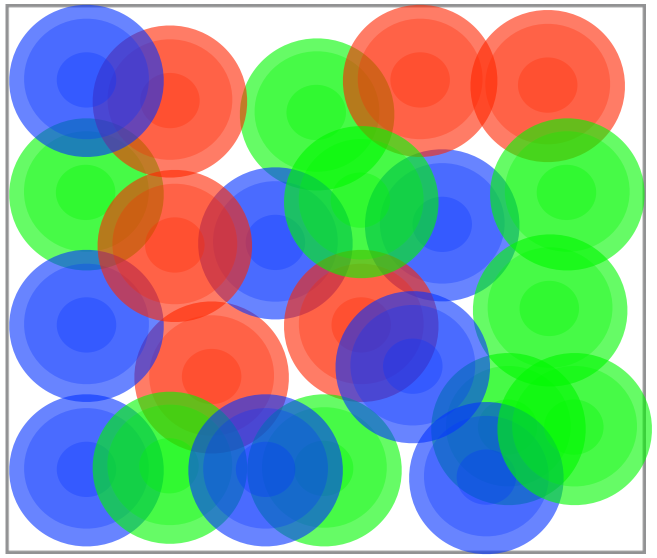

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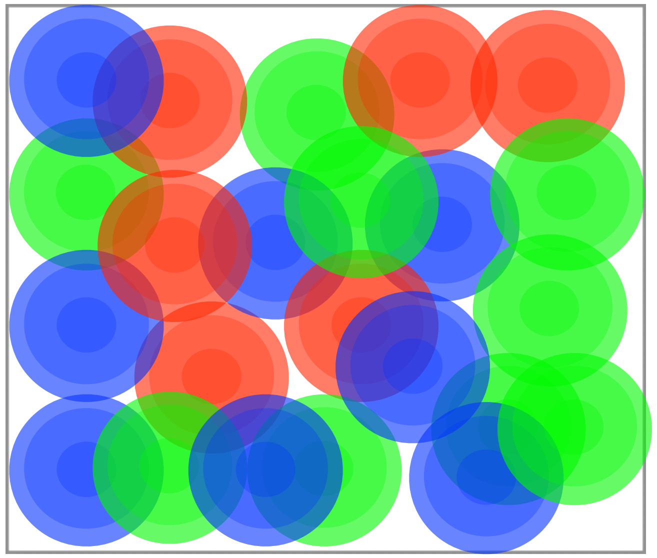

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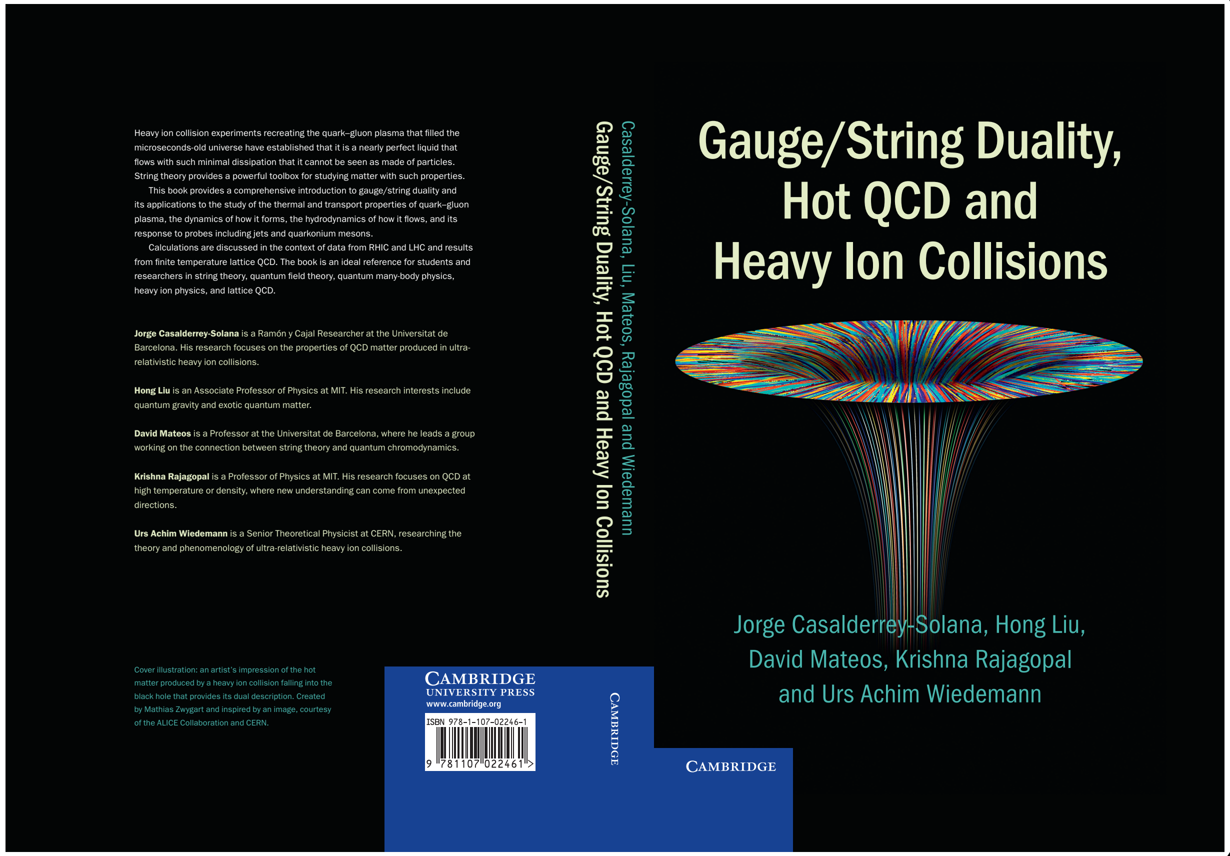
$$\frac{\eta}{s} = \frac{1}{4\pi} = 0.08$$

... but for a large class of non-abelian gauge theories at infinite coupling via holography

Holography

- Gauge Theories in the limit

$$\lambda = 4\pi\alpha_s N_c \to \infty$$



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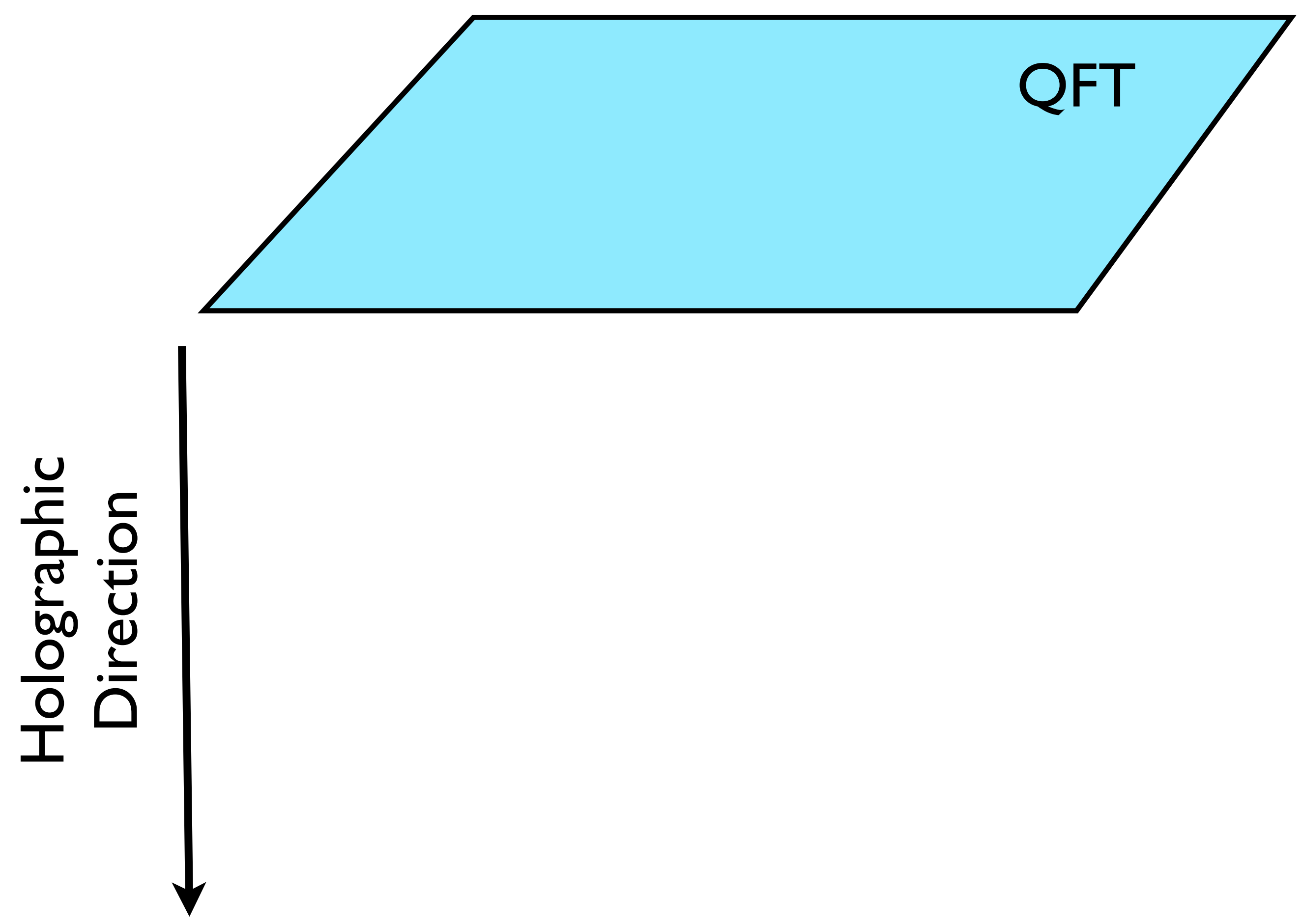

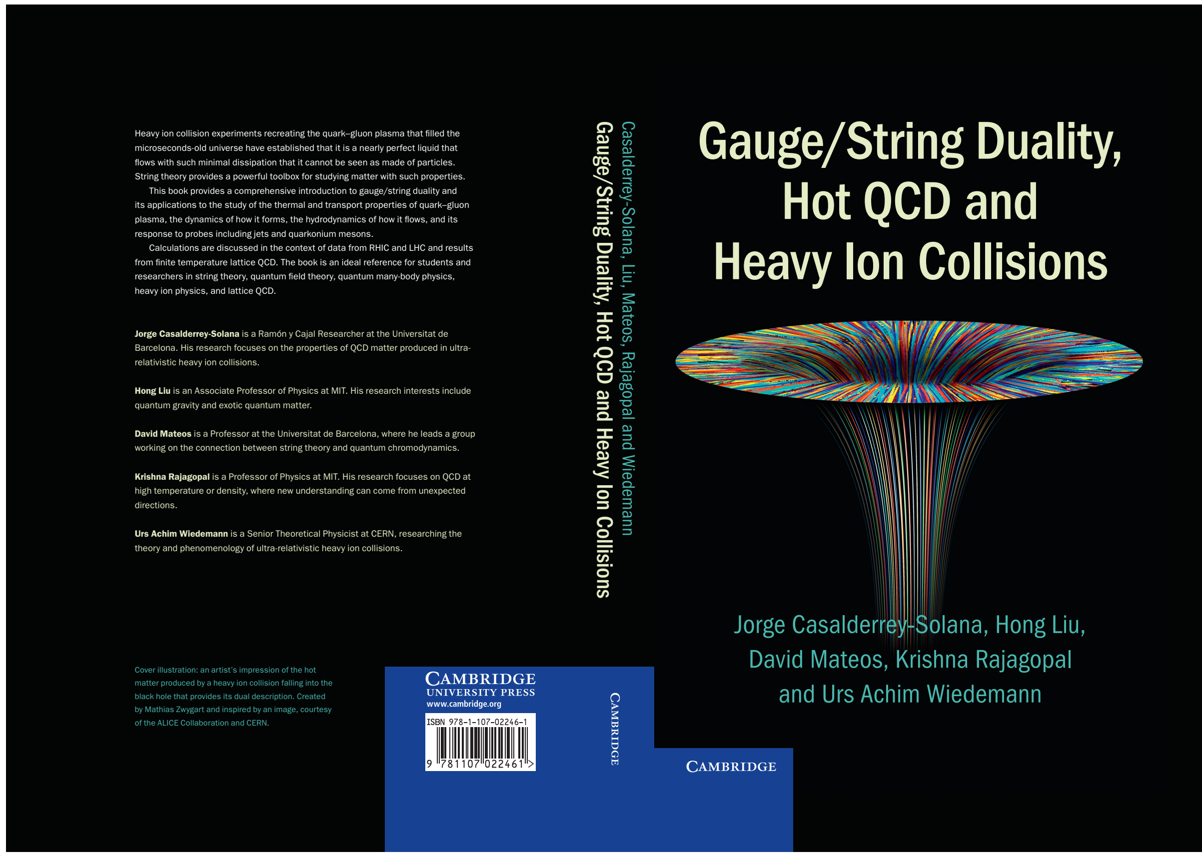

J. M. Maldacena, *Adv. Theor. Math. Phys* 2, 231 *(1998)*

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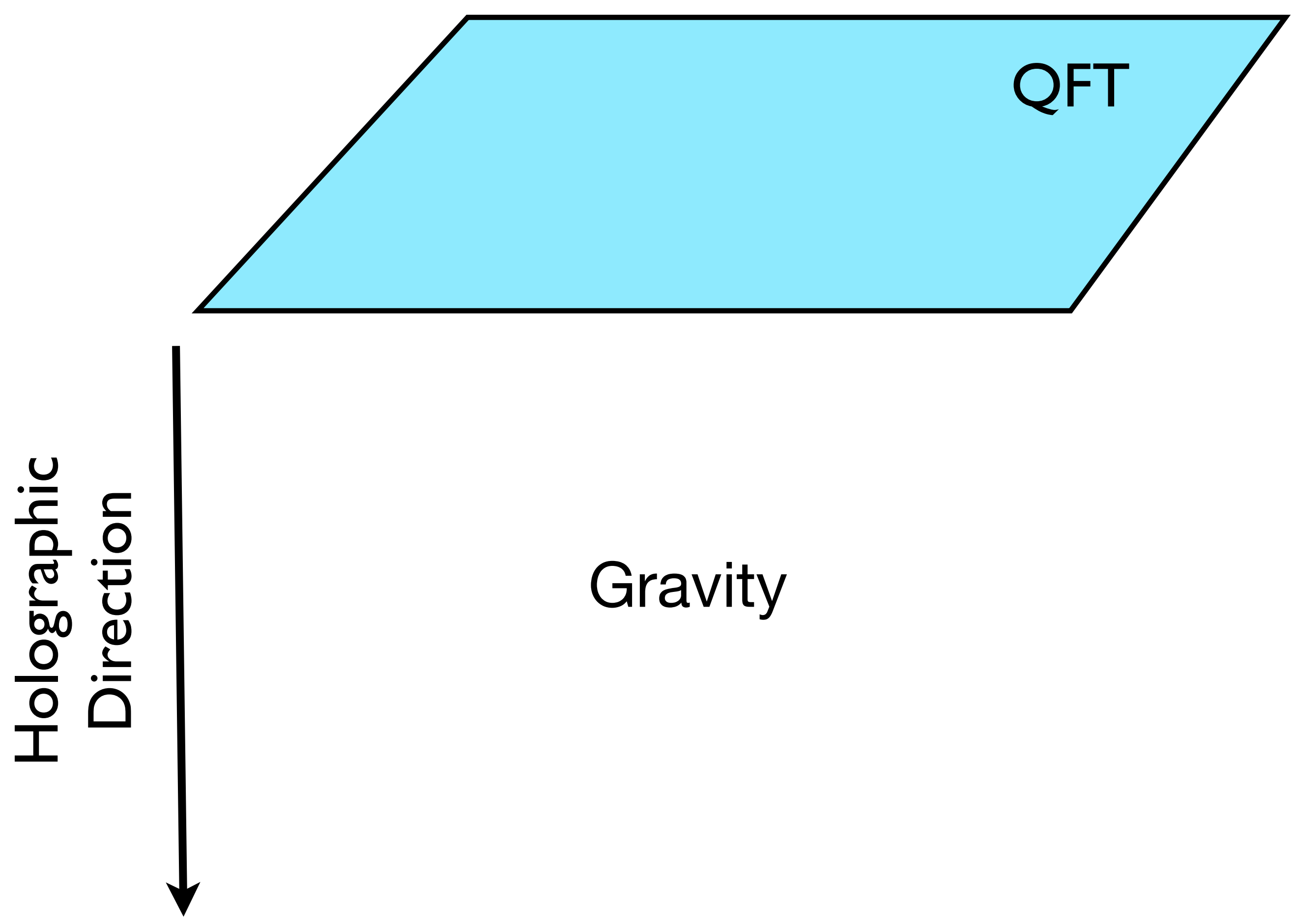

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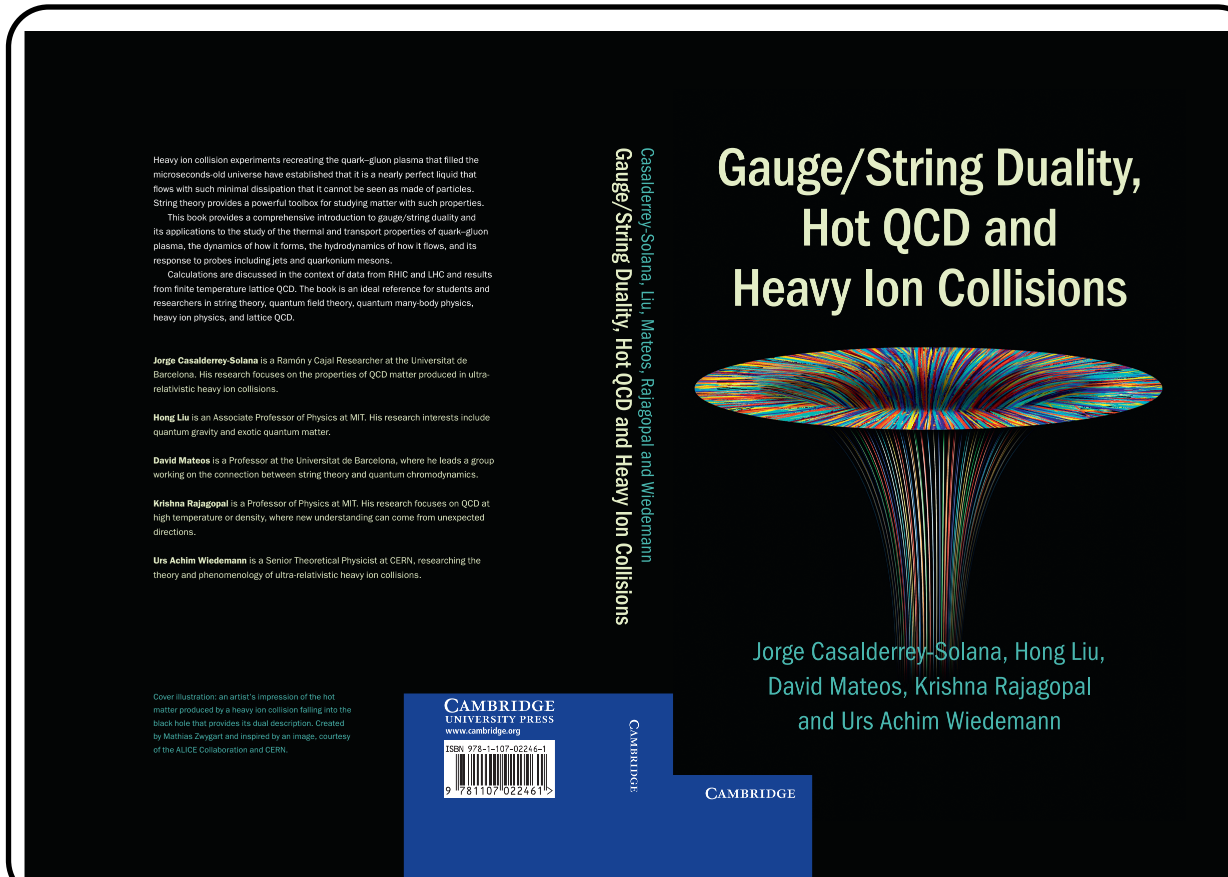

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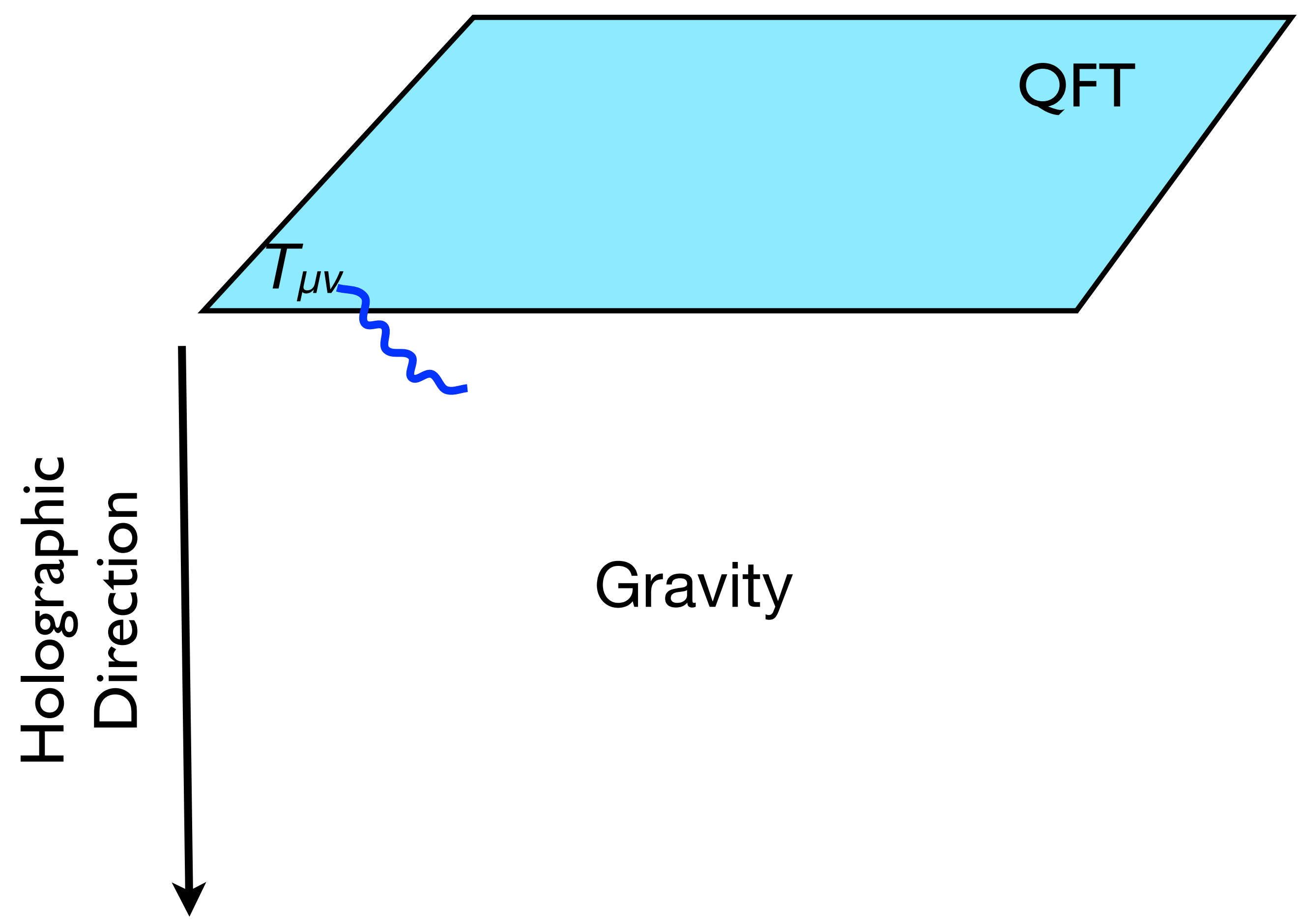

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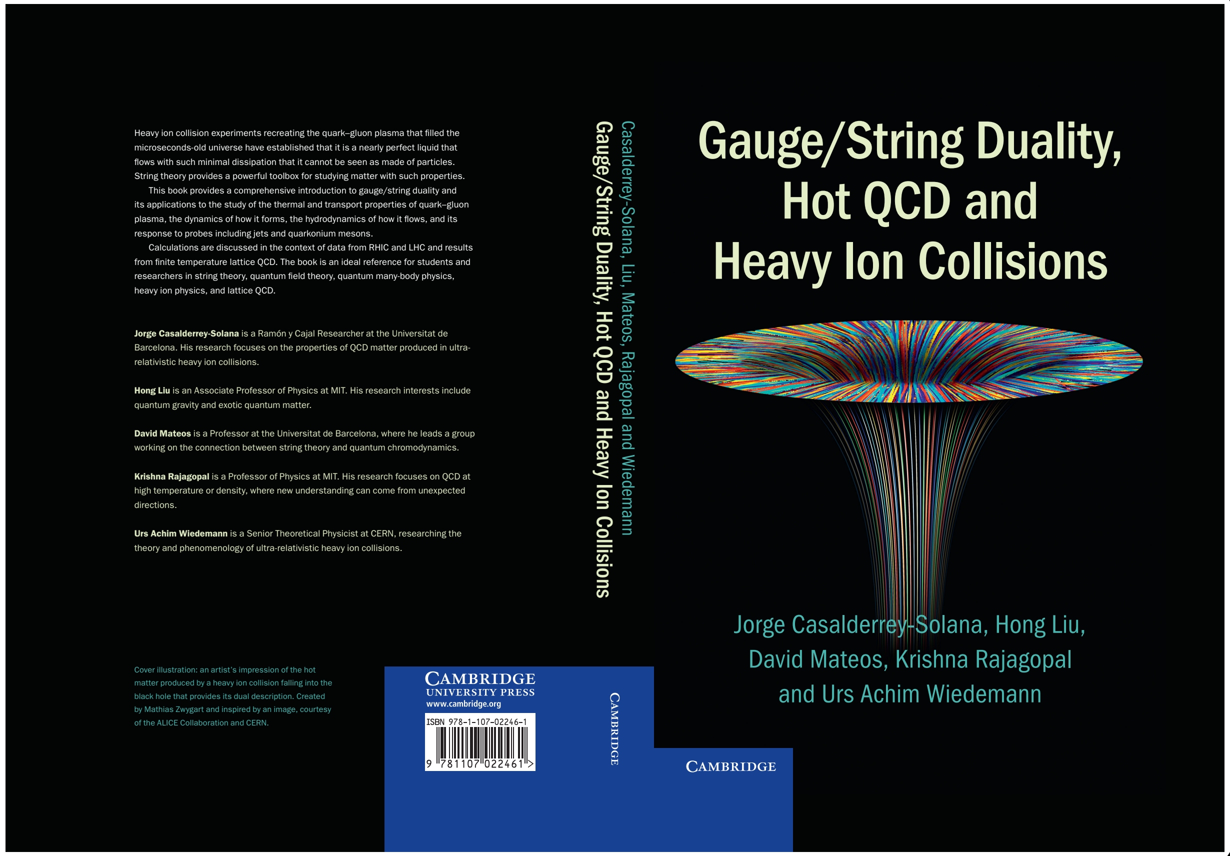

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$$T_{\mu\nu}\leftrightarrow g_{\mu\nu}$$

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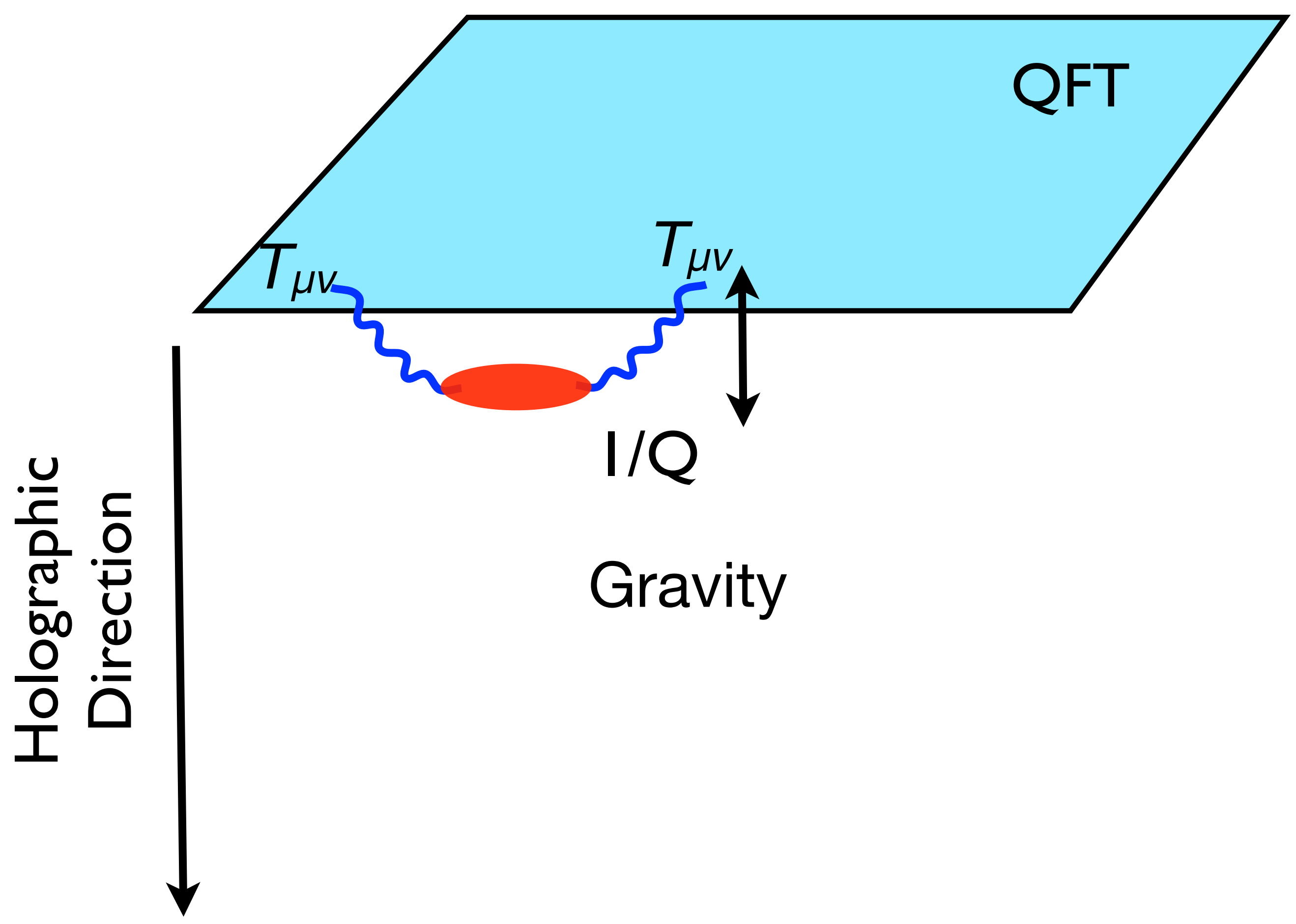

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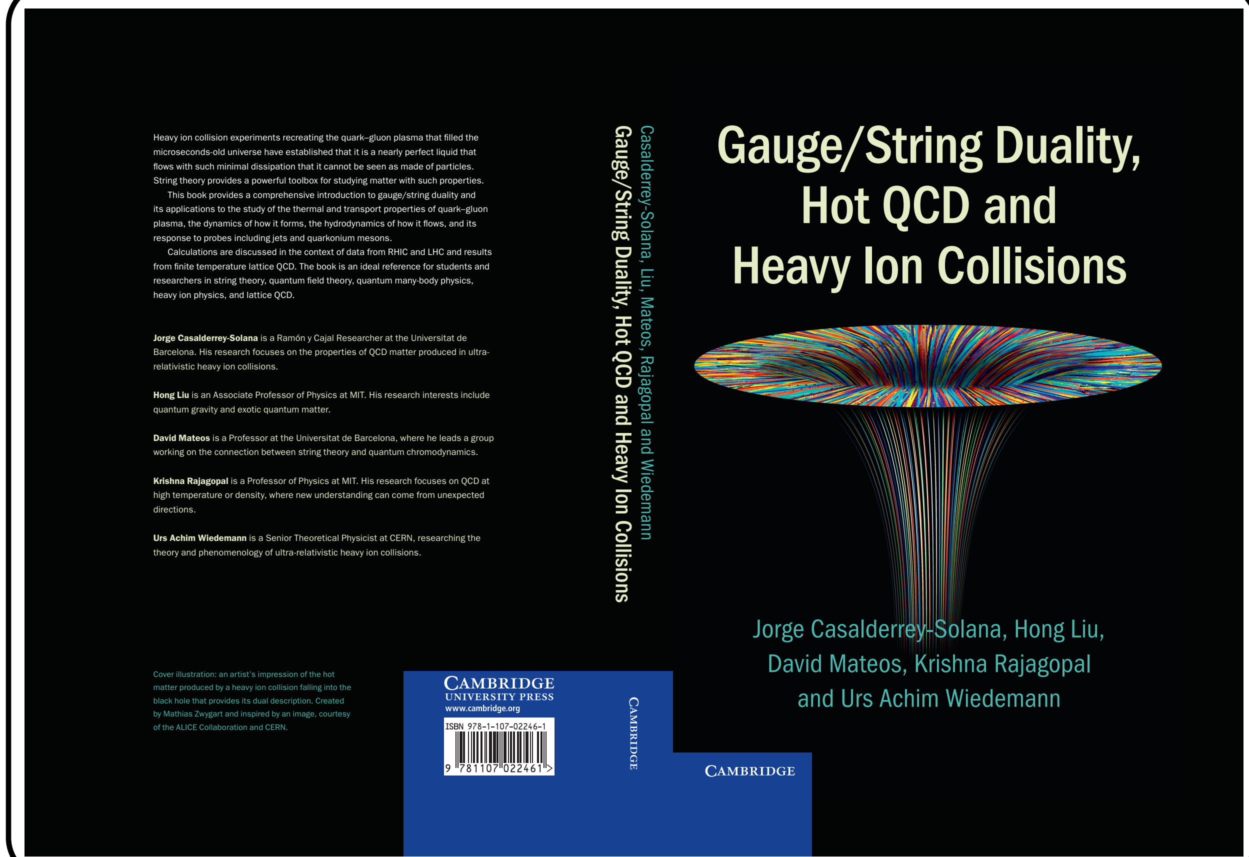

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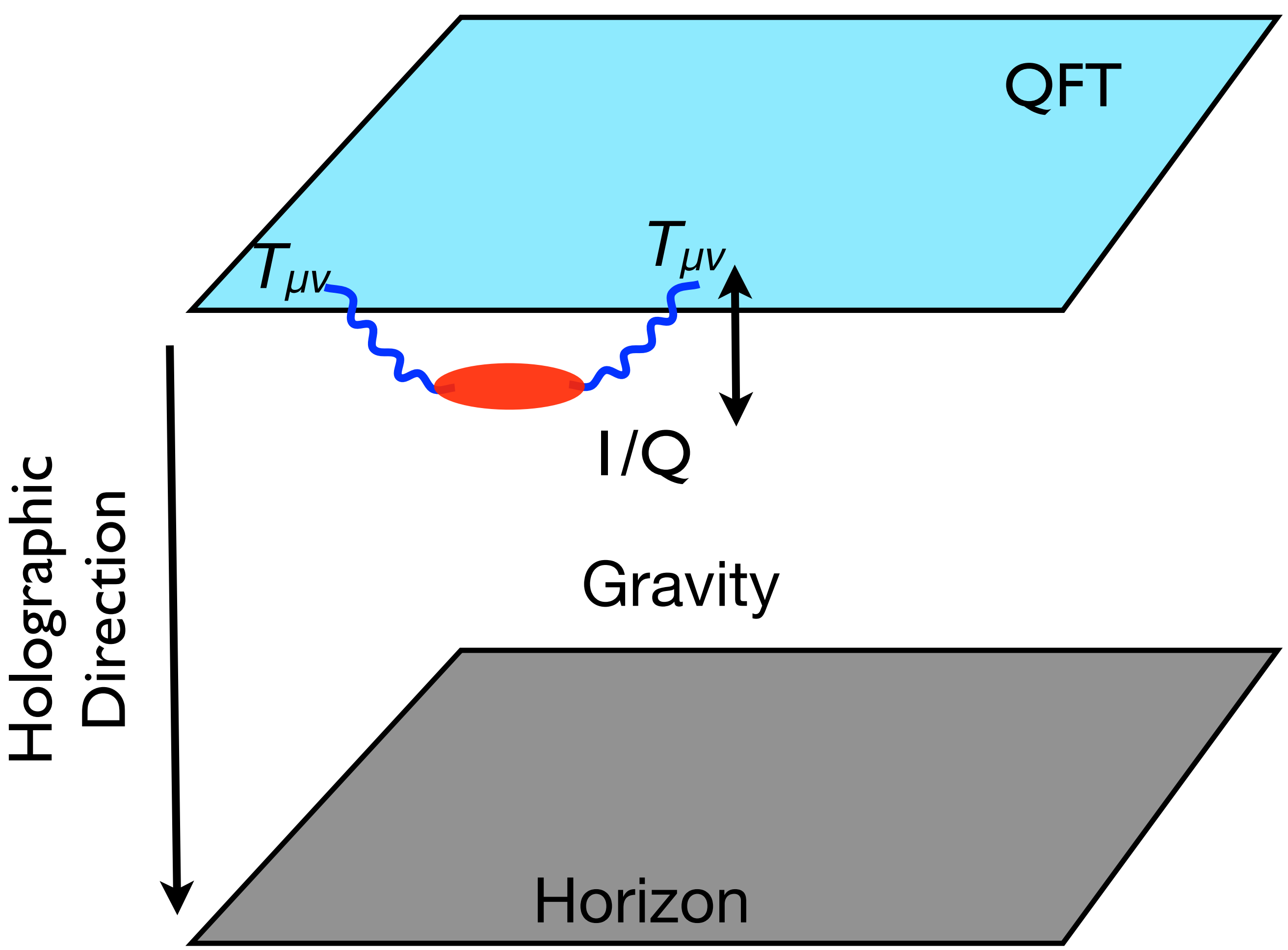

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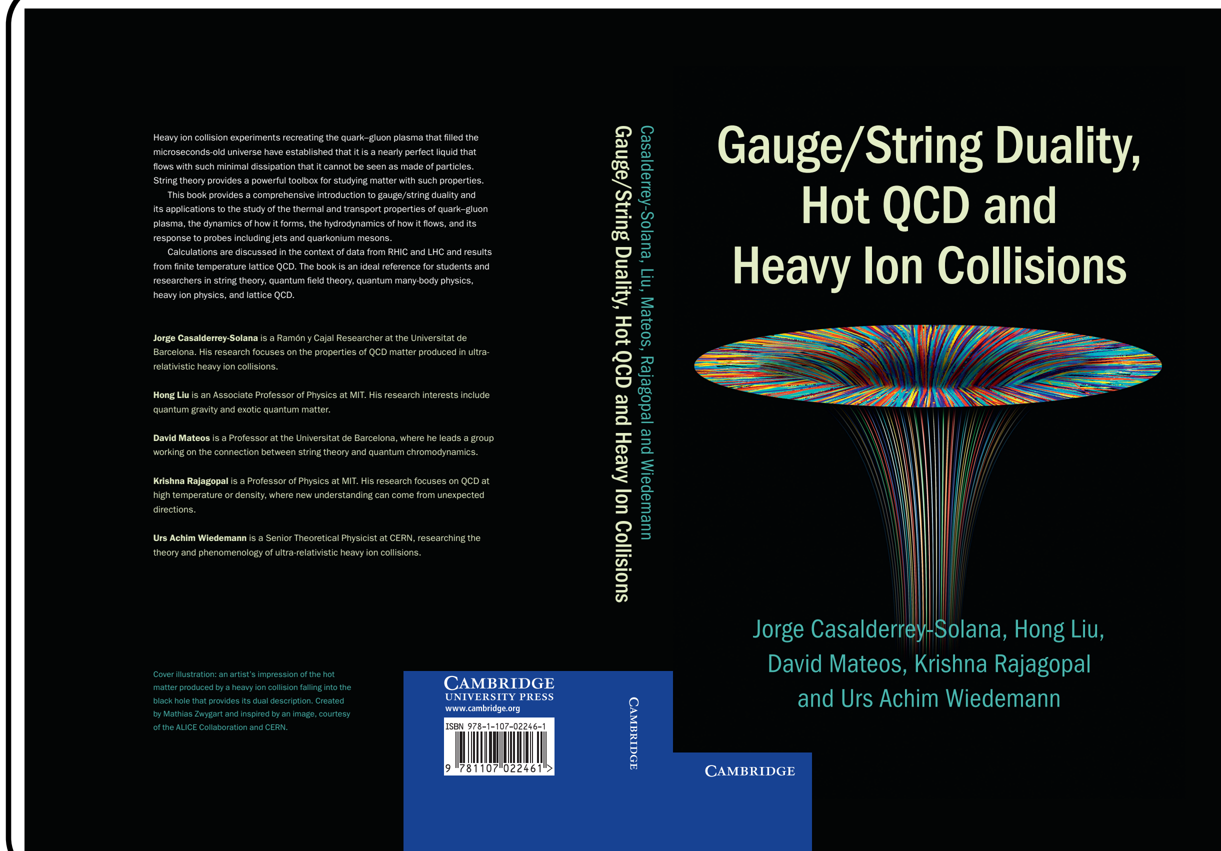

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T ↔ *black hole*

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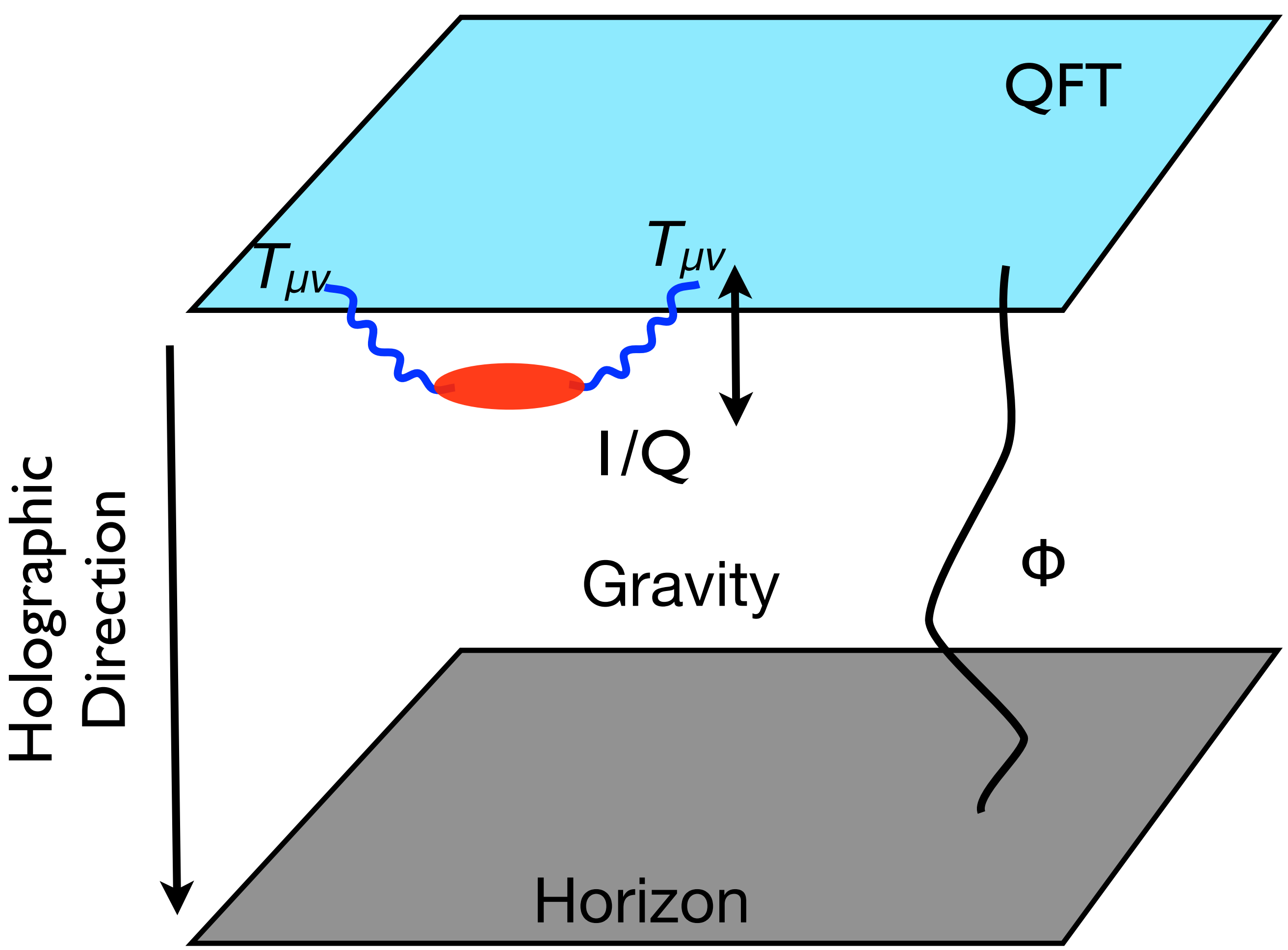

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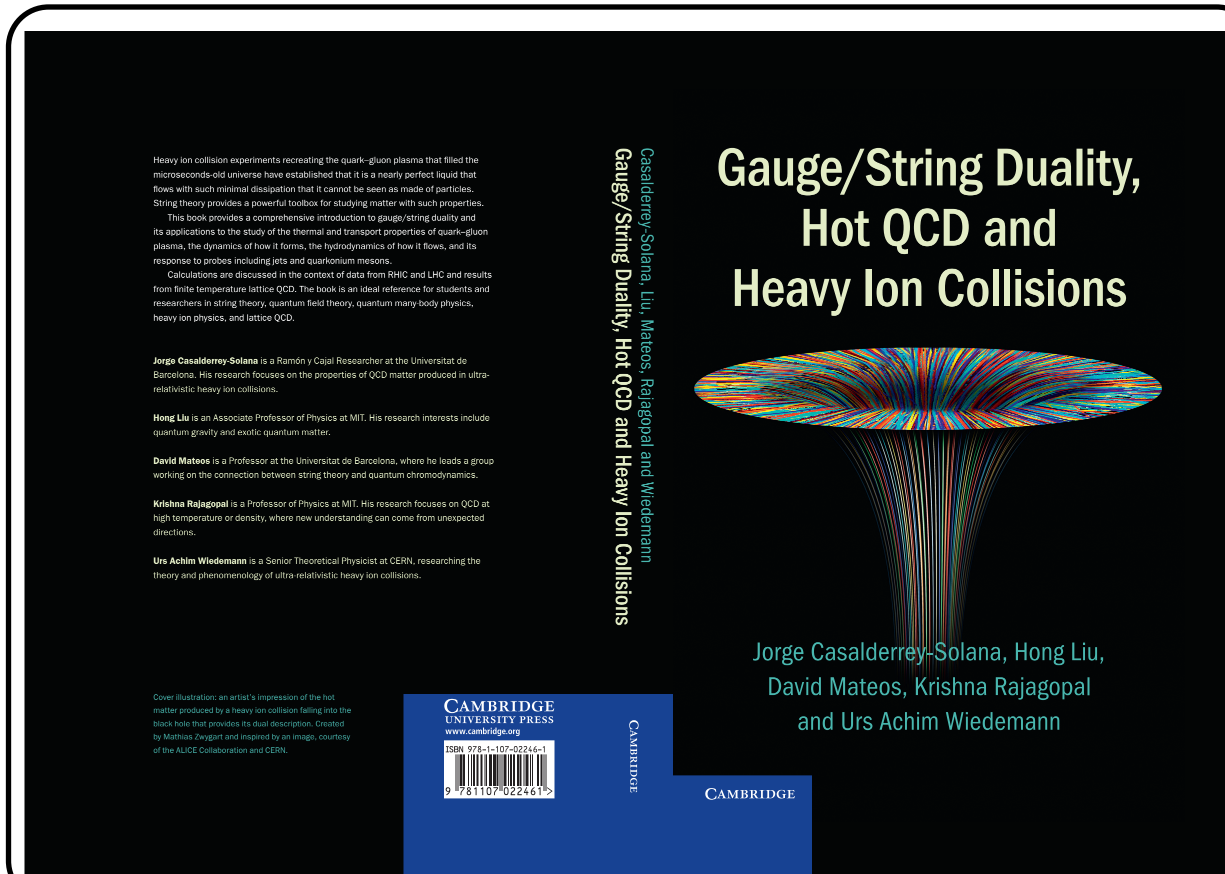

Dictionary

$T_{\mu\nu} \leftrightarrow g_{\mu\nu}$

T ↔ *black hole*

operator ↔ *classical field*

A Bottom-up Non-Conformal Model

- Einstein gravity + Scalar

$$S = \frac{2}{\kappa_5^2}\int d^5x\sqrt{-g}\left[\frac{1}{4}\mathcal{R} - \frac{1}{2}\left(\nabla\phi\right)^2 - V(\phi)\right]$$

- Phenomenological (family of) potential(s)

$$V = -3 - \frac{3}{2}\phi^2 - \frac{1}{3}\phi^4 + \left(\frac{1}{3\phi_M^2} + \frac{1}{2\phi_M^4}\right)\phi^6 - \frac{1}{12\phi_M^4}\phi^8$$

→ parameter

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ϕ_M → parameter

- Dual field theory: "mimics" a deformation of N=4 SYM with a dimension 3 operator

$$S_{\text{Gauge Theory}} = S_{\text{conformal}} + \int d^4x\,\Lambda\,\mathcal{O} \qquad \mathcal{O} \sim \bar{\psi}\psi + \cdots$$

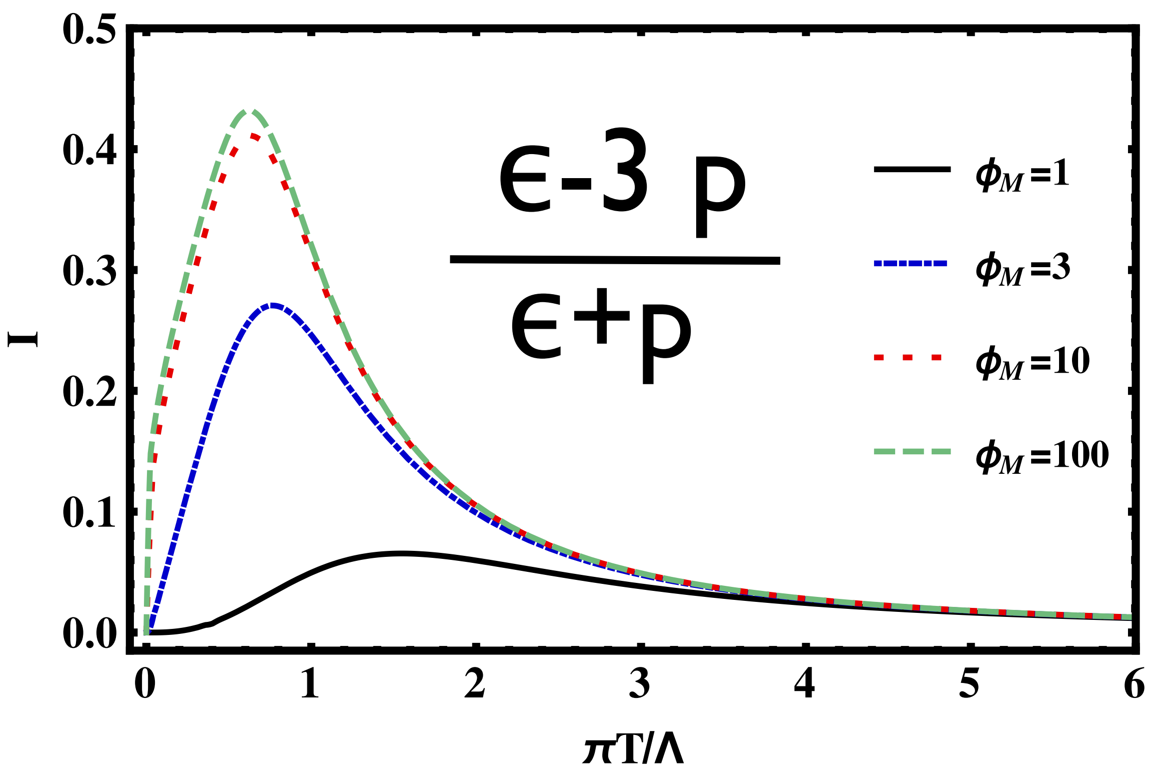
Λ ↓ "mass"

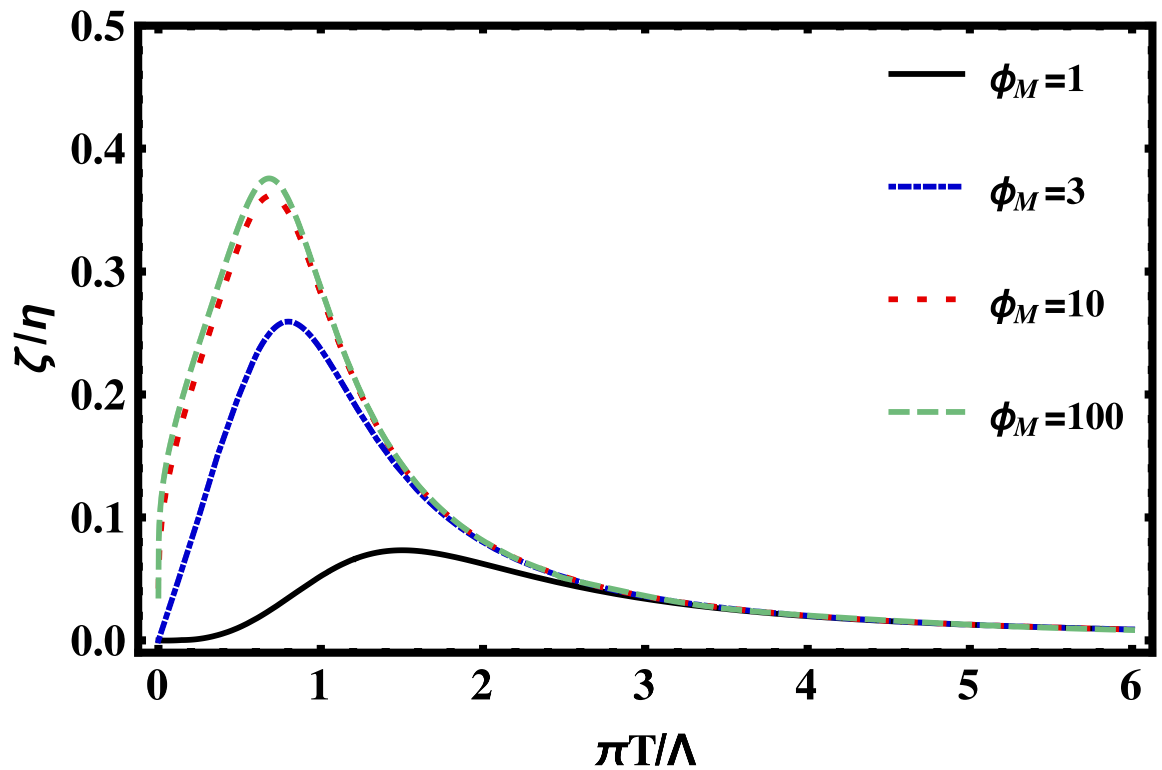
- Rich thermodynamic and transport properties

Attems, JCS, Mateos, Papadimitriou, Santos, Sopuerta, Triana, Zilhao, 16

Thermo and Transport

- Non conformal (bottom-up) holographic model: Einstein + Scalar



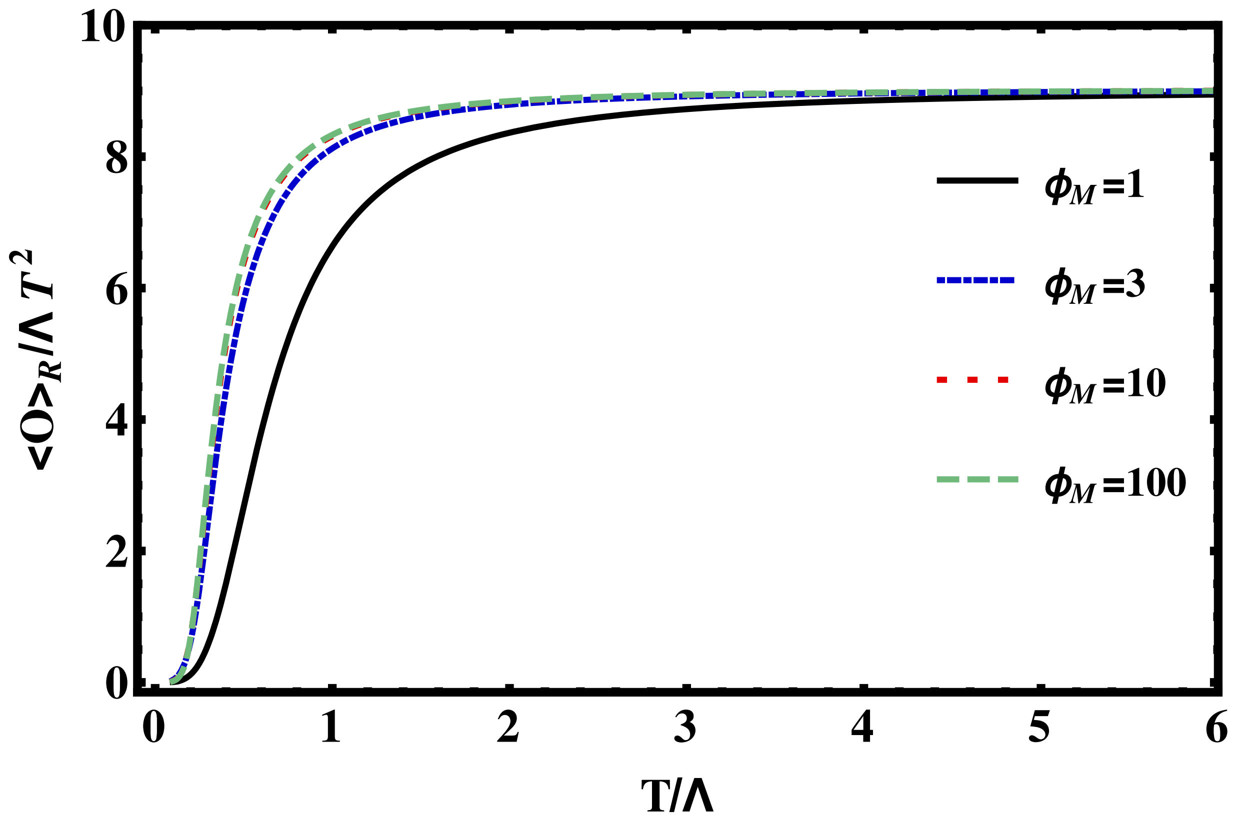


- Universality: $$\frac{\eta}{s} = \frac{1}{4\pi}$$

Attems, JCS, Mateos, Papadimitriou, Santos,Sopuerta, Triana, Zilhao, 16

EOS and VeV's

- Non-thermodynamic one point function



- Non-trivial T-dependence ⇒ non-trivial e.o.s

$$\left\langle T^{\mu}_{\mu} \right\rangle = -\Lambda \left\langle \mathcal{O} \right\rangle \xrightarrow{\text{equilibrium}} P_{\rm eq}\left(\mathcal{E}\right) = \frac{1}{3}\left[\mathcal{E} - \Lambda \left\langle \mathcal{O} \right\rangle_{\rm eq}\left(\mathcal{E}\right)\right]$$

↑ ward identity

Small perturbations off equilibrium

- How do small off-equilibrium excitations of the plasma relax?

Focus on non-hydro mode: homogeneous excitations

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon) & 0 & 0 \\ 0 & 0 & p(\varepsilon) & 0 \\ 0 & 0 & 0 & p(\varepsilon) \end{pmatrix}$$

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$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon) & \delta(t) & 0 \\ 0 & 0 & p(\varepsilon) & 0 \\ 0 & 0 & 0 & p(\varepsilon) \end{pmatrix}$$

tensor

Small perturbations off equilibrium

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Focus on non-hydro mode: homogeneous excitations

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon)+\delta(t) & 0 & 0 \\ 0 & 0 & p(\varepsilon)-\delta(t)/2 & 0 \\ 0 & 0 & 0 & p(\varepsilon)-\delta(t)/2 \end{pmatrix}$$ anisotropic

Small perturbations off equilibrium

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Focus on non-hydro mode: homogeneous excitations

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon) + \delta(t)/3 & 0 & 0 \\ 0 & 0 & p(\varepsilon) + \delta(t)/3 & 0 \\ 0 & 0 & 0 & p(\varepsilon) + \delta(t)/3 \end{pmatrix}$$ bulk

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$$\langle \mathcal{O} \rangle - \Lambda \delta(t)$$

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bulk

$$\langle \mathcal{O} \rangle - \Lambda \delta(t)$$

- Relaxation controlled by retarded greens functions

$$\delta(t) \sim G_R(t-t_0) S(t_0)$$

Small (homogeneous) fluctuations

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon) & \delta & 0 \\ 0 & 0 & p(\varepsilon) & 0 \\ 0 & 0 & 0 & p(\varepsilon) \end{pmatrix}$$

(Local) Stress tensor becomes diagonal

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon)+\delta & 0 & 0 \\ 0 & 0 & p(\varepsilon)-\delta/2 & 0 \\ 0 & 0 & 0 & p(\varepsilon)-\delta/2 \end{pmatrix}$$

(Local) Stress tensor becomes isotropic

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon)+\delta/3 & 0 & 0 \\ 0 & 0 & p(\varepsilon)+\delta/3 & 0 \\ 0 & 0 & 0 & p(\varepsilon)+\delta/3 \end{pmatrix}$$

(Local) Stress tensor satisfies equation of state

Small (homogeneous) fluctuations

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon) & \delta & 0 \\ 0 & 0 & p(\varepsilon) & 0 \\ 0 & 0 & 0 & p(\varepsilon) \end{pmatrix}$$

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon)+\delta & 0 & 0 \\ 0 & 0 & p(\varepsilon)-\delta/2 & 0 \\ 0 & 0 & 0 & p(\varepsilon)-\delta/2 \end{pmatrix}$$

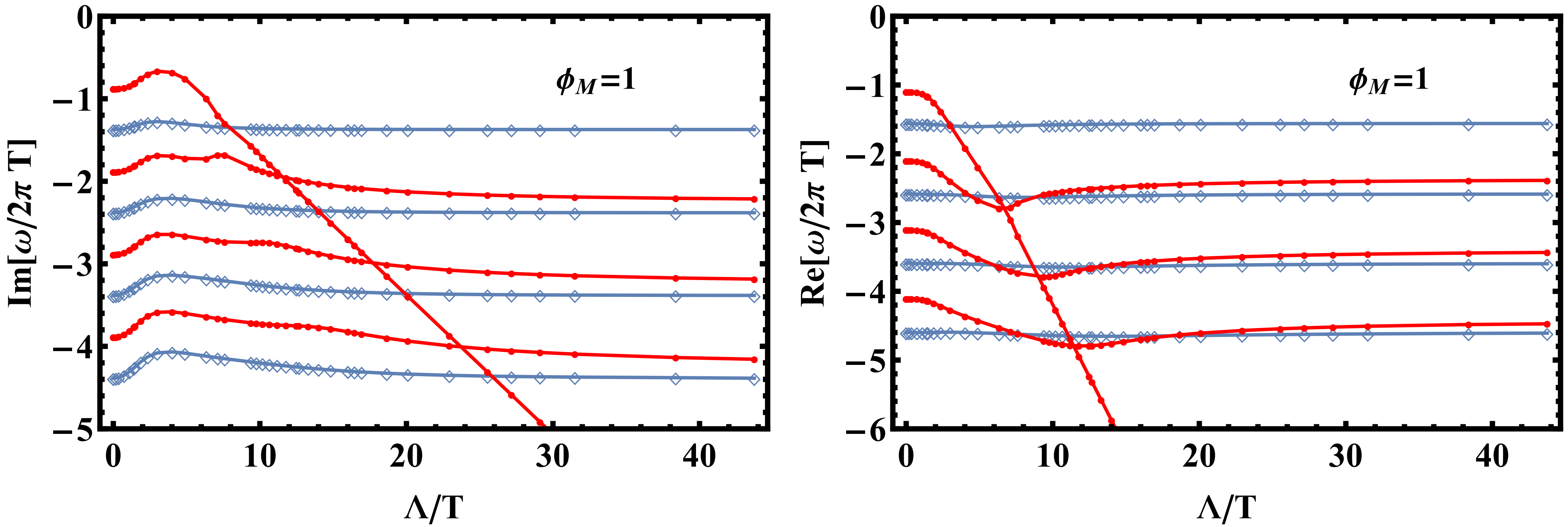
Which one is faster?

$$\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & p(\varepsilon)+\delta/3 & 0 & 0 \\ 0 & 0 & p(\varepsilon)+\delta/3 & 0 \\ 0 & 0 & 0 & p(\varepsilon)+\delta/3 \end{pmatrix}$$

- Holography: relaxation of fluctuations ⇔ relaxation of black brane

  Discrete set of (complex) characteristic frequencies (quasi-normal modes)

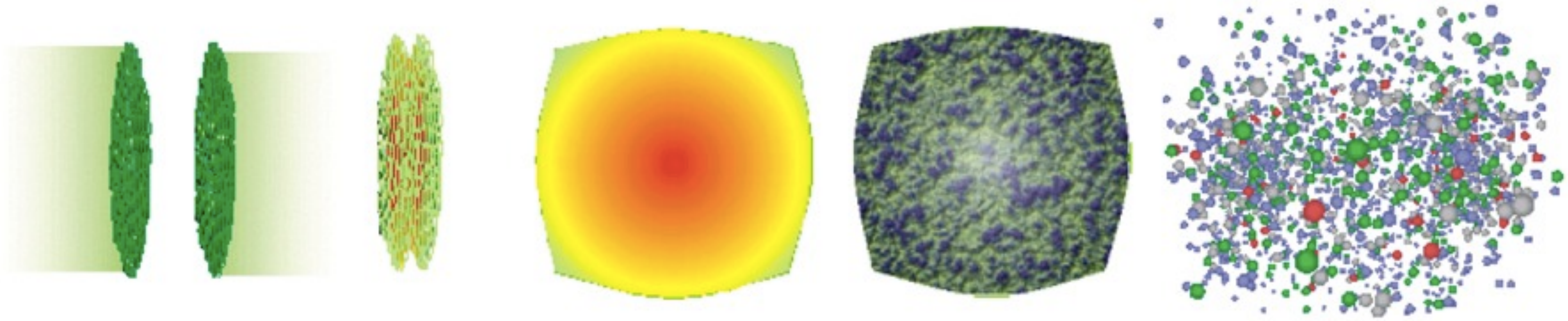
$$G_R(t) \sim \mathrm{Re} \sum \left(e^{-i\omega t}\right) \sim \sum e^{-\Gamma t} \cos \Omega t$$



- Quasi-normal modes depend on the channel
- The ordering of different relaxation processes is not unique.

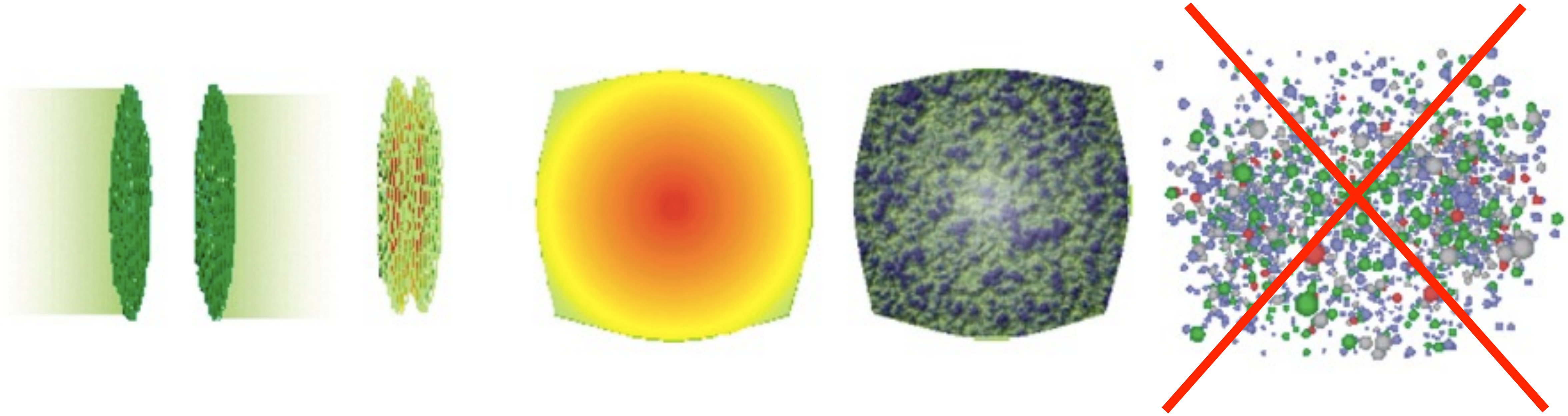
What happens at non-linear level?

From Initial to Final State in Holography



- Can we describes all these stages in a single framework?

From Initial to Final State in Holography



- Can we describes all these stages in a single framework?
  - Holography says: yes! (up to the last one)

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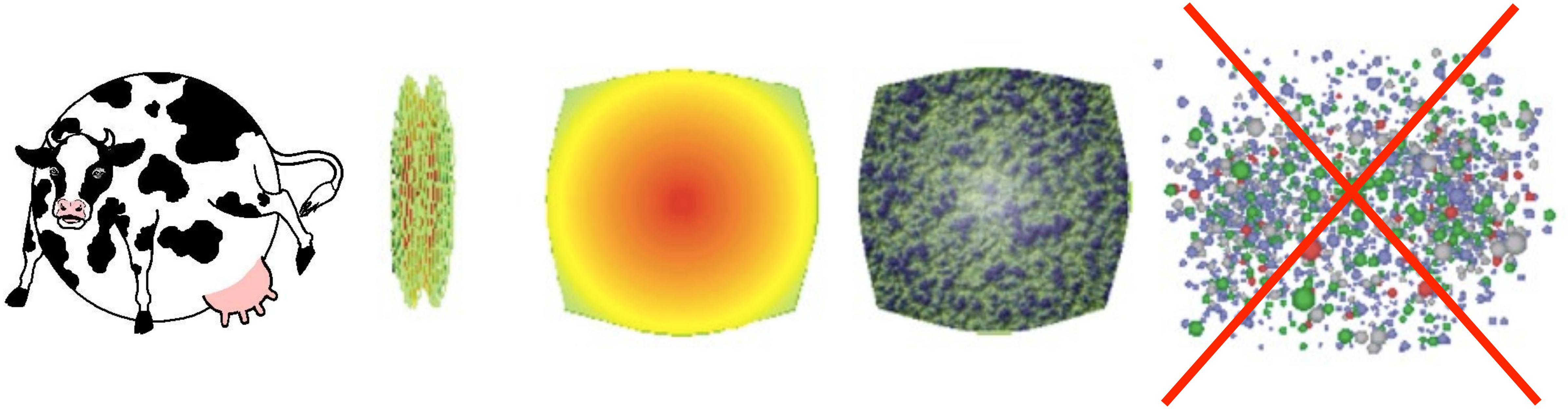

- Can we describes all these stages in a single framework?
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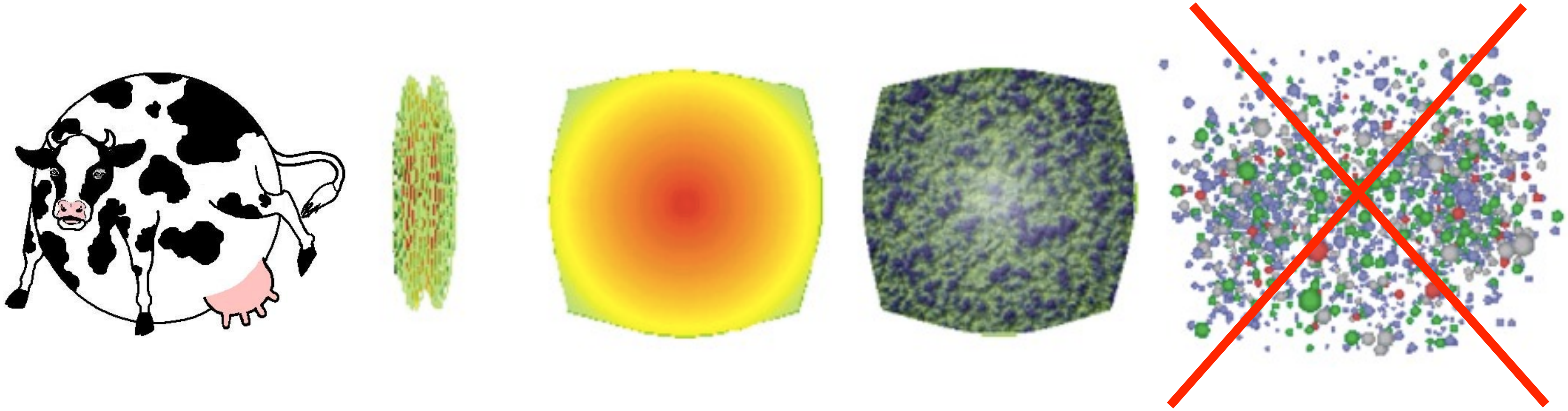
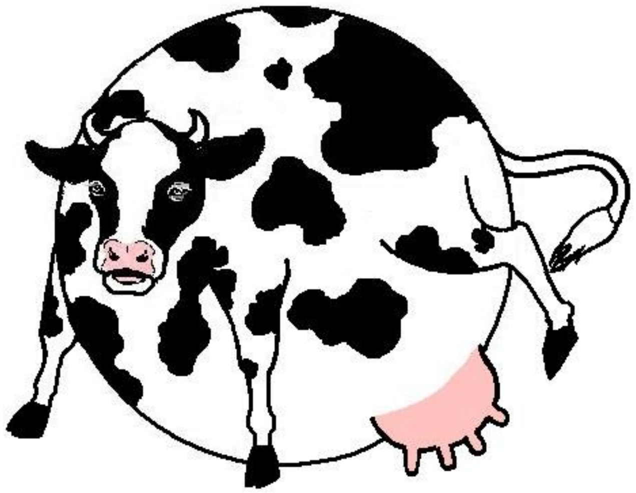
  - As long as we are happy with an oversimplified “nucleus”

From Initial to Final State in Holography

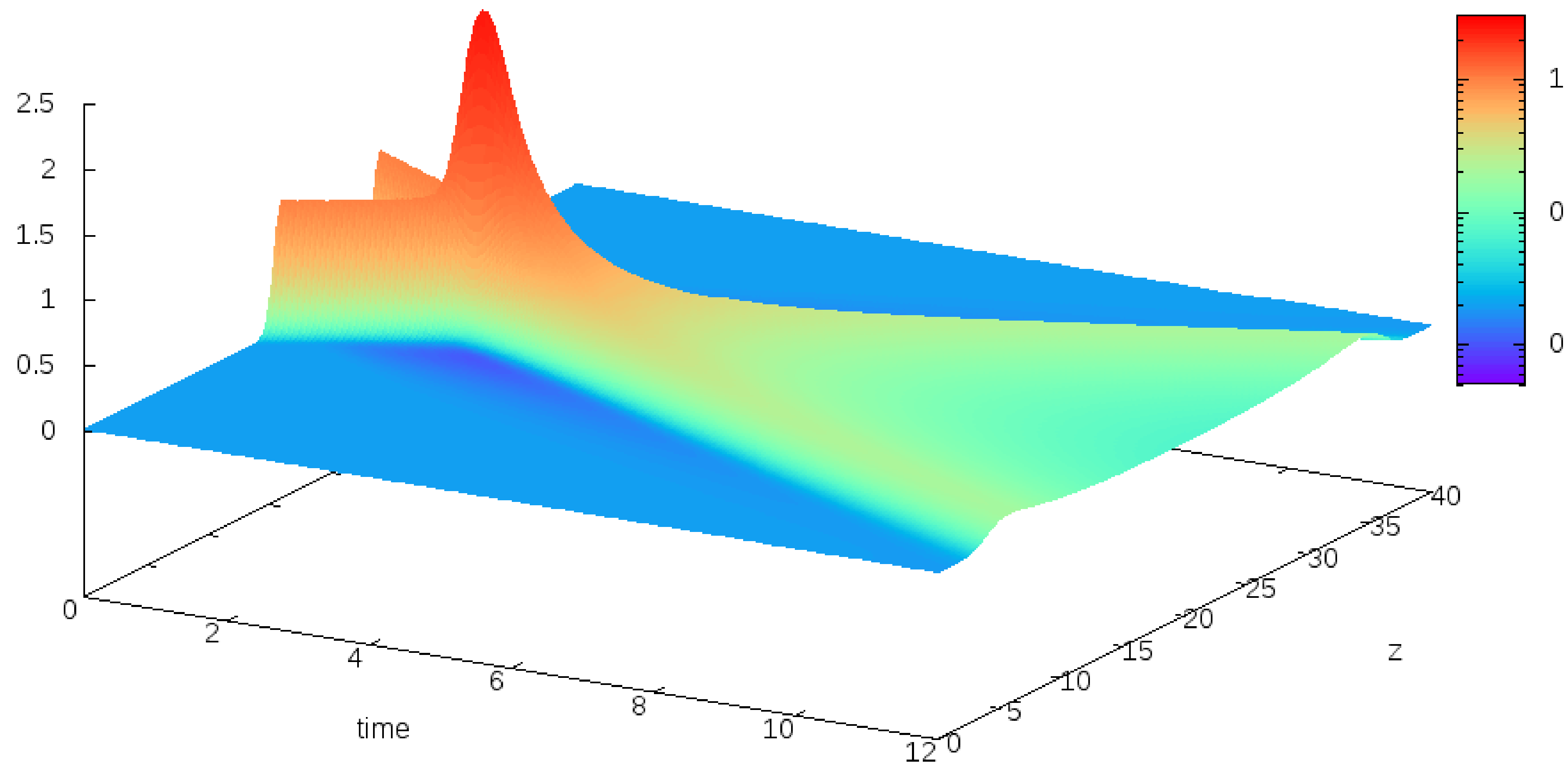


- Can we describes all these stages in a single framework?
  - Holography says: yes! (up to the last one)
  - As long as we are happy with an oversimplified “nucleus”
  - As long as we are happy with other strongly coupled theory



Non conformal Shock Collisions

Attems, JCS, Mateos, Santos, Sopuerta, Triana, Zilhao, 16



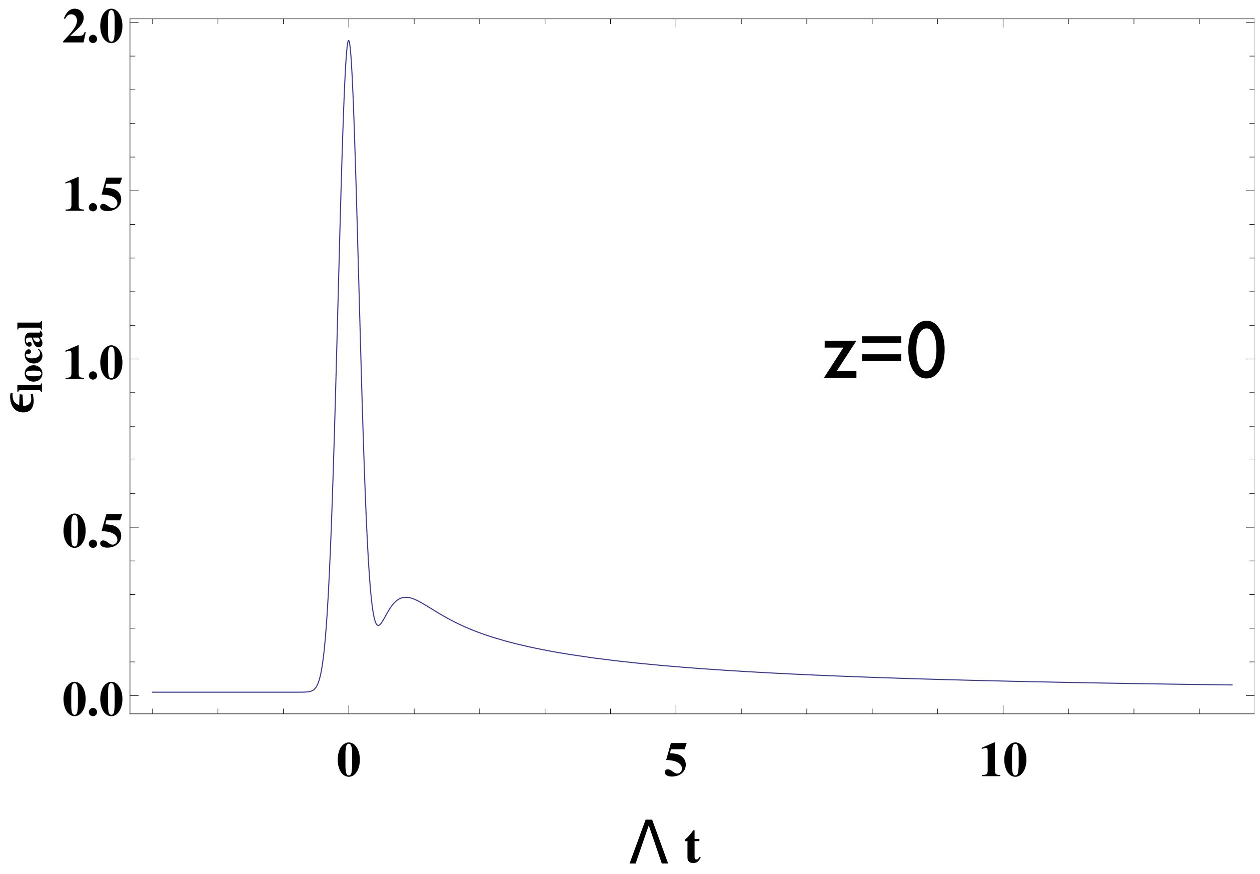
Same techniques as for conformal collisions

Chesler & Yaffe 2011

JCS, Heller, Mateos, van der Schee, 2013

See D. Mateos talk

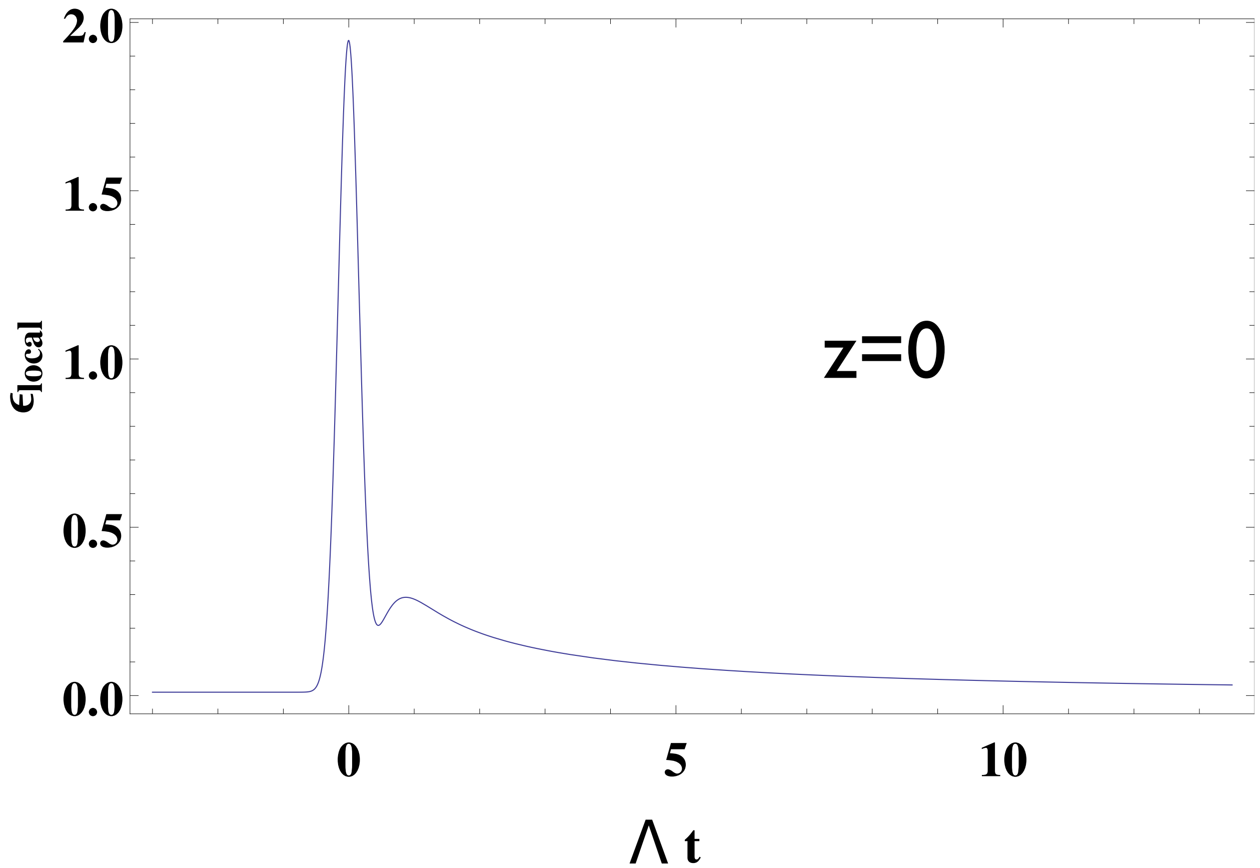
Evolution of Expectation Values



- Stress tensor components

$$\begin{pmatrix} \varepsilon_{\rm local}(t,z) & 0 & 0 & 0 \\ 0 & p_{\rm L}(t,z) & 0 & 0 \\ 0 & 0 & p_{\rm T}(t,z) & 0 \\ 0 & 0 & 0 & p_{\rm T}(t,z) \end{pmatrix}$$

Evolution of Expectation Values



- Stress tensor components

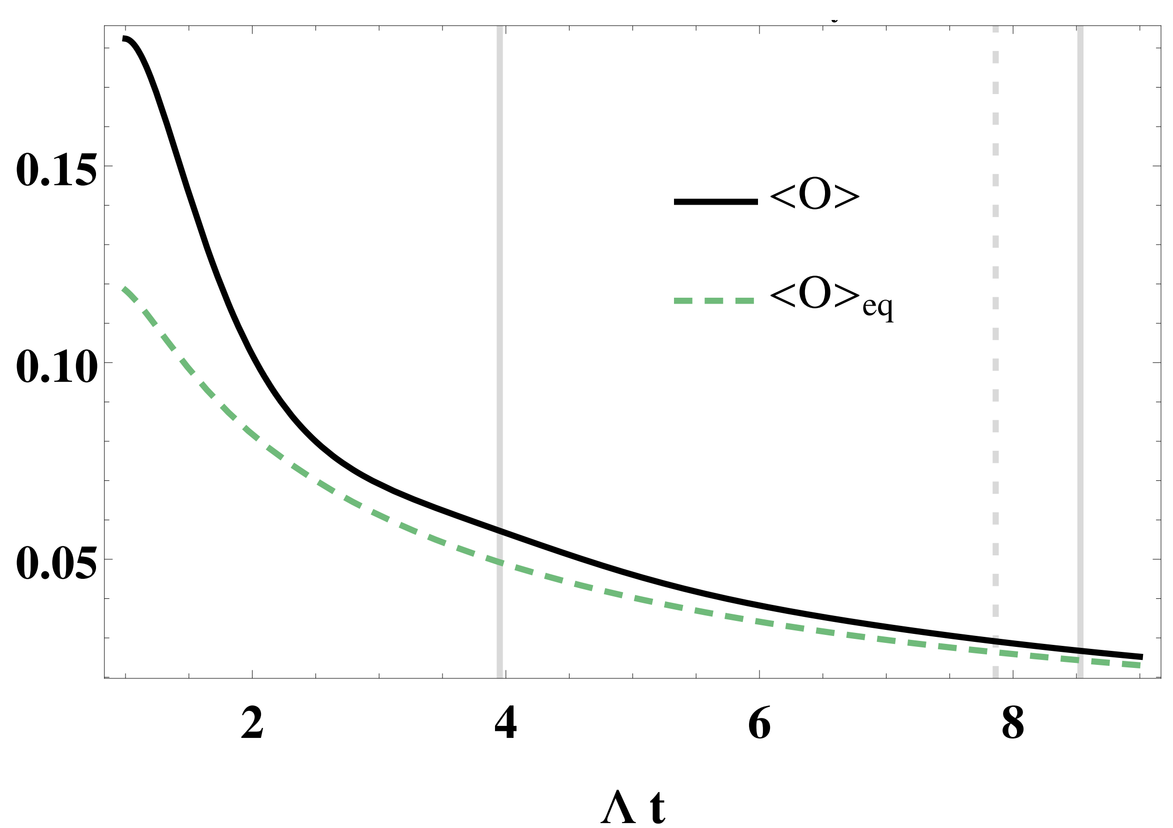
$$\begin{pmatrix} \varepsilon_{\rm local}(t,z) & 0 & 0 & 0 \\ 0 & p_{\rm L}(t,z) & 0 & 0 \\ 0 & 0 & p_{\rm T}(t,z) & 0 \\ 0 & 0 & 0 & p_{\rm T}(t,z) \end{pmatrix}$$

- Condensate (non-conformal)

  Differs from its equilibrium value

  Does not have hydro description

  Does it affect hydrodynamics?



Hydrodynamization

- Does hydrodynamics describe the evolution?

  Are constitutive hydro-relations satisfied?

$$u^\mu T^\nu_\mu \equiv \varepsilon_{\rm local} u^\nu \quad \longrightarrow \quad \begin{aligned} P_L^{\rm hyd} &= P_{\rm eq} + P_\eta + P_\zeta\,, \\ P_T^{\rm hyd} &= P_{\rm eq} - \tfrac{1}{2} P_\eta + P_\zeta \end{aligned}$$

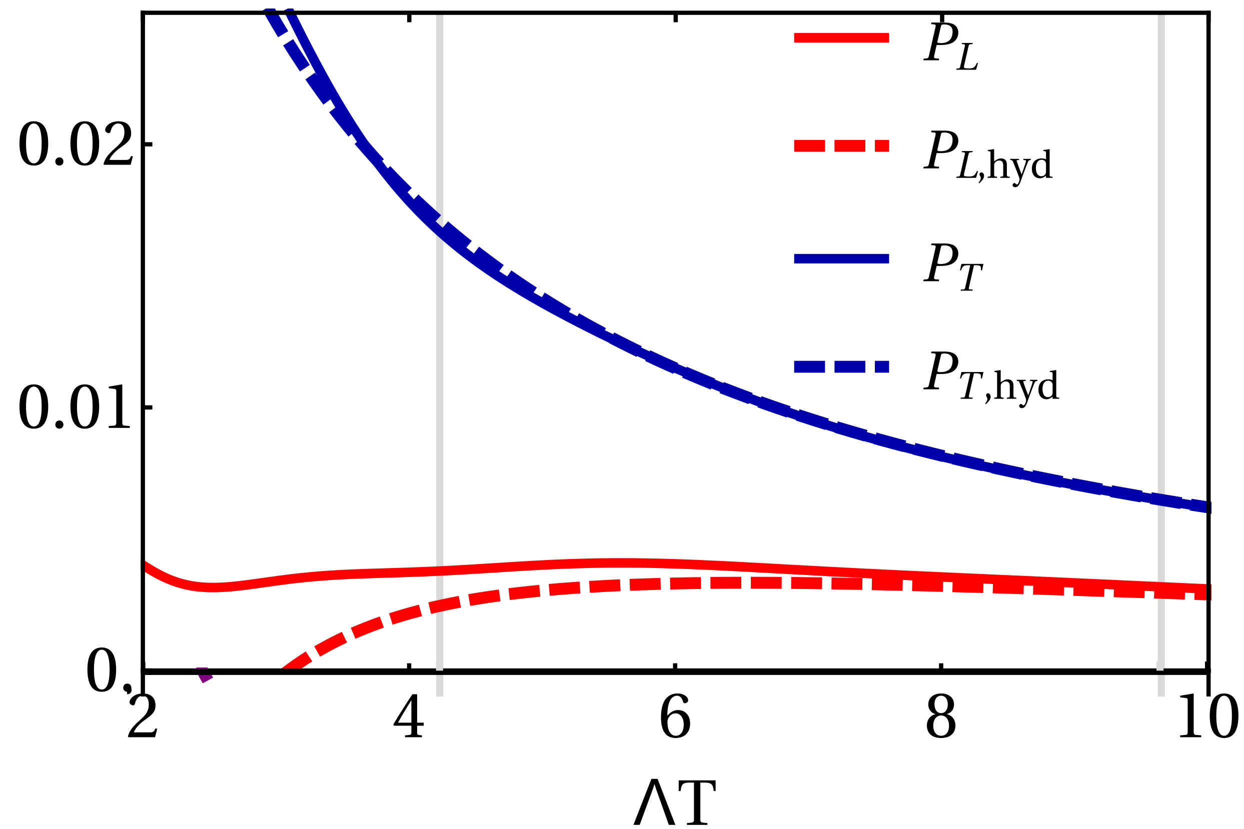
P_η → η ∂u, P_ζ → ζ ∂u

Hydrodynamization criterium

$$\frac{\left|P_{L,T} - P^{\rm hyd}_{LT}\right|}{\bar{P}} < 0.1 \qquad t_{\rm hyd} T_{\rm hyd} = 1.0$$

Fast hydrodynamization!

(hydro with large gradients)



Chesler & Yaffe, Wu & Romatschke, Heller, Janik & Witaszczyk, Heller, Mateos, van der Schee, Trancanelli

Kurkela and Zhu 15, Keegan, Kurkela, Mazeliausksa and Teaney 16

"Equilibration"

- Non-trivial VeV dynamics influence the stress tensor

$$\left\langle T^{\mu}_{\mu} \right\rangle = -\Lambda \left\langle \mathcal{O} \right\rangle$$

↑ ward identity

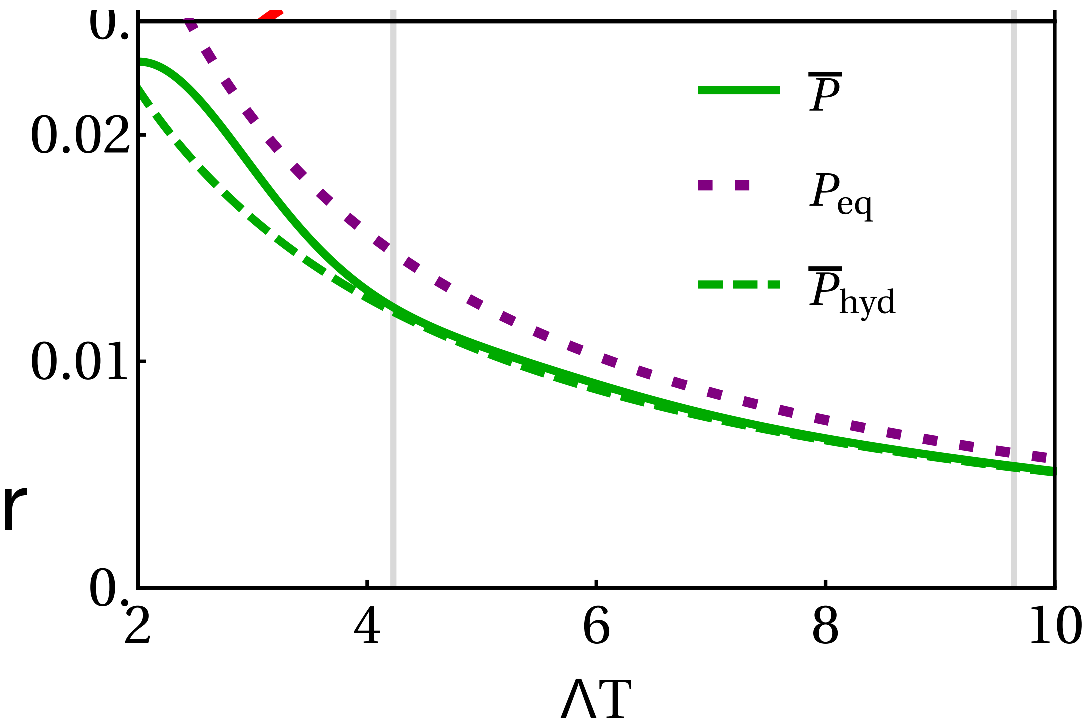
equilibrium →
$$P_{\rm eq}(\mathcal{E}) = \frac{1}{3}\left[\mathcal{E} - \Lambda \left\langle \mathcal{O} \right\rangle_{\rm eq}(\mathcal{E})\right]$$

off-equilibrium →
$$\bar{P} \equiv \frac{1}{3}\left[P_{\rm L} + 2P_{\rm T}\right] = \frac{1}{3}\left[\mathcal{E} - \Lambda \left\langle \mathcal{O} \right\rangle\right]$$

- The average pressure is a proxy of how well de e.o.s is satisfied

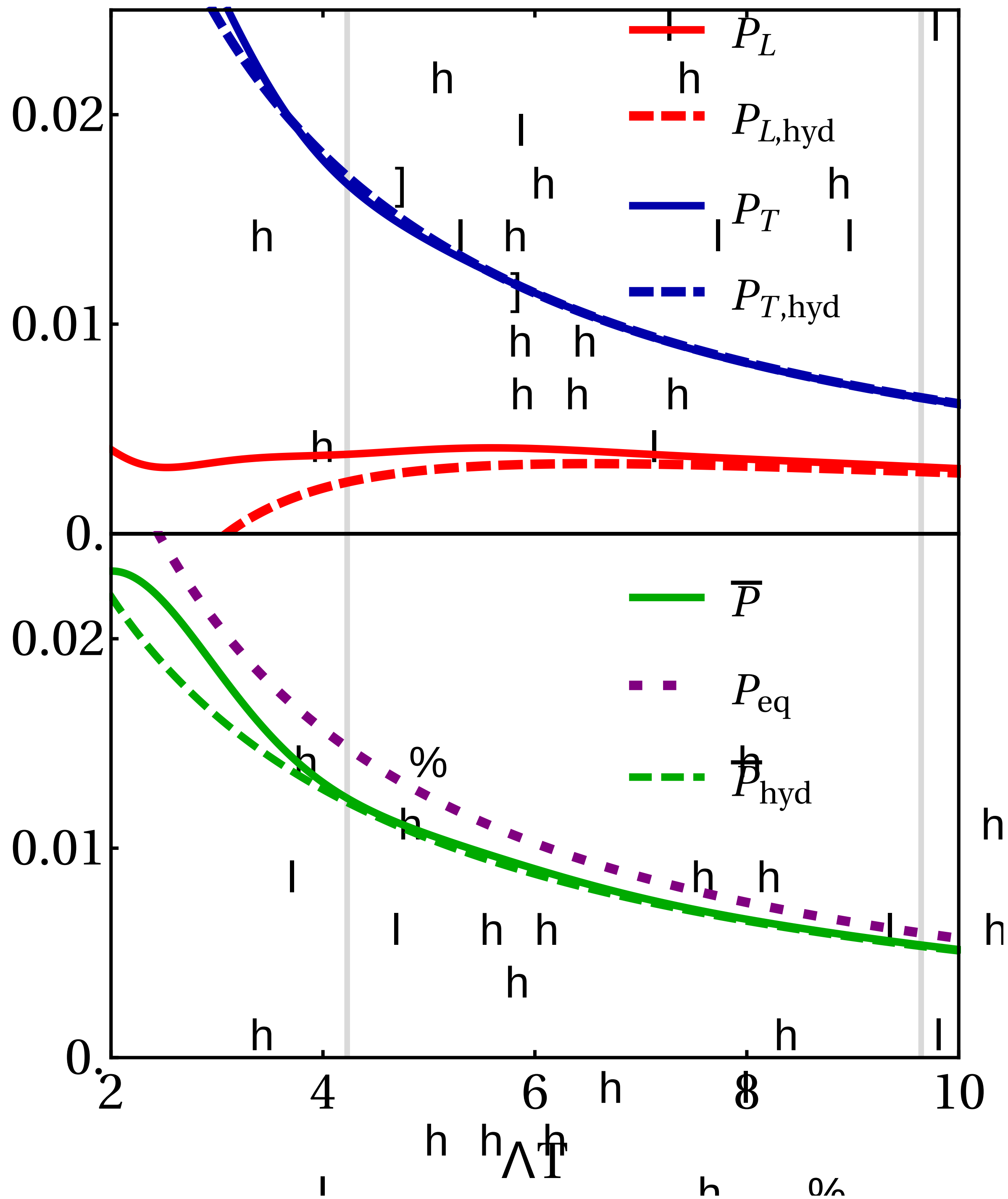
$$\frac{\left|\bar{P} - \bar{P}^{\rm hyd}\right|}{\bar{P}} < 0.1$$ "equilibration"

- Large bulk corrections responsible for deviations from equilibrium!

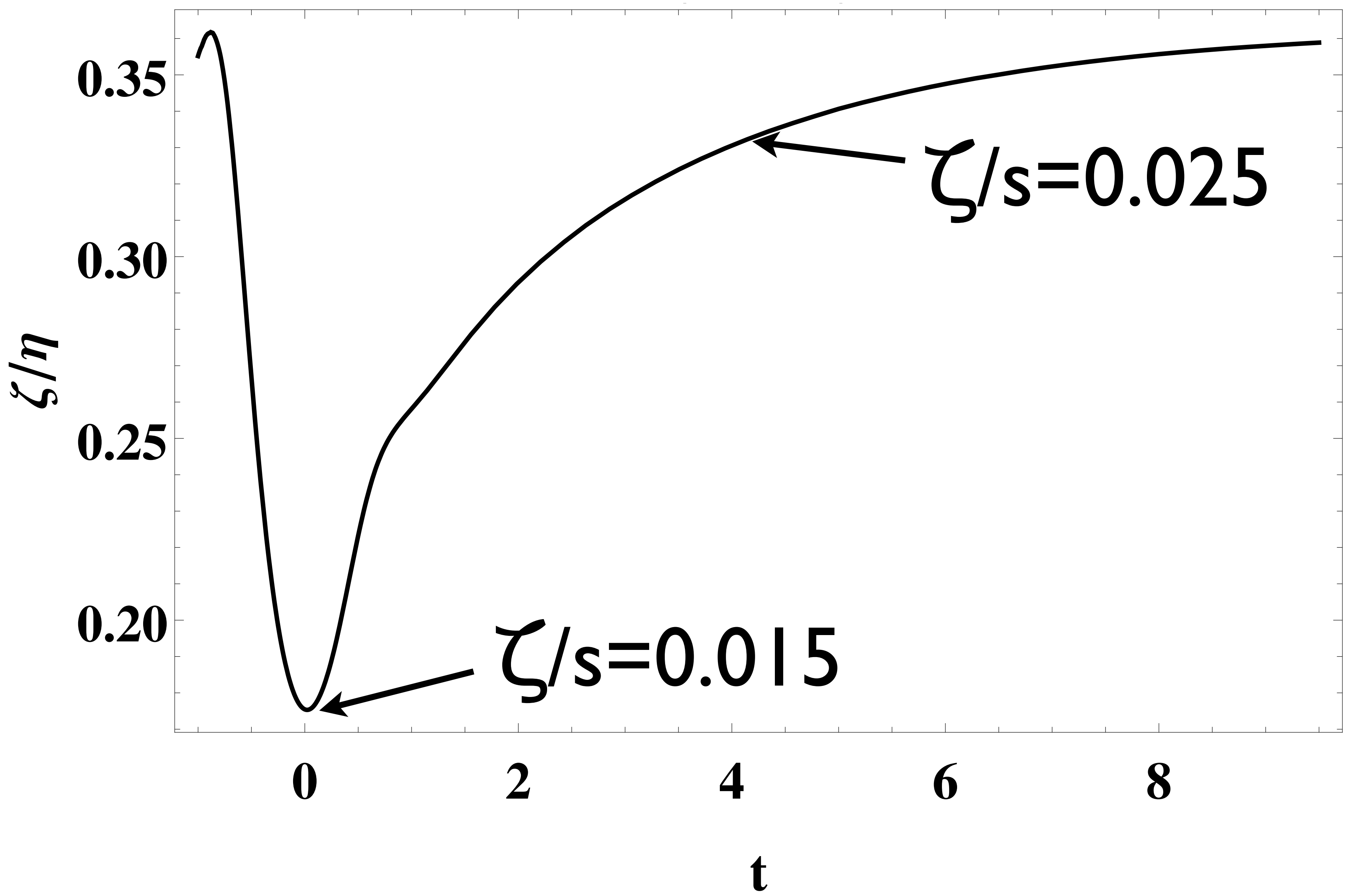


Hydrodynamics without equilibration

$$t_{\rm eq} = 2.3\, t_{\rm hyd}$$

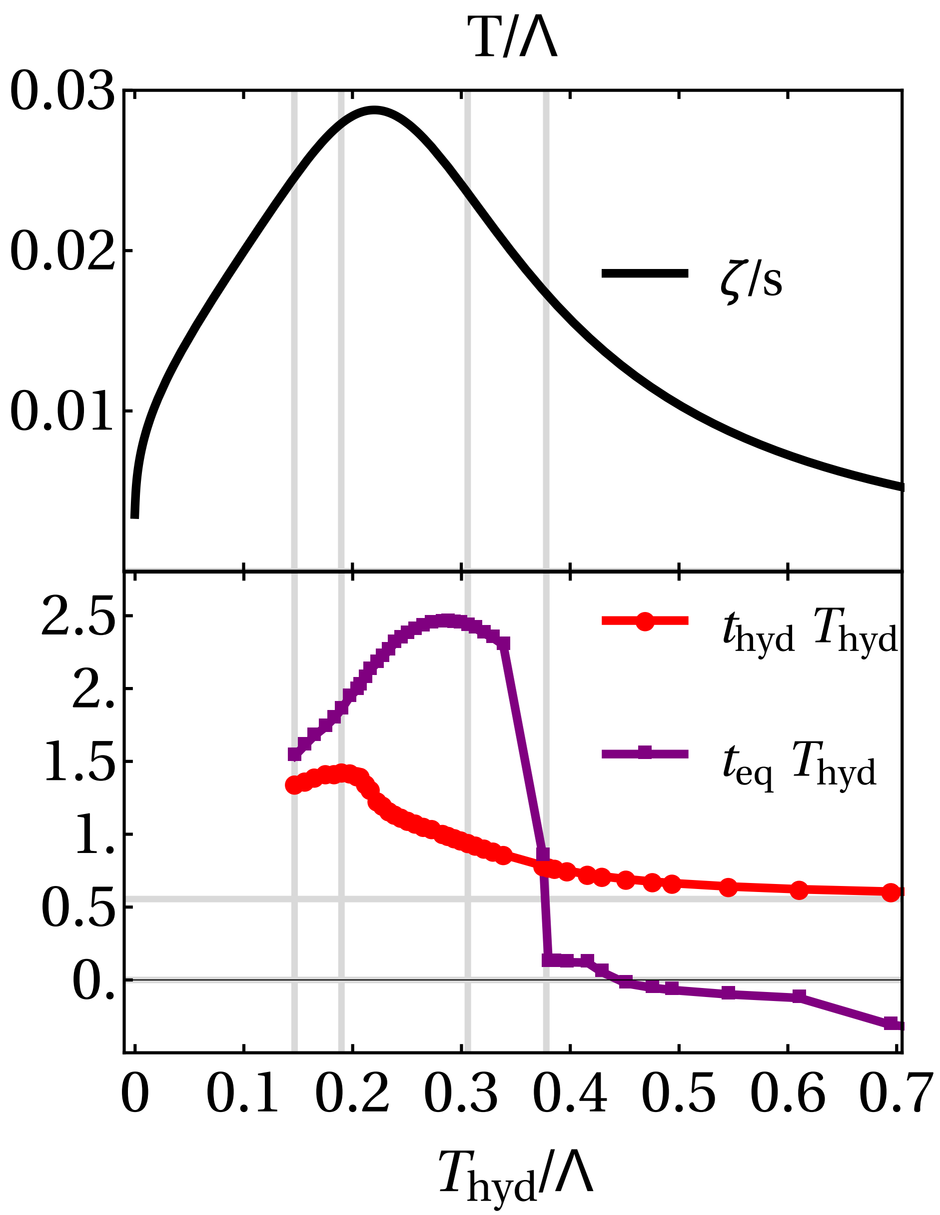


Time evolution of viscosity



Exploring Different Conditions

- Non-conformal theories take longer to hydrodynamise



- For ζ/s>0.025 hydrodynamisation occurs first.

Exploring Different Conditions

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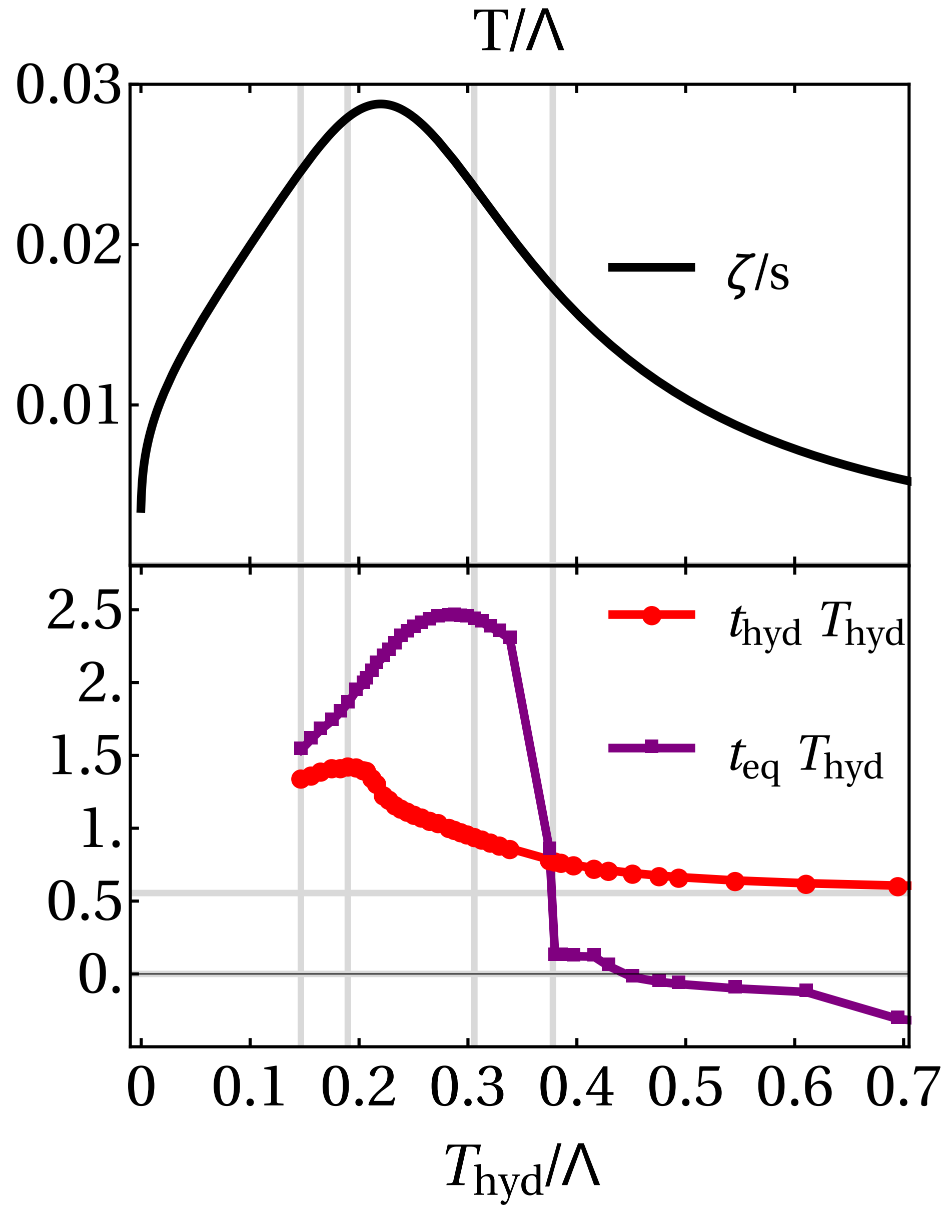

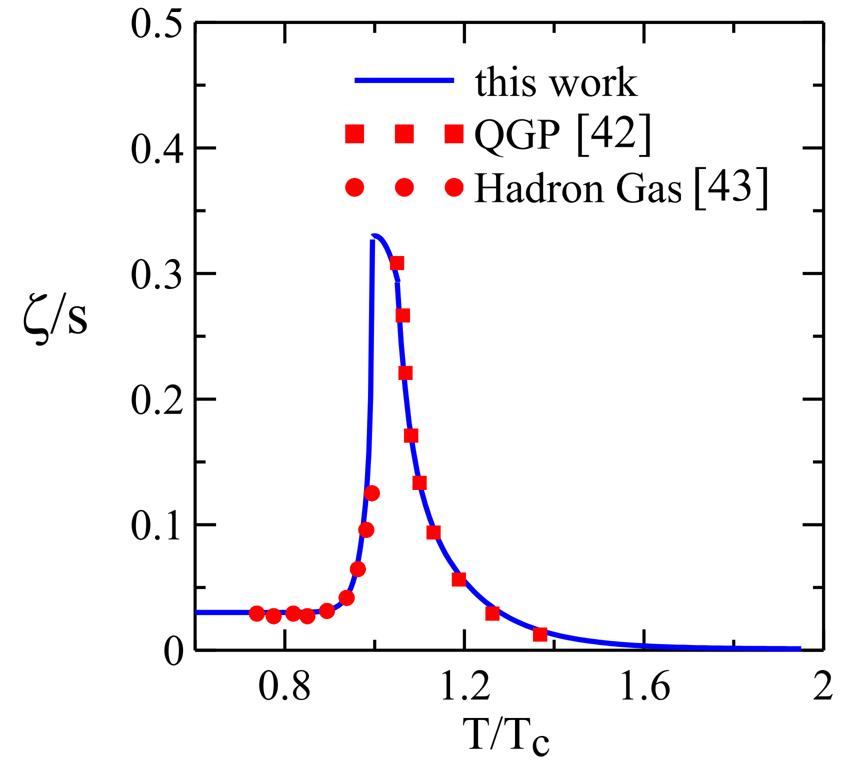

- For ζ/s>0.025 hydrodynamisation occurs first.

(similar to the estimated bulk viscosity about 1.3-1.4 Tc)

Conclusions

- First analysis of ultra-relativistic collision dynamics in non-conformal gauge theories.

- Hydrodynamics provides an (unreasonably) good description of dynamics
    - Large anisotropies
    - Large deviation from equilibrium
    - What controls the applicability of hydro?

- Heavy Ion collisions allow us to explore the different paths for the onset of hydrodynamic behavior